Isospin splittings of doubly heavy baryons

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Abstract

The SELEX Collaboration has reported a very large isospin splitting of doubly charmed baryons. We show that this effect would imply that the doubly charmed baryons are very compact. One intriguing possibility is that such baryons have a linear geometry $Q - q - Q$ where the light quark q oscillates between the two heavy quarks Q, analogous to a linear molecule such as carbon dioxide. However, using conventional arguments, the size of a heavy-light hadron is expected to be around 0.5 fm, much larger than the size needed to explain the observed large isospin splitting. Assuming the distance between two heavy quarks is much smaller than that between the light quark and a heavy one, the doubly heavy baryons are related to the heavy mesons via heavy quark–diquark symmetry. Based on this symmetry, we predict the isospin splittings for doubly heavy baryons including Ξ_{cc}, Ξ_{bb} and Ξ_{bc}. The prediction for the Ξ_{cc} is much smaller than the SELEX value. On the other hand, the Ξ_{bb} baryons are predicted to have an isospin splitting as large as (6.3 ± 1.7) MeV. An experimental study of doubly bottomed baryons is therefore very important to better understand the structure of baryons with heavy quarks.

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1 Introduction

A key prediction of QCD is the existence of baryons with two or three charm or bottom quarks. Several years ago, evidence for the hadroproduction of five different baryons with two charm quarks was reported by the SELEX Collaboration at Fermilab [1, 2, 3, 4, 5, 6]. Two singly charged states $\Xi_{cc}^+(3443)$ and $\Xi_{cc}^+(3520)$ were observed in the $\Lambda_c^+ K^- \pi^+$ mass distribution, and three doubly charged states $\Xi_{cc}^{++}(3460)$, $\Xi_{cc}^{++}(3541)$ and $\Xi_{cc}^{++}(3780)$ were observed decaying into $\Lambda_c^+ K^- \pi^+ \pi^+$ [1, 2, 3]. The $\Xi_{cc}^+(3520)$ was also observed in the $pD^+ K^-$ [4] and $\Xi_c^+ \pi^+ \pi^-$ final states [6]. An analysis of the helicity angular distribution support the assignments that the $\Xi_{cc}^+(3443)$ and $\Xi_{cc}^{++}(3460)$ form an isospin doublet, and the $\Xi_{cc}^+(3520)$ and $\Xi_{cc}^{++}(3541)$ form another. The preliminary isospin mass splittings were reported to be 17 MeV and 21 MeV, respectively [3]. This observation is very puzzling because such values are much larger than all isospin mass splitting of hadrons known so far. For instance, the mass difference between the proton and neutron is $m_n - m_p = 1.29$ MeV, and between the charged and neutral D mesons is $M_{D^+} - M_{D^0} = 4.77 \pm 0.10$ MeV [7]. The largest isospin splitting ever observed is the double-strange baryons $M_{\Xi^-} - M_{\Xi^0} = 6.85 \pm 0.21$ MeV [7].

Recently, the mass of the lowest Ξ_{cc}^{++} state was updated from 3460 to 3452 MeV [8]. Although the isospin splitting is decreased from 17 MeV to 9 MeV for the lower doubly charmed baryon isospin-doublet, it is still larger than all the other known isospin splittings. It is thus interesting to see whether it is possible to obtain the observed rather large values from known physical principles with controlled uncertainty. In Section 2, it will be shown that the SELEX observations would imply the Ξ_{cc} to be very compact, which, however, cannot be understood by any known mechanism of the strong interactions.

Predictions for the isospin splittings of doubly heavy baryons will be presented in Section 3 based on the conventional assumption that the two heavy quarks constitute a compact diquark. We then can apply the ansatz of heavy quark–diquark symmetry. Our predictions for the doubly charm baryons are similar in magnitude to the isospin splittings for other hadron isospin multiplets, but considerably smaller than the SELEX data. We will also give predictions for the isospin splittings of the $ccc\bar{q}$ and $bbbb\bar{q}$ pentaquark states based on a heavy quark–"quadra-quark" symmetry in Section 4.

