Introduction to Extra Dimensions

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Abstract. Extra dimensions provide a very useful tool in addressing a number of the fundamental problems faced by the Standard Model. The following provides a very basic introduction to this very broad subject area as given at the VIII School of the Gravitational and Mathematical Physics Division of the Mexican Physical Society in December 2009. Some prospects for extra dimensional searches at the 7 TeV LHC with $\sim$1 fb^{-1} of integrated luminosity are provided.

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INTRODUCTION: WHY STUDY EXTRA DIMENSIONS?

Most particle physicists agree that some form of New Physics (NP) must exist beyond the Standard Model(SM)–we simply don't know what it is yet. Though there are many prejudices based on preconceived ideas about the form this NP may take, it will be up to experiments at future colliders, such as the LHC and the ILC, to reveal its true nature. While we are all familiar with the list of these theoretical possibilities one must keep in mind that nature may prove to be more creative than we are and that something completely unexpected may be discovered. After all, we certainly have not yet explored more than a fraction of the theory landscape. With the turn on of the LHC at 7 TeV in a matter of days we may hope to finally get some answers to at least some of our questions.

One now much-discussed possibility is that extra spatial dimensions will begin to show themselves at or near the TeV scale. Only a dozen years or so ago not many of us would have thought this was even a remote possibility, yet the discovery of extra dimensions(EDs) would produce a fundamental change in how we view the universe. The study of the physics of TeV-scale EDs that has taken place over the past dozen years has its origins in the ground breaking work of Arkani-Hamed, Dimopoulos and Dvali(ADD)[1]. Since that time EDs has evolved from a single idea to a new general paradigm with many authors employing EDs as a tool to address the large number of outstanding issues that remain unanswerable within the SM context. This in turn has lead to other phenomenological implications which should be testable at colliders and elsewhere. A partial and (very) incomplete list of some of these ideas includes, *e.g.*,

- addressing the hierarchy problem[1, 2]
- producing electroweak symmetry breaking without a Higgs boson[3]
- the generation of the ordinary fermion and neutrino mass hierarchy, the CKM matrix and new sources of CP violation[4]

- TeV scale grand unification or unification without SUSY while suppressing proton decay[5]
- new Dark Matter candidates and a new cosmological perspective[6, 7]
- black hole production at future colliders as a window on quantum gravity[8]

This list hardly does justice to the wide range of issues that have been considered in the ED context. Clearly a discussion of all these ideas is beyond the scope of the present introduction and only some of them will be briefly considered in the text which follows. However it is clear from this list that ED ideas have found their way into essentially every area of interest in high energy physics which certainly makes them worthy of detailed study.

Of course for many the real reason to study extra dimensions is that they are fun to think about and almost always lead to some surprising and unanticipated results.

THINKING ABOUT EXTRA DIMENSIONS

Most analyses of EDs are within the context of quantum field theory. We might first ask if we can learn anything about EDs from purely 'classical' considerations and some general principles without going into the complexities of field theory. Consider a single massless particle moving in 5D 'Cartesian' co-ordinates and assume that 5D Lorentz invariance holds. Then the square of the 5D momentum for this particle is given by $p^2 = 0 = g_{AB}p^A p^B = p_0^2 - \mathbf{p}^2 \pm p_5^2$ where I employ $g_{AB} = diag(1, -1, -1, -1, \pm 1)$ as the 5D metric tensor (*i.e.*, defined by the invariant interval $ds^2 = g_{AB}dx^A dx^B$), p_0 can be identified as the usual particle energy, $\mathbf{p}^2$ is the square of the particle 3-momentum and p_5 is its momentum along the 5th dimension. (Note that here the indices A, B run over all 5D. We will sometimes denote the 5th dimension as x_5 and sometimes just as y.) The 'zero' in the equality above arises from the fact that the particle is assumed to be massless in 5D. Note that, a priori, we do not know the sign of the metric tensor for the 5th dimension but as we will now see that some basic physics dictates a preference; the choice of the +(-) sign corresponds to either a time- or space-like ED. We can re-write the equation above in a more traditional particle physics form as $p_0^2 - \mathbf{p}^2 = p_\mu p^\mu = \mp p_5^2$ and we recall, for all the familiar particles we know of which satisfy 4D Lorentz invariance, that $p_\mu p^\mu = m^2$, which is just the square of the particle mass. (Note that Greek indices will be assumed to run only over 4D here.) Notice that if we choose the sign for a time-like extra dimension that the corresponding sign of the square of the mass of the particle will appear to be *negative*, *i.e.*, the particle is a *tachyon*! Tachyons are well known to be very dangerous in most theories, even classically, as they can cause severe causality problems[9]–something we'd like to avoid in any theory–provided they interact with SM particles. This seems to imply that we should only pick the space-like solution. Generally, it turns out that to avoid tachyons appearing in our ED theory we must always choose EDs to be space-like and therefore we assume there will always be only one time dimension even though we could all use some extra time.[1].

