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The Nature of Accelerating Modes in PBG Fibers

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Abstract. Transverse magnetic (TM) modes with phase velocities at or just below the speed of light, c, are intended to accelerate relativistic particles in hollow-core, photonic band gap (PBG) fibers. These are so-called "surface defect modes", being lattice modes perturbed by the defect to have their frequencies shifted into the band gap, and they can have any phase velocity. PBG fibers also support so-called "core defect modes" which are characterized as having phase velocities always greater than c and never cross the light line. In this paper we explore the nature of these two classes of accelerating modes and compare their properties.

Keywords: Photonic band gap fiber, defect modes, particle acceleration.
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INTRODUCTION

Photonic band gap (PBG) structures have been suggested for use as particle accelerators [1, 2]. A photonic crystal is a structure with a permittivity which is periodic in one or more of its dimensions. Analogous to a crystalline solid with its electronic band structure, the electromagnetic modes of a photonic crystal lie in a set of allowed bands (pass bands). A photonic crystal will similarly exhibit one or more band gaps (stop bands), with the frequencies in the gap corresponding to modes unable to propagate in the lattice. Defect modes (also called trapped modes in the literature) can be obtained by breaking the periodicity of the lattice in a localized region, a so-called “defect”. Since frequencies in the band gap represent modes forbidden to propagate in the perfect crystal, these modes are spatially confined to remain near the defect. Confinement of the mode is obtained by an interference effect due to the periodicity of the surrounding lattice rather than by use of a metallic boundary typical of microwave devices.

The PBG accelerator idea was introduced by Kroll et al [1] using a transverse array of dielectric rods (relative permittivity = 9) in air, separated by periodic conducting plates in the longitudinal direction. The central rod was removed to introduce an open defect, leaving space for a particle beam to propagate. The rods provided the transverse mode confinement and the conducting plates permitted a selection of the wave-number, and hence phase velocity for a transverse magnetic (TM) mode in the defect region. In the work of Kroll et al, PBG structures at 11 GHz (~1.5 cm lattice period) were fabricated, and the presence of confined modes was confirmed.

The desire to achieve ever higher gradients to make compact high energy particle accelerators has led to research into all-dielectric structures without any metallic components. Dielectrics have a higher breakdown field (~ 10^9 V/m) compared to metals at picosecond durations. Efficient, high repetition rate optical lasers have been developed for the telecom industry, making a readily available power source for an all-dielectric accelerator. A hollow-core, photonic band gap fiber is drawn as a uniform dielectric waveguide along which the light propagates longitudinally. The geometry is typically a hexagonal array of hollow cylinders in a silica matrix, making up a two-dimensional PBG lattice which confines modes transversely. The central defect is hole of larger radius than the holes which make up the matrix. The structure is not periodic in the longitudinal dimension, and the defect acts simultaneously as an optical waveguide for a TM mode and a channel for the charged particles. An example described by Lin [2] is shown in Figure 1, which is a silica PBG fiber designed to support a TM defect mode at 1 micron wavelength. The dispersion relations of modes relative to the propagation direction are of primary interest for particle acceleration, and typically the band-gap diagram is projected along the longitudinal wave-number k_z. The gap diagram was calculated with R-Soft BandSolve [3], a commercial plane-wave expansion code, and the TM mode profile was calculated with CUDOS [4], a Fourier-Bessel expansion code from the University of Sydney.

