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Precise Predictions for $W + 4$ Jet Production at the Large Hadron Collider

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We present the first next-to-leading order QCD results for W + 4-jet production at hadron colliders. Total cross sections, as well as distributions in the jet transverse momenta and in the total transverse energy H_T, are provided for the initial LHC energy of $\sqrt{s} = 7$ TeV. We use a leading-color approximation, known to be accurate to 3% for W production with fewer jets. The virtual matrix elements and the most complicated real-emission matrix elements are handled by the BlackHat library, based on on-shell methods. The remaining parts of the calculation, including the integration over phase space, are performed by the SHERPA package.

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The first data and analyses emerging from experiments at the Large Hadron Collider (LHC) emphasize the need for reliable theoretical calculations in searches for new physics beyond the Standard Model. In many channels, new-physics signals can hide in broad distributions underneath Standard Model backgrounds. Extraction of a signal will require accurate predictions for the background processes, for which next-to-leading order (NLO) cross sections in perturbative QCD are crucial.

In this Letter, we present the first NLO results for inclusive W + 4-jet production, using a leading-color approximation for the virtual terms that has been validated for processes with fewer jets. This process has been studied since the early days of the Tevatron, where it was the dominant background to top-quark pair production. At the LHC, it will be an important background to many new physics searches involving missing energy, as well as to precise top-quark measurements.

In previous papers [1, 2] we presented the first complete results for hadron-collider production of a W or Z boson in association with three jets at NLO in the strong coupling α_s. (Other NLO results for W + 3 jets have used various leading-color approximations [3–5].) We performed detailed comparisons to Tevatron data [6]. The sensitivity to the unphysical scale used to define α_s and the parton distributions is reduced from around 40% at leading order (LO) to 10∼20% at NLO, and the NLO results agree well with the data. At the LHC, a much wider range of kinematics will be probed, making NLO studies even more important.

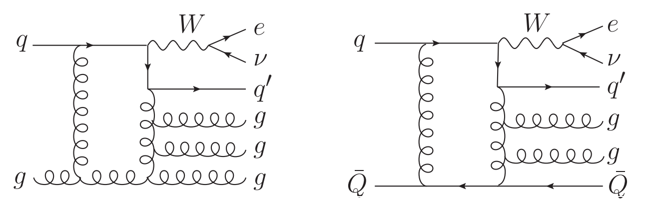

FIG. 1: Sample diagrams for the seven-point loop amplitudes for $qg \to W\, q'ggg$ and $q\bar{Q} \to W\, q'gg\bar{Q}$, followed by $W \to e\nu$.

The past few years have seen rapid progress in NLO QCD for the LHC. Several important processes involving four final-state objects (including jets) have been computed [1–5, 7]. The process we present here is the first of a new class, involving five final-state objects.

The computation of hadron collider processes with complex final states at NLO has long been a challenge to theorists. The evaluation of the one-loop (virtual) corrections has been a longstanding bottleneck. Feynman-diagram techniques suffer from rapid growth in complexity as the number of legs increases. On-shell methods [8–13], in contrast, rely on the unitarity and factorization properties of scattering amplitudes to generate new amplitudes from previously-computed ones. Such methods scale very well as the number of external legs increases,

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offering a solution to these difficulties.

We use the same basic setup as in our earlier computations [1, 2] of $W+$ 3-jet and $Z, \gamma^* +$ 3-jet production. The virtual contributions are computed using on-shell methods via the BlackHat package [14]. We show representative virtual diagrams in fig. 1. We use a leading-color approximation in the finite virtual contributions[1], while keeping the full color dependence in all other contributions. We have confirmed that this approximation is an excellent one for $W +$ 1, 2, 3-jet production, shifting the total cross section by about 3%, which is significantly smaller than uncertainties from parton distributions or higher-order terms in α_s. Subleading-color corrections to $W +$ 4-jet production should also be small.

The remaining NLO ingredients, the real-emission and dipole-subtraction terms [15], are computed by AMEGIC++ [16], part of the SHERPA package [17]. We also use SHERPA to perform phase-space integration. The efficiency of the integrator has been improved significantly with respect to ref. [1] through the use of QCD antenna structures [18, 19]. BlackHat computes the real-emission tree amplitudes using on-shell recursion relations [10], along with efficient analytic forms extracted from $\mathcal{N} = 4$ super-Yang-Mills theory [20].

