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Gauge/Gravity Duality and Hadron Physics at the Light-Front

Guy F. de Téramond[a] and Stanley J. Brodsky[b,c]

[a]Universidad de Costa Rica, San José, Costa Rica
[b]SLAC National Accelerator Laboratory Stanford University, Stanford, CA 94309, USA
[c]CP^3-Origins, University of Southern Denmark, Odense, 5230 M, Denmark

Abstract

We discuss some remarkable features of the light-front holographic mapping of classical gravity in anti-de Sitter space modified by a confining dilaton background. In particular, we show that a positive-sign dilaton solution $\exp(+\kappa^2 z^2)$ has better chances to describe the correct hadronic phenomenology than the negative solution $\exp\left(-\kappa^2 z^2\right)$ extensively studied in the literature. We also show that the use of twist-scaling dimensions, instead of canonical dimensions, is required to give a good description of the spectrum and form factors of hadrons. Another key element is the explicit connection of AdS modes of total angular momentum J with the internal structure of hadrons and the proper identification of the orbital angular momentum of the constituents.

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1 Introduction

The AdS/CFT correspondence [1] between a gravity or string theory on a higher dimensional Anti-de Sitter (AdS) spacetime and conformal gauge field theories in physical spacetime, modified by color confinement, has led to a semiclassical approximation for strongly-coupled QCD [2], which provides analytical insights into its inherently nonperturbative nature including hadronic spectra, form factors and, very recently, the nonperturbative behavior of the QCD coupling in the infrared region [3].

Five dimensional AdS_5 spacetime has negative curvature and a four dimensional spacetime boundary. The most general group of transformations that leave the AdS metric

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \tag{1}$$

invariant (R the AdS radius), the isometry group, has 15 dimensions, in agreement with the number of generators of the conformal group in four dimensions $SO(4,2)$. This isomorphism is the basic principle underlying the AdS/CFT approach to conformal gauge theories. Since the metric (1) is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space: different values of z correspond to different length scales.

In order to describe a confining theory, the conformal invariance of AdS_5 must be broken. A simple way to impose confinement and discrete normalizable modes is to truncate the regime where the string modes can propagate by introducing an IR cutoff at a finite value $z_0 \sim 1/\Lambda_{\mathrm{QCD}}$. Thus, the "hard-wall" at z_0 breaks conformal invariance and allows the introduction of the QCD scale and a spectrum of particle states [4]. As first shown by Polchinski and Strassler, [4] the AdS/CFT duality, modified to incorporate a mass scale, provides a nonperturbative derivation of dimensional counting rules [5] for the leading power-law falloff of hard scattering.

The conformal metric of AdS space can be modified within the gauge/gravity framework to simulate confinement forces by the introduction of an additional warp factor or, equivalently, with a dilaton background $\varphi(z)$, which introduces an energy scale in the five-dimensional Lagrangian, thus breaking the conformal invariance. A particularly interesting case is a dilaton profile $\exp\left(\pm\kappa^2 z^2\right)$ of either sign, since it leads to linear Regge trajectories [6] and avoids the ambiguities in the choice of boundary conditions at the infrared wall. The modified metric induced by the dilaton can be interpreted in AdS space as a gravitational potential for an object of mass m in the fifth dimension: $V(z) = mc^2\sqrt{g_{00}} = mc^2 R\, e^{\pm\kappa^2 z^2/2}/z$. In the case of the negative solution the potential decreases monotonically, and thus an object in AdS will fall to infinitely large values of z. For the positive solution, the potential is nonmonotonic and has an absolute minimum at $z_0 = 1/\kappa$. Furthermore, for large values of z the gravitational potential increases exponentially, thus confining any object to distances $\langle z\rangle \sim 1/\kappa$ [7].

An important part of this paper is to show that the positive confining dilaton solution $\exp\left(+\kappa^2 z^2\right)$ found in Ref. [6], and subsequently used in [8] to describe the confining potential between two heavy quarks, has better chances to describe the correct hadronic phenomenology than the negative solution $\exp(-\kappa^2 z^2)$, extensively studied in the literature and known as the "soft-wall model".[1] Another key element is the explicit connection [9] of AdS string modes of total angular momentum J with the internal structure of hadrons and the proper identification of the orbital angular momentum of the constituents [2].

