

Applications of AdS/QCD and Light-Front Holography to Baryon Physics

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Abstract

The correspondence between theories in anti-de Sitter space and field theories in physical space-time leads to an analytic, semiclassical model for strongly-coupled QCD which has scale invariance at short distances and color confinement at large distances. These equations, for both mesons and baryons, give a very good representation of the observed hadronic spectrum, including a zero mass pion. Light-front holography allows hadronic amplitudes in the AdS fifth dimension to be mapped to frame-independent light-front wavefunctions of hadrons in physical space-time, thus providing a relativistic description of hadrons at the amplitude level. The meson and baryon wavefunctions derived from light-front holography and AdS/QCD also have remarkable phenomenological features, including predictions for the electromagnetic form factors and decay constants. The approach can be systematically improved using light-front Hamiltonian methods. Some novel features of QCD for baryon physics are also discussed.

1 Introduction

The light-front wavefunctions (LFWFs) of relativistic bound states in quantum chromodynamics provide a fundamental description of the structure and internal dynamics of hadronic states in terms of their constituent quark and gluons at a fixed light-front time $\tau = t + z/c$, the time marked by the front of a light wave, [1, 2] – rather than at instant time t , the ordinary time. Unlike instant time quantization, the Hamiltonian equation of motion in the light front (LF) is frame independent. The simple structure of the light-front vacuum allows an unambiguous definition of the partonic content of a hadron in QCD and of hadronic light-front wavefunctions which relate its quark and gluon degrees of freedom to their asymptotic hadronic state. For example, the proton's eigenstate $|p\rangle$, the lightest $Q = +1$, $B = +1$ eigenstate of the QCD LF Hamiltonian, can be expanded in terms of the light front Fock components: $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ corresponding to its $\langle p|uud\rangle, \langle p|uudg\rangle, \langle p|uudQ\bar{Q}\rangle$, etc. projections. Here $x_i = k_i^+/P^+ = (k_i^0 + k_i^z)/(P^0 + P^z)$ are the light-front momentum fractions of the constituents. The plus momentum and transverse momenta are conserved at fixed τ , $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \mathbf{k}_{\perp i} = 1$ in each n -parton wavefunction. Remarkably, the $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ are independent of the hadrons 4-momentum P^μ . In light-cone gauge $A^+ = 0$, the gluon quanta have $S^z = \pm 1$ and there are no ghosts. Angular momentum on the LF is simply $J^z = \sum_{i=1}^n S_i^z + \sum_{i=1}^{n-1} L_i^z$ since there are only $n - 1$ independent orbital angular momentum. The angular momentum carried by the gluon in the proton as measured by experiment is simply the mean value of $S_g^z + L_g^z$ summed over Fock states. The structure functions measured by experiment in deep inelastic proton scattering (DIS) are related to the absolute square of the LFWFs summed over Fock states. The proton's Dirac and Pauli form factors have an exact representation as the spin-conserving and spin-flip overlaps of initial and final wavefunctions, respectively. Unlike the ordinary instant form, there are no diagrams where the current couples to vacuum currents, and the boost of the LFWFs is trivial. Given, the LFWFs, one can compute hadronization at the amplitude level from the coalescence of a set of comoving color-singlet quarks and gluons, [3] in analogy to the formation of moving atoms in quantum electrodynamics. [4]

There are many novel features of baryons which are illuminated using the LFWF representation:

1. The higher Fock states of baryons contain intrinsic heavy quark [5] such as $|uudc\bar{c}\rangle$; these can arise from gluon splitting, but also from multi-connected amplitudes. The intrinsic heavy quark (IQ) contributions are maximal at minimum off-shellness of the LFWF; i.e, when the partons have the same rapidity. This corresponds to $x_i \propto \sqrt{\mathbf{k}_{\perp i}^2 + m_i^2}$, so that the heavy quark have most of the momentum. In a collision the states are materialized and the quarks with the same rapidity such as $|Qud\rangle$ coalesce to high x_F baryons such as the $\Lambda_c(cud)$ and the $\Lambda_b(bud)$ observed at high x_F at the ISR. [6] SELEX [7] has also discovered double charm baryons $|ccu\rangle$ and $|ccd\rangle$ at high x_F this way. This suggests using the LHC

beam in a fixed target mode to observe very heavy baryons at high x_F such as the $\Omega_b(bbb)$. [8] Intrinsic heavy quarks also provide a novel mechanism to produce the Higgs meson and Z^0 at high x_F . [9, 10]

