# Observation of $\boldsymbol{\eta}_{\boldsymbol{c}}(1 S)$ and $\boldsymbol{\eta}_{\boldsymbol{c}}(2 S)$ decays to $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ in two-photon interactions 

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We study the processes $\gamma \gamma \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $\gamma \gamma \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ using a data sample of $519.2 \mathrm{fb}^{-1}$ recorded by the BABAR detector at the PEP-II asymmetric-energy $e^{+} e^{-}$collider at center-of-mass energies near the $\Upsilon(n S)(n=2,3,4)$ resonances. We observe the $\eta_{c}(1 S), \chi_{c 0}(1 P)$ and $\eta_{c}(2 S)$ resonances produced in two-photon interactions and decaying to $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$, with significances of 18.1, 5.4 and 5.3 standard deviations (including systematic errors), respectively, and report $4.0 \sigma$ evidence of the $\chi_{c 2}(1 P)$ decay to this final state. We measure the $\eta_{c}(2 S)$ mass and width in $K_{S}^{0} K^{ \pm} \pi^{\mp}$ decays, and obtain the values $m\left(\eta_{c}(2 S)\right)=3638.5 \pm 1.5 \pm 0.8 \mathrm{MeV} / c^{2}$ and $\Gamma\left(\eta_{c}(2 S)\right)=$ $13.4 \pm 4.6 \pm 3.2 \mathrm{MeV}$, where the first uncertainty is statistical and the second is systematic. We measure the two-photon width times branching fraction for the reported resonance signals, and search for the $\chi_{c 2}(2 P)$ resonance, but no significant signal is observed.

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The first radial excitation $\eta_{c}(2 S)$ of the $\eta_{c}(1 S)$ charmonium ground state was observed at $B$-factories $[1-4]$. The only observed exclusive decay of this state to date is to $K \bar{K} \pi$ [5]. Decays to $p \bar{p}$ and $h^{+} h^{-} h^{\prime+} h^{\prime-}$, with $h^{(\prime)}=K, \pi$, have been observed for the $\eta_{c}(1 S)$ [5], but
not for the $\eta_{c}(2 S)[6,7]$. Precise determination of the $\eta_{c}(2 S)$ mass may discriminate among models that predict the $\psi(2 S)-\eta_{c}(2 S)$ mass splitting [8].

After the discovery of the $X(3872)$ state [9] and its confirmation by different experiments [10], charmonium
spectroscopy above the open-charm threshold received renewed attention. Many new states have been established to date [11-13]. The $Z(3930)$ resonance was discovered by Belle in the $\gamma \gamma \rightarrow D \bar{D}$ process [12] and subsequently confirmed by $B A B A R$ [13]. Its interpretation as the $\chi_{c 2}(2 P)$, the first radial excitation of the ${ }^{3} P_{2}$ charmonium ground state, is commonly accepted [5].

In this paper we study charmonium resonances produced in the two-photon process $e^{+} e^{-} \rightarrow \gamma \gamma e^{+} e^{-} \rightarrow f e^{+} e^{-}$, where $f$ denotes the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ or $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ final state. Two-photon events where the interacting photons are not quasi-real are strongly suppressed by the selection criteria described below. This implies that the allowed $J^{P C}$ values of the initial state are $0^{ \pm+}, 2^{ \pm+}, 4^{ \pm+}, \ldots ; 3^{++}, 5^{++}, \ldots[16]$. Angular momentum conservation, parity conservation, and charge conjugation invariance, then imply that these quantum numbers apply to the final states $f$ also, except that the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ state cannot have $J^{P}=0^{+}$.

The results presented here are based on data collected with the $B A B A R$ detector at the PEP-II asymmetricenergy $e^{+} e^{-}$collider, corresponding to an integrated luminosity of $519.2 \mathrm{fb}^{-1}$, recorded at center-of-mass (CM) energies near the $\Upsilon(n S)(n=2,3,4)$ resonances.

The BABAR detector is described in detail elsewhere [14]. Charged-particles resulting from the interaction are detected, and their momenta are measured, by a combination of five layers of double-sided silicon microstrip detectors and a 40-layer drift chamber. Both systems operate in the 1.5 T magnetic field of a superconducting solenoid. Photons and electrons are identified in a $\mathrm{CsI}(\mathrm{Tl})$ crystal electromagnetic calorimeter. Chargedparticle identification (PID) is provided by the specific energy loss $(\mathrm{d} E / \mathrm{d} x)$ in the tracking devices, and by an internally reflecting, ring-imaging Cherenkov detector. Samples of Monte Carlo (MC) simulated events [15], which are more than 10 times larger than the corresponding data samples, are used to study signals and backgrounds. Two-photon events are generated using the GamGam generator [13].