Light-front (LF) quantization is the ideal framework for describing the structure of hadrons in terms of their quark and gluon degrees of freedom. Light-front wave functions (LFWFs) play the same role in hadron physics that Schrödinger wave functions play in atomic physics. The simple structure of the LF vacuum provides an unambiguous definition of the partonic content of a hadron in QCD. Light-front holography provides a remarkable connection between the equations of motion in AdS space and the Hamiltonian formulation of QCD in physical spacetime quantized on the light front at fixed LF time $\tau = x^+ = x^0 + x^3$ [2]. This correspondence provides a direct connection between the hadronic amplitudes $\Phi(z)$ in AdS space with LFWFs $\phi(\zeta)$ describing the quark and gluon constituent structure of hadrons in physical spacetime. The mapping between the LF invariant variable ζ and the fifth-dimension AdS coordinate z was originally obtained by matching the expression for electromagnetic (EM) current matrix elements in AdS space with the corresponding expression for the current matrix element, using LF theory in physical spacetime [10, 11]. It has also been shown that one obtains the identical holographic mapping using the matrix elements of the energy-momentum tensor [12], thus verifying the consistency of the holographic mapping from AdS to physical observables defined on the light front.

2 Higher Spin Modes in AdS Space

Our starting point is the Lagrangian for a scalar field in AdS$_{d+1}$ spacetime in presence of a dilaton background field $\varphi(z)$

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \left(g^{\ell m} \partial_\ell \Phi^* \partial_m \Phi - \mu^2 \Phi^* \Phi \right), \tag{2}$$

where $\varphi(z)$ is a function of the holographic coordinate z which vanishes in the conformal ultraviolet limit $z \to 0$. The coordinates of AdS are the Minkowski coordinates x^ℓ and the holographic variable z labeled $x^\ell = \left(x^\ell, z\right)$. Taking the variation of (2) and factoring out plane waves along the Poincaré coordinates, $\Phi_P(x^\mu, z) = e^{-iP\cdot x}\Phi(z)$, we obtain the eigenvalue equation

$$\left[-\frac{z^{d-1}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi(z) = \mathcal{M}^2 \Phi(z), \tag{3}$$

[1]The positive dilaton solution was discarded as non-physical in Ref. [6] as it leads to a massless ρ meson for the specific AdS/QCD model discussed in Ref. [6].

where $P_\mu P^\mu = \mathcal{M}^2$ is the invariant mass of a physical hadron with four-momentum P_μ.

We define a spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 with shifted dimensions $\Phi_J(z) = (z/R)^{-J}\Phi(z)$ and normalization

$$R^{d-1-2J} \int_0^\infty \frac{dz}{z^{d-1-2J}}\, e^{\varphi(z)} \Phi_J^2(z) = 1. \tag{4}$$

The shifted field Φ_J obeys the wave equation

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi(z) = \mathcal{M}^2\Phi(z), \tag{5}$$

which follows from (3) upon mass rescaling $(\mu R)^2 \to (\mu R)^2 - J(d-J)$ and $\mathcal{M}^2 \to \mathcal{M}^2 - Jz^{-1}\partial_z\varphi$. It is useful to introduce fields with tangent indices

$$\tilde{\Phi}_{i_1 i_2 \cdots i_J} = e_{i_1}^{\ell_1} e_{i_2}^{\ell_2} \cdots e_{i_J}^{\ell_J}\, \Phi_{\ell_1 \ell_2 \cdots \ell_J} = \left(\frac{z}{R}\right)^J \Phi_{i_1 i_2 \cdots i_J}, \tag{6}$$

with scaling behavior $\tilde{\Phi}_J(z \to 0) \sim z^\tau$ and scaling dimension τ given by the relation $(\mu R)^2 = (\tau - J)(\tau - d + J)$. The vielbein e_ℓ^i is defined by $g_{\ell m} = e_\ell^i e_m^j \eta_{ij}$, where $i, j = 1, \cdots, d+1$ are tangent AdS space indices.

2.1 Light-Front Holographic Mapping

In light-front QCD a physical hadron in four-dimensional Minkowski space has four-momentum P_μ and invariant hadronic mass states, $P_\mu P^\mu = \mathcal{M}^2$, determined by the Lorentz-invariant Hamiltonian equation for the relativistic bound-state system [13]

$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right)|\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle. \tag{7}$$

The hadron four-momentum generator is $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, and the hadronic state $|\psi\rangle$ is an expansion in multiparticle Fock eigenstates $|n\rangle$ of the free LF Hamiltonian: $|\psi\rangle = \sum_n \psi_n |n\rangle$. The internal partonic coordinates of the hadron are the momentum fractions $x_i = k_i^+/P^+$ and the transverse momenta $\mathbf{k}_{\perp i}$, $i = 1, 2, \ldots, n$, where n is the number of partons in a given Fock state. Momentum conservation requires $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \mathbf{k}_{\perp i} = 0$. It is useful to employ a mixed representation [14] in terms of $n-1$ independent momentum fraction variables x_j and position coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$, so that $\sum_{i=1}^n \mathbf{b}_{\perp i} = 0$. The relative transverse variables $\mathbf{b}_{\perp i}$ are Fourier conjugates of the momentum variables $\mathbf{k}_{\perp i}$.