SELEX used $p_{\text{lab}} = 600$ GeV/c π^-, Σ^-, and proton beams on a nuclear target to produce the doubly charm baryons. A striking feature of the SELEX measurements is the fact that the observed doubly charmed baryons are all produced at $x_F > 0.1$ (SELEX only has sensitivity in that region); i.e., at a significant fraction of the projectile momentum. This is consistent with the ISR measurements of the Λ_c [9] and the Λ_b [10] at high x_F, as well as NA3 measurements at CERN [11] which showed that two J/ψ's are hadro-produced at high x_F in pion-nucleus collisions; in fact, each $\pi A \to J/\psi J/\psi X$ event measured by NA3 has four charmed quarks with a flat longitudinal momentum distribution for $x_F > 0.4$. [12]. The SELEX and NA3 measurements cannot be explained if the heavy quarks only arise from gluon splitting; however, this is a natural consequence of the existence of intrinsic heavy quarks in the projectile [13, 14, 15, 16], such as the rare $|uudc\bar{c}c\bar{c}\rangle$ Fock state in the proton or the $|\bar{u}dc\bar{c}c\bar{c}\rangle$ Fock state in the π^-. Since the momentum distribution in such Fock states is maximized at low invariant mass, all of the quarks tend to have the same rapidity and small transverse momentum. The heavy quarks have the maximum momentum fractions in such configurations since equal rapidity implies $x_i \propto \sqrt{m_i^2 + k_{\perp i}^2}$. The doubly charmed ccq baryons are then formed in a collision by the coalescence [17, 18] of the comoving heavy quarks with

a light quark of the projectile — the domain where the wave function of the produced doubly charm hadron is maximal. This mechanism also explains why doubly charmed baryons are not readily produced in e^+e^- annihilation; in that case it is rare for the two charmed quarks to be in the same kinematic domain. The intrinsic charm mechanism also accounts for the non-factorized nuclear-target dependence [19, 20] of J/ψ hadroproduction [17, 18]. It also points to the high x_F domain of hadroproduction as the best kinematic region to search for heavy hadron systems in general. Thus the best opportunity to create superheavy hadrons and test their properties is in hadron-hadron collisions at high x_F using the intrinsic heavy quark Fock state mechanism — for example, at the LHCb, or at future fixed-target experiments using the 7 TeV LHC beam.

The last section contains a brief summary of our results.

2   Implication of a large isospin splitting

It is instructive to ask the question: What does a large isospin splitting imply for the doubly heavy baryons? Isospin splittings originate from two sources — the u and d quark mass difference as well as electromagnetic contributions. The interference pattern of the two different contributions to the mass differences can be easily understood. The repulsive (attractive) Coulomb interaction gives positive (negative) contribution to the electromagnetic (em) self-energy of the baryons, so that the baryon with more absolute electric charge has more em self-energy. The sign of the quark mass difference contribution reflects the fact that the down quark is heavier than the up quark. Hence, for the Ξ_{cc} and Ξ_{bc}, the interference is destructive while for the Ξ_{bb} it is constructive. The sign of the em contribution to $M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+}$ is the same as the SELEX data, however, the em and the quark mass term interfere destructively substantially reducing the em effect. Thus, in order to quantitatively understand the SELEX data an unusually large em contribution to the mass differences is necessary.

In the case of heavy particles the em effect is Coulombic since the magnetic contribution is negligible. To quantify the em contribution, one may employ the Cottingham formula to analyze the contribution of virtual photons. It can be used to relate the em self-energy to the em form factor of a particle, see e.g. [21]. Then the em self-energy is given by [21] (neglecting the inelastic contributions)

$$M^{\mathrm{em}} = \frac{\alpha Q^2}{4\pi^2} \int \frac{d^3q}{\mathbf{q}^2} [G_E(-\mathbf{q}^2)]^2, \qquad (1)$$

where $\alpha = 1/137.06$ is the fine structure constant, Q is the total electric charge in units of the proton charge, and $G_E(t)$ is the Fourier transform of the charge distribution of the particle. Taking a dipole distribution with a mass parameter m in units of GeV (which is sufficient for our purpose)

$$G_E(t) = \frac{1}{(1 - t/m^2)^2}, \qquad (2)$$

one can perform the integration analytically, and gets

$$M^{\mathrm{em}} = \frac{5}{32} \alpha Q^2 m. \qquad (3)$$

For a first estimate, let us consider the ccq as a two-particle system with charge 4/3 and e_q. Therefore, the em contribution to the isospin splitting of the doubly charmed baryons