Neutral pions and kaons are reconstructed through the decays $\pi^{0} \rightarrow \gamma \gamma$ and $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$. Photons from $\pi^{0}$ decays are required to have laboratory energy larger than 30 MeV . We require the invariant mass of a $\pi^{0}\left(K_{S}^{0}\right)$ candidate to be in the range $[100-160]([470-520]) \mathrm{MeV} / c^{2}$. Neutral pions reconstructed with these criteria are used to veto events with multiple $\pi^{0}$ mesons, as described below. For the $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ mode, we refine the selection of the $\pi^{0}$ by requiring the laboratory energy of the lower-energy photon from the signal $\pi^{0}$ decay to be larger than 50 MeV . Furthermore, we require $\left|\cos \mathcal{H}_{\pi^{0}}\right|<0.95$, where $\mathcal{H}_{\pi^{0}}$ is the angle between the signal $\pi^{0}$ flight direction in the laboratory frame and the direction of one of its daughters in the $\pi^{0}$ rest frame. These requirements are optimized by maximizing $S / \sqrt{S+B}$, where $S$ is the number of MC signal events with a well-reconstructed
$\pi^{0}$, and $B$ is the combinatorial background in the signal region. Primary charged-particle tracks are required to satisfy PID requirements consistent with a kaon or pion hypothesis. A candidate event is constructed by fitting the $\pi^{0}\left(K_{S}^{0}\right)$ candidate and four (two) charged-particle tracks of zero net charge coming from the interaction region to a common vertex. In this fit the $\pi^{0}$ and $K_{S}^{0}$ masses are constrained to their nominal values [5]. We require the vertex fit probability of the charmonium candidate to be larger than $0.1 \%$. The outgoing $e^{ \pm}$are not detected.

Background arises mainly from random combinations of particles from $e^{+} e^{-}$annihilation, other two-photon collisions, and initial state radiation (ISR) processes. To suppress these backgrounds, we require that each event have exactly four charged-particle tracks. The candidate event is rejected if the number of additional reconstructed photons is larger than 6 (5) for $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ $\left(K_{S}^{0} K^{ \pm} \pi^{\mp}\right)$. Similarly, the event is rejected if the number of additional reconstructed $\pi^{0}$ 's is larger than 1 (3) for a $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\left(K_{S}^{0} K^{ \pm} \pi^{\mp}\right)$ candidate event. We discriminate against ISR background by requiring $M_{\text {miss }}^{2}=\left(p_{e^{+} e^{-}}-p_{\text {rec }}\right)^{2}>2\left(\mathrm{GeV} / c^{2}\right)^{2}$, where $p_{e^{+} e^{-}}$ ( $p_{\text {rec }}$ ) is the four momentum of the initial state (reconstructed final state). The effect of this requirement on the signal efficiency is studied using a $K^{+} K^{-} \pi^{+} \pi^{-}$control sample that contains large $\eta_{c}(1 S), J / \psi$, and $\chi_{c 0,2}(1 P)$ signals. Two-photon events are expected to have low transverse momentum $\left(p_{T}\right)$ with respect to the collision axis. In Fig. 1, we show the $p_{T}$ distribution for selected candidates with the above requirements. The distribution is fitted with the signal $p_{T}$ shape obtained from MC simulation plus a combinatorial background component, modeled using a sixth-order polynomial function. We require $p_{T}<0.15 \mathrm{GeV} / c$.

The average number of surviving candidates per selected event is 1.003 (1.09) for the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ $\left(K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ final state. Candidates that are rejected by a possible best-candidate selection do not lead to any peaking structures in the mass spectra, and so no best-candidate selection is performed. The $K_{S}^{0} K^{+} \pi^{-}$ and $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ mass spectra are shown in Fig. 2. We observe prominent peaks at the position of the $\eta_{c}(1 S)$ resonance. We also observe signals at the positions of the $J / \psi, \chi_{c 0}(1 P), \chi_{c 2}(1 P)$, and $\eta_{c}(2 S)$ states.

