

# Laser assisted emittance transfer for storage ring lasing

Dao Xiang\*

*SLAC National Accelerator Laboratory, Menlo Park, CA, 94025, USA*

(Dated: February 15, 2011)

## Abstract

In modern storage rings the transverse emittance of electron beams can be comparable to that from state-of-art photoinjectors, but the intrinsic low peak current and large energy spread precludes the possibility of realizing short-wavelength high-gain free electron lasers (FELs) in storage rings. In this note I propose a technique to significantly increase beam peak current without greatly increasing beam energy spread, which is achieved by transferring part of the longitudinal emittance to transverse plane. It is shown that by properly repartitioning the emittance in 6-D phase space, the beam from a large storage ring may be used to drive a single-pass high-gain FEL in soft x-ray wavelength range.

---

\* dxiang@slac.stanford.edu

## I. INTRODUCTION

It is generally believed that high quality electron beams ( $\sim 1 \mu\text{m}$  transverse emittance,  $\sim \text{kA}$  peak current and  $\sim 1 \text{ MeV}$  energy spread) required in x-ray free electron lasers (FELs) can only be provided by linear accelerators with low emittance injectors and bunch compressors. It is the purpose of this note to show that the electron beam from a large storage ring after appropriate manipulation may be used to drive a single-pass high-gain x-ray FEL as well.

While the beam in storage rings is stable and has high repetition rate, its quality is not suitable for single-pass x-ray FELs. In storage rings, the peak current of the steady-state beam is typically far less than kA because of collective instabilities, and the energy spread is typically a few MeV due to quantum diffusion in bends and wigglers. For a beam with small transverse emittance and large energy spread, the FEL power gain length may be approximately estimated as  $L_G = L_{G0}[1 + (\sigma_\delta/\rho)^2]$  [1], where  $L_{G0} = \lambda_u/4\pi\rho$  is the 1D gain length for a beam with vanishing energy spread,  $\lambda_u$  is the undulator period,  $\rho$  is the FEL Pierce parameter [2] that has a strong dependence on beam peak current and  $\sigma_\delta$  is the relative energy spread. Note, the FEL gain length quickly increases when beam energy spread is larger than  $\rho$ , and in storage rings  $\rho$  is typically much smaller than  $\sigma_\delta$  because of the low peak current and large energy spread. This may rule out the possibility of using the storage ring beams to drive single-pass high-gain x-ray FELs.

Several methods have been proposed to enhance the performance of linac-based x-ray FELs and some of them may be applied to storage ring FELs as well [3-6]. The emittance exchange technique [3,4] can be used in storage rings to exchange the large longitudinal emittance with the small transverse emittance to reduce the beam energy spread. However, due to the relatively long bunch length (a few mm), the longitudinal emittance of the beam in storage rings is typically quite large (a few tens of thousand  $\mu\text{m}$ ). If the standard emittance exchange technique which exchanges the projected emittance for the whole bunch is used to initiate the emittance exchange, the beam will end up with an unacceptably large transverse emittance which is as harmful to the FEL gain as the large energy spread. Alternatively, if one employs the recently proposed laser assisted emittance exchange technique [6] in which the TEM01 mode laser is used to interact with the beam in a dispersive region to initiate the emittance exchange for particles within short slices, the longitudinal emittance

for the emittance exchanged beam is reduced to a few  $\mu\text{m}$  and the FEL performance can be enhanced [7].

In addition to reducing the beam energy spread, one may also increase the beam peak current to enhance the FEL gain. An efficient way to do so is the current-enhanced self-amplified spontaneous emission technique (E-SASE [5]) that uses a high power laser to modulate beam energy and after passing through a chicane the current for part of the beam is increased. In E-SASE the longitudinal emittance of the beam is conserved, so the energy spread will increase by the same factor. For the beams in storage rings, because the energy spread is large, simultaneously increasing beam current and energy spread does not improve FEL performance. Significant improvements in FEL performance may be achieved by increasing the beam peak current while keeping the energy spread almost unchanged (in other words the longitudinal emittance is reduced). To achieve this, one must transfer longitudinal emittance to transverse plane in order not to violate Liouville's theorem.