Compared to LO, NLO cross sections and distributions generally depend much less on the common (unphysical) renormalization and factorization scale μ. However, it is still important to select a scale characteristic of the typical kinematics. A scale that performs well for many distributions is the total partonic transverse energy. We set $\mu = \hat{H}'_T/2$, where $\hat{H}'_T = \sum_j p_T^j + E_T^W$; the sum runs over all final-state partons j, and $E_T^W = \sqrt{M_W^2 + (p_T^W)^2}$ is the transverse energy of the W boson[2]. Refs. [5, 22] present other satisfactory choices. We follow the conventional procedure of varying the chosen central scale up and down by a factor of two to construct scale-dependence bands, taking the minimum and maximum of the observable evaluated at five values: $\mu/2, \mu/\sqrt{2}, \mu, \sqrt{2}\mu, 2\mu$.

Fixed-order perturbation theory may break down in special kinematic regions, where large logarithms of scale ratios emerge. For instance, threshold logarithms can affect production at very large mass scales, which can be reached in inclusive single-jet production [24]. Using this study one can argue [2] that at the mass scales probed in $W +$ 4-jet production, such logarithms should remain quite modest. Similarly, the sort of large logarithms arising in vector-boson production in association with a single jet [25] do not appear in the case of multiple jets.

In our study, we consider the inclusive process $pp \to W +$ 4 jets at an LHC center-of-mass energy of $\sqrt{s} =$ 7 TeV. We impose the following cuts: $E_T^e > 20$ GeV, $|\eta^e| < 2.5$, $\not{E}_T > 20$ GeV, $p_T^{\rm jet} > 25$ GeV, $|\eta^{\rm jet}| < 3$, and $M_T^W > 20$ GeV. Here, p_T are transverse momenta; η, pseudorapidities; and M_T^W, the transverse mass of the $e\nu$ pair. The missing transverse energy, $\not{E}_T$, corresponds to the neutrino transverse energy, E_T^ν. Jets are defined using the anti-k_T algorithm [26] with parameter $R =$ 0.5, and are ordered in p_T. (We also quote results for $R = 0.4$.) We use the CTEQ6M [27] parton distribution functions and α_s at NLO, and the CTEQ6L1 set at LO. Electroweak couplings are as in ref. [1].

In table I, we present LO and NLO parton-level cross sections for inclusive W^--boson production accompanied by zero through four jets. We include all subprocesses, using the leading-color virtual approximation only in $W+$ 4-jet production. The upward scale-variation figures for the NLO cross sections are quite small for $W +$ 3- and $W +$ 4-jet production, because the values at the central scale choice are close to the maximum values across scale variations. We also display the ratios of the W^+ to W^- cross sections, and the "jet-production" ratios of $W^- + n$-jet to $W^- + (n-1)$-jet production. Both kinds of ratios should be less sensitive to experimental and theoretical systematics than the absolute cross sections.

The W^+/W^- ratios are greater than one because the LHC is a pp machine, and because the parton luminosity ratio $u(x)/d(x)$ exceeds one. As the number of jets increases, production of a W requires a larger value of x, driving $u(x)/d(x)$ and hence the W^+/W^- ratio to larger values. These ratios have been discussed recently [28] as a probe of certain new-physics processes; our results extend the NLO analysis to W production accompanied by three jets. This ratio changes very little under correlated variations of the scale in numerator and denominator; hence we do not exhibit such scale variation here. The NLO $W^+ +$ 4-jet cross section needed for the four-jet ratio will be reported on elsewhere.

Standard lore [29] says that the jet-production ratio should be roughly independent of the number of jets. The results for the ratios displayed here for $n > 1$ are indeed consistent with this lore. However, they are rather sensitive to the experimental cuts, and can depend strongly on n when binned in the vector-boson p_T [2]. The $W +$ 1-jet/$W +$ 0-jet ratio is much smaller because of the restricted kinematics of the leading contribution to $W +$ 0-jet production.

In table II, we give cross sections for narrower jets, using the anti-k_T jet algorithm with $R = 0.4$. For two or more jets, the LO cross sections are larger than for $R = 0.5$, and the effect increases with the number of jets. However, at NLO, the effect is greatly diminished; only for four jets is the NLO cross section for $R = 0.4$ significantly above that for $R = 0.5$. The NLO jet-production ratio is somewhat larger for $R = 0.4$, for $n > 2$; in con-

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[1] Our definition of leading-color terms follows ref. [2]; it includes virtual quark loops in addition to the terms identified in ref. [3].