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[1] See, however, [10] for a discussion of time-like EDs.

Now lets think about the simplest quantum field, *i.e.*, a real massless scalar field in a flat 5D-space (assuming a space-like ED!) which is a solution of the 5D Klein-Gordon equation: $(\partial_A \partial^A)\Phi = (\partial_\mu \partial^\mu - \partial_y^2)\Phi(x,y) = 0$, where y here represents the extra dimension. We can do a fast and dirty trick by performing something like separation of variables, *e.g.*, take $\Phi = \sum_n \chi_n(y)\phi_n(x)$ and plug it into the Klein-Gordon equation above giving us $\sum_n(\chi_n \partial_\mu \partial^\mu \phi_n - \phi_n \partial_y^2 \chi_n) = 0$. Now we note that if $\partial_y^2 \chi_n = -m_n^2 \chi_n$, we obtain a set of equations that appears like $\sum_n \chi_n(\partial_\mu \partial^\mu + m_n^2)\phi_n = 0$ which looks similar to an infinite set of equations for a collection of distinct 4D scalar fields ϕ_n with masses m_n. This collection of states with different masses is called a Kaluza-Klein(KK) tower. Note that we labeled the states by the set of integers (n) so that the levels are discrete; we could just as well have replaced the sum by an integral and treat n as a continuous variable. The difference between these two possibilities and the link to the nature of the 5D space with the associated boundary conditions will be made clear below by considering the action for the 5D scalar field.

Now we have pulled a bit of a fast one in performing this quick and dirty analysis so let us return and do a somewhat better job; we will still assume, however, that n is a distinct integer label for reasons to be clarified below. Let us start from the action (*i.e.*, the 5D volume integral of the Lagrangian) for the massless 5D scalar assuming y is constrained to an interval $y_1 \leq y \leq y_2$ with $y_{1,2}$ for now treated as arbitrary:

$$S = \int d^4x \int_{y_1}^{y_2} dy\, \frac{1}{2}\partial_A \Phi \partial^A \Phi. \tag{1}$$

Now we recall $\partial_A \Phi \partial^A \Phi = \partial_\mu \Phi \partial^\mu \Phi - \partial_y \Phi \partial_y \Phi$ and substitute this as well as the decomposition $\Phi = \sum_n \chi_n(y)\phi_n(x)$ as we did above. Then the integrand of the action becomes a double sum proportional to $\sum_{nm}[\chi_n \chi_m \partial_\mu \phi_n \partial^\mu \phi_m - \phi_n \phi_m \partial_y \chi_n \partial_y \chi_m]$. This appears to be a mess but we can 'diagonalize' this equation in a few steps. First if we choose to orthonormalize the χ_n such that

$$\int_{y_1}^{y_2} dy\, \chi_n \chi_m = \delta_{nm}, \tag{2}$$

then the kinetic term of the ϕ_n (the first one in the bracket above) reduces to a single sum after the y integration becoming simply $\sum_n \partial_\mu \phi_n \partial^\mu \phi_n$. This is essentially just sum of the kinetic terms for an infinite set of distinct 4D scalars. To handle the second term in the brackets we integrate by-parts and note that *if* we take the boundary conditions to be of the form

$$\chi_m \partial_y \chi_n |_{y_1}^{y_2} = 0, \tag{3}$$

and also require that the χ_n satisfy

$$\partial_y^2 \chi_n = -m_n^2 \chi_n, \tag{4}$$

as above, we can then integrate the entire action over y and obtain an effective 4D theory:

$$S = \int d^4x\, \frac{1}{2}\sum_n [\partial_\mu \phi_n \partial^\mu \phi_n - m_n^2 \phi_n^2], \tag{5}$$

which is just the (infinite) sum of the actions of the independent 4D scalars labeled by n with masses m_n, *i.e.*, the KK tower states. One sees that in this derivation it was important for the above boundary conditions(BCs) to hold in order to obtain this result. It is important to note that in fact the various masses that we observe in 4D correspond to (apparently) quantized values of 5D momentum p_y for the different ϕ_n.

The fields χ_n can be thought of as the wave functions of the various KK states in the 5th dimension and in this simple, flat 5D scenario are most generally simple harmonic functions: $\chi_n = A_n e^{im_n y} + B_n e^{-im_n y}$. What are the m_n? What are A_n and B_n? To say more we must discuss the BCs a bit further. In thinking about BCs in this kind of model it is good to recall one's experience with one-dimensional Quantum Mechanics(QM) that we all learned (too) many years ago.

First recall the Schrödinger Equation for a free particle moving along the Cartesian x direction. It has the same form as Eq.4 above and since the x direction is infinite in extent, *i.e.*, *noncompact*, the solution is just $\psi \sim A' e^{ipx} + B' e^{-ipx}$ where p is the particle momentum which can take on an infinite set of continuous eigenvalues. We say that in this case the momentum p is not quantized and this is due to the fact that the space is noncompact. Now let us consider a slightly different problem, a particle in a box, *i.e.*, a situation where the 'potential' is zero for $0 \leq y \leq \pi L$ but is infinite elsewhere so that the wavefunction vanishes outside this region confining the states to a finite interval. Since the physical region is of a finite size, *i.e.*, volume, this is called a *compact* dimension. We know that the solution inside the box takes the same general form as does the case of a free particle or χ_n above but it must also vanish at the boundaries. These BCs tell us A' and B' so that the solutions actually takes the unique form $\sim \sin ny/L$ and that the momenta are quantized, *i.e.*, $p = n/L$ with $n = 1, 2, ...$. Clearly these two situations are completely analogous to having a 5th dimension which is either infinite (*i.e.*, noncompact) or finite (*i.e.*, compact) in size. *Under almost all circumstances* we will assume that extra dimensions are compact in our discussions below. For a flat 5th dimension of length πL the analysis above tells us that the KK masses are just given by $m_n = n/L$, *i.e.*, the masses are clearly large if the size of the extra dimension is very small. Perhaps it is natural to think that the reason that we have not seen EDs is that they are very small and the corresponding KK states are then too massive to be produced at any of the existing colliders (except for the LHC!). In fact, the observation of KK excitations is the hallmark of EDs. It is interesting to observe that there are no solutions in the 'particle in a box' example corresponding to massless particles, *i.e.*, those with $n = 0$, the so-called zero modes.

There are other sorts of BCs that can be important. In introductory QM we also examine the case of a particle moving on a circle of radius R where we find that angular momentum is quantized. In the 5D case we can, by analogy, imagine that the 5th dimension is curled up into a circle, S^1–a one dimensional sphere of radius R–so that the points $y = -\pi R$ and $y = \pi R$ are identified as the same, *i.e.*, we then have periodic boundary conditions. Here the KK masses are found to be given by $m_n = n/R$ so that we still see the correlation between the KK masses and the inverse size of the extra dimensions but the solutions now will take the form of $A_n \cos ny/R + B_n \sin ny/R$ with $n = 0, 1, 2, ...$. Note that here a massless mode does exist due to the periodicity of the BCs. We can change this solution slightly by imagining defining a parity operation on the interval $-\pi R \leq y \leq \pi R$ which maps $y \rightarrow -y$. There are now 2 special points on