In this paper, we propose a scheme to significantly increase the beam peak current without greatly increasing the energy spread. In the scheme a TEM01 mode laser is first used to interact with the electron beam to modulate the vertical angular distribution. After passing through a special chicane with non-zero transfer matrix element  $R_{54}$ , the angular modulation is converted to density modulation. It is shown that the current for about half of the beam can be greatly increased while the energy spread is almost unchanged. This is achieved at the cost of transferring part of the longitudinal emittance to vertical plane, and as a result the vertical emittance is increased. Fortunately, the beam in storage ring typically has an extremely small vertical emittance. So the phase space in vertical plane is almost 'empty' and it has the room to admit the longitudinal emittance. By properly choosing the parameters for the laser and the chicane system, the final vertical emittance of the beam can be controlled within permissible level.

For these current-enhanced beam slices, the FEL gain length is greatly reduced and they can lead to effective lasing with a long undulator. The generated x-ray radiation consists of pulse trains equally separated by the laser wavelength. The x-ray pulse generated in the proposed scheme is naturally synchronized with the laser, and thus can be conveniently used for laser pump x-ray probe experiments. Simulations using the parameters of the proposed PEP-X storage ring (beam energy 4.5 GeV) at SLAC [8] confirmed the feasibility that the electron beam from a large storage ring after appropriate manipulation may be used to

drive a single-pass high-gain x-ray FEL. Together with a long bypass straight section, the technique may be applied to other storage rings (for instance, APS at ANL, NSLS-II at BNL, etc.) to achieve x-ray FELs as well.

## II. METHODS

We would like to first emphasize here that the 6-D brightness of the beam in storage rings actually can be comparable to that in linear accelerators. The 6-D beam brightness can be defined as,

$$B = \frac{I}{\epsilon_{nx}\epsilon_{ny}\sigma_\delta}, \quad (1)$$

where  $I$  is the peak current,  $\epsilon_{nx}$  and  $\epsilon_{ny}$  are the normalized horizontal and vertical emittance, and  $\sigma_\delta$  is the energy spread of the beam. The normalized transverse emittance is defined as  $\epsilon_{nx} = \gamma\beta\epsilon_x = \gamma\beta\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$ , where  $\gamma$  is the relativistic energy of the beam,  $\beta$  is the beam velocity normalized to the speed of light,  $\epsilon_x$  is the geometric emittance and  $\langle \rangle$  is the ensemble average. Similarly, the normalized longitudinal emittance is defined as  $\epsilon_{nz} = \gamma\beta\epsilon_z = \gamma\beta\sqrt{\langle z^2 \rangle \langle \delta^2 \rangle - \langle z\delta \rangle^2}$ , where  $\epsilon_z$  is the geometric longitudinal emittance,  $z$  and  $\delta$  are relative longitudinal position and energy deviation with respect to the reference particle.

Let's first compare the brightness of the beam from LCLS [9] with that from the proposed ultimate storage ring PEP-X [8]. For the x-ray FEL at 1.5 nm, the LCLS beam energy is 4.3 GeV, transverse emittance is about 0.5  $\mu\text{m}$  for both planes and beam energy spread is about 1.5 MeV when beam peak current is 2 kA. For PEP-X, when the coupling parameter (the ratio of the vertical emittance to the horizontal emittance) is unity, the transverse emittance is 0.6  $\mu\text{m}$  in both planes, beam energy spread is about 5 MeV and peak current is about 300 A. From Eq. (1) one can see that the brightness of PEP-X beam is one order of magnitude lower than LCLS beam. However, if the storage ring is operated at a low coupling mode which is the nominal mode for most of the rings, the beam brightness actually can be higher, taking advantage of the extremely low vertical emittance from radiation damping. For instance, when the coupling parameter is 1%, the normalized horizontal emittance for PEP-X beam is about 1.76  $\mu\text{m}$  when beam peak current is 300 A. In this case, the beam brightness is comparable to the LCLS beam and in principle it could be used to drive an x-ray FEL as well.

However, it should also be pointed out that the beam in storage rings is not suitable for x-ray FELs, because the extremely small vertical emittance does not help much and the large energy spread is too harmful to FEL gain. Therefore, it is advantageous to repartition the emittance in vertical and longitudinal planes.