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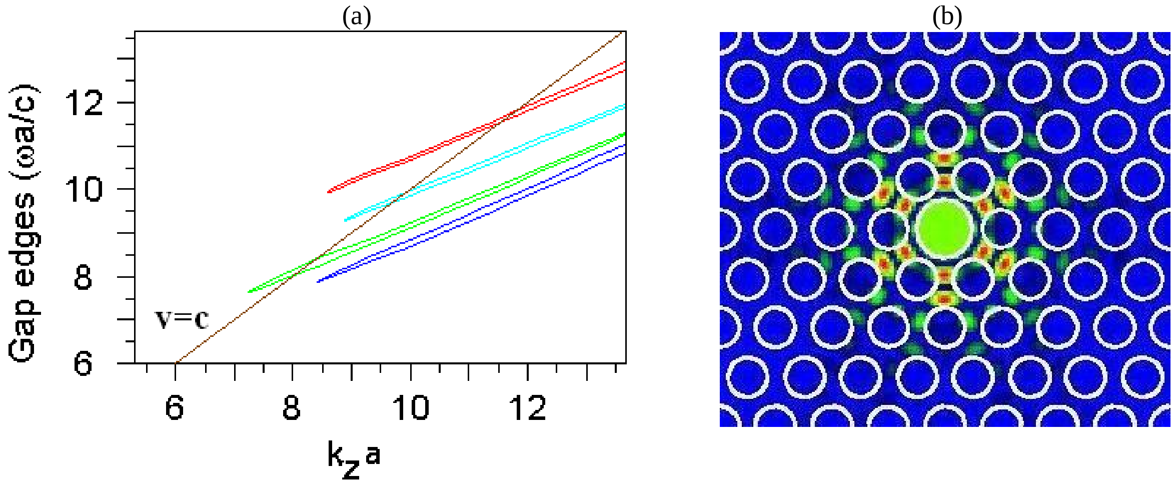


FIGURE 1. (a) Band gap diagram (ω vs k_z) calculated with R-Soft BandSolve [3] and (b) accelerating mode calculated with CUDOS [4] in the Lin PBG fiber with lattice period a = 1.31 microns. In (a), the lines are the gap edges with the enclosed region of each being the frequencies of modes which cannot propagate in the perfect lattice. In (b) the accelerating field intensity of the TM surface defect mode crossing the light line near $k_0a = 8.2$ (in second band gap from bottom) is overlaid on the fiber geometry with central defect. White circles are the hole edges, and their interiors represent vacuum surrounded by the glass matrix. The longitudinal direction is out of the page. The color scale is relative with red being the highest field and blue the lowest.

The defect modes in hollow-core PBG fibers have become recognized by photonic researchers as belonging to two classes: “core modes” with Poynting flux almost entirely contained within the hollow core and “surface modes” which are localized at the boundary separating the defect and matrix [5], [6], [7]. Core modes exhibit properties similar to TM, TE (transverse electric) and TEM (transverse electric and magnetic) eigenmodes of circular waveguides, since most of their field energy is within the circular defect region, which mimics a finite, copper cavity structure. These modes are the dielectric analog of eigenmodes in a conducting waveguide. Core modes are distinguished by the fact that their dispersion relation never crosses the speed of light (SOL) line. Their effective index, n_{eff} (= mode’s longitudinal wave-number, k_z in the fiber divided by the free-space wave number k_0 (= ω/c), is always less than one, and their phase velocity $v_p = c/n_{eff}$, is greater than the speed of light, c. The physical reason for this is the limited overlap of the mode fields and the surrounding high index material, which prevents the mode’s effective index from being raised above one. An optical communication mode is an example of a TEM core defect mode which has a transversely polarized, dipole-like field pattern.

The dispersion relation of a surface mode on the other hand can cross the light line, exhibiting an effective index that can be greater than, equal to, or less than one. There is large overlap of the fields and surrounding material, which can act to slow the wave below the speed of light. Surface modes generally occur whenever a periodic lattice is terminated at a boundary. Surface modes are in fact lattice modes of the perfect structure with frequencies that have been sufficiently perturbed by the defect to lie in the band gap. Within the hollow defect region, surface modes do exhibit some properties of TM, TE, and TEM modes, but outside the defect their spatial pattern does not conform to that simple behavior, being greatly affected by the matrix symmetry. A TM surface defect mode with phase velocity near c is well suited for high-energy acceleration since it can remain in phase with a relativistic particle over arbitrarily long distances.