2. High p_T hadrons can be created directly from hard subprocesses such as $gq \rightarrow Mq$ and $q\bar{q} \rightarrow B + \bar{q}$, rather than from quark or gluon fragmentation. [11] Since the direct hadrons are produced with small transverse size, they are color transparent and can traverse the nuclear medium without absorption. The direct processes have been seen in many experiments. They also account for the remarkable baryon anomaly observed in ion-ion collisions, [12] as well as the anomalously large power-law falloff of the inclusive cross sections $\frac{E d\sigma}{d^3p}(pp \rightarrow HX)$ for meson and baryon production at fixed θ_{cm} and $x_T = 2p_t/\sqrt{s}$. Since there are no same-side hadrons, the direct processes are energy efficient, requiring the minimum incident parton momentum fractions where the parton distributions are maximal.
3. The Siverts effect in deep inelastic lepton-polarized proton scattering is most easily computed as an interference of the proton's $L^z = 0$ and $L^z = \pm 1$ LFWFs. [13] The Siverts spin-correlation $\vec{S}_p \cdot \vec{p}_q \times \vec{q}$ is $T - odd$, which reflects the different phases of the $L^z = 0$ and $L^z = \pm 1$ amplitudes due to final state interaction of the scattered quark with the proton's spectators. The Siverts correlation for each quark is thus seen to be proportional to that quark's contribution to the proton orbital angular momentum. The Siverts correlation has the opposite sign in Drell-Yan reactions [14, 15] because it measures initial state scattering of the annihilating quark. There are many other novel factorization-breaking effects which arise due to initial state or final state scattering of the active quark with the spectators, such as the double Bohr-Mulders correlation which leads to a breakdown of the PQCD Lam-Tung relation in Drell-Yan reactions. [16]
4. The quark condensate, normally identified as a vacuum expectation value $\langle 0|\bar{\psi}\psi|0\rangle$ is actually an "in-hadron" condensate $\langle 0|\bar{\psi}\psi|H\rangle$ associated with the $q\bar{q}$ sea quark excitations in the hadron's higher particle number Fock states. [17, 18] The QCD vacuum is trivial - equal to the vacuum of the free theory in the front form, and there is thus no contribution to the cosmological constant within this framework.

2 AdS/QCD and Light Front Holography

The AdS/CFT correspondence [19] between string states on anti-de Sitter (AdS) space-time and conformal gauge field theories (CFT) in physical space-time has brought a new set of tools for studying the dynamics of strongly coupled quantum field theories, and it has led to new analytical insights into the confining dynamics of QCD which is difficult to realize using other methods. Most important, it provides an initial approximation to QCD which is analytically tractable and which can be systematically improved. The

original conformal theory can be modified in the far infrared region of AdS space, for example by the introduction of a dilaton background, which yields a confining potential between the colored quarks. The resulting model is usually called AdS/QCD.

One of the most remarkable features of AdS/QCD is the connection between the description of hadronic modes in AdS space and the Hamiltonian formulation of QCD in physical space-time quantized on the light-front; i.e., at equal light-front time τ . The first step for establishing the correspondence of light-front QCD in physical 3+1 space with AdS space is to observe that the LF bound state Hamiltonian equation of motion in QCD has an essential dependence in the invariant transverse variable ζ , [20] which measures the separation of the quark and gluonic constituents within the hadron at the same LF time. The variable ζ plays the role of the radial coordinate r in atomic systems. The result is a single-variable light-front relativistic Schrödinger equation. This first approximation to relativistic QCD bound-state systems is equivalent to the equations of motion that describe the propagation of spin- J modes in a fixed gravitational background asymptotic to AdS space. [20] The eigenvalues of the LF Schrödinger equation give the hadronic spectrum and its eigenmodes represent the probability amplitudes of the hadronic constituents. By using the correspondence between ζ in the LF theory and z in AdS space, one can identify the terms in the dual gravity AdS equations that correspond to the kinetic energy terms of the partons inside a hadron and the interaction terms that build confinement. [20] The identification of orbital angular momentum of the constituents in the light-front is also a key element in our description of the internal structure of hadrons using holographic principles. This mapping was originally obtained by matching the expression for electromagnetic current matrix elements in AdS space with the corresponding expression for the current matrix element using LF theory in physical space time. [21] More recently we have shown that one obtains the identical holographic mapping using the matrix elements of the energy-momentum tensor, [22] thus providing a consistency test and verification of holographic mapping from AdS to physical observables defined on the light front.

3 A SEMICLASSICAL LIGHT-FRONT APPROXIMATION TO QCD

The eigenmass M^2 of hadrons in light-front theory is determined from the eigenvalue equation

$$\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = M^2\langle\psi(P')|\psi(P)\rangle, \quad (1)$$

where one can expand the initial and final hadronic state in terms of its Fock components. The computation is simplified in the frame $P = (P^+, M^2/P^+, \vec{0}_\perp)$ where $P^2 = P^+P^-$. We find

$$M^2 = \sum_n \int [dx_i] [d^2\mathbf{k}_{\perp i}] \sum_q \left(\frac{\mathbf{k}_{\perp q}^2 + m_q^2}{x_q} \right) |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 + (\text{interactions}), \quad (2)$$

plus similar terms for antiquarks and gluons ($m_g = 0$). The integrals in (2) are over the internal coordinates of the n constituents for each Fock state

$$\int [dx_i] \equiv \prod_{i=1}^n \int dx_i \delta\left(1 - \sum_{j=1}^n x_j\right), \quad \int [d^2\mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \int \frac{d^2\mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right), \quad (3)$$

with phase space normalization $\sum_n \int [dx_i] [d^2\mathbf{k}_{\perp i}] |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 = 1$.