Here we propose a scheme to transfer part of the longitudinal emittance to vertical plane, so that one can significantly increase the beam peak current without greatly increasing the energy spread. The schematic of the technique is shown in Fig. 1. A laser operating in TEM01 mode is first used to interact with the electron beam in a short undulator to generate angular modulation in beam phase space. A special chicane that consists of four dipole magnets (red triangles in Fig. 1) and four quadrupole magnets (green diamonds in Fig. 1) is used to convert the angular modulation into density modulation so that the current for part of the beam is significantly increased. For these current-enhanced beam slices, the FEL gain length is greatly reduced and they may be used to drive a single-pass x-ray FEL in a long undulator.

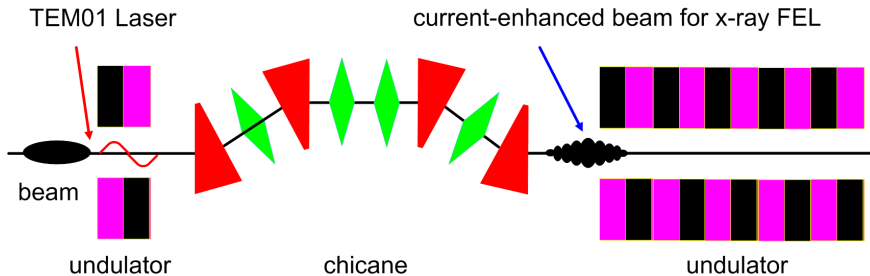


FIG. 1: Scheme to realize x-ray FEL in storage rings using the laser assisted emittance transfer technique.

Hereafter we will only consider the beam dynamics in vertical and longitudinal planes, and neglect the uncoupled motion in horizontal plane. Consider an electron with initial state  $\vec{g}_0 = (y_0, y'_0, z_0, \delta_0)^T$ , where  $y_0$  is the vertical position,  $y'_0$  is the vertical divergence,  $z_0$  and  $\delta_0$  are relative longitudinal position and energy deviation with respect to the reference particle, respectively. After passing through a linear Hamiltonian system, electron's state  $\vec{g}_1$  is related to its initial state  $\vec{g}_0$  by  $\vec{g}_1 = R\vec{g}_0$ , where  $R$  is a symplectic  $4 \times 4$  transport matrix that describes the beam dynamics associated with the system. We assume the beam is initially uncoupled in  $y$  and  $z$  planes, as is the typical case in storage rings. For simplicity we further assume the beam ellipses are upright in both planes. In this case the  $4 \times 4$  beam

matrix is simply,

$$\sigma_0 = \begin{bmatrix} \sigma_{y0} & 0 \\ 0 & \sigma_{z0} \end{bmatrix} = \begin{bmatrix} \epsilon_{y0}\beta_y & 0 & 0 & 0 \\ 0 & \epsilon_{y0}\gamma_y & 0 & 0 \\ 0 & 0 & \epsilon_{z0}\beta_z & 0 \\ 0 & 0 & 0 & \epsilon_{z0}\gamma_z \end{bmatrix} \quad (2)$$

where  $\beta_{y,z}, \gamma_{y,z}$  are the Twiss parameters,  $\epsilon_{y0}$  and  $\epsilon_{z0}$  are the geometric vertical and longitudinal emittance, respectively. It should be pointed out that this simplification does not limit the generality of the results, because for general case when beam ellipse is tilted there always exist symplectic uncoupled transformations that can transform the beam to uncoupled upright ellipses.

It has been shown in [6,10] that the undulator together with the TEM01 mode laser is equivalent to the rf dipole mode cavity for the electrons around the zero crossing of the laser: particles get energy kick linearly proportional to vertical position ( $\delta_1 = \delta_0 + ky_0$ ), and get vertical angular kick linearly proportional to longitudinal position ( $y'_1 = y'_0 + kz_0$ ), where  $k$  is the dimensionless kick strength. For these particles the transfer matrix that describes the beam dynamics for the interaction with the TEM01 mode laser in the undulator can be written as,

$$R_u = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

where  $k = 2\pi\Lambda/\lambda$ ,  $\Lambda = (2K/\gamma^2)\sqrt{P/P_0}[JJ]f$ ,  $P$  is the laser power,  $P_0 = I_A mc^2/e$ ,  $I_A$  is the Alfvén current,  $[JJ] = J_0(\xi) - J_1(\xi)$  with  $\xi = K^2/(4 + 2K^2)$ ,  $K$  is the undulator parameter,  $f$  is a function depending on the undulator length and the Rayleigh length [6,10].