SYNCHRONOUS ACCELERATING MODES

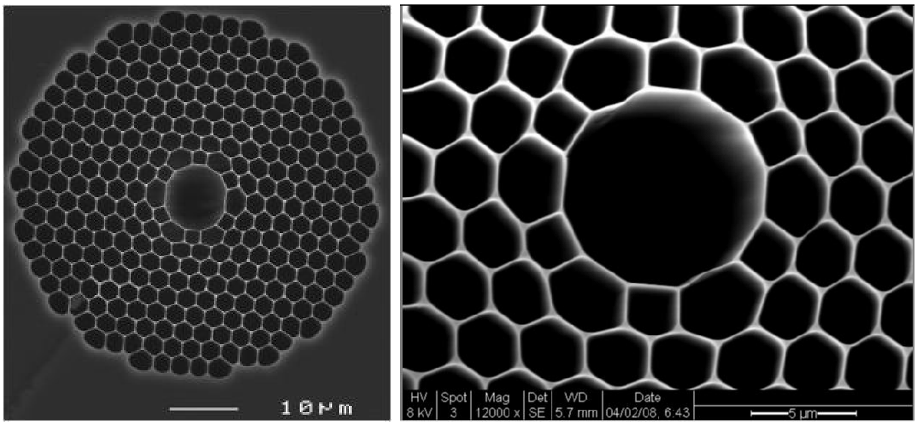
The three basic requirements for a travelling wave particle accelerator are a longitudinal electric field (field parallel to particle velocity vector), synchronization of particle and wave phase velocity, and confinement of field energy. The first two are needed to insure energy transfer to the particle over long distances. The final requirement is one of efficiency, insuring that input power overlaps strongly with the region where particle will absorb energy and be accelerated. As noted in the Introduction, a TM surface defect mode with phase velocity near c is suitable for relativistic particle acceleration since it can remain in phase with the particle over long distances. The dielectric structure must be designed to support such a confined mode with a high gradient G (longitudinal field in the defect) and a low power flow P. The accelerator figure of merit relating gradient to mode power is the so-called characteristic impedance, $Z_c = G^2\lambda^2/P$, where λ is the mode wavelength.

The Lin fiber lattice in Figure 1 is defined by four constants: the transverse hole period of the lattice a, which simply scales the wavelengths of the fiber modes, the ratio of hole radius to lattice period r/a, the ratio of central defect radius to lattice period R/a, and the relative permittivity of the dielectric matrix ε_r. For this fiber, the parameters are r/a = 0.35, R/a = 0.52, and $\varepsilon_r = 2.13$ for silica. The lattice period was chosen to be a = 1.31 µm to yield a speed-of-light (SOL) accelerating mode at 1 µm with the specified defect. The band diagram of a perfect lattice is determined only by the ratio r/a and the matrix permittivity. The Lin fiber is about half silica in volume, which results in the band diagram consisting of several well separated gaps at relatively small values of $\omega a/c$. The band gap diagram for the perfect lattice was calculated with the code R-Soft BandSolve [3]. This is an eigenmode solver using a plane-wave expansion technique with supercell periodic boundary conditions. As long as the supercell is large enough to reduce periodic mode overlap at the supercell boundaries, the code can also accurately calculate defect modes as though they were physically isolated in a single-defect lattice.

The SOL accelerating mode occurs at a normalized frequency of $\omega a/c = k_0 a = 8.2$ and resides in the second bandgap from the bottom in Figure 1. Its frequency within the gap is set primarily by the defect radius R, increasing linearly with larger radius. The ratio of the accelerating gradient G on axis to the maximum field E_{max} in the lattice is $G/E_{max} = 0.48$, which is very good and comparable to the ratio of gradient to peak field in copper radio-frequency linacs. But the breakdown fields in silica are much higher than copper structures so accelerating fields of about a GeV/m are achievable. The characteristic impedance of this mode is $Z_c \approx 18$ ohms and its group velocity is 0.58 c, being relatively low because of the large glass fraction.

FAST-WAVE ACCELERATING MODES

The recognition of synchronous accelerating modes as surface modes implies that the details of the surface boundary separating the defect from the surrounding matrix are the critical ingredients for optimizing the accelerator mode properties [7]. As a consequence, the designs of PBG fibers for telecom applications and particle acceleration actually have opposite goals. Telecom fibers are designed with few or no surface modes since these have higher diffractive losses, and by mixing with core modes due to perturbations, they can degrade the fiber's performance. A particle accelerator fiber is optimized to support a particular surface mode, and core modes are deleterious in that they may absorb input power. One does not expect industrially produced telecom fibers to normally support any useful accelerating modes, and we must design our fibers specifically for this new purpose.