The structure of the QCD Hamiltonian equation (7) is similar to the structure of the AdS wave equation (5); they are both frame-independent and have identical eigenvalues $\mathcal{M}^2$, the mass spectrum of the color-singlet states of QCD, a possible indication of a more profound connection between physical QCD and the physics of hadronic modes in AdS space. However, important differences are also apparent: Eq. (7) is a linear quantum-mechanical equation of states in Hilbert space, whereas Eq. (5) is a classical gravity equation stemming from general relativity or string theory; its solutions describe spin-J modes propagating in a higher dimensional warped space. Physical hadrons are composite and thus inexorably endowed of orbital angular momentum. Thus, the identification of orbital angular momentum is of primary interest in finding a connection between both approaches. In fact, to a first semiclassical approximation, light-front QCD is formally equivalent to the equations of motion on a fixed gravitational background [2] asymptotic to AdS_5 where the prominent properties of confinement are encoded in the dilaton profile $\varphi(z)$. One can indeed systematically reduce the LF Hamiltonian eigenvalue Eq. (7) to an effective relativistic wave equation [2], analogous to the AdS equations, by observing that each n-particle Fock state has an essential dependence on the invariant mass of the system $\mathcal{M}_n^2 = \left(\sum_{a=1}^n k_a^\mu\right)^2$ and thus, to a first approximation, LF dynamics depend only on $\mathcal{M}_n^2$ [15]. In impact space the relevant variable is the boost invariant transverse variable ζ which measures the separation of the quark and gluonic constituents within the hadron at the same LF time and which also allows one to separate the dynamics of quark and gluon binding from the kinematics of the constituent internal angular momentum. In the case of two constituents, $\zeta = \sqrt{x(1-x)}|\mathbf{b}_\perp|$ where $x = k^+/P^+$ is the LF fraction.

Following [2] we compute $\mathcal{M}^2$ from the hadronic amplitude $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2\langle\psi(P')|\psi(P)\rangle$, expanding the initial and final hadronic states in terms of its Fock components. In the limit of zero quark mass, the longitudinal and transverse modes decouple and we obtain for a quark-antiquark hadronic bound state the result

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta) U(\zeta)\phi(\zeta), \tag{8}$$

where all the complexity of the confining interaction terms in the QCD Lagrangian is summed up in the effective potential $U(\zeta)$. The LF eigenvalue equation $P_\mu P^\mu|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is thus a light-front wave equation for ϕ [2]

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta), \tag{9}$$

an effective single-variable light-front Schrödinger equation which is relativistic, frame independent and analytically tractable.

Upon the substitution $z \to \zeta$ and $\phi_J(\zeta) = (\zeta/R)^{-3/2+J}\, e^{\varphi(z)/2}\, \Phi_J(\zeta)$, in (5), we find for $d = 4$ the QCD light-front wave equation (9) with effective potential

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{z}\varphi'(z), \tag{10}$$

where the fifth dimensional mass is not a free parameter but scales according to $(\mu R)^2 = -(2-J)^2 + L^2$. The LFWFs are normalized $\langle\phi|\phi\rangle = \int d\zeta \phi^2(\zeta) = 1$. If $L^2 < 0$ the LF Hamiltonian is unbounded from below $\langle\phi|H_{LF}|\phi\rangle < 0$ and thus $\mathcal{M}^2 < 0$; the particle "falls towards the center" as the effective potential is conformal at small ζ. The critical value corresponds to $L = 0$. For $J = 0$ the five dimensional mass μ is related to the orbital momentum of the hadronic bound state by $(\mu R)^2 = -4 + L^2$ and thus $(\mu R)^2 \geq -4$. The quantum mechanical stability condition $L^2 \geq 0$ is thus equivalent to the Breitenlohner-Freedman stability bound in AdS [16]. The scaling dimensions are $2 + L$ independent of J in agreement with the twist-scaling dimension of a two-parton bound state in QCD. It is important to notice that in the light-front the $SO(2)$ Casimir for orbital angular momentum L^2 is a kinematical quantity, in contrast with the usual $SO(3)$ Casimir $\ell(\ell+1)$ from non-relativistic physics which is rotational, but not boost invariant.