The resonance signal yields and the mass and width of the $\eta_{c}(1 S)$ and $\eta_{c}(2 S)$ are extracted using a binned, extended maximum likelihood fit to the invariant mass distributions. The bin width is $4 \mathrm{MeV} / c^{2}$. In the likelihood function, several components are present: $\eta_{c}(1 S)$, $\chi_{c 0}(1 P), \chi_{c 2}(1 P)$, and $\eta_{c}(2 S)$ signal, combinatorial background, and $J / \psi$ ISR background. The $\chi_{c 0}(1 P)$ component is not present in the fit to the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ invariant mass spectrum, since $J^{P}=0^{+}$is forbidden for this final state.

We parameterize each signal PDF as a convolution of


FIG. 1: The $p_{T}$ distributions for selected (a) $K_{S}^{0} K^{ \pm} \pi^{\mp}$ and (b) $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ candidates (data points). The solid histogram represent the result of a fit to the sum of the simulated signal (dashed) and background (dotted) contributions.
a non-relativistic Breit-Wigner and the detector resolution function. The $J / \psi$ ISR background is parameterized with a Gaussian shape, and the combinatorial background PDF is a fourth-order polynomial. The free parameters of the fit are the yields of the resonances and the background, the peak masses and widths of the $\eta_{c}(1 S)$ and $\eta_{c}(2 S)$ signals, the width of the Gaussian describing the $J / \psi$ ISR background, and the background shape parameters. The mass and width of the $\chi_{c 0,2}(1 P)$ states (and the mass of the $J / \psi$ in the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ channel), are fixed to their nominal values [5]. For the $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ channel, the $\eta_{c}(2 S)$ width is fixed to the value found in the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ channel.

We define a MC event as "MC-Truth" (MCT) if the reconstructed decay chain matches the generated one. We use MCT signal and MCT ISR-background events to determine the detector mass resolution function. This function is described by the sum of a Gaussian plus powerlaw tails [17]. The width of the resolution function at half-maximum for the $\eta_{c}(1 S)$ is $8.1(11.8) \mathrm{MeV} / c^{2}$ in the $K_{S}^{0} K^{ \pm} \pi^{\mp}\left(K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ decay mode. For the $\eta_{c}(2 S)$ decay it is 10.6 (13.1) $\mathrm{MeV} / c^{2}$ in the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ ( $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ ) decay mode. The parameter values for the resolution functions, are fixed to their MC values in the fit.

Fit results are reported in Table I and shown in Fig. 2. We correct the fitted $\eta_{c}(1 S)$ yields by subtracting the number of peaking-background events originating from
the $J / \psi \rightarrow \gamma \eta_{c}(1 S)$ decay, estimated below. The statistical significances of the signal yields are computed from the ratio of the number of observed events to the sum in quadrature of the statistical and systematic uncertainties. The $\chi^{2} / n d f$ of the fit is 1.07 (1.03), where $n d f$ is the number of degrees of freedom, which is $361(360)$ for the fit to $K_{S}^{0} K^{ \pm} \pi^{\mp}\left(K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$.

To search for the $\chi_{c 2}(2 P)$, we add to the fit described above a signal component with the mass and width fixed to the values reported in Ref. [13]. No significant changes are observed in the fit results. Several processes, includ-


FIG. 2: Fit to (a) the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ and (c) the $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ mass spectrum. The solid curves represent the total fit functions and the dashed curves show the combinatorial background contributions. The background-subtracted distributions are shown in (b) and (d), where the solid curves indicate the signal components.
ing ISR, continuum $e^{+} e^{-}$annihilation, and two-photon events with a final state different from the one studied, may produce irreducible-peaking-background events, containing real $\eta_{c}(1 S), \eta_{c}(2 S), \chi_{c 0}(1 P)$ or $\chi_{c 2}(1 P)$. Well-reconstructed signal and $J / \psi$ ISR background are expected to peak at $p_{T} \sim 0 \mathrm{GeV} / c$. Final states with similar masses are expected to have similar $p_{T}$ distributions. Non-ISR background events mainly originate from events with a number of particles in the final state larger than the one in signal events. Such extra particles are lost in the reconstruction. Thus, non-ISR background

TABLE I: Extraction of event yields and mass and width of the $\eta_{c}(1 S)$ and $\eta_{c}(2 S)$ resonances: average signal efficiency for phase-space MCT events, corrected signal yield with statistical and systematic uncertainties, number of peaking-background events estimated with the $p_{T}$ fit ( $N_{\text {peak }}$ ), number of peaking-background events from $J / \psi$ and $\psi(2 S)$ radiative decays $\left(N_{\psi}\right)$, significance (including systematic uncertainty), corrected mass, and fitted width for each decay mode. We do not report $N_{\psi}$ for modes where it is negligible.