(a) (b)

FIGURE 2. Scanning electrom microscope image of the HC-1060 hollow-core PBG fiber. The 10 micron scale is shown for reference in the left image (a) of the full matrix. The right image (b) shows the detail of the defect with the central hole being about 9.5 microns in diameter.

At SLAC, we are designing PBG fibers especially for particle acceleration and working with industry (Incom Inc., Charlton, MA) to fabricate these. But until these fibers are available, we can use telecom fibers to benchmark our codes and investigate the nature of fiber modes in laboratory experiments. Telecom fibers are not totally free of surface modes, and some of these can be TM modes. We have numerically modeled the HC-1060 fiber made by NKT Photonics A/S (formerly Crystal Fibre) [8] using R-Soft BandSolve. The HC-1060 fiber cross section is shown in Figure 2.

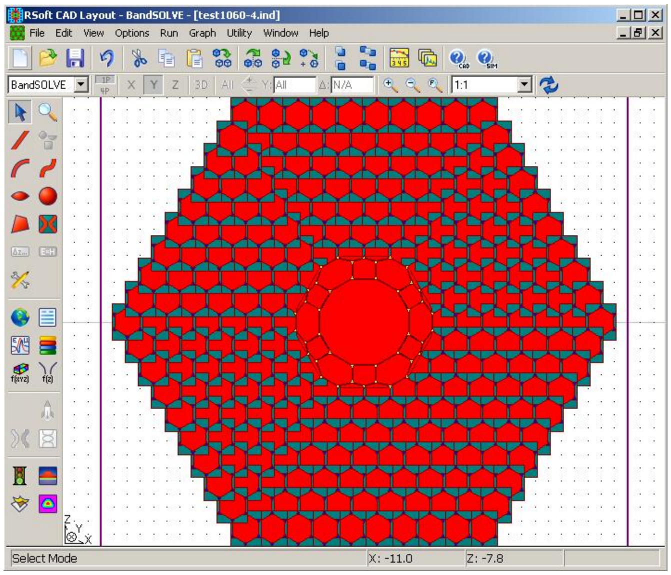
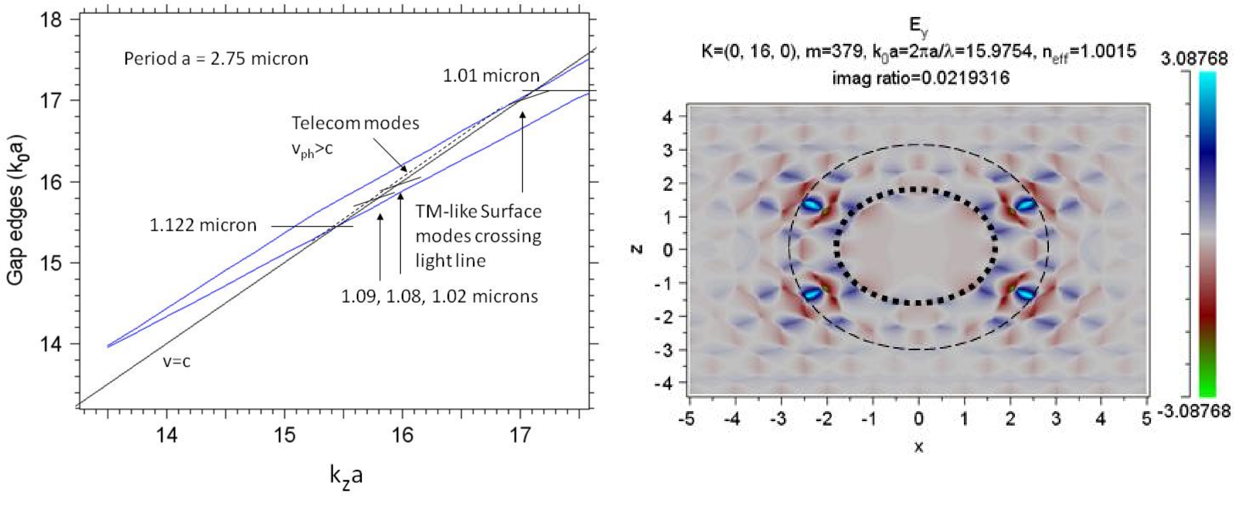


FIGURE 3. R-Soft CAD layout of the HC-1060 fiber. The hierarchal tiling tool was used to create the regular hexagonal lattice (period a= 2.75 micron), and the central defect region was modeled with silica veins and vertices following Figure 2.