[2] In refs. [1, 2] we used the scalar sum of the decay leptons' transverse energies instead of E_T^W. The present choice is preferred for studies of W polarization effects [1, 23].

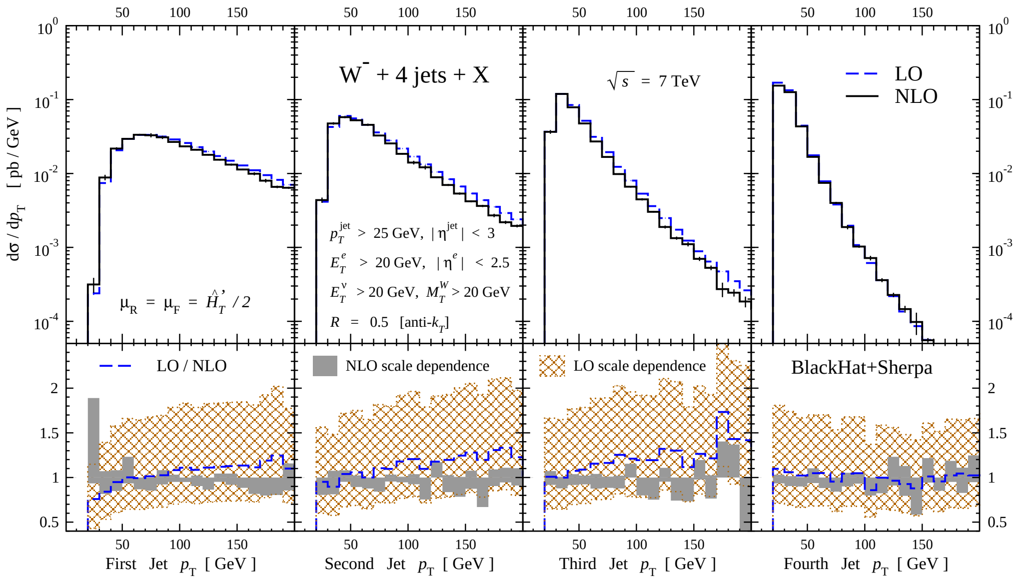

FIG. 2: A comparison of the p_T distributions of the leading four jets in $W^- + 4$-jet production at the LHC at $\sqrt{s} = 7$ TeV. In the upper panels the NLO distribution is the solid (black) histogram and the LO predictions are shown as dashed (blue) lines. The thin vertical line in the center of each bin (where visible) gives its numerical integration error. The lower panels show the distribution normalized to the central NLO prediction. The scale-dependence bands are shaded (gray) for NLO and cross-hatched (brown) for LO.

no. jets	W^- LO	W^- NLO	W^+/W^- LO	W^+/W^- NLO	$W^-n/(n-1)$ LO	$W^-n/(n-1)$ NLO
0	$1614.0(0.5)^{+208.5}_{-235.2}$	$2077(2)^{+40}_{-31}$	1.656(0.001)	1.580(0.004)	—	—
1	$264.4(0.2)^{+22.6}_{-21.4}$	$331(1)^{+15}_{-12}$	1.507(0.002)	1.498(0.009)	$0.1638(0.0001)^{+0.044}_{-0.031}$	0.159(0.001)
2	$73.14(0.09)^{+20.81}_{-14.92}$	$78.1(0.5)^{+1.5}_{-4.1}$	1.596(0.003)	1.57(0.02)	$0.2766(0.0004)^{+0.051}_{-0.037}$	0.236(0.002)
3	$17.22(0.03)^{+8.07}_{-4.95}$	$16.9(0.1)^{+0.2}_{-1.3}$	1.694(0.005)	1.66(0.02)	$0.2354(0.0005)^{+0.034}_{-0.025}$	0.216(0.002)
4	$3.81(0.01)^{+2.44}_{-1.34}$	$3.56(0.03)^{+0.08}_{-0.30}$	1.817(0.003)	—	$0.2212(0.0004)^{+0.026}_{-0.020}$	0.211(0.003)

TABLE I: Total cross sections in pb for $W + n$ jet production at the LHC at $\sqrt{s} = 7$ TeV, using the anti-k_T jet algorithm with $R = 0.5$. The NLO result for $W + 4$ jets uses the leading-color approximation discussed in the text. The fourth and fifth columns give the cross-section ratios for W^+ production to W^- production. The last column gives the ratios of the cross section for the given process to that with one jet less. The numerical integration uncertainty is in parentheses, and the scale dependence is quoted in super- and subscripts.

trast, the ratios of W^+ to W^- cross sections are unchanged within errors.