The transfer matrix for the special chicane system depends on the parameters of the dipoles and quadrupoles. By properly choosing the strength for the quadrupoles, one can make all the elements  $R_{3i}$  of the chicane transfer matrix vanishing except for  $i = 2$  and  $i = 3$  ( $R_{33} = 1$ ). In this case, due to the symplectic condition and the mirror symmetry, the

transfer matrix for the chicane system is [11]

$$R_c = \begin{bmatrix} -1 & R_{12} & 0 & \eta \\ 0 & -1 & 0 & 0 \\ 0 & -\eta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4)$$

where  $R_{12}$  is the angular-to-spatial element and  $\eta$  is the dispersion of the system. It is straightforward to obtain the transfer matrix for the undulator and the chicane system,

$$R_s = R_c R_u = \begin{bmatrix} -(1 - \eta k) & R_{12} & kR_{12} & \eta \\ 0 & -1 & -k & 0 \\ 0 & -\eta & 1 - \eta k & 0 \\ k & 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

When the dispersion is chosen to satisfy  $\eta k = 1$  the transfer matrix for the system is,

$$R = \begin{bmatrix} 0 & R_{12} & R_{12}/\eta & \eta \\ 0 & -1 & -1/\eta & 0 \\ 0 & -\eta & 0 & 0 \\ 1/\eta & 0 & 0 & 1 \end{bmatrix}. \quad (6)$$

The final beam matrix is then found to be,

$$\begin{aligned} \sigma_1 &= R\sigma_0 R^T = \begin{bmatrix} \sigma_{y1} & \sigma_{yz1} \\ \sigma_{yz1}^T & \sigma_{z1} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\epsilon_{z0}\beta_z R_{12}^2}{\eta^2} + \epsilon_{y0}\gamma_y R_{12}^2 + \eta^2 \epsilon_{z0}\gamma_z & -\frac{\epsilon_{z0}\beta_z R_{12}}{\eta^2} - \epsilon_{y0}\gamma_y R_{12} & -\eta\epsilon_{y0}\gamma_y R_{12} & \eta\epsilon_{z0}\gamma_z \\ -\frac{\epsilon_{z0}\beta_z R_{12}}{\eta^2} - \epsilon_{y0}\gamma_y R_{12} & \frac{\epsilon_{z0}\beta_z}{\eta^2} + \epsilon_{y0}\gamma_y & \eta\epsilon_{y0}\gamma_y & 0 \\ -\eta\epsilon_{y0}\gamma_y R_{12} & \eta\epsilon_{y0}\gamma_y & \eta^2 \epsilon_{y0}\gamma_y & 0 \\ \eta\epsilon_{z0}\gamma_z & 0 & 0 & \frac{\epsilon_{y0}\beta_y}{\eta^2} + \epsilon_{z0}\gamma_z \end{bmatrix} \end{aligned} \quad (7)$$

From Eq. (7) it follows that the final vertical and longitudinal emittance for the beam around the zero crossing of the laser is,

$$\begin{aligned} \epsilon_{y1}^2 &= |\sigma_{y1}| = \epsilon_{z0}^2 + \eta^2 \epsilon_{z0}\epsilon_{y0}\gamma_y\gamma_z, \\ \epsilon_{z1}^2 &= |\sigma_{z1}| = \epsilon_{y0}^2 + \eta^2 \epsilon_{z0}\epsilon_{y0}\gamma_y\gamma_z, \end{aligned} \quad (8)$$