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$\delta_{\Xi_{cc}} \equiv M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+}$ with $\Delta Q^2 = 3$ is

$$\delta_{\Xi_{cc}}^{\text{em}} = \frac{15}{32}\alpha m = 0.0034 \ m. \tag{4}$$

The mean square radius of a particle is obtained by taking the first derivative of its em form factor. For a heavy particle with the em form factor given by Eq. (2), it is

$$\langle r^2 \rangle = 6 \frac{dG_E(t)}{dt}\bigg|_{t=0} = \frac{12}{m^2}. \tag{5}$$

Since the Ξ_{cc}^{++} and Ξ_{cc}^+ contain ccu and ccd quarks, respectively, the contribution to $\delta_{\Xi_{cc}}$ from the quark mass difference is negative, i.e. $\delta_{\Xi_{cc}}^{\text{strong}} < 0$ because the u quark is lighter than the d quark. Hence, the em contribution must be larger than the total isospin splitting of the doubly charmed baryons, $\delta_{\Xi_{cc}}^{\text{em}} > \delta_{\Xi_{cc}}$. If we take 9 MeV as the isospin splitting, one gets $m > 2.65$ GeV from Eq. (4), and

$$\sqrt{\langle r^2 \rangle} < 0.26 \text{ fm}. \tag{6}$$

This value is much smaller than the typical size of a hadron containing light quark(s). In fact, it is of similar size as the distance between the two heavy quarks $\sim 1/(m_Q v)$, with m_Q and v the mass and the velocity of the heavy quark, respectively. If we use a larger isospin splitting, e.g. 17 MeV, instead, the resulting $\sqrt{\langle r^2 \rangle} < 0.14$ fm is even smaller.

Therefore, we conclude that a large isospin splitting would imply the doubly heavy baryon to be very compact — the larger the splitting, the smaller the size. This conclusion can be easily understood because a large isospin splitting would mean a large em self-energy which, being proportional to $\langle 1/r \rangle$, in turn would mean a small size of the doubly heavy baryon.

One intriguing possibility is that doubly charm baryon states have a linear geometry $Q - q - Q$ where the light quark q oscillates between the two heavy quarks Q, analogous to a linear molecule such as carbon dioxide $CO_2 = O - C - O$. In this case the overall size of the baryon would be relatively small. A lattice gauge theory investigation of this possibility would be interesting. However, we are not aware of a mechanism in quantum chromodynamics (QCD) which can keep the light quark in line and close to the heavy quarks. In particular, if we take a Coulomb plus linear potential as the interquark interaction, the distance between the light quark and a heavy quark scales as $(\sigma m_{\text{cons}})^{-1/3} \sim 1/\Lambda_{\text{QCD}}$ with the flavor-independent string tension σ being the coefficient in front of the linear potential and m_{cons} the constituent light quark mass. The numerical value for $\sqrt{\sigma}$ is about 430 MeV from Regge trajectories of light mesons and also from heavy quarkonia spectrum, see e.g. [22]. Thus the size of a heavy-light hadron is expected to be around 0.5 fm, much larger than the size needed to explain the observed large isospin splitting.

3 Isospin splittings of doubly heavy baryons – a quantitative analysis

In the following we will assume that the distance between the light quark and a heavy quark is much larger than the distance between the two heavy quarks inside the doubly heavy baryon. Based on this conventional assumption, it was proposed that there is a heavy quark–diquark

symmetry [23] which relates a doubly heavy baryon containing two heavy quarks to a heavy meson containing a heavy anti-quark.

The distance between the two heavy quarks in a doubly heavy baryon is characterized by $r_{QQ} \sim 1/(m_Q v)$, with v being the heavy quark velocity. It is much larger than the length scale for the light quark $r_{qQ} \sim 1/\Lambda_{\rm QCD}$, as already discussed in the previous section. Hence, one may perform an expansion in r_{QQ}/r_{qQ} with a controlled uncertainty. In the heavy quark limit, only the leading term is necessary, which means the diquark formed by the two heavy quarks is point-like. Furthermore, the diquark has the same color charge as a heavy anti-quark. Hence, there is an approximate U(5) symmetry relating the spin-1/2 heavy anti-quark and the spin-1 heavy diquark, called heavy quark-diquark symmetry [23]. Using this symmetry, the doubly heavy baryons can be studied by relating them to the heavy mesons. In this section, we will calculate the em as well as quark mass difference contribution to the isospin splitting of doubly heavy baryon masses at next-to-leading order (NLO) in the chiral expansion. This is the lowest order at which isospin breaking operators appear. In view of the present knowledge of the masses of the doubly heavy baryons, this should suffice. Both effects can be taken into account systematically up to a given order using chiral perturbation theory with virtual photons [24, 25]. We will use the formalism proposed in Ref. [26], which combines heavy mesons and doubly heavy baryons into the same field, and construct the NLO chiral Lagrangian which is responsible for the isospin mass splittings.

