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The Principle of Maximum Conformality

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A key problem in making precise perturbative QCD predictions is the uncertainty in determining the renormalization scale of the running coupling $\alpha_s(\mu^2)$. It is common practice to guess a physical scale $\mu = Q$ which is of order of a typical momentum transfer Q in the process, and then vary the scale over a range $Q/2$ and $2Q$. This procedure is clearly problematic since the resulting fixed-order pQCD prediction will depend on the renormalization scheme, and it can even predict negative QCD cross sections at next-to-leading-order [1]. Other heuristic methods to set the renormalization scale, such as the "principle of minimal sensitivity" [2], give unphysical results [3] for jet physics, sum physics into the running coupling not associated with renormalization, and violate the transitivity property of the renormalization group [4]. Such scale-setting methods also give incorrect results when applied to Abelian QED. Note that the *factorization scale* in QCD is introduced to match nonperturbative and perturbative aspects of the parton distributions in hadrons; it is present even in conformal theory and thus is a completely separate issue from *renormalization scale* setting.

Scales in QED: There is no ambiguity in setting the renormalization scale in quantum electrodynamics: In the standard Gell-Mann–Low scheme for QED, the renormalization scale is simply the virtuality of the virtual photon. For example, in electron-muon elastic scattering, the renormalization scale is the momentum transfer t; i.e., $\alpha(t) = \alpha(t_0)/(1 - \Pi(t, t_0))$ where $\Pi(t, t_0) = (\Pi(t) - \Pi(t_0))/(1 - \Pi(t_0))$ sums all vacuum contributions in the dressed photon propagator, proper and improper. Although the *initial* choice of renormalization scale t_0 is arbitrary, the *final* scale t is not. In the case of muonic atoms, the modified muon-nucleus Coulomb potential is precisely $\alpha(\vec{q}^{\,2})/\vec{q}^{\,2}$. One can use other renormalization schemes in QED, such as $\overline{MS}$ scheme, but the physical result will be the same after allowing for the displacement of scales. For example, if $Q^2 >> m_\ell^2$, $\alpha_{\overline{MS}}(e^{-5/3}t) = \alpha_{GM-L}(t)$. The same underlying principle for scale setting must hold in QCD since the n_F terms in the QCD β function have the same role as the lepton N_ℓ vacuum polarization contributions in QED.

PMC and BLM: The purpose of the running coupling in gauge theory is to sum all terms involving the β function; when the renormalization scale μ is set properly, all nonconformal $\beta \neq 0$ terms in a perturbative expansion arising from renormalization are summed into the running coupling. The remaining terms in the perturbative series are then identical to that of a conformal theory; i.e., the theory with $\beta = 0$. The divergent "renormalon" series of order $\alpha_s^n \beta^n n!$ does not appear in the conformal series. Thus as in QED, the renormalization scale μ is determined unambiguously by the "Principle of Maximal Conformality (PMC)". This is the principle underlying BLM scale setting [5] An important feature of PMC is that its QCD predictions are independent of the choice of renormalization scheme. The PMC procedure also agrees with QED in the $N_C \to 0$ limit. In the case of e^+e^- annihilation to three jets, the BLM/PMC scale is set by the gluon jet virtuality.

Global PMC Scale: Ideally, as in the BLM method, one should allow for separate scales for each skeleton graph; e.g., for to electron-electron scattering, one takes $\alpha(t)$ and $\alpha(u)$ for the t-channel and u-channel amplitudes, respectively. Setting separate scales can be a challenging task for complicated processes in QCD where there are many final-state

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particles and thus many possible Lorentz scalars q_i^2. However, one can obtain a useful first approximation to the full BLM-PMC scale-setting procedure using a single *global* scale $\hat{\mu}$ which appropriately weights the individual BLM scales. The global scale [6] can be determined by varying the subprocess amplitude with respect to each invariant, thus determining the coefficients f_i of $\log q_i^2/\mu_0^2$ in the amplitude; the global PMC scale is then $\hat{\mu}^2 = C\Pi_i(q_i^2)^{w_i}$, where the weight $w_i = f_i/\sum_j f_j$. C is the scheme displacement; e.g., $C = e^{-5/3}$ for $\overline{MS}$.

Commensurate Scale Relations (CSR) [7]: Relations between observables must be independent of the choice of scale and renormalization scheme. CSRs are thus fundamental tests of theory, devoid of theoretical conventions. For example, the PMC relates the effective charge $\alpha_{g_1}(Q^2)$ determined by measurements of the Bjorken sum rule, to the effective charge $\alpha_R(s)$ measured in the total e^+e^- annihilation cross section: $[1-\alpha_{g_1}(Q^2)/\pi]\times[1+\alpha_R(s^*)/\pi] = 1$. Because all $\beta \neq 0$ nonconformal terms are absorbed into the running couplings using PMC, one recovers the conformal prediction [8]; in this case, it is the Crewther relation [9]. The ratio of PMC scales $\sqrt{s}^*/Q \simeq 0.52$ is set by physics; it guarantees that each observable goes through each quark flavor threshold simultaneously as Q^2 and s are raised . Thus by applying PMC, the conformal commensurate scale relations between observables, such as the Crewther relation, become valid for non-conformal QCD at leading twist.

Conclusions: The PMC provides a consistent method for determining the renormalization scale in pQCD. The PMC scale-fixed prediction is independent of the choice of renormalization scheme, a key requirement of renormalization group invariance. The results avoid renormalon resummation and agree with QED scale-setting in the Abelian limit. The PMC global scale can be derived efficiently at NLO from basic properties of the PQCD cross section. The elimination of the renormalization scheme ambiguity using the PMC will not only increases the precision of QCD tests, but it will also increase the sensitivity of colliders to new physics beyond the Standard Model.

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