In order to greatly increase the peak current, the amplitude of the angular modulation  $\Delta y'$  should be much larger than beam's intrinsic divergence  $\sigma'_{y0}$ . In this case the final vertical

and longitudinal emittance can be approximately written as,

$$\begin{aligned}\epsilon_{y1}^2 &\approx \epsilon_{z0}^2 + \left(\frac{\lambda}{2\pi n}\right)^2 \sigma_{\delta 0}^2, \\ \epsilon_{z1}^2 &\approx \epsilon_{y0}^2 + \left(\frac{\lambda}{2\pi n}\right)^2 \sigma_{\delta 0}^2,\end{aligned}\tag{9}$$

where  $n = \Delta y' / \sigma_{y0}'$ ,  $\lambda$  is the laser wavelength and  $\sigma_{\delta 0} = \sqrt{\epsilon_{z0} \gamma_z}$  is the initial rms beam energy spread. Typically the initial vertical emittance is much smaller than the longitudinal emittance, so after this manipulation the longitudinal emittance will be reduced and the vertical emittance will be increased. The amount of longitudinal emittance transferred to vertical plane can be controlled to permissible level by properly choosing the laser wavelength and modulation amplitude. From Eq. (7) it is straightforward to get the final duration and energy spread for the particles after the manipulation,

$$\begin{aligned}\sqrt{\langle z_1^2 \rangle} &= \sqrt{\eta^2 \epsilon_{y0} \gamma_y} \approx \frac{\lambda}{2\pi n}, \\ \sqrt{\langle \delta_1^2 \rangle} &= \sqrt{\frac{\epsilon_{y0} \beta_y}{\eta^2} + \epsilon_{z0} \gamma_z} \approx \sqrt{\sigma_{\delta 0}^2 + \left(\frac{2\pi n}{\lambda}\right)^2 \epsilon_{y0}^2},\end{aligned}\tag{10}$$

Eq. (10) implies that after the emittance transfer, the peak current for the beam slices around the zero crossing of the laser will be increased by a factor of  $n$ . Taking advantage of the low vertical emittance in storage rings, the beam energy spread will grow slightly while simultaneously the beam current is greatly increased.

### III. SIMULATION

To show the potential of the proposed scheme in enhancing the x-ray FEL performance in storage rings, we use the beam parameters for the proposed PEP-X at SLAC [8] where long straight sections ( $> 100$  m) are available. The beam parameters are listed in Table. 1. To have a small vertical emittance, we assume that the coupling parameter is 1% and the normalized vertical emittance is about  $0.0176 \mu\text{m}$  when beam peak current is 300 A [1].

In our example we assume the modulator has 10 periods with a period length of 40 cm and a  $K$  value of 39, and the seed laser has a wavelength of  $2 \mu\text{m}$ . The dipole magnet in the chicane has a length of 0.5 m and a bending angle of about 1.38 degrees. The distance between the dipoles and quadrupoles are all 0.5 m. The quadrupole strengths are properly



TABLE I: Main parameters for the FEL in PEP-X storage ring

Electron beam energy	4.5 GeV
Peak current	300 A
Normalized horizontal emittance	1.76 $\mu\text{m}$
Normalized vertical emittance	0.0176 $\mu\text{m}$
Relative energy spread	1.14e-3
Intrinsic vertical divergence $\sigma'_y$	0.13 $\mu\text{rad}$
$R_{54}$ of the beam line	-15.5 cm
$N_p \times \lambda_u$ for the modulator	10 $\times$ 40 cm
Laser wavelength in the modulator	2 $\mu\text{m}$
Laser waist size in the modulator	0.8 mm
Laser power in the modulator	58 GW

adjusted to achieve vanishing  $R_{53}$  and  $R_{56}$  (this is achieved with the "optimization" function in ELEGANT code [12]). The corresponding  $R_{54}$  for the chicane is about -15.5 cm.

The beam-laser interaction and tracking of the particles through the chicane are both simulated with ELEGANT, including 3-D effects and second order transport effects. The details of TEM01 mode laser and its interaction with beam in the undulator can be found in [10]. The laser waist is assumed to be 0.8 mm, and the laser power of about 58 GW is needed in order to generate an angular modulation amplitude  $\sim 12$  times larger than beam's intrinsic vertical divergence. Such high power laser with TEM01 mode is also of great interest in laser acceleration [13-14] and is under fast development. In reality the required TEM01 mode can be excited with standard mode-matching technique and the required 2  $\mu\text{m}$  high power laser can be obtained from an optical parametric amplifier pumped by a Ti:Sapphire laser. It should be pointed out that a TEM11 mode (doughnut-shape) laser can be used to modulate the beam vertical divergence as well, because only the vertical polarization component will interact with the beam in the undulator. From a practical point of view, generation of TEM11 mode laser may be easier than TEM01 mode laser because it is axisymmetric.