2.2 A Linear Confining Dilaton Background

A particularly interesting analytical example of a dilaton background is that of a Gaussian dilaton profile $\varphi(z) = \pm\kappa z^2$ [6], which corresponds to a transverse oscillator in the light-front. From (10) we obtain for the positive sign solution the effective potential [7] $U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$ where $J_z = L_z + S_z$. Equation (9) has eigenfunctions

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2), \tag{11}$$

and eigenvalues

$$\mathcal{M}^2_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2}\right). \tag{12}$$

The lowest possible solution for $n = L = S = 0$ has eigenvalue $\mathcal{M}^2 = 0$. This is a chiral symmetric bound state of two massless quarks with scaling dimension 2 and size $\langle\zeta^2\rangle \sim 1/\kappa^2$, which we identify with the lowest state, the pion. Thus one can compute the hadron spectrum by simply adding $4\kappa^2$ for a unit change in the radial quantum number, $4\kappa^2$ for a change in one unit in the orbital quantum number and $2\kappa^2$ for a change of one unit of spin to the ground state value of $\mathcal{M}^2$. Remarkably, the same rule holds for baryons [7], thus predicting the same multiplicity of states for mesons and baryons, which is observed experimentally [17]. The LFWFs (11) for different orbital and radial excitations are depicted in Fig. 1. Constituent quark and antiquark separate from each other as the orbital and radial quantum numbers increase.

Individual hadron states can be identified by their interpolating operator at $z \to 0$. For example, the vector-meson (VM) operator $\mathcal{O}^\mu_{2+L} = \bar{q}\gamma^\mu D_{\{\ell_1}\cdots D_{\ell_m\}}q$ with total internal orbital momentum $L = \sum_{i=1}^m \ell_i$, is a twist $\tau = 2 + L$ operator with canonical dimension $\Delta = 3 + L$. The scaling of $\tilde{\Phi}(z)_\mu \sim z^\tau$ at $z \to 0$ is precisely the scaling required to match the scaling dimension of the local vector-meson interpolating operators. The spectral predictions for light VM states are compared with experimental data in Fig. 2 for the positive sign dilaton model discussed here. Only confirmed PDG states [18] are shown.

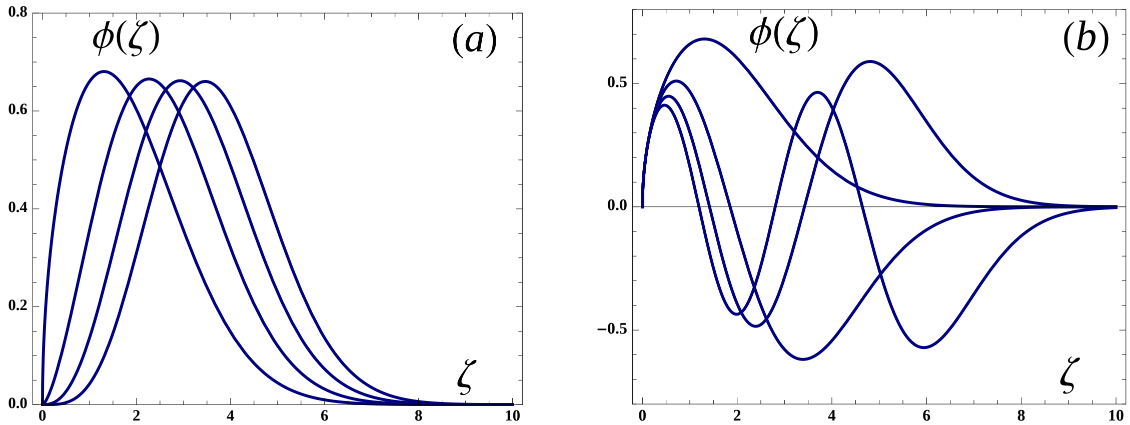

Figure 1: Light-front wavefunctions $\phi_{n,L}(\zeta)$ is physical spacetime corresponding to a dilaton $\exp(\kappa^2 z^2)$: a) orbital modes ($n = 0$) and b) radial modes ($L = 0$).

2.3 Twist versus Canonical Conformal Dimensions

The short-distance behavior of a hadronic state is characterized by its twist (dimension minus spin) $\tau = \Delta - \sigma$, where σ is the sum over the constituent's spin $\sigma = \sum_{i=1}^{n} \sigma_i$. The twist of the interpolating operator ensures dimensional counting rules for form factors and other hard exclusive processes [5], consistent with conformal invariance at short distances as well as the scaling expected from supersymmetry [19], since the scalar, the spinor and the gluon fields G all have twist one. Thus, twist is also equal to the number of partons $\tau = N$.