The super-flavor multiplet of heavy mesons and doubly heavy baryons can be collected into a single 5×2 matrix field [26], which can be written in components as

$$\mathcal{H}_{a,\mu\beta} = H_{a,\alpha\beta} + T_{a,i\beta} \tag{7}$$

where $a = u, d, s$ is the light flavor index, μ runs from 1 to 5, $\alpha, \beta = 1, 2$ and $i = 1, 2, 3$. The fields for the heavy mesons, H_a, and and doubly heavy baryons, T_a, are given by

$$H_{a,\alpha\beta} = \vec{P}_a^* \cdot \vec{\sigma}_{\alpha\beta} + P_a \delta_{\alpha\beta},$$
$$T_{a,i\beta} = \sqrt{2} \left(\Xi_{a,i\beta}^* + \frac{1}{\sqrt{3}} \Xi_{a,\alpha} \sigma_{\alpha\beta}^i \right), \tag{8}$$

where $P_a^{(*)}$ and $\Xi_a^{(*)}$ are the fields annihilating the vector (pseudoscalar) heavy mesons and the spin-1/2 (3/2) doubly heavy baryons, respectively. The field for the spin-3/2 baryon is constrained by $\Xi_{a,i\beta}^* \sigma_{\beta\gamma}^i = 0$.

One can construct the effective Lagrangian for the mass terms assuming the heavy quark–diquark symmetry. At leading order, there is no isospin splitting within the same multiplet as can been from the Lagrangian constructed in Ref. [26]. At NLO, the Lagrangian relevant for the isospin and SU(3) mass differences reads

$$\mathcal{L}_{\rm ISV} = -c\,{\rm Tr}\left[\mathcal{H}_a^\dagger \mathcal{H}_b (\chi_+)_{ba} \right] - d_0\,{\rm Tr}\left[\mathcal{H}_a^\dagger \hat{q} \mathcal{H}_b (Q_+)_{ba} \right]$$
$$- {\rm Tr}\left\{ \mathcal{H}_a^\dagger \mathcal{H}_b \left[d_1 (Q_+^2 - Q_-^2)_{ba} + d_2 (Q_+ \langle Q_+ \rangle)_{ba} \right] \right\}, \tag{9}$$

where the heavy-flavor charge operator $\hat{q}$ is defined as $\hat{q} H_a = q_Q H_a$ and $\hat{q} T_a = -2 q_Q T_a$, with q_Q being the charge of the heavy quark in a heavy meson. It is similar to the isospin breaking terms in the Lagrangians constructed for heavy mesons [27] and singly heavy baryons [28]. The operators χ_+ and $Q_\pm$ contain the Goldstone boson fields which are needed for higher order calculations

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi u, \qquad Q_\pm = \frac{1}{2}\left(u^\dagger Q u \pm u Q u^\dagger \right), \tag{10}$$

where $u = \sqrt{U}$, $U = \exp\left(\sqrt{2}i\phi/F_\pi\right)$, with F_π the pion decay constant, and ϕ collects the pseudoscalar mesons,

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{11}$$

The light quark mass and charge matrices are

$$\chi = 2B_0 \cdot \text{diag}\left\{m_u, m_d, m_s\right\}, \quad Q = e \cdot \text{diag}\left\{\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right\}. \tag{12}$$

The light quark mass and em contributions to the heavy meson and doubly heavy baryon masses can be easily worked out using this Lagrangian. At NLO, the Goldstone bosons are not needed since the chiral loop corrections to the hadron masses start from higher order. In this case, physically, the d_1 term describes the effect of the virtual photons coupled to the light quark. Because there is only one light quark in both heavy mesons and doubly heavy baryons, these virtual photons merely contribute to the self-energy of the light quark. So they can be absorbed into a redefinition of the quark masses

$$\tilde{m}_u = m_u + \frac{d_1 e^2}{9cB_0}, \qquad \tilde{m}_{d(s)} = m_{d(s)} + \frac{d_1 e^2}{36cB_0}. \tag{13}$$

There is no d_2 term because $\langle Q_+ \rangle = \langle Q \rangle$ vanishes with the charge matrix given in Eq. (12). We remark that we are well aware of the subtleties concerning em corrections to quark masses (see e.g. Ref. [29]) but these can be ignored to the accuracy we are working.