The initial  $y' - z$  phase space distribution, beam current, vertical emittance and beam energy spread are shown in Fig. 2. The distribution is periodic, so only the particles within one laser wavelength are shown. The initial beam peak current is 300 A with a normalized

vertical emittance of  $0.0176 \mu\text{m}$  and an energy spread of  $5.13 \text{ MeV}$ .

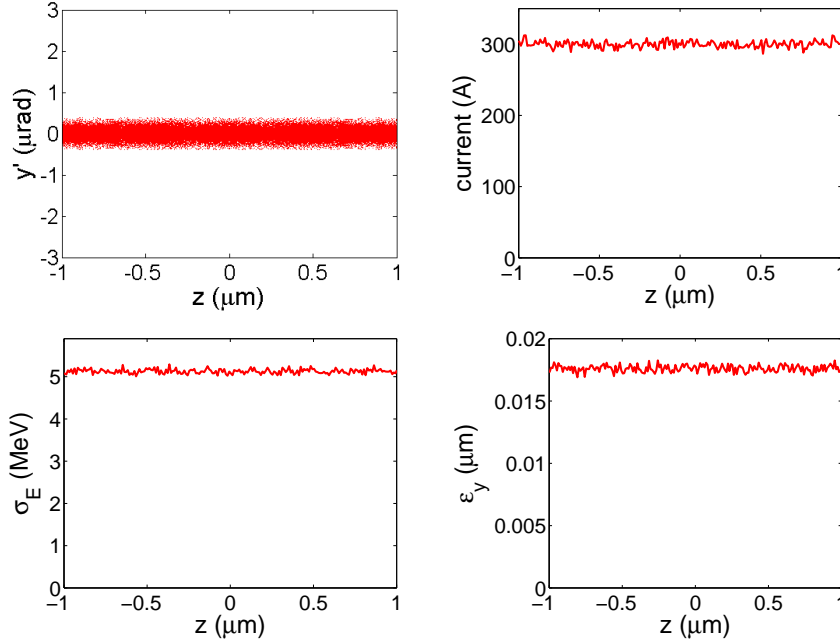


FIG. 2: Initial beam distribution. (top-left):  $y' - z$  phase space; (top-right): current distribution; (bottom-left): energy spread; (bottom-right): vertical emittance.

After interaction with the TEM01 laser, the distributions are shown in Fig. 3. While the vertical angular distribution is modulated by the TEM01 laser, the beam current remains the same. The energy spread growth is very small due to the extremely small vertical emittance. The emittance growth depends on the laser phase. The emittance growth is roughly proportional to the chirp of the angular kick where it achieves maximum at the zero crossing phases and is minimized at the on-crest phases.

After passing through the chicane, the beam distributions are shown in Fig. 4. One can clearly see that the angular modulation is converted to density modulation, and the beam peak current is increased accordingly. The energy spread growth is small, compared to the initial energy spread. Therefore, the longitudinal emittance for the beam slice where beam current is significantly enhanced is greatly reduced. As a result of this emittance transfer, the vertical emittance for the beam slice is increased by about two orders of magnitude (still on the order of  $\sim 1 \mu\text{m}$ ).

The FEL parameter for these current-enhanced slices is about  $\rho \approx 1.2 \times 10^{-3}$ , and the estimated power gain length at  $1 \text{ nm}$  using Xie's formula [15] is about  $L_G \approx 4.8 \text{ m}$  (assuming