| Decay <br> Mode | Efficiency <br> $(\%)$ | Corrected <br> Yield (Evts.) | $N_{\text {peak }}$ <br> (Evts.) | $N_{\psi}$ <br> (Evts.) | Significance <br> $(\sigma)$ | Corrected <br> Mass (MeV $\left./ c^{2}\right)$ | Fitted <br> Width (MeV) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\eta_{c}(1 S) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 10.7 | $12096 \pm 235 \pm 274$ | $189 \pm 18$ | $214 \pm 82$ | 33.5 | $2982.5 \pm 0.4 \pm 1.4$ | $32.1 \pm 1.1 \pm 1.3$ |
| $\chi_{c 2}(1 P) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 13.1 | $126 \pm 37 \pm 14$ | $-45 \pm 11$ | - | 3.2 | 3556.2 (fixed) | 2 (fixed) |
| $\eta_{c}(2 S) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 13.3 | $624 \pm 72 \pm 34$ | $25 \pm 5$ | - | 7.8 | $3638.5 \pm 1.5 \pm 0.813 .4 \pm 4.6 \pm 3.2$ |  |
| $\eta_{c}(1 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | 4.2 | $11132 \pm 430 \pm 442$ | $118 \pm 32$ | $26 \pm 9$ | 18.1 | $2984.5 \pm 0.8 \pm 3.1$ | $36.2 \pm 2.8 \pm 3.0$ |
| $\chi_{c 0}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | 5.6 | $1094 \pm 143 \pm 143$ | $-39 \pm 19$ | $75 \pm 21$ | 5.4 | 3415.8 (fixed) | 10.2 (fixed) |
| $\chi_{c 2}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | 5.8 | $1250 \pm 118 \pm 290$ | $14 \pm 24$ | $233 \pm 73$ | 4.0 | 3556.2 (fixed) | 2 (fixed) |
| $\eta_{c}(2 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | 5.9 | $1201 \pm 133 \pm 185$ | $-46 \pm 17$ | - | 5.3 | $3640.5 \pm 3.2 \pm 2.5$ | 13.4 (fixed) |

events are expected to have a nearly flat $p_{T}$ distribution, as observed in MC simulation.

To estimate the number of such events, we fit the invariant mass distribution in intervals of $p_{T}$, thus obtaining the signal yield for each resonance as a function of $p_{T}$. The signal yield distribution is then fitted using the signal $p_{T}$ shape from MCT events plus a flat background. The yield of peaking-background events originating from $\psi$ radiative decays $(\psi=J / \psi, \psi(2 S))$ is estimated using the number of $\psi$ events fitted in data, and the knowledge of branching fractions [5] and MC reconstruction efficiencies for the different decays involved. The number of peaking-background events for each resonance is reported in Table I. The value of $\mathcal{B}\left(\chi_{c 0,2} \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$, which is needed to estimate the number of peakingbackground events from $\psi(2 S) \rightarrow \gamma \chi_{c 0,2}(1 P)$ decays, is obtained using the results reported in this paper and the world-average values of $\Gamma_{\gamma \gamma}\left(\chi_{c 0,2}\right)$ [5]. We obtain $\mathcal{B}\left(\chi_{c 0}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)=(1.14 \pm 0.27) \%$, and $\mathcal{B}\left(\chi_{c 2}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)=(1.30 \pm 0.36) \%$, where statistical and systematic errors have been summed in quadrature. The value of $\mathcal{B}\left(\chi_{c 2}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ is in agreement with a preliminary result reported by CLEO [18]. The number of peaking background events from $\psi$ radiative decays for $\eta_{c}(2 S)$ and $\chi_{c 2}(1 P) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ (denoted by "-" in Table I) is negligible.