The LFWF $\psi_n(x_i, \mathbf{k}_{\perp i})$ can be expanded in terms of $n - 1$ independent position coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \dots, n - 1$, conjugate to the relative coordinates $\mathbf{k}_{\perp i}$, with $\sum_{i=1}^n \mathbf{b}_{\perp i} = 0$. We can also express (2) in terms of the internal impact coordinates $\mathbf{b}_{\perp j}$ with the result

$$M^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_j, \mathbf{b}_{\perp j}) \sum_q \left(\frac{-\nabla_{\mathbf{b}_{\perp q}}^2 + m_q^2}{x_q} \right) \psi_n(x_j, \mathbf{b}_{\perp j}) + (\text{interactions}). \quad (4)$$

The normalization is defined by $\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} |\psi_n(x_j, \mathbf{b}_{\perp j})|^2 = 1$. To simplify the discussion we will consider a two-parton hadronic bound state. In the limit of zero quark mass $m_q \rightarrow 0$

$$M^2 = \int_0^1 \frac{dx}{x(1-x)} \int d^2\mathbf{b}_{\perp} \psi^*(x, \mathbf{b}_{\perp}) (-\nabla_{\mathbf{b}_{\perp}}^2) \psi(x, \mathbf{b}_{\perp}) + (\text{interactions}). \quad (5)$$

The functional dependence for a given Fock state is given in terms of the invariant mass

$$M_n^2 = \left(\sum_{a=1}^n k_a^\mu \right)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a} \rightarrow \frac{\mathbf{k}_{\perp}^2}{x(1-x)}, \quad (6)$$

giving the measure of the off-energy shell of the bound state, $M^2 - M_n^2$. Similarly in impact space the relevant variable for a two-parton state is $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$. Thus, to first approximation LF dynamics depend only on the boost invariant variable M_n or ζ , and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from the relation

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}, \quad (7)$$

thus factoring out the angular dependence φ and the longitudinal, $X(x)$, and transverse mode $\phi(\zeta)$ with normalization $\langle \phi | \phi \rangle = \int d\zeta |\langle \zeta | \phi \rangle|^2 = 1$.

We can write the Laplacian operator in (5) in circular cylindrical coordinates (ζ, φ) and factor out the angular dependence of the modes in terms of the $SO(2)$ Casimir representation L^2 of orbital angular momentum in the transverse plane. Using (7) we find [20]

$$M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta), \quad (8)$$

where all the complexity of the interaction terms in the QCD Lagrangian is summed in the effective potential $U(\zeta)$. The LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$ is thus a light-front wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta), \quad (9)$$

a relativistic single-variable LF Schrödinger equation. Its eigenmodes $\phi(\zeta) = \langle \zeta | \phi \rangle$ determine the hadronic mass spectrum and represent the probability amplitude to find n -partons at transverse impact separation ζ , the invariant separation between pointlike constituents within the hadron [21] at equal LF time. Extension of the results to arbitrary n follows from the x -weighted definition of the transverse impact variable of the $n - 1$ spectator system [21]: $\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$, where $x = x_n$ is the longitudinal momentum fraction of the active quark. One can also generalize the equations to allow for the kinetic energy of massive quarks using Eqs. (2) or (4). In this case, however, the longitudinal mode $X(x)$ does not decouple from the effective LF bound-state equations.

4 A Soft-Wall AdS/QCD Model for Mesons

The conformal algebraic structure of AdS/CFT can be extended to include a scale κ . This procedure breaks conformal invariance and provides a solution for the confinement of modes, while maintaining an integrable algebraic structure. It also allows one to determine the stability conditions for the solutions. The resulting model resembles the soft wall model of Ref. [23]. We write the bound-state LF Hamiltonian as a bilinear product of operators plus a constant $C(\kappa^2)$ to be determined:

$$H_{LF}^\nu(\zeta) = \Pi_\nu^\dagger(\zeta) \Pi_\nu(\zeta) + C, \quad \nu^2 \geq 0, \quad (10)$$

where the LF generator Π and its adjoint Π^\dagger

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} - \kappa^2 \zeta \right), \quad \Pi_\nu^\dagger(\zeta) = -i \left(\frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta \right), \quad (11)$$

obey the commutation relation

$$[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)] = \frac{2\nu + 1}{\zeta^2} - 2\kappa^2. \quad (12)$$

For $\nu^2 \geq 0$ and $C \geq -4\kappa^2$, the Hamiltonian is positive definite, $\langle \phi | H_{LC}^\nu | \phi \rangle = \int d\zeta |\Pi_\nu \phi(z)|^2 + C \geq 0$ and $M^2 \geq 0$. For $\nu^2 < 0$ the Hamiltonian cannot be written as a bilinear product and the Hamiltonian is unbounded from below. The lowest stable solution of the extended LF Hamiltonian corresponds to $C = -4\kappa^2$ and $\nu = 0$ and it