Consider a hadronic state composed of an arbitrary number of quark and gluons $|qqq\cdots\bar{q}q\cdots GG\ldots\rangle$. As each individual quark or gluon state has dimension $[\,|p_i\rangle] = [L]$, the hadronic state of momentum P, measured at an energy scale Q, should scale as $|\psi(P)\rangle_Q \sim (1/Q)^N$, since the dimension is set by the energy scale $1/Q$ at large Q. The gauge/gravity correspondence implies a duality between the hadronic state $|\psi(P)\rangle$ and the normalizable mode $\Phi_P(x^\mu, z)$ in AdS space. Consequently, the relevant scaling dimension of hadronic modes in AdS is dictated by the twist and not the naive conformal dimension [9]; thus $\tilde{\Phi}(z) \sim z^N$, for twist $\tau = N$.[2] This is in fact consistent with the scaling of observables. Consider for example a form factor in AdS space [20, 21]

$$F(Q^2) = R^{3-2J} \int \frac{dz}{z^{3-2J}}\, e^{\varphi(z)} V(Q,z)\, \Phi_J^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau-1}, \tag{13}$$

and the ultraviolet pointlike behavior [22] responsible for the power law scaling [5] is recovered. The scaling of the form factor at large Q follows from integration in the region near $z \sim 1/Q$ where $\tilde{\Phi}_J(z) = (z/R)^J \Phi(z) \sim z^\tau$. At large Q, the bulk-to-boundary propagator $V(Q,z) \sim zQK_1(zQ)$, and consequently the power-law falloff of

[2]The scaling dimension of a hadronic state with N partons and orbital angular momentum L is $\tau = N + L$.

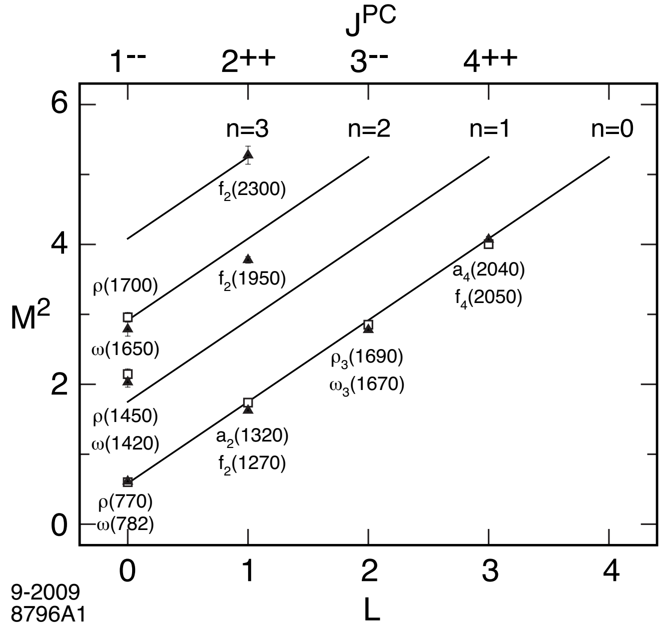

Figure 2: Regge trajectories for the $I = 1$ ρ-meson and the $I = 0$ ω-meson families for $\kappa = 0.54$ GeV.

the form factors only depends on the twist-scaling behavior of the hadronic modes and not on the electromagnetic current.

Conserved currents are not renormalized and correspond to five dimensional massless fields propagating in AdS according to the relation $(\mu R)^2 = (\Delta - p)(\Delta + p - 4)$ for a p form in $d = 4$. Thus for an electromagnetic current the wave equation

$$\left[\frac{z}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z}\partial_z\right) - Q^2\right] A_\mu(Q, z) = 0, \tag{14}$$

corresponds to $\Delta = 3$ or 1, which are precisely the canonical dimensions of an EM current and field strength respectively. How can we reconcile this assignment with the twist-scaling behavior of fundamental hadronic constituents which is required to account for hard scattering? [4] For massless quarks, currents do not flip spin. Thus a $\bar{q}q$ state produced by an electromagnetic current is an $L_z = \pm 1$ state with the q and $\bar{q}$ with opposite spins. Consequently, the electromagnetic current is dual to hadronic states with components $L_z = 1$ and twist $\tau = 3$. From the LF mapping relation $(\mu R)^2 = -(2 - J)^2 + L^2$ there follows that $\mu R = 0$ for $J = L = 1$ and the wave equation (14) can also be derived from (5) for $J = 1$ and $\mathcal{M}^2 \to -Q^2$. The result is consistent with conformal dimension $\Delta = 3$, the usual assignment in AdS/QCD models [23, 24].