The ratios of the branching fractions of the two modes are obtained from

$$
\begin{equation*}
\frac{\mathcal{B}\left(\eta_{c}(n S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)}{\mathcal{B}\left(\eta_{c}(n S) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}\right)}=\frac{N_{K K 3 \pi}^{\eta_{c}(n S)}}{N_{K_{S}^{0} K \pi}^{\eta_{c}(n S)}} \cdot \frac{\epsilon_{K_{S}^{0} K \pi}^{\eta_{c}(n S)}}{\epsilon_{K K 3 \pi}^{\eta_{c}(n S)}} \tag{1}
\end{equation*}
$$

where $\eta_{c}(n S)$ denotes $\eta_{c}(1 S), \quad \eta_{c}(2 S) ; \quad N_{K K 3 \pi}^{\eta_{c}(n S)}$ and $N_{K_{S}^{0} K \pi}^{\eta_{c}(n S)}\left(\epsilon_{K K 3 \pi}^{\eta_{c}(n S)}\right.$ and $\left.\epsilon_{K_{S}^{0} K \pi}^{\eta_{c}(n S)}\right)$ represent the peaking-background-subtracted $\eta_{c}(n S)$ yield (the efficiency) for the $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ and $K_{S}^{0} K^{ \pm} \pi^{\mp}$ channels, respectively. The efficiencies are parameterized using MCT events. The $K_{S}^{0} K^{ \pm} \pi^{\mp}$ efficiency is parameterized as a twodimensional histogram of the invariant $K \pi$ mass versus
the angle between the direction of the $K^{+}$in the $K \pi$ rest frame and that of the $K \pi$ system in the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ reference frame. The $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ efficiency is parameterized as a function of the $K^{+} K^{-}, \pi^{+} \pi^{-}$, and $\pi^{+} \pi^{-} \pi^{0}$ $(3 \pi)$ masses, and the five angular variables, $\cos \theta_{K}, \cos \Theta$, $\Phi, \cos \theta_{\pi \pi}$, and $\theta_{\pi}$, as defined in Fig. 3; $\theta_{K}$ is the angle


FIG. 3: Angles used to describe the $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ decay kinematics
between the $K^{+}$and the $3 \pi$ recoil direction in the $K^{+} K^{-}$ rest frame. The angles $\Theta$ and $\Phi$ describe the orientation of the normal $\hat{n}$ to the $3 \pi$ decay plane with respect to the $K^{+} K^{-}$recoil direction in the $3 \pi$ rest frame; $\theta_{\pi}$ is the angle describing a rotation of the $3 \pi$ system about its decay plane normal; $\theta_{\pi \pi}$ is the angle between the $\pi^{+}$and $\pi^{-}$directions in the $3 \pi$ reference frame. The correlations between $\cos \theta_{K}, \Theta, \Phi$, and $\theta_{\pi}$ and the invariant masses are negligible. The correlation between $\cos \theta_{\pi \pi}$ and $m_{\pi \pi}$ is -0.70 and is not considered in the efficiency parameterization. Neglecting such a correlation introduces a change in the efficiency of $1.4 \%(1.1 \%)$ for the $\eta_{c}(1 S)\left(\eta_{c}(2 S)\right)$, which is taken as a systematic uncertainty. The efficiency dependence on $\cos \theta_{K}, \cos \theta_{\pi \pi}$, and $\Phi(\cos \Theta$ and $\theta_{\pi}$ ) is parameterized using uncorrelated fourth(second)order polynomial shapes. A three-dimensional histogram
is used to parameterize the dependence on the invariant masses. The efficiency is calculated as the ratio of the number of MCT events surviving the selection to the number of generated events in each bin, in both channels. We assign null efficiency to bins with less than 10 reconstructed events. The fraction of data falling in these bins is $0.5 \%(3.0 \%)$ in the $K_{S}^{0} K^{ \pm} \pi^{\mp}\left(K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ channel. We assign a systematic uncertainty to cover this effect. The average efficiency $\bar{\epsilon}$ for each decay, computed using flat phase-space simulation, is reported in Table I.

The ratio $N_{f}^{\eta} / \epsilon_{f}^{\eta}$ of Eq. (1) is equal to $N_{f}^{\eta} /\left(\epsilon_{f}^{* \eta} \times \bar{\epsilon}_{f}^{\eta}\right)$, where we have defined $\epsilon_{f}^{* \eta}=\epsilon_{f}^{\eta} / \bar{\epsilon}_{f}^{\eta}$. The value of $N_{f}^{\eta} / \epsilon_{f}^{* \eta}$ is extracted from an unbinned maximum likelihood fit to the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ invariant mass distributions, where each event is weighted by the inverse of $\epsilon_{f}^{* \eta}$. We use $\epsilon_{f}^{* \eta}$ instead of $\epsilon_{f}^{\eta}$ to weight the events since weights far from one might result in incorrect errors for the signal yield obtained in the maximum likelihood fit [19]. Since the kinematics of peaking-background events are similar to those of the signal, we assume the signal to peaking-background ratio to be unaffected by the weighting technique. The fit is performed independently in the $\eta_{c}(1 S)\left([2.5,3.3] \mathrm{GeV} / c^{2}\right)$ and $\eta_{c}(2 S)$ $\left([3.2,3.9] \mathrm{GeV} / c^{2}\right)$ mass regions. The mass and width for each signal PDF are fixed to the values reported in Table I. The free parameters of the fit are the yields of the background and the signal resonances, the mean and the width of the Gaussian describing the $J / \psi$ background, and the background shape parameters. We compute a $\chi^{2}$ using the total fit function and the binned $K_{s}^{0} K^{ \pm} \pi^{\mp}\left(K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ mass distribution obtained after weighting. The values of $\chi^{2} / n d f$ are 1.16 (1.15) and $1.20(1.00)$ in the $\eta_{c}(1 S)$ and $\eta_{c}(2 S)$ mass regions, in the $K_{S}^{0} K^{ \pm} \pi^{\mp}\left(K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ channel.