Therefore, at NLO there are only two parameters describing the mass corrections of the heavy mesons and doubly heavy baryons. One is for the light quark mass difference, and the other one is for the em effects which originate from the virtual photons exchanged between the light quark and the heavy quarks. These two effects can also be parameterized by two parameters in quark models once neglecting the spin-dependent interactions which are suppressed by $1/m_Q$, see e.g. [30, 31]. The explicit expressions for the mass corrections can be found in the Appendix. The former parameter can be determined through the measured SU(3) mass splitting between the heavy mesons, and the latter one can be determined from the isospin mass splitting between the charged and neutral heavy mesons [27]. Defining $\tilde{c} \equiv 4cB_0(\tilde{m}_s - \tilde{m}_d)$ and $\tilde{d} \equiv d_3 e^2/3$, using the mass differences among the pseudoscalar charmed mesons D^0, D^+, D_s^+ [7], one gets

$$\tilde{c} = (98.99 \pm 0.30) \text{ MeV}. \tag{14}$$

The value of $\tilde{d}$ can be extracted via

$$\tilde{d} = \frac{1}{2}[(M_{D_s^+} - M_{D^+})\lambda - (M_{D^+} - M_{D^0})] = (-1.05 \pm 0.16) \text{ MeV}, \tag{15}$$

where $\lambda = (\tilde{m}_d - \tilde{m}_u)/(\tilde{m}_s - \tilde{m}_d) = 0.027 \pm 0.003$ is calculated from the recent FLAG average [32]. This value for λ is consistent with the latest precise lattice determinations of the light quark masses, $\lambda = 0.029 \pm 0.002$ [33]. Note that in both cases the value for λ refers to the masses without the em shift. In what follows, we will ignore this difference, as it is expected to be a minor effect. Therefore, these values correspond to

$$(M_{D^+} - M_{D^0})^{\text{strong}} = (2.67 \pm 0.30) \text{ MeV}, \qquad (M_{D^+} - M_{D^0})^{\text{em}} = (2.10 \pm 0.32) \text{ MeV}, \tag{16}$$

	$M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}}$	$M_{\Xi_{bb}^{-}} - M_{\Xi_{bb}^{0}}$	$M_{\Xi_{bc}^{+}} - M_{\Xi_{bc}^{0}}$
EM	4.2 ± 2.3	4.0 ± 1.5	1.6 ± 1.1
Strong	-2.7 ± 1.5	2.3 ± 0.8	-2.5 ± 1.4
Total	1.5 ± 2.7	6.3 ± 1.7	-0.9 ± 1.8
Ref. [31]	2.3 ± 1.7	5.3 ± 1.1	-1.5 ± 0.9

Table 1: Predicted isospin splitting of the doubly heavy baryons. The electromagnetic and quark mass difference contributions are given in the second and third rows, respectively. The final results are shown in the fourth row. The results in Ref. [31] are given in the last row for comparison. All values are given in units of MeV.

where the superscript "strong" and "em" denote the effects from the u, d quark mass difference and the virtual photons, respectively. If we use the mass differences among the pseudoscalar bottom mesons, $M_{B_s^0} - M_B = 87.0 \pm 0.6$ MeV and $M_{B^0} - M_{B^+} = 0.33 \pm 0.06$ MeV [7], then we get different values

$$\tilde{c} = (87.0 \pm 0.6) \text{ MeV}, \qquad \tilde{d} = (-2.0 \pm 0.3) \text{ MeV}, \tag{17}$$

which corresponds to the decomposition

$$(M_{B^0} - M_{B^+})^{\text{strong}} = (2.3 \pm 0.3) \text{ MeV}, \qquad (M_{B^0} - M_{B^+})^{\text{em}} = -(2.0 \pm 0.3) \text{ MeV}. \tag{18}$$

The values agree within uncertainties — the differences between these two sets of values may be understood as flavor symmetry breaking corrections of order $\mathcal{O}(\Lambda_{\text{QCD}}/m_c)$.