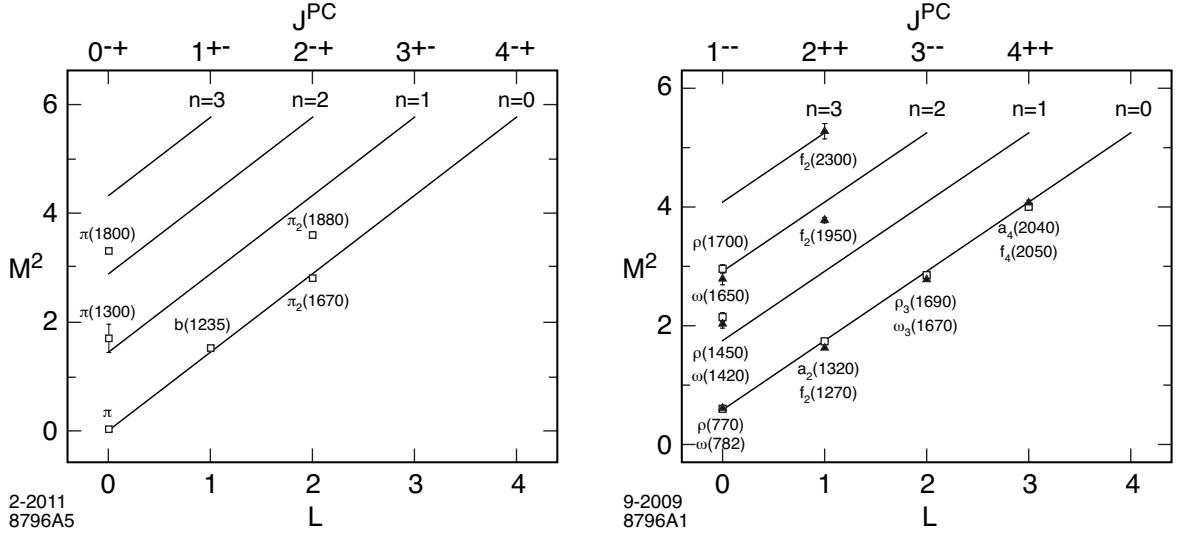


Figure 1: Parent and daughter Regge trajectories for the π -meson family for $\kappa = 0.6$ GeV (left) and for the $I=1$ ρ -meson and the $I=0$ ω -meson families for $\kappa = 0.54$ GeV (right). The data are from Ref. [24].

is massless, $M^2 = 0$. We impose chiral symmetry by choosing $C = -4\kappa^2$ and thus identifying the ground state with the pion. With this choice of the constant C , the LF Hamiltonian (10) is

$$H_{LF}(\zeta) = -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L - 1), \quad (13)$$

with eigenfunctions

$$\phi_L(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2), \quad (14)$$

and eigenvalues $\mathcal{M}^2 = 4\kappa^2(n + L)$. This is illustrated in Fig. 1 for the pseudoscalar meson spectra.

The confining model has also an effective classical gravity description corresponding to an AdS_5 geometry modified by a positive-sign dilaton background $\exp(+\kappa^2 z^2)$, with sign opposite to that of reference of Ref. [23]. The positive dilaton solution has interesting physical implications, since it leads to a confining potential between heavy quarks [25] and to a convenient framework for describing chiral symmetry breaking. [26] It also leads to the identification of a nonperturbative effective strong coupling α_s and β -functions which are in agreement with available data and lattice simulations.[27] In the presence of a dilaton profile $\exp(+\kappa^2 z^2)$ the wave equation for a spin J mode $\Phi(z)_{\mu_1 \dots \mu_J}$ is given by [28]

$$[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 M^2 - (\mu R)^2] \Phi_J = 0. \quad (15)$$

Upon the substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2} \Phi_J(\zeta)$, we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L+S-1) \right) \phi_{\mu_1 \dots \mu_J} = M^2 \phi_{\mu_1 \dots \mu_J}, \quad (16)$$

with $J_z = L_z + S_z$ and $(\mu R)^2 = -(2-J)^2 + L^2$. Equation (16) has eigenfunctions given by (14) and eigenvalues $M_{n,L,S}^2 = 4\kappa^2(n+L+S/2)$. The results for $S=1$ vector mesons is illustrated in Fig. 1, where the spectrum is built by simply adding $4\kappa^2$ for a unit change in the radial quantum number, $4\kappa^2$ for a change in one unit in the orbital quantum number and $2\kappa^2$ for a change of one unit of spin to the ground state value of M^2 . Remarkably, the same rule holds for baryons as shown below.