Several sources contribute to systematic uncertainties on the resonance yields and parameters. Systematic uncertainties due to PDF parameterization and fixed parameters in the fit are estimated to be the sum in quadrature of the changes observed when repeating the fit after varying the fixed parameters by $\pm 1$ standard deviation $(\sigma)$. The uncertainty associated with the peaking background is taken to be $\sqrt{\left(\max \left[0, N_{\text {peak }}\right]\right)^{2}+\sigma_{N_{\text {peak }}}^{2}}$, where $N_{\text {peak }}$ is the estimated number of peaking-background events reported in Table I, and $\sigma_{N_{\text {peak }}}$ is its uncertainty. The systematic errors on the $\chi_{c 0,2}(1 P)$ yields are taken to be $\sqrt{\left(\max \left[0, N_{\text {peak }}\right]\right)^{2}+\sigma_{N_{\text {peak }}}^{2}+N_{\psi}^{2}+\sigma_{N_{\psi}}^{2}}$, where $N_{\psi}$ is the number of peaking-background events from the $\psi(2 S) \rightarrow \gamma \chi_{c 0,2}(1 P)$ processes. The uncertainty on $N_{\text {peak }}$ due to differences in signal and ISR background $p_{T}$ distribution is estimated by adding an ISR background component to the fit to the $p_{T}$ yield distribution described above. The ISR background $p_{T}$ shape is taken from MC simulation and its yield is fixed to $N_{\psi}$. This uncertainty is found to be negligible. We take the systematic error due to the $J / \psi \rightarrow \gamma \eta_{c}(1 S)$ peaking-background subtrac-
tion to be the uncertainty on the estimated number of events originating from this process. We assign an uncertainty due to the background shape, taking the changes in results observed when using a sixth-order polynomial as the background PDF in the fit.

An ISR-enriched sample is obtained by reversing the $M_{\text {miss }}^{2}$ selection criterion. The ISR-enriched sample is fitted to obtain the shift $\Delta M$ between the measured and the nominal $J / \psi$ mass [5], and the difference in mass resolution between MC and data. The corrected masses in Table I are $m^{\text {corr }}=m^{f i t}-\Delta M$, where $m^{f i t}$ is the mass determined by the fit. The mass shift is $-0.5 \pm 0.2 \mathrm{MeV} / c^{2}$ in $K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $-1.1 \pm 0.8 \mathrm{MeV} / c^{2}$ in $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$. We assign the statistical error on $\Delta M$ as a systematic uncertainty on $m^{\text {corr }}$. The difference in mass resolution is $(24 \pm 5) \%$ in $K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $(9 \pm 5) \%$ in $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$. We take the difference in fit results observed when including this correction in the $\eta_{c}(1 S), \chi_{c 0}(1 P), \chi_{c 2}(1 P)$, and $\eta_{c}(2 S)$ resolution functions as the systematic uncertainty due to the mass-resolution difference between data and MC. A systematic uncertainty on the mass accounts for the different kinematics of two-photon signal and ISR $J / \psi$ events.