Using these parameter values, the mass difference between the Ξ_{cc}^{++} and Ξ_{cc}^{+} can be easily obtained. One can also get the isospin splittings for the doubly bottom baryons Ξ_{bb} and the bottom-charm baryons Ξ_{bc}. All of the predictions are listed in Table 1. To minimize the uncertainty from heavy quark flavor symmetry, the results for the Ξ_{cc} and Ξ_{bb} are given using the values of $\tilde{c}$ and $\tilde{d}$ extracted from the charmed and bottom mesons, respectively. The results for the Ξ_{bc} cover the values obtained using both sets of parameter values. Now let us discuss the other possible uncertainties. Except for the ones from $\tilde{c}$ and $\tilde{d}$, there are still uncertainties from neglecting higher order counterterms and loops in the chiral expansion. Since our predictions concentrate on the isospin splittings, the relevant corrections from higher order terms in the chiral expansion should be of order $\mathcal{O}(M_\pi/\Lambda_\chi) \sim 15\%$, with M_π the pion mass and $\Lambda_\chi \simeq 1$ GeV the chiral symmetry breaking scale. In addition, there should also be corrections to the heavy quark-diquark U(5) symmetry. These corrections should be of order $\mathcal{O}(r_{QQ}/r_{qQ}) = \mathcal{O}(\Lambda_{\text{QCD}}/(m_Q v))$ which describes the relative size of the neglected heavy quark separation with respect to the distance between the light quark and the heavy diquark. Conservatively, we take $\mathcal{O}(\Lambda_{\text{QCD}}/(m_Q v)) \sim 50\%, 30\%$ and 50% for the Ξ_{cc}, Ξ_{bb} and Ξ_{bc} baryons, respectively. The same isospin splittings were also calculated in an approach based on a parameterization inspired by heavy quark effective theory and utilizing some data to fix the parameters [31]. For comparison, their results are given in the last row.

One can get both spin-3/2 and 1/2 doubly heavy baryons from binding a spin-1 heavy diquark and a light quark. Because the spin of the diquark is the same in both cases, they are related to each other by the heavy quark spin symmetry, and have the same isospin splittings

as given in Table 1. Corrections to the spin symmetry are suppressed by $\Lambda_{\mathrm{QCD}}/m_Q$. These corrections are expected to be small as confirmed by a comparison with the results of a quark model [30] which takes into account the spin-dependent interactions, namely $M_{\Xi_{bb}^-} - M_{\Xi_{bb}^0} = 6.24 \pm 0.21$ MeV or 6.4 ± 1.6 MeV using different inputs, which is quite close to ours.

4   Isospin splittings of quadratically heavy pentaquarks

The analysis can be extended to pentaquarks containing four heavy quarks and one light quark. One should notice that the size of the four heavy quark cluster is not four times larger than the distance between two heavy quarks, r_{QQ}. Being in an S-wave, if the four quarks are of the same flavor, they should be spatially symmetric. Hence, the size of the cluster is the same as $r_{QQ} \sim 1/(m_Q v)$, which is again much smaller than $1/\Lambda_{\mathrm{QCD}}$ in the heavy quark limit. Moreover, in a pentaquark, the four heavy quarks are in a fundamental representation of the SU(3) color symmetry group, which is the same as one quark. Hence, to a first approximation they can be treated as one object, to be called quadra-quark in the following. For the four heavy quarks of the same flavor being in an S-wave, Fermi statistics requires their spin wave function to be symmetric. Hence, the quadra-quark is a spin-2 object. Analogous to the U(5) symmetry between the heavy diquark and heavy anti-quark, the symmetry for the heavy quadra-quark and heavy quark is U(7). One may find an interesting phenomenology for the pentaquarks using the U(7) symmetry. Here, we are only interested in the isospin splittings. It is easy to find the following relations to lowest order in isospin breaking:

$$M_{ccc c\bar{d}} - M_{cccc\bar{u}} = 4(M_{D^+} - M_{D^0})^{\mathrm{em}} + (M_{D^+} - M_{D^0})^{\mathrm{strong}} = (11 \pm 5) \text{ MeV},$$
$$M_{bbbb\bar{d}} - M_{bbbb\bar{u}} = 4(M_{B^0} - M_{B^+})^{\mathrm{em}} + (M_{B^0} - M_{B^+})^{\mathrm{strong}} = -(6 \pm 3) \text{ MeV}. \qquad (19)$$

The splittings are large, but certainly more of an intellectual curiosity at present.