The analysis is similar for a graviton. The canonical conformal dimension in this case is the dimension of the energy-momentum tensor, thus $\Delta = 4$. We can identify a

$J = 2$ field $\Phi_{\mu\nu}$ with an external graviton $h_{\mu\nu}$ propagating in AdS, provided that we take into account the proper normalization of the action for pure gravity. This means that $h_{\mu\nu} \sim z^2 \Phi_{\mu\nu}$. From (5) we obtain the wave equation

$$\left[\frac{z^3}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^3}\partial_z\right) - Q^2\right] h_\mu^\nu(Q,z) = 0, \tag{15}$$

which coincides with the result obtained by Abidin and Carlson from the linearized gravity action [25]. This is again consistent with a $\bar{q}q$ state with $L_z = \pm 2$ and opposite spins produced by a gravitational current.

2.4 Positive vs Negative Sign Dilaton Background

2.4.1 Hadronic Spectrum

The soft-wall model of Ref. [6] uses the AdS/QCD framework of Refs. [23] and [24], where bulk fields are introduced to match the $SU(2)_L \times SU(2)_R$ chiral symmetry of QCD and its spontaneous breaking, but without an explicit connection to the internal constituent structure of hadrons as done in this article. Instead, axial and vector currents become the primary entities as in an effective chiral theory. Comparison of both approaches is not straightforward and could be misleading, but one would expect that the results are rather similar for both approaches. Comparison of the results for the ρ principal Regge trajectory of radial excitations are however significantly different as shown in Fig (3), where we compare the predictions of Ref. [6] with the results from Eq. (12). This particular example does not require a discussion of the orbital angular momentum and it is particularly relevant for the computation of hadronic form factors. An AdS mode with a node in the coordinate z should correspond to a radial resonance with a node in the interquark separation. The lowest state, the $\rho(770)$, has no nodes in the wavefunction and corresponds to $n = 0$. For both models we fix the scale κ at the $\rho(770)$ mass.

2.4.2 Hadronic Form Factors

For the soft wall model of Ref. [6] the form factor of a hadron of arbitrary twist τ can be expressed in terms of gamma functions [11] by using the integral representation of the bulk-to-boundary propagator found in Ref. [27]. In absence of anomalous dimensions the twist is an integer, $\tau = N$ and the result for either sign dilaton is expressed as an $N - 1$ product of poles along the vector radial trajectory [11]

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}. \tag{16}$$

As expected, the results for the form factor are sensitive to the location of the VM mass poles along the Regge trajectory. This is more noticeable in the time-like region which

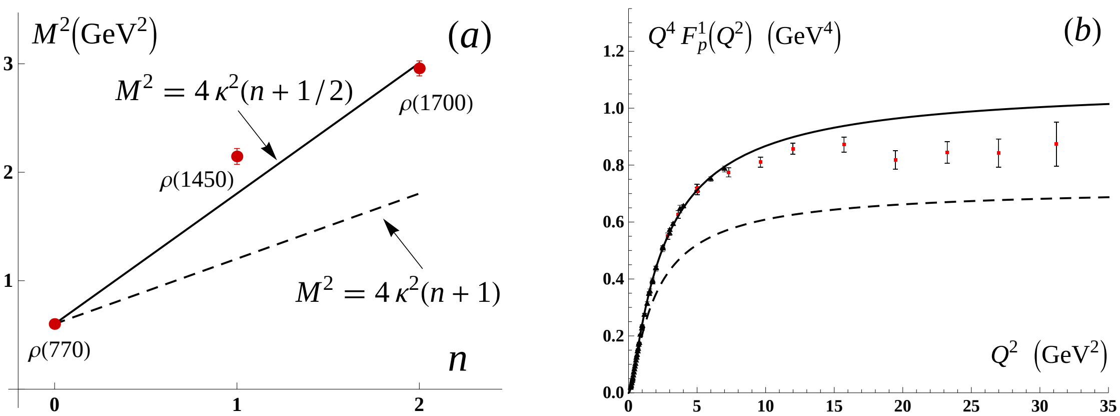

Figure 3: Comparison of predictions for hadronic observables in a negative (dashed line) and positive (continuous line) dilaton backgrounds: a) vector meson radial trajectories and b) space-like scaling behavior of $Q^4 F_1^p(Q^2)$ as function of Q^2. The proton form factor data compilation is from Diehl [26]. The values of κ are $\mathcal{M}_\rho/2$ ($\varphi = -\kappa^2 z^2$) and $\mathcal{M}_\rho/\sqrt{2}$ ($\varphi = \kappa^2 z^2$).