The distortion of the resolution function due to differences between the invariant mass distributions of the decay products in data and MC produces negligible changes in the results. We take as systematic uncertainty the changes in the resonance parameters observed by including in the fit the effect of the efficiency dependence on the invariant mass and on the decay dynamics. The effect of the interference of the $\eta_{c}(1 S)$ signal with a possible $J^{P C}=0^{-+}$contribution in the $\gamma \gamma$ background is considered. We model the mass distribution of the $J^{P C}=0^{-+}$ background component with the PDF describing combinatorial background. The changes in the fitted signal yields are negligible. The changes of the values of the $\eta_{c}(1 S)$ mass and width with respect to the nominal results are $+1.2 \mathrm{MeV} / c^{2}$ and +0.2 MeV for $K_{S}^{0} K^{ \pm} \pi^{\mp}$, and $+2.9 \mathrm{MeV} / c^{2}$ and +0.6 MeV for $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$. We take these changes as estimates of systematic uncertainty due to interference. The effect of the interference on the $\eta_{c}(2 S)$ parameter values cannot be determined due to the small signal to background ratio and the smallness of the signal sample. We therefore do not include any systematic uncertainty due to this effect for the $\eta_{c}(2 S)$.
Systematic uncertainties on the efficiency due to tracking ( $0.2 \%$ per track), $K_{S}^{0}$ reconstruction ( $1.7 \%$ ), $\pi^{0}$ reconstruction (3.0\%) and PID ( $0.5 \%$ per track) are obtained from auxiliary studies. The statistical uncertainty of the efficiency parameterization is estimated with simulated parameterized experiments. In each experiment, the efficiency in each histogram bin and the coefficients of the functions describing the dependence on $\cos \theta_{K}, \cos \theta_{\pi \pi}$, $\cos \Theta, \theta_{\pi}$ and $\Phi$ are varied within their statistical uncertainties. We take as systematic uncertainty the width of the resulting yield distribution. The fit bias is negligi-
ble. The small impact of the presence of events falling in bins with zero efficiency is accounted for as an additional systematic uncertainty.

Using the efficiency-weighted yields of the $\eta_{c}(1 S)$ and $\eta_{c}(2 S)$ resonances, the number of peaking-background events, and $\mathcal{B}\left(K_{S}^{0} \rightarrow \pi^{+} \pi^{-}\right)=(69.20 \pm 0.05) \%$ [5], we find the branching fraction ratios

$$
\begin{align*}
& \frac{\mathcal{B}\left(\eta_{c}(1 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)}{\mathcal{B}\left(\eta_{c}(1 S) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}\right)}=1.43 \pm 0.05 \pm 0.21  \tag{2}\\
& \frac{\mathcal{B}\left(\eta_{c}(2 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)}{\mathcal{B}\left(\eta_{c}(2 S) \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}\right)}=2.2 \pm 0.5 \pm 0.5 \tag{3}
\end{align*}
$$

where the first error is statistical and the second is systematic. The uncertainty in the efficiency parameterization is the main contribution to the systematic uncertainties and is equal to 0.17 and 0.3 , in Eqs. (2) and (3), respectively. Using Eqs. (2)-(3), $\mathcal{B}\left(\eta_{c}(1 S) \rightarrow K \bar{K} \pi\right)=(7.0 \pm 1.2) \%$ and $\mathcal{B}\left(\eta_{c}(2 S) \rightarrow K \bar{K} \pi\right)=(1.9 \pm 1.2) \%$ [5] , and isospin relations, we obtain $\mathcal{B}\left(\eta_{c}(1 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)=(3.3 \pm$ $0.8) \%$, and $\mathcal{B}\left(\eta_{c}(2 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}\right)=(1.4 \pm 1.0) \%$, where we have summed in quadrature the statistical and systematic errors.