5   Summary

Using the Cottingham formula to compute the Coulomb electromagnetic shift, we have shown that the large SELEX value of the isospin splitting of the Ξ_{cc} states implies that double charm baryons are very compact; i.e., the light quark must be very close to the two heavy quarks. A novel possibility is that the quarks in the doubly charm baryons are arranged as a compact state $c-q-c$ with a linear geometry. This possibility could be checked using lattice gauge theory simulations. However, the infrared behavior of the light quark is expected to be governed by the non-perturbative confining interaction, and thus the size of any hadron containing a light quark should be of order $\mathcal{O}(1/\Lambda_{\mathrm{QCD}})$. A conventional approach exploiting this is based on quark-diquark symmetry. It allows us to predict the isospin splitting for doubly heavy baryons Ξ_{cc}, Ξ_{bb} and Ξ_{bc} at NLO in the chiral expansion. These predictions for the doubly charm baryons give isospin separations much smaller than the SELEX measurements. Therefore, the compactness implied by the SELEX data appears to call for a significant violation of heavy quark–diquark symmetry — today no mechanism is known that can provide this. However, it should be noticed that among all the four Ξ_{cc} states in the two reported isospin doublets only the mass of the $\Xi_{cc}^+(3520)$ has been measured with certainty.

In order to resolve the discrepancy between the experiment and theory, further precise experimental investigations are clearly needed. Our prediction for the Ξ_{bb} isospin splitting is

6.3 ± 1.7 MeV based on heavy quark diquark symmetry. It is of similar size as $M_{\Xi^-} - M_{\Xi^0}$ which contain two strange quarks. (Because $m_s < \Lambda_{\text{QCD}}$, we have refrained from making use of the same method to calculate the isospin splittings containing strange quark(s).) Any configuration that leads to isospin splittings as large as those reported by SELEX for the doubly charmed baryons would lead to significantly larger splittings for the doubly bottomed baryons, because the em self-energy would be much larger.

We have also made predictions for the isospin splittings of the pentaquarks $cccc\bar{q}$ and $bbbb\bar{q}$ and have found that the value for the $cccc\bar{q}$ is as large as (11 ± 5) MeV using a generalization of heavy quark diquark symmetry.

As we have noted, the best chance to create these super-heavy hadrons and test their properties is in hadron-hadron collisions at high x_F using the intrinsic heavy quark Fock state mechanism — for example, at the LHCb, or at future fixed-target experiments using the 7 TeV LHC beam.

Acknowledgments

A   Expressions for the mass corrections at NLO

In the appendix, we give explicit expressions for the NLO corrections of the masses of the heavy mesons and doubly heavy baryons. In the heavy quark limit, the QCD Lagrangian does not depend on the heavy quark mass, which results in the heavy quark flavor symmetry, see e.g. [34]. In the following, the formula are given in terms of general heavy quark flavors,

$$
\begin{aligned}
Q\bar{u} &: 4cB_0\tilde{m}_u + \frac{2}{3}e_Q d_0 e^2, & Q_1 Q_2 u &: 4cB_0\tilde{m}_u - \frac{2}{3}(e_{Q_1} + e_{Q_2})d_0 e^2, \\
Q\bar{d} &: 4cB_0\tilde{m}_d - \frac{1}{3}e_Q d_0 e^2, & Q_1 Q_2 d &: 4cB_0\tilde{m}_d + \frac{1}{3}(e_{Q_1} + e_{Q_2})d_0 e^2, \\
Q\bar{s} &: 4cB_0\tilde{m}_s - \frac{1}{3}e_Q d_0 e^2, & Q_1 Q_2 s &: 4cB_0\tilde{m}_s + \frac{1}{3}(e_{Q_1} + e_{Q_2})d_0 e^2, \qquad \text{(A.1)}
\end{aligned}
$$

where $Q_{(1,2)}$ represents the heavy quark flavor, and $e_{Q_{(1,2)}}$ its charge in unit of the elementary charge e with $e > 0$.

References

[1] M. Mattson *et al.* [SELEX Collaboration], Phys. Rev. Lett. **89**, 112001 (2002) [arXiv:hep-ex/0208014].

[2] M. A. Moinester *et al.* [SELEX Collaboration], Czech. J. Phys. **53**, B201 (2003) [arXiv:hep-ex/0212029].

[3] J. S. Russ, "The double charm baryon family at SELEX: An update", Fermilab Joint Experimental and Theoretical Physics Seminar, June 13, 2003, `http://www-selex.fnal.gov/Welcome.html` (unpublished).

[4] A. Ocherashvili *et al.* [SELEX Collaboration], Phys. Lett. B **628**, 18 (2005) [arXiv:hep-ex/0406033].