this interval, call the fixed points, which are left invariant by this discrete Z_2 operation when combined with the translation $y \rightarrow y + 2\pi R$ and the periodicity property; these are the points $y = 0, \pi R$. (Note that $\pm \pi R$ are already identified as the same point due to periodicity.) The eigenfunctions of our 'wave equation' must now respect the discrete, Z_2, parity symmetry so that our solutions can only be *either* Z_2-even, $\sim \cos ny/R$ or Z_2-odd, $\sim \sin ny/R$. Note that only Z_2-even states will have a zero-mode amongst them. This geometry is called S^1/Z_2 and is the simplest example of an *orbifold*, a manifold with a discrete symmetry that identifies different points in the manifold, here y and $-y$. The S^1/Z_2 orbifold is of particular interest in model building as we will see below. By the way, we note that all of the BCs of interest to us above are such as to satisfy the conditions following from Eq.3.

So far we have seen that a 5D scalar field decomposes into a tower of 4D scalars when going from the 5D to 4D framework. What about other 5D fields? In a way we have encountered a somewhat similar question before when we first learned Special Relativity, *i.e.*, how do 3-vectors and scalars get embedded into 4D fields? Just as a 4-vector contains a 3-vector and a 3-scalar, one finds that, *e.g.*, a 5D massless gauge field (which has 3 polarization states!) contains two KK towers, one corresponding to a 4D gauge field (with 2 polarization states) and the other to a 4D scalar field. In fact in (4+n)-dimensions a gauge field will decompose into a 4D gauge KK tower plus n distinct scalar towers.

At this point a subtlety exists. We know from our previous discussion that KK tower states above the zero mode are massive. How can there be massive 4D gauge fields with only 2 transverse polarization states? Consider for simplicity the 5D case. It turns out that due to gauge invariance (that we assume from the beginning) we can make a gauge transformation to eliminate the scalar KK tower fields and have them 'eaten' by the gauge field–in a manner similar to the Higgs-Goldstone mechanism–thus becoming their longitudinal components. In a way this is a geometric Goldstone mechanism. The scalar KK tower is then identified to be just the set of Goldstone bosons eaten by the gauge fields to acquire masses. Thus in the *unitary* or physical gauge the massless 5D gauge field becomes a massive tower of 4D gauge fields. In (4+n)-dimensions only one linear combination of the scalars is eaten so that a massless (4+n)-dimensional gauge field produces a massive tower of 4D gauge fields together with $n - 1$ scalar towers in the unitary gauge. Note, however, that the zero mode gauge field *does not* necessarily eat its corresponding scalar partner. This depends on the BC that are applied.

To see how this 5D decomposition for gauge fields works in practice consider a massless 5D gauge field with the 5th dimension compactified on S^1/Z_2. In the first step in the KK decomposition one can show that the 2-component vector KK's are Z_2-even with 5D wavefunctions like $\sim \cos ny/R$, whereas the KK scalars are Z_2-odd with wavefunctions like $\sim \sin ny/R$. Note the absence of an $n = 0$ scalar mode due to the Z_2 orbifold symmetry. Once we employ the KK version of the Higgs-Goldstone trick all the $n > 0$ gauge KK tower fields become massive 3-component gauge fields with $m_n = n/R$, having eaten their scalar partners. However, the zero mode remains massless, *i.e.*, there was no Goldstone boson for it to eat. In 4D the masslessness of the zero mode tells us that gauge invariance has not been broken. We see that orbifold BCs are useful at generating massless zero modes and maintaining gauge invariance. One can translate these orbifold BCs for the gauge fields into the simple relations $\partial_y A_n^\mu| = 0$, $A_n^5| = 0$ for

all n and where '|' stands for a boundary.

Interestingly the physics changes completely if we change the BCs in this case. Instead of an orbifold, consider compactifying on a line segment or interval, $0 \le y \le \pi R$, and taking as BCs $A_n^\mu = \partial_y A_n^5 = 0$ at $y = 0$ and $\partial_y A_n^\mu = A_n^5 = 0$ at $y = \pi R$. Here one now finds that $A^\mu \sim \sin m_n y / R$ and no massless zero mode exists. After the Higgs-Goldstone trick *all* the 4D gauge KK tower fields become massive (leaving no remaining scalars) with $m_n = (n + 1/2)/R$ thus implying that gauge invariance is now broken. Here we see a simple example that demonstrates that we can use BCs to break gauge symmetries. The possibility that such techniques can be successfully employed to break the symmetries of the SM without the introduction of fundamental Higgs fields has been quite extensively discussed in the literature[3].