In fig. 2, we show the p_T distributions of the leading four jets in $W^- + 4$-jet production at LO and NLO; the predictions are normalized to the central NLO prediction in the lower panels. With our central scale choice, there is a noticeable shape difference between the LO and NLO distributions for the first three leading jets, while fourth-jet distribution is very similar at LO and NLO. Similarly, in $W + 3$-jet production, the p_T distributions of the leading two jets exhibit shape changes from LO to NLO, while the third-jet distribution does not [1].

Fig. 3 shows the distribution of the total transverse energy H_T, given by the scalar sum of the jet and lepton transverse energies, $H_T = \sum_j E_{T,j}^{\rm jet} + E_T^e + E_T^\nu$. We show the NLO and LO predictions, along with their scale-dependence bands. As in the p_T distributions, the NLO band is narrower. The shapes at LO and NLO are similar above 200 GeV, where the integration errors are small.

In order to compare our parton-level results to forthcoming experimental data, the size of non-perturbative effects (such as hadronization and the underlying event) need to be estimated, for example using LO parton-shower Monte Carlo programs. As NLO parton-shower programs are developed [30], the virtual corrections computed here should be incorporated into them.

A related process that contributes an irreducible background to certain missing energy signals of new physics is $Z+$ 4-jet production. We expect that the current BlackHat along with SHERPA will allow us to compute NLO corrections to it, as well as to other complex processes, thereby providing an unprecedented level of theoretical

no. jets	W^- LO	W^- NLO	W^+/W^- LO	W^+/W^- NLO	$W^-n/(n-1)$ LO	$W^-n/(n-1)$ NLO
0	$1614.0(0.5)^{+208.5}_{-235.2}$	$2077(2)^{+40}_{-31}$	1.656(0.001)	1.580(0.004)	—	—
1	$264.4(0.2)^{+22.6}_{-21.4}$	$324(1)^{+14}_{-11}$	1.507(0.002)	1.499(0.009)	$0.1638(0.0001)^{+0.044}_{-0.031}$	0.156(0.001)
2	$74.17(0.09)^{+21.08}_{-15.12}$	$76.2(0.5)^{+0.8}_{-3.4}$	1.597(0.003)	1.56(0.02)	$0.2805(0.0004)^{+0.051}_{-0.038}$	0.235(0.002)
3	$18.42(0.03)^{+8.61}_{-5.29}$	$17.0(0.1)^{+0.0}_{-1.0}$	1.694(0.005)	1.66(0.02)	$0.2483(0.0005)^{+0.036}_{-0.026}$	0.223(0.002)
4	$4.41(0.01)^{+2.82}_{-1.55}$	$3.84(0.04)^{+0.00}_{-0.44}$	1.817(0.003)	—	$0.2394(0.0004)^{+0.028}_{-0.021}$	0.226(0.003)

TABLE II: The same quantities as in table I, but with $R = 0.4$.

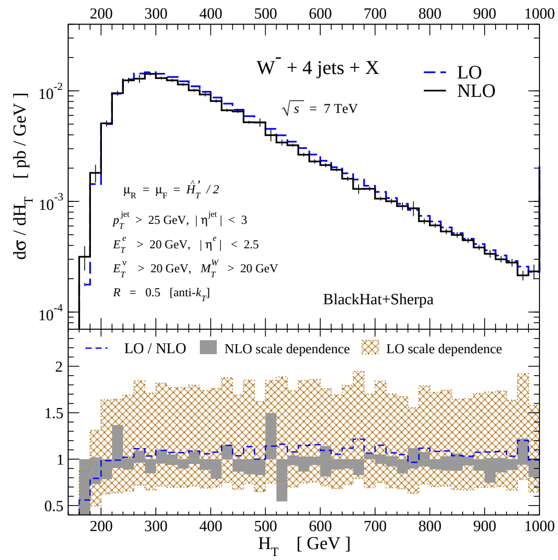

FIG. 3: The H_T distribution for $W^- + 4$ jets.

precision for such backgrounds at the LHC.

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