[5] J. Engelfried [SELEX Collaboration], Nucl. Phys. A **752**, 121c (2005).

[6] J. Engelfried [SELEX Collaboration], *In the Proceedings of International Conference on Heavy Quarks and Leptons (HQL 06), Munich, Germany, 16-20 Oct 2006, pp 003* [arXiv:hep-ex/0702001].

[7] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).

[8] J. Engelfried, "The Status of Double Charm Baryons", The Third Workshop of the APS Topical Group in Hadron Physics April 30, 2009, `www.fz-juelich.de/ikp/ghp2009/Talks/talk_engelfried_jurgen.pdf` (unpublished).

[9] G. Bari, M. Basile, G. Bruni *et al.*, Nuovo Cim. **A104**, 571-599 (1991).

[10] G. Bari, M. Basile, G. Bruni *et al.*, Nuovo Cim. **A104**, 1787-1800 (1991).

[11] J. Badier *et al.* [NA3 Collaboration], Phys. Lett. **B114**, 457 (1982).

[12] R. Vogt, S. J. Brodsky, Phys. Lett. **B349**, 569-575 (1995). [hep-ph/9503206].

[13] S. J. Brodsky, P. Hoyer, C. Peterson and N. Sakai, Phys. Lett. B **93**, 451 (1980).

[14] S. J. Brodsky, J. C. Collins, S. D. Ellis, J. F. Gunion and A. H. Mueller, published in *Design and Utilization of the SSC, Snowmass, 1984*: proceedings. Edited by Rene Donaldson and Jorge G. Morfin. Batavia, Ill., Fermilab, 1985. 851p.

[15] B. W. Harris, J. Smith and R. Vogt, Nucl. Phys. B **461**, 181 (1996) [arXiv:hep-ph/9508403].

[16] M. Franz, M. V. Polyakov and K. Goeke, Phys. Rev. D **62**, 074024 (2000) [arXiv:hep-ph/0002240].

[17] S. J. Brodsky, A. S. Goldhaber, B. Z. Kopeliovich *et al.*, Nucl. Phys. **B807**, 334-347 (2009). [arXiv:0707.4658 [hep-ph]].

[18] S. J. Brodsky, B. Kopeliovich, I. Schmidt and J. Soffer, Phys. Rev. D **73**, 113005 (2006) [arXiv:hep-ph/0603238].

[19] P. Hoyer, M. Vanttinen, U. Sukhatme, Phys. Lett. **B246**, 217-220 (1990).

[20] M. J. Leitch [FNAL E866/NuSea and PHENIX Collaboration], Eur. Phys. J. **A19S1**, 129-132 (2004).

[21] J. Gasser and H. Leutwyler, Phys. Rept. **87**, 77 (1982).

[22] G. S. Bali, Phys. Rept. **343**, 1 (2001). [hep-ph/0001312].

[23] M. J. Savage and M. B. Wise, Phys. Lett. B **248**, 177 (1990).

[24] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321**, 311 (1989).

[25] R. Urech, Nucl. Phys. B **433**, 234 (1995) [arXiv:hep-ph/9405341].

[26] J. Hu and T. Mehen, Phys. Rev. D **73**, 054003 (2006) [arXiv:hep-ph/0511321].

[27] F. K. Guo, C. Hanhart, S. Krewald and U.-G. Meißner, Phys. Lett. B **666**, 251 (2008) [arXiv:0806.3374 [hep-ph]].

[28] F. K. Guo, C. Hanhart and U.-G. Meißner, JHEP **0809**, 136 (2008) [arXiv:0809.2359 [hep-ph]].

[29] J. Gasser, A. Rusetsky and I. Scimemi, Eur. Phys. J. C **32**, 97 (2003) [arXiv:hep-ph/0305260].

[30] M. Karliner, B. Keren-Zur, H. J. Lipkin and J. L. Rosner, Annals Phys. **324**, 2 (2009) [arXiv:0804.1575 [hep-ph]].

[31] C.-W. Hwang, C.-H. Chung, Phys. Rev. D **78**, 073013 (2008). [arXiv:0804.4044 [hep-ph]].

[32] G. Colangelo *et al.*, arXiv:1011.4408 [hep-lat].

[33] S. Durr, Z. Fodor, C. Hoelbling *et al.*, arXiv:1011.2403 [hep-lat].

[34] M. Neubert, Phys. Rept. **245**, 259 (1994) [arXiv:hep-ph/9306320].