The R-Soft CAD model of the HC-1060 fiber is shown in Figure 3. The lattice has a period of 2.75 microns along the horizontal axis, is more than 90% air by volume, and the hollow defect is about 9.5 microns in diameter. A relatively large aperture like this is good for transporting particle beams with low loss but only if the fiber supports a suitable accelerating mode, which is not guaranteed. The defect region is very complicated and includes deformed cells in the two layers surrounding the central hole. The band gap diagram is determined by the perfect lattice without defect. We find that the silica vein thickness sets the gap's central value of ωa/c. Thinner veins push the gap diagram to higher frequencies. The glass vertex size (the point where three veins meet) sets the band gap width. Larger vertices widen the band gap. We used the vendor's transmission band width (~110 nm) for the telecom mode as a reasonable approximation for the width of the band gap along the light line, since outside this range a defect mode will mix with lattice modes and suffer large diffractive losses.

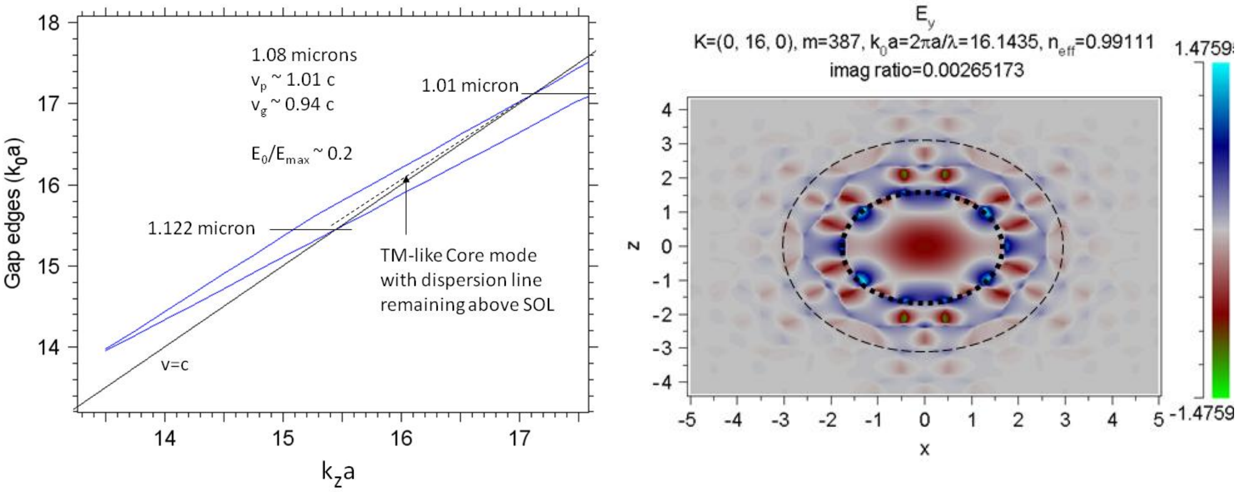


(a) (b)

FIGURE 4. Band gap diagram for the HC-1060 fiber and a TM surface mode at 1.08 micron wavelength as modeled by R-Soft BandSolve. In (a) the short lines indicate where the surface mode dispersion lines cross the light line, and the dashed line above the light line is the dispersion line of the telecom modes. In (b) the longitudinal field intensity of the 1.08 micron mode is shown with the heavy dotted ellipse marking the edge of the central hole, and the thin dashed ellipse enclosing the two layers of deformed cells surrounding the central hole visible in Figure 2.