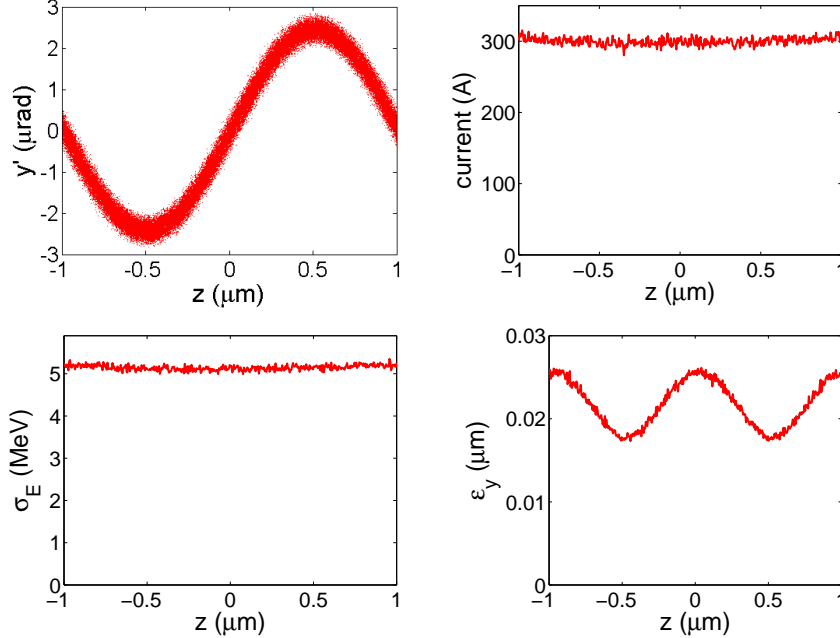


FIG. 3: Beam distribution after interaction with the TEM01 laser. (top-left): phase space; (top-right): current distribution; (bottom-left): energy spread; (bottom-right): vertical emittance.

the undulator period is  $\lambda_u = 3$  cm and average beta function is 5 m). With the long straight section, FEL at 1 nm may achieve saturation and the peak FEL power is estimated to be  $P \approx 1.6\rho(L_{G0}/L_G)^2 P_b \approx 1.6$  GW, where  $P_b$  is the peak beam power. The peak power is about 4 to 5 orders of magnitude higher than the nominal case without laser manipulation.

It is worth mentioning that the duration of the beam slices where the current is enhanced is only a few hundred nm, shorter than the FEL slippage length which is a few  $\mu\text{m}$  in our example. To have sustained interaction, one needs to delay the electron beam by approximately the laser wavelength after a few hundred undulator periods to make the radiation interact with the upstream current bump. The main concerns in delaying the electron beam are the preservation of the micro-bunches developed in upstream undulators and the matching of the radiation phase to the micro-bunches. In a simple 4-dipole chicane, the path length difference between the electron beam and the radiation is approximately  $R_{56}/2$ . For our case the  $R_{56}$  should be about  $4 \mu\text{m}$  assuming the laser wavelength is  $2 \mu\text{m}$ . Considering the fact that the slice energy spread is about  $\sigma_\delta \approx 10^{-3}$ , after passing through the simple chicane all the micro-structures that are finer than  $\sigma_\delta R_{56} \approx 4$  nm will be washed out. So isochronous chicanes are needed in order to preserve the micro-structures. As for the match-