is particularly sensitive to the detailed pole structure and the interference effects from the phase of the amplitude. We show in Fig. 3 the results for the Dirac spin-non-flip proton form factor. Since the proton is twist three the results follow from the first two terms in Eq. (16). The dashed line corresponds to the minus sign dilaton $\varphi = -\kappa^2 z^2$ with $M^2 = \kappa^2(n+1)$ and the continuous line to the positive sign dilaton $\varphi = \kappa^2 z^2$ with VM masses $M^2 = 4\kappa^2(n+1/2)$. For both models the scale κ is fixed at the $\rho(770)$ mass. The positive dilaton solution gives a better result when we use the mass eigenvalues of the Hamiltonian equation which are close to the observed masses, thus shifting the poles from twist-three (the dimension required to have zero fifth dimensional mass in the conserved current) to twist two, the twist of a two-component object. How can this procedure be justified?

In the limit of zero quark mass, states such as e^+e^-, $\bar{q}q$ or GG are produced with opposite spin and non-zero orbital angular momentum. The bound states, however, can have a zero L_z component, which mix with states with non-zero orbital. For example, the vector meson $\rho(770)$ state has multiple components $S = 1, L_z = 0$ as well as $S = 0, L_z = \pm 1$. The mass of the state can be read off from the lowest twist ($L_z = 0$) bound state. For example, the solution for a spin-$\frac{1}{2}$ field in AdS has two components: a plus component ψ_+ which represent a state with $S_z = \pm\frac{1}{2}, L_z = 0$, and a minus component ψ_- which represents a state with $S_z = \mp\frac{1}{2}, L_z = \pm 1$. Since both components mix in the AdS wave equation they have the same mass: the value of the mass eigenstate is the same for the lower twist $L_z = 0$ component than for the higher $L_z = \pm 1$ component [7]. Thus to take into account the different components of a VM state, one requires a multiple-component equation in AdS space, as for the spin-$\frac{1}{2}$ case. This multiple-component equation (a Kummer-Duffin-Petiau-like equation) for vector mesons has not been derived, to our knowledge, in AdS space, since one usually compute the mass spectrum from a single-component wave equation (like Eq. (5)), where the eigenmass corresponds to the lowest ($L_z = 0$) state. On the other hand, the mass poles in the Green's function for the current propagator of a VM AdS field $A(z)$ is [3]

$$G(z, z'; q) = R \sum_n \frac{e^{\kappa^2 z'^2}}{z'} \frac{A_n(z') A_n(z)}{M_n^2 - q^2 - i\epsilon}, \tag{17}$$

corresponding to the higher-twist component and not to the physical mass (which corresponds to the lowest-twist component in the eigenvalue equation) when proper component mixing is allowed in the equations of motion. Consequently the location of the poles has to be shifted to their lowest-twist (physical) mass to describe correctly the form factors in a positive dilaton background.

2.4.3 Nonperturbative QCD coupling

Very recently we have examined with Alexandre Deur the behavior of nonperturbative effective couplings in QCD from the perspective of light-front holography [3]. The infrared (IR) results for the strong coupling are markedly different according to the sign of the dilaton chosen. The positive dilaton $\varphi(z) = \kappa^2 z^2$ leads to an IR fixed-point. In contrast, the negative solution $\varphi(z) = -\kappa^2 z^2$ leads to a coupling which blows up in the IR. Following [3], we consider a five-dimensional gauge field F propagating in AdS_5 in the presence of dilaton $\varphi(z)$. At quadratic order in the field strength the action is

$$S = -\frac{1}{4} \int d^4x \, dz \sqrt{g} \, e^{\varphi(z)} \frac{1}{g_5^2} F^2, \tag{18}$$

where we identify the prefactor $g_5^{-2}(z) = e^{\varphi(z)} g_5^{-2}$, as the effective coupling of the theory at the length scale z. The coupling $g_5(z)$ incorporates the nonconformal dynamics of confinement. The five-dimensional coupling $g_5(z)$ is mapped, modulo a constant, onto the Yang-Mills (YM) coupling g_{YM} of the confining theory in physical space-time using light-front holography. One identifies z with the invariant impact separation variable ζ: $g_5(z) \to g_{\mathrm{YM}}(\zeta)$. Thus $\alpha_s^{AdS}(\zeta) = g_{\mathrm{YM}}^2(\zeta)/4\pi \propto e^{-\kappa^2 \zeta^2}$. The physical coupling measured at the scale Q is the two-dimensional Fourier transform of the LF transverse coupling $\alpha_s^{AdS}(\zeta)$. Integration over the azimuthal angle gives

$$\alpha_s^{AdS}(Q^2) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta) \sim e^{-Q^2/4\kappa^2}. \tag{19}$$

The strong coupling $\alpha^{AdS}(Q^2)$ is compared in Fig. 4 with experimental and lattice data. The falloff of $\alpha_s^{AdS}(Q^2)$ at large Q^2 is exponential: $\alpha_s^{AdS}(Q^2) \sim e^{-Q^2/\kappa^2}$, rather

[3] In terms of the Green's function (17) the bulk-to-boundary propagator $V(q, z)$ in (13) is $V(q, z) = V(q, 0) \lim_{z' \to 0} e^{\varphi(z')} \frac{R}{z'} \partial_{z'} G(z, z'; q)$ [21].