In our simulation of the HC-1060 fiber, we found the expected telecom TEM core modes (phase velocity above light line) as well as both TM surface modes and TM core modes. The complicated nature of the defect region with deformed cells gives rise to very intricate field patterns for the surface and core modes, as shown in Figure 4. Several TM surface modes were found in the calculation. Telecom fibers are not designed to optimize any accelerating mode, and it is only by chance that these modes occur in this fiber. The surface mode with the highest accelerating gradient relative to peak field in the glass is at 1.08 micron wavelength. But even this mode would be poor for acceleration, having a ratio $G/E_{max} = 0.006$ and characteristic impedance 0.005 ohm. The field flatness is $\Delta E/G \approx 25\%$ in the central region $r < 0.5$ a, with the field tending to increase along the horizontal axis and decrease along the vertical axis. With so little glass in the fiber, the mode's group velocity v_g is relatively high at 0.81 c on the light line. This is a good attribute of modes in a highly porous fiber since the accelerated particles ($v \approx c$) can remain within the laser pulse τ_p and gain energy over a longer distance. The slip distance $c\tau_p/(1-v_g/c)$ is ideally set to the accelerator section length so that particles do not outrun the laser pulse within the accelerator.

In addition to the TM surface modes, a TM core defect mode was found in the BandSolve calculation for the HC-1060 fiber. This mode's dispersion line has a phase velocity above the light line over the entire band gap with a nearly constant group velocity of 0.94 c. Figure 5 illustrates the dispersion line and field intensity of this TM core mode. Note that the field pattern is much more concentrated toward the central region than for the TM surface mode in Figure 4. The on-axis accelerating field is much higher than the TM surface mode, with a ratio $G/E_{max} = 0.2$ at $k_0a = 16$ (1.08 microns), and the characteristic impedance is 2.5 ohms, or 500 times greater than the impedance of the 1.08 micron surface mode.



(a) (b)

FIGURE 5. Band gap diagram for the HC-1060 fiber and a TM core mode at 1.08 microns as modeled by R-Soft BandSolve. In (a) the dotted line above the light line denotes the dispersion line of this core mode, which runs over the entire band gap. In (b) the longitudinal field intensity of the 1.08 micron mode is shown with the heavy dotted ellipse marking the edge of the central hole, and the thin dashed ellipse enclosing the two layers of deformed cells surrounding the central hole visible in Figure 2.

In rf accelerator engineering, EM modes with $v_p > c$ are often called "fast waves" but in photonics these may be referred to as "superluminal" waves. The particular fast-wave TM core modes just above the light line which can be partially synchronous with relativistic particles we call by the special shorthand "tachyon" modes (from the Greek *takhus*, meaning "swift"). These fast modes can accelerate a relativistic particle over a limited distance before synchronism is lost. The mode's phase velocity at 1.08 microns is 1.01 c. In principle this core mode could remain synchronous with a relativistic particle and suffer only π phase shift over 50 wavelengths. At this point, the fields would have to be re-phased to continue the acceleration. The accelerator section length corresponding to π phase shift for a fast wave is equal to $0.5\lambda/(v_p/c-1)$. If this is also set to the laser pulse slip distance due to finite group velocity, then the laser pulse length $c\tau_p$ needs to be $0.5\lambda(1-v_g/c)/(v_p/c-1)$ for this mode. An interesting property of the tachyon mode is that with increasing frequency the phase velocity gets closer to c, the accelerating field decreases in strength, and the characteristic impedance is reduced. The mode tends to become more "transverse" with stronger fields perpendicular to the acceleration axis as it approaches the light line near the top of the band gap. At the bottom of the band gap, the longitudinal field increases, the phase velocity increases, and the characteristic impedance is close to that of the Lin mode.

SUMMARY

Although only TM surface defect modes can synchronously accelerate particles over arbitrarily long distances in PBG fibers, TM core modes with phase velocities just above the light line can provide acceleration over many tens to hundreds of wavelengths. This would require periodic re-phasing of the fields to maintain synchronism, but there is already a recognized need for such re-phasing between accelerator sections. The accelerating gradient tends to get weaker as the core mode's phase velocity approaches c, but in principle modifying the defect geometry can concentrate more field intensity in the core and also make the field more uniform, as was done in previous work on surface mode optimization [7]. For the HC-1060 fiber studied, the accelerating gradient of the TM core mode is more than ten times higher than for a surface mode. For large aperture fibers, the TM core mode may be preferred for particle acceleration if the field quality can be improved near the light line.

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