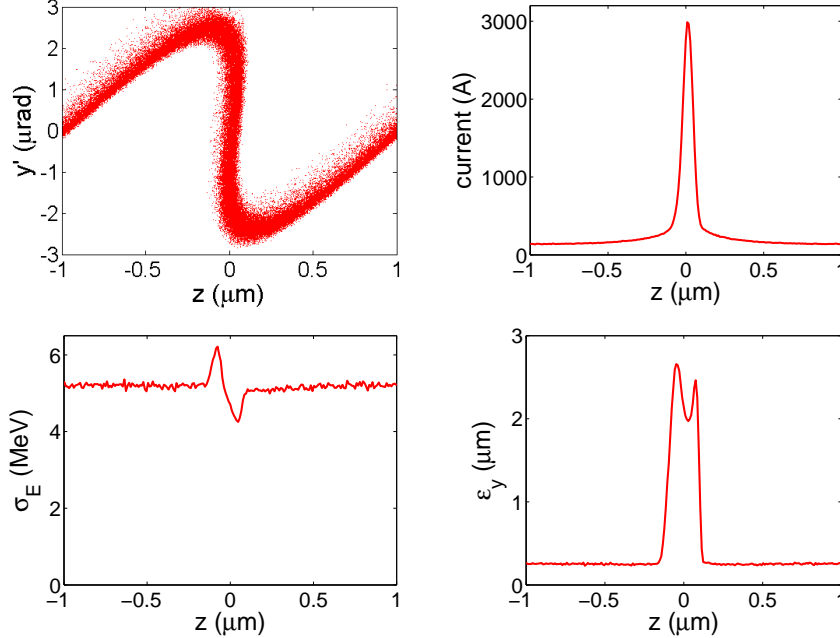


FIG. 4: Beam distribution after passing through the chicane. (top-left): phase space; (top-right): current distribution; (bottom-left): energy spread; (bottom-right): vertical emittance.

ing of the radiation phase to the micro-bunches, since the path difference in the chicane is much larger than the cooperation length of the FEL, the averaged FEL interaction is not sensitive to the radiation phase [16]. Nevertheless, further studies with fully self-consistent FEL simulations are needed to confirm the performance of the proposed technique.

It should be pointed out that the utilization of the high power laser in the proposed scheme may limit the repetition rate of the FEL to  $\sim$ kHz, which is limited by the laser technology. Furthermore, the beam energy spread will be increased after the FEL process, and the beam may get damped or get lost, depending on the momentum aperture of the storage ring. The energy spread growth is about  $\rho E$  after FEL interaction. For the above example the final beam energy spread is below 10 MeV. For the proposed PEP-X storage ring, the momentum aperture is about 3% [8] in the long straight sections, therefore the beam loss from FEL interaction is negligible.

## IV. CONCLUSIONS

In summary, we proposed a technique to increase the peak current of the electron beam without greatly increasing the energy spread, which is achieved by transferring part of the longitudinal emittance to transverse plane. It is shown that by properly repartitioning the emittance in 6-D phase space, the beam from a large storage ring may be used to drive a single-pass high-gain FEL in the soft x-ray wavelength range.

## V. ACKNOWLEDGEMENTS

We thank M. Borland for adding the options of higher order mode laser in ELEGANT code and Y. Ding, Z. Huang for helpful discussions. This work was supported by the US DOE under Contract No. DE-AC02-76SF00515.

- 
- [1] Z. Huang, K. Bane, Y. Cai, A. Chao, R. Hettel and C. Pellegrini, Nucl. Instrum. Methods Phys. Res., Sect. A 593, 120 (2008).
  - [2] R. Bonifacio, C. Pellegrini, and L. M. Narducci, Opt. Commun. 50, 373 (1984).
  - [3] M. Cornacchia and P. Emma, Phys. Rev. ST Accel. Beams, 5, 084001 (2002).
  - [4] P. Emma Z. Huang, K.-J. Kim and P. Piot, Phys. Rev. ST Accel. Beams, 9, 100702 (2006).
  - [5] A. Zholents, Phys. Rev. ST Accel. Beams, 8, 040701 (2005).
  - [6] D. Xiang, Phys. Rev. ST Accel. Beams, 13, 010701 (2010).
  - [7] D. Xiang, Proceedings of the FLS 2010, ICFA Beam Dynamic Workshop, Menlo Park, California, (2010).
  - [8] K. Bane *et al.*, SLAC-PUB-13999, (2010).
  - [9] P. Emma, *et al.*, Nature Photon., 4, 641 (2010).
  - [10] A. Zholents and M. Zolotarev, New J. Phys, 10, 025005 (2008).
  - [11] D. Xiang and W. Wan, Phys. Rev. Lett, 104, 084803 (2010).
  - [12] M. Borland, "Elegant: A flexible SDDS-compliant code for accelerator simulation," Advanced Photon Source LS-287, September, (2000).
  - [13] R. Tikhoplav, A. Melissinos, N. Barov and J. Santucci, Proceedings of EPAC 2002, Paris, France, p. 984.

- [14] K. Nakajima, Proceedings of APAC 2004, Gyeongju, Korea, p. 523 (2004).
- [15] M. Xie, Proceedings of PAC95, p. 183 (1995).
- [16] Y. Ding, P. Emma, Z. Huang and V. Kumar, Phys. Rev. ST Accel. Beams, 9, 070702 (2006).