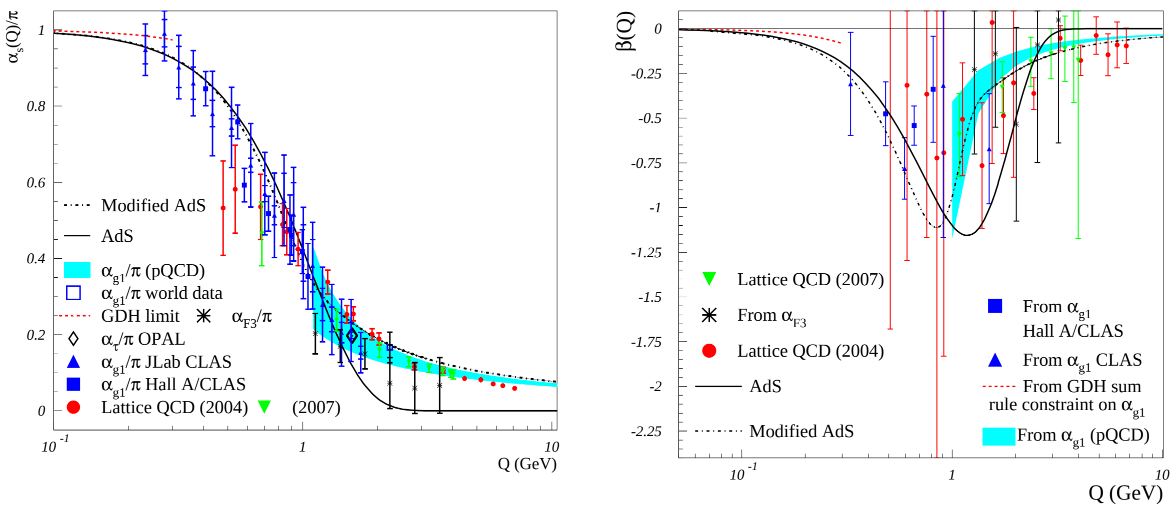

Figure 4: The strong coupling (left) and β function (right) from LF holographic mapping (continuous line) for $\kappa = 0.54$ GeV are compared with effective QCD results extracted from different observables and lattice simulations. Details of the comparison and normalization used are given in Ref. [3].

than the perturbative QCD (pQCD) logarithmic falloff, since effects from gluon creation and absorption are not included in the semiclassical theory. The corresponding beta function in Fig. 4 is conformal in the infrared and ultraviolet (UV) regions. It becomes significantly negative in the nonperturbative regime $Q^2 \sim \kappa^2$, where it reaches a minimum, signaling the transition region from the IR conformal region, characterized by hadronic degrees of freedom, to a pQCD conformal UV regime where the relevant degrees of freedom are the quark and gluon constituents. The β function is always negative; it vanishes at large Q^2 consistent with asymptotic freedom, and it vanishes at small Q^2 consistent with an infrared fixed point.

3 Conclusion

Light-front holography provides a direct correspondence between an effective gravity theory defined in a fifth-dimensional warped space and a physical description of hadrons in $3 + 1$ spacetime. The relativistic light-front wave equations which follow from the semiclassical approximation to the gauge/gravity correspondence in light-front QCD provide remarkably successful predictions for the light-quark meson and baryon spectra as a function of hadron spin, quark angular momentum, and radial quantum number. The predictions for form factors are also remarkably successful, and the predicted power law fall-off agrees with dimensional counting rules as required by conformal invariance at small distances. The use of twist-scaling dimensions and the proper identification of the orbital angular momentum of the constituents are key elements to describe the observed hadronic data. As in the Schrödinger equation, the semiclassical approximation to light-front QCD described in this paper does not account for particle creation and absorption; it is thus expected to break down at short distances where hard gluon exchange and quantum corrections become important. However, one can systematically improve the semiclassical approximation, for example by introducing nonzero quark masses and short-range Coulomb corrections.

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