How do other higher dimensional fields correspondingly decompose? As the simplest example consider the gravitational field in 5D, represented as by symmetric tensor h^{AB} which decomposes as $h^{AB} \to (h^{\mu\nu}, h^{\mu 5}, h^{55})_n$ when going to 4D; as before we'll use n as a KK tower index. If we compactify on S^1/Z_2 to keep the zero mode $h_0^{\mu\nu}$ massless (and which we will identify as the ordinary graviton of General Relativity) then for $n > 0$ all of the $h_n^{\mu 5}$ and h_n^{55} fields get eaten to generate a massive KK graviton tower with fields that have 5 polarization states (as they should). For $n = 0$ there is no $h_0^{\mu 5}$ to eat due to the orbifold symmetry yet a massless h^{55} scalar remains in addition to the massless graviton. This field is the *radion*, which corresponds to an fluctuation of the size of the extra dimension, and whose vacuum value must be stabilized by other new physics to keep radius the extra dimension stable[11] against perturbations. The physical spectrum is then just the radion and the graviton KK tower in this case which, as we will see below, corresponds to what happens in the RS model.

In terms of manifolds on which to compactify higher dimensional fields it is easy to imagine that as we go to higher and higher dimensions the types of manifolds and the complexity of the possible orbifold symmetries grows rapidly. Typical manifolds that are most commonly considered are torii, T^n, which are simply products of circles, and spheres, S^n.

Let us now turn to two important representative ED models.

LARGE EXTRA DIMENSIONS

The Large Extra Dimensions scenario of Arkani-Hamed, Dimopoulos and Dvali(ADD)[1] was proposed as a potential solution to the hierarchy problem, *i.e.*, the question of why the (reduced) Planck scale, $\overline{M}_{Pl} \simeq 2.4 \cdot 10^{18}$ GeV, is so much larger than the weak scale ~ 1 TeV. ADD propose that we (and all other SM particles!) live on an assumed to be rigid 4D hypersurface (sometimes called a wall or brane). On the other hand gravity is allowed to propagate in a (4+n)-dimensional 'bulk' which is, *e.g.*, an $n-$torus, T^n. This 4D brane is conveniently located at the origin in the EDs, *i.e.*, **y**=0. Gauss' Law then tells us that the Planck scale we measure in 4D, $\overline{M}_{Pl}$, is related to the (4+n)-dimensional fundamental scale, M_*, that appears in the higher dimensional General Relativistic action, via the relation

$$\overline{M}_{Pl}^2 = V_n M_*^{n+2} \,, \tag{6}$$

where V_n is the volume of the n-dimensional compactified space. M_* (sometimes denoted as M_D in the literature up to an O(1) factor) can be thought of as the *true* Planck scale since it appears in the higher dimensional action which is assumed to describe 'ordinary' General Relativity but extended to (4+n)-dimensions. We then can ask if is possible that M_* could be as small as $\sim$ a few TeV thus essentially 'eliminating' the hierarchy problem? Could we have been fooled in our extrapolation of the behavior of gravity from what we know up to the TeV scale and beyond and that gravity, becomes strong at M_* and not at $\overline{M}_{Pl}$? To get an idea whether this unusually sounding scenario can work at all we need to get some idea about the size of V_n, the volume of the compactified space. As a simple example, and the one most often considered in the literature, imagine that this space is a torus T^n all of whose radii are equal to R. Then it is easy to see that $V_n = (2\pi R)^n$; knowing the value of $\overline{M}_{Pl}$ and assuming $M_* \sim$ a few TeV we can estimate the value of R. Before we do this, however, we need to think about how gravity behaves in EDs.

FIGURE 1. Regions in the $\alpha - \lambda$ plane excluded by table top searches for deviations from Newtonian gravity from Adelberger *et al.*[12]. The ADD prediction with $n = 2$ and $M_* = 1$ TeV is also shown.

If one considers two masses separated by a distance r in $(n+4)$-dimensions the force of attraction will now depend on the relative magnitudes of r and the compactification radius R. To see