5 Baryons in Light-Front Holography

The effective light-front wave equation which describes baryonic states in holographic QCD is a linear equation determined by the LF transformation properties of spin 1/2 states. We write

$$D_{LF}(\zeta)\psi(\zeta) = M\psi(\zeta), \quad (17)$$

where D_{LF} is a hermitian operator, $D_{LF} = D_{LF}^\dagger$, thus $D_{LF}^2 = M^2$. We write D_{LF} as a product $D_{LF} = \alpha\Pi$, where Π is the matrix valued (non-hermitian) generator

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma \right). \quad (18)$$

It follows from the square of D_{LF} , $D_{LF}^2 = M^2$, that the matrices α and γ are 4×4 anti-commuting hermitian matrices with unit square. The operator Π and its adjoint Π^\dagger thus satisfy the commutation relation

$$[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)] = \frac{2\nu+1}{\zeta^2} \gamma. \quad (19)$$

The light front Hamiltonian H_{LF} is

$$H_{LF}^\nu(\zeta) = \Pi_\nu(\zeta)^\dagger \Pi_\nu(\zeta) = -\frac{d^2}{d\zeta^2} + \frac{(\nu + \frac{1}{2})^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2} \gamma. \quad (20)$$

The LF equation $H_{LF}\psi_\pm = M^2\psi_\pm$, has a two-component solution

$$\psi_+(\zeta) \sim \sqrt{\zeta} J_\nu(\zeta M), \quad \psi_-(\zeta) \sim \sqrt{\zeta} J_{\nu+1}(\zeta M), \quad (21)$$

where $\gamma\psi_\pm = \pm\psi_\pm$. Thus γ is the four dimensional chirality operator γ_5 . In the Weyl representation

$$\gamma = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \text{and} \quad i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}. \quad (22)$$

The effective LF equation for baryons (17) is equivalent to the Dirac equation describing the propagation of spin-1/2 hadronic modes, on AdS₅ space $\Psi_P(x^\mu, z) = e^{-iP \cdot x} \Psi(z)$

$$[i(z\eta^{MN}\Gamma_M\partial_N + 2\Gamma_z) + \mu R] \Psi = 0, \quad (23)$$

where M, N represent the indices of the full space with coordinates x^μ and z . Upon the transformation $\Psi(z) \sim z^2\psi(z)$, $z \rightarrow \zeta$, we recover (17) with $\mu R = \nu + 1/2$ and $\Gamma_z = -i\gamma$. Higher spin fermionic modes $\Psi_{\mu_1 \dots \mu_{J-1/2}}$, $J > 1/2$, with all polarization indices along the 3+1 coordinates follow by shifting dimensions as shown for the case of mesons.

6 A Soft-Wall Light-Front Model for Baryons

An effective LF equation for baryons with a mass gap κ is constructed by extending the conformal algebraic structure for baryons described above, following the analogy with the mesons. We write the effective LF Dirac equation (17) in terms of the matrix-valued operator Π and its adjoint Π^\dagger

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma - \kappa^2 \zeta \gamma \right), \quad \Pi_\nu^\dagger(\zeta) = -i \left(\frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} \gamma + \kappa^2 \zeta \gamma \right), \quad (24)$$

with the commutation relation

$$[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma. \quad (25)$$

The extended baryonic model also has a geometric interpretation. It corresponds to the Dirac equation in AdS₅ space in presence of a linear potential $\kappa^2 z$

$$[i(z\eta^{MN}\Gamma_M\partial_N + 2\Gamma_z) + \kappa^2 z + \mu R] \Psi = 0, \quad (26)$$

as can be shown directly by using the transformation $\Psi(z) \sim z^2\psi(z)$, $z \rightarrow \zeta$.

As for the case of the mesons Eq. (10) we write the LF Hamiltonian $H_{LF}^\nu = \Pi_\nu^\dagger \Pi_\nu + C$ and chose the same value for C : $C = -4\kappa^2$, effectively modifying the wave equation (26). With this choice for C the LF Hamiltonian is

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{(\nu + \frac{1}{2})^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2} \gamma_5 + \kappa^4 \zeta^2 + \kappa^2(2\nu - 3) + \kappa^2 \gamma_5. \quad (27)$$

The LF equation $H_{LF}\psi_\pm = M^2\psi_\pm$, has a two-component solution

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2/2} L_n^\nu(\kappa^2 \zeta^2), \quad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2/2} L_n^{\nu+1}(\kappa^2 \zeta^2), \quad (28)$$

and eigenvalues $M^2 = 4\kappa^2(n + \nu)$, identical for both plus and minus eigenfunctions.