For each resonance and each final state, we compute the product between the two-photon coupling $\Gamma_{\gamma \gamma}$ and the resonance branching fraction $\mathcal{B}$ to the final state, using $473.8 \mathrm{fb}^{-1}$ of data collected near the $\Upsilon(4 S)$ energy. The efficiency-weighted yields for the resonances, and the integrated luminosity near the $\Upsilon(4 S)$ energy are used to obtain $\Gamma_{\gamma \gamma} \times \mathcal{B}$ with the GamGam generator [13]. The mass and width of the resonances are fixed to the values reported in Table I. The uncertainties on the luminosity ( $1.1 \%$ ) and on the GamGam calculation (3\%) [13] are included in the systematic uncertainty of $\Gamma_{\gamma \gamma} \times \mathcal{B}$. For the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ decay mode, we give the results for the isospin-related $K \bar{K} \pi$ final state, taking into account $\mathcal{B}\left(K_{S}^{0} \rightarrow \pi^{+} \pi^{-}\right)=(69.20 \pm 0.05) \%$ [5] and isospin relations. For the $\chi_{c 2}(2 P)$, we compute $\Gamma_{\gamma \gamma} \times \mathcal{B}$ using the fitted $\chi_{c 2}(2 P)$ yield, the integrated luminosity near the $\Upsilon(4 S)$ energy, and the average detection efficiency for the relevant process. The average detection efficiency is equal to $13.9 \%$ and $6.4 \%$ for the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ and $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ modes, respectively. The mass and width of the $\chi_{c 2}(2 P)$ resonance are fixed to the values reported in [13]. Since no significant $\chi_{c 2}(2 P)$ signal is observed, we determine a Bayesian upper limit (UL) at $90 \%$ confidence level (CL) on $\Gamma_{\gamma \gamma} \times \mathcal{B}$, assuming a uniform prior probability distribution. We compute the UL by finding the value of $\Gamma_{\gamma \gamma} \times \mathcal{B}$ below which lies $90 \%$ of the total of the likelihood integral in the $\left(\Gamma_{\gamma \gamma} \times \mathcal{B}\right) \geq 0$ region. Systematic uncertainties are taken into account in the UL calculation. Results for $\Gamma_{\gamma \gamma} \times \mathcal{B}$ for each resonance and final state are reported in Table II. The $\eta_{c}(1 S) \rightarrow K \bar{K} \pi$ measurement is consistent with, but slightly more precise than, the PDG value [5]; the other entries are first measurements.

TABLE II: Results for $\Gamma_{\gamma \gamma} \times \mathcal{B}$ for each resonance in $K \bar{K} \pi$ and $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ final states. The first uncertainty is statistical, the second systematic. Upper limits are computed at $90 \%$ confidence level.

| Process | $\Gamma_{\gamma \gamma} \times \mathcal{B}(\mathrm{keV})$ |
| :--- | :---: |
| $\eta_{c}(1 S) \rightarrow K \bar{K} \pi$ | $0.386 \pm 0.008 \pm 0.021$ |
| $\chi_{c 2}(1 P) \rightarrow K \bar{K} \pi$ | $(1.8 \pm 0.5 \pm 0.2) \times 10^{-3}$ |
| $\eta_{c}(2 S) \rightarrow K \bar{K} \pi$ | $0.041 \pm 0.004 \pm 0.006$ |
| $\chi_{c 2}(2 P) \rightarrow K \bar{K} \pi$ | $<2.1 \times 10^{-3}$ |
| $\eta_{c}(1 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | $0.190 \pm 0.006 \pm 0.028$ |
| $\chi_{c 0}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | $0.026 \pm 0.004 \pm 0.004$ |
| $\chi_{c 2}(1 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | $(6.5 \pm 0.9 \pm 1.5) \times 10^{-3}$ |
| $\eta_{c}(2 S) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | $0.030 \pm 0.006 \pm 0.005$ |
| $\chi_{c 2}(2 P) \rightarrow K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ | $<3.4 \times 10^{-3}$ |

In conclusion, we report the first observation of $\eta_{c}(1 S)$, $\chi_{c 0}(1 P)$ and $\eta_{c}(2 S)$ decays to $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$, with significances (including systematic uncertainties) of $18 \sigma$, $5.4 \sigma$ and $5.3 \sigma$, respectively. This is the first observation of an exclusive hadronic decay of $\eta_{c}(2 S)$ other than $K \bar{K} \pi$. We also report the first evidence of $\chi_{c 2}(1 P)$ decays to $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$, with a significance (including systematic uncertainties) of $4.0 \sigma$, and have obtained first measurements of the $\chi_{c 0}(1 P)$ and $\chi_{c 2}(1 P)$ branching fractions to $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$. The measurements reported in this paper are consistent with previous BABAR results [3, 20], and with world average values [5]. The measurement of the $\eta_{c}(2 S)$ mass and width in the the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ decay supersedes the previous $B A B A R$ measurement [3]. The measurement of the $\eta_{c}(1 S)$ mass and width in the the $K_{S}^{0} K^{ \pm} \pi^{\mp}$ decay does not supersede the previous BABAR measurement [20]. The value of $\Gamma_{\gamma \gamma} \times \mathcal{B}$ is measured for each observed resonance for both $K \bar{K} \pi$ and $K^{+} K^{-} \pi^{+} \pi^{-} \pi^{0}$ decay modes. We provide an UL at $90 \% \mathrm{CL}$ on $\Gamma_{\gamma \gamma} \times \mathcal{B}$ for the $\chi_{c 2}(2 P)$ resonance.

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