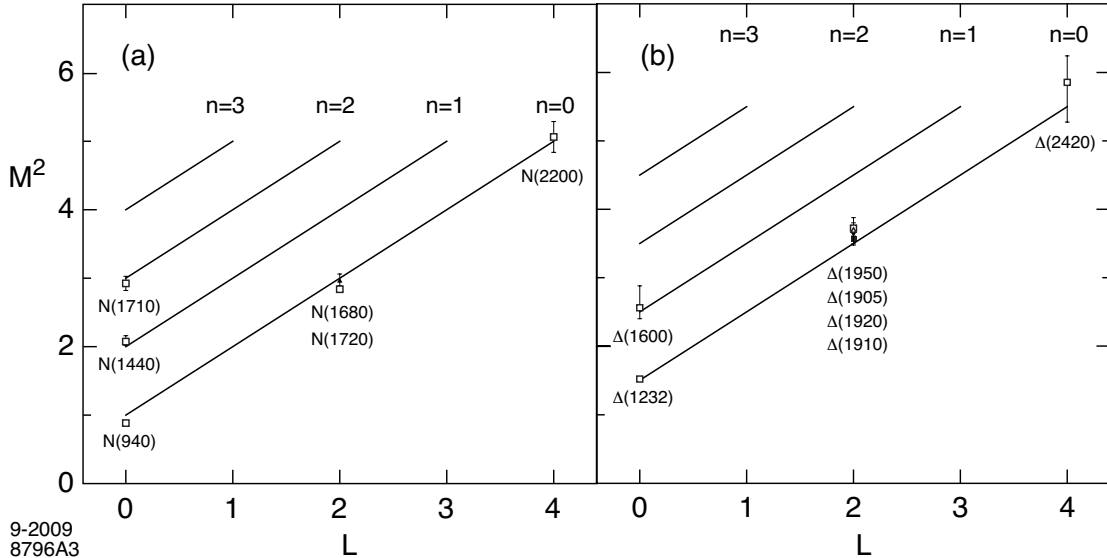


Figure 2: **56** Parent and daughter Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV. Data from [24].

The baryon interpolating operator $\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots \ell_q\}} \psi D_{\{\ell_{q+1} \dots \ell_m\}} \psi$, $L = \sum_{i=1}^m \ell_i$, is a twist 3, dimension $9/2 + L$ operator with scaling behavior given by its twist-dimension $3 + L$. We thus require $\nu = L + 1$ to match the short distance scaling behavior. Higher spin fermionic modes $\Psi_{\mu_1 \dots \mu_{J-1/2}}$, $J > 1/2$, are obtained by shifting dimensions for the fields as in the bosonic case. Thus, as in the meson sector, the increase in the mass squared for higher baryonic states is $\Delta n = 4\kappa^2$, $\Delta L = 4\kappa^2$ and $\Delta S = 2\kappa^2$, relative to the lowest ground state, the proton.

The predictions for the positive parity light baryons are shown in Fig. 2. As for the predictions for mesons in Fig. 1, only confirmed PDG [24] states are shown. The Roper state $N(1440)$ and the $N(1710)$ are well accounted for in this model as the first and second radial states. Likewise the $\Delta(1660)$ corresponds to the first radial state of the Δ family. The model is successful in explaining the important parity degeneracy observed in the light baryon spectrum, such as the $L=2$, $N(1680) - N(1720)$ pair and the $\Delta(1905)$, $\Delta(1910)$, $\Delta(1920)$, $\Delta(1950)$ states which are degenerate within error bars. The parity degeneracy of baryons is also a property of the hard wall model, but radial states are not well described by this model. [29] For other recent calculations of the hadronic spectrum based on AdS/QCD, see Refs. [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44].

The proton eigenstate in light-front holography

$$\psi(\zeta) = \psi_+(\zeta)u_+ + \psi_-(\zeta)u_-, \quad (29)$$

has $L^z = 0$ and $L^z = +1$ orbital components combined with spin components $S^z = +1/2$ and $S^z = -1/2$ respectively. An interesting feature of light front holography for

baryons and massless quarks is that the lowest valence Fock states with $L^z = 0$ and $L^z = \pm 1$ have the same probability

$$\int d\zeta |\psi_+(\zeta)|^2 = \int d\zeta |\psi_-(\zeta)|^2, \quad (30)$$

a manifestation of the chiral invariance of the theory for massless quarks. [45] This implies that the quarks carry zero angular momentum $\langle S_z = 0 \rangle$ in the proton with $J^z = \pm 1/2$ and $\langle L^z = 1/2 \rangle$.

There are many other interesting predictions for baryons using AdS/QCD and light front holographic methods such as space-like and time-like Dirac and Pauli form factors, valence structure functions, etc. The AdS/QCD LF Hamiltonian also creates Fock states with extra quark-antiquark pairs [46] as in QCD(1+1).

7 Conclusions

We have derived a correspondence between a semiclassical first approximation to QCD quantized on the light-front and hadronic modes propagating on a fixed AdS background. This provides a duality between the bosonic and fermionic wave equations in AdS higher dimensional space and the corresponding LF equations in physical $3 + 1$ space. The duality leads to Schrödinger and Dirac-like equations for hadronic bound states in physical space-time when one identifies the AdS fifth dimension coordinate z with the LF coordinate ζ . The light-front equations of motion, which are dual to an effective classical gravity theory, possess remarkable algebraic and integrability properties which follow from the underlying conformal properties of the theory. We also extend the algebraic construction to include a confining potential while preserving the integrability of the mesonic and baryonic bound-state equations.

Light-Front Holography is one of the most remarkable features of AdS/CFT. It allows one to project the functional dependence of the wavefunction $\Phi(z)$ computed in the AdS fifth dimension to the hadronic frame-independent light-front wavefunction $\psi(x_i, \mathbf{b}_{\perp i})$ in $3 + 1$ physical space-time. The variable z maps to $\zeta(x_i, \mathbf{b}_{\perp i})$. To confirm this, we have shown that there exists a correspondence between the matrix elements of the energy-momentum tensor of the fundamental hadronic constituents in QCD with the transition amplitudes describing the interaction of string modes in anti-de Sitter space with an external graviton field which propagates in the AdS interior. The agreement of the results for both electromagnetic and gravitational hadronic transition amplitudes provides an important consistency test and verification of holographic mapping from AdS to physical observables defined on the light-front. As we have discussed, this correspondence is a consequence of the fact that the metric ds^2 for AdS₅ at fixed light-front time τ is invariant under the simultaneous scale change $\mathbf{x}_{\perp}^2 \rightarrow \lambda^2 \mathbf{x}_{\perp}^2$ in transverse space and $z^2 \rightarrow \lambda^2 z^2$. The transverse coordinate ζ is closely related to the invariant mass squared of the constituents in the LFWF and its off-shellness in the light-front kinetic energy, and it is thus the natural variable to characterize

the hadronic wavefunction. In fact ζ is the only variable to appear in the light-front Schrödinger equations predicted from AdS/QCD. These equations for both meson and baryons give a good representation of the observed hadronic spectrum. The resulting LFWFs also have remarkable phenomenological features, including predictions for the electromagnetic form factors and decay constants. We have also shown that the LF Hamiltonian formulation of quantum field theory provides a natural formalism to compute hadronization at the amplitude level. [3]

The light-front holographic theory provides successful predictions for the light-quark meson and baryon spectra, as function of hadron spin, quark angular momentum, and radial quantum number. Using the positive dilaton background $\exp(+\kappa^2 z^2)$ the pion is massless, corresponding to zero mass quarks, in agreement with chiral invariance arguments. Higher spin light-front equations can be derived by shifting dimensions in the AdS wave equations. [28] Unlike the top-down string theory approach, one is not limited to hadrons of maximum spin $J \leq 2$, and one can study baryons with finite color $N_C = 3$. Both the hard and soft-wall models predict similar multiplicity of states for mesons and baryons as it is observed experimentally. [47] In the hard-wall model the dependence has the form: $M \sim 2n + L$. However, in the soft-wall model the observed Regge behavior is found: $M^2 \sim n + L$, which has the same slope in radial quantum number and orbital angular momentum.

The semiclassical AdS/QCD approximation to light-front QCD described in this talk breaks down at short distances where hard gluon exchange and quantum corrections become important. However, one can systematically improve the semiclassical approximation by introducing nonzero quark masses and short-range Coulomb corrections, thus extending the predictions of the model to the dynamics and spectra of heavy and heavy-light quark systems. One can also diagonalize the LF Hamiltonian as in DLCQ, but on the orthonormal basis states provided by AdS/QCD. [48] One could also employ Lippmann-Schwinger perturbation theory, systematically correcting the AdS/QCD eigensolutions.

Acknowledgments

Presented by SJB at the International Conference on the Structure of Baryons BARYONS'10, December 7-11, 2010, Osaka, Japan. This research was supported by the Department of Energy contract DE-AC02-76SF00515. SLAC-PUB-14381

References

- [1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
- [2] For a review of light-front quantization, see S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys. Rept. **301**, 299-486 (1998) [hep-ph/9705477].

- [3] S. J. Brodsky, G. de Teramond, R. Shrock, AIP Conf. Proc. **1056**, 3-14 (2008) [arXiv:0807.2484 [hep-ph]].
- [4] C. T. Munger, S. J. Brodsky, I. Schmidt, Phys. Rev. **D49**, 3228-3235 (1994).
- [5] S. J. Brodsky, P. Hoyer, C. Peterson, N. Sakai, Phys. Lett. **B93**, 451-455 (1980).
- [6] G. Bari, M. Basile, G. Bruni *et al.*, Nuovo Cim. **A104**, 1787-1800 (1991).
- [7] M. Mattson *et al.* [SELEX Collaboration], Phys. Rev. Lett. **89**, 112001 (2002) [hep-ex/0208014].
- [8] S. J. Brodsky, .J Fleuret, J. P. Lansberg, C. Lorce (in progress).
- [9] S. J. Brodsky, B. Kopeliovich, I. Schmidt, J.-Soffer, Phys. Rev. **D73**, 113005 (2006) [hep-ph/0603238].
- [10] S. J. Brodsky, A. S. Goldhaber, B. Z. Kopeliovich, I. Schmidt, Nucl. Phys. **B807**, 334-347 (2009) [arXiv:0707.4658 [hep-ph]].
- [11] F. Arleo, S. J. Brodsky, D. S. Hwang, A. Sickles, Phys. Rev. Lett. **105**, 062002 (2010) [arXiv:0911.4604 [hep-ph]].
- [12] S. J. Brodsky, A. Sickles, Phys. Lett. **B668**, 111-115 (2008) [arXiv:0804.4608 [hep-ph]].
- [13] S. J. Brodsky, D. S. Hwang, I. Schmidt, Phys. Lett. **B530**, 99-107 (2002) [hep-ph/0201296].
- [14] J. C. Collins, Phys. Lett. **B536**, 43-48 (2002) [hep-ph/0204004].
- [15] S. J. Brodsky, D. S. Hwang, I. Schmidt, Nucl. Phys. **B642**, 344-356 (2002) [hep-ph/0206259].
- [16] D. Boer, S. J. Brodsky, D. S. Hwang, Phys. Rev. **D67**, 054003 (2003). [hep-ph/0211110].
- [17] S. J. Brodsky, R. Shrock, Proc. Nat. Acad. Sci. **108**, 45-50 (2011) [arXiv:0905.1151 [hep-th]].
- [18] S. J. Brodsky, C. D. Roberts, R. Shrock , P. Tandy, Phys. Rev. **C82**, 022201 (2010) [arXiv:1005.4610 [nucl-th]].
- [19] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
- [20] G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **102**, 081601 (2009) [arXiv:0809.4899 [hep-ph]].

- [21] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. **96**, 201601 (2006) [arXiv:hep-ph/0602252]; Phys. Rev. **D77**, 056007 (2008) [arXiv:0707.3859 [hep-ph]].
- [22] S. J. Brodsky and G. F. de Teramond, Phys. Rev. D **78**, 025032 (2008) [arXiv:0804.0452 [hep-ph]].
- [23] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [arXiv:hep-ph/0602229].
- [24] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667** (2008) 1.
- [25] O. Andreev, Phys. Rev. D **73**, 107901 (2006) [arXiv:hep-th/0603170].
- [26] F. Zuo, Phys. Rev. D **82**, 086011 (2010) [arXiv:0909.4240 [hep-ph]].
- [27] S. J. Brodsky, G. F. de Teramond, A. Deur, Phys. Rev. **D81**, 096010 (2010) [arXiv:1002.3948 [hep-ph]].
- [28] G. F. de Teramond, S. J. Brodsky, Nucl. Phys. Proc. Suppl. **199**, 89-96 (2010) [arXiv:0909.3900 [hep-ph]].
- [29] G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **94**, 201601 (2005) [arXiv:hep-th/0501022].
- [30] H. Boschi-Filho and N. R. F. Braga, JHEP **0305**, 009 (2003) [arXiv:hep-th/0212207].
- [31] H. Boschi-Filho, N. R. F. Braga and H. L. Carrion, Phys. Rev. D **73**, 047901 (2006) [arXiv:hep-th/0507063].
- [32] N. Evans and A. Tedder, Phys. Lett. B **642**, 546 (2006) [arXiv:hep-ph/0609112].
- [33] D. K. Hong, T. Inami and H. U. Yee, Phys. Lett. B **646**, 165 (2007) [arXiv:hep-ph/0609270].
- [34] P. Colangelo, F. De Fazio, F. Jugeau and S. Nicotri, Phys. Lett. B **652**, 73 (2007) [arXiv:hep-ph/0703316].
- [35] H. Forkel, Phys. Rev. D **78**, 025001 (2008) [arXiv:0711.1179 [hep-ph]].
- [36] A. Vega and I. Schmidt, Phys. Rev. D **78**, 017703 (2008) [arXiv:0806.2267 [hep-ph]].
- [37] K. Nawa, H. Suganuma and T. Kojo, Mod. Phys. Lett. A **23**, 2364 (2008) [arXiv:0806.3040 [hep-th]].

- [38] W. de Paula, T. Frederico, H. Forkel and M. Beyer, Phys. Rev. D **79**, 075019 (2009) [arXiv:0806.3830 [hep-ph]].
- [39] P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D **78**, 055009 (2008) [arXiv:0807.1054 [hep-ph]].
- [40] H. Forkel and E. Klempt, Phys. Lett. B **679**, 77 (2009) [arXiv:0810.2959 [hep-ph]].
- [41] H. C. Ahn, D. K. Hong, C. Park and S. Siwach, Phys. Rev. D **80**, 054001 (2009) [arXiv:0904.3731 [hep-ph]].
- [42] Y. Q. Sui, Y. L. Wu, Z. F. Xie and Y. B. Yang, Phys. Rev. D **81**, 014024 (2010) [arXiv:0909.3887 [hep-ph]].
- [43] M. Kirchbach, C. B. Compean, Phys. Rev. **D82**, 034008 (2010) [arXiv:1003.1747 [hep-ph]].
- [44] T. Branz, T. Gutsche, V. E. Lyubovitskij *et al.*, Phys. Rev. **D82**, 074022 (2010) [arXiv:1008.0268 [hep-ph]].
- [45] S. J. Brodsky, G. F. de Teramond, Acta Phys. Pol. B **41**, 2605 (2010) [arXiv:1009.4232 [hep-ph]].
- [46] G. F. de Teramond, S. J. Brodsky [arXiv:1010.1204 [hep-ph]].
- [47] E. Klempt and A. Zaitsev, Phys. Rept. **454**, 1 (2007) [arXiv:0708.4016 [hep-ph]].
- [48] J. P. Vary, H. Honkanen, J. Li *et al.*, Phys. Rev. **C81**, 035205 (2010) [arXiv:0905.1411 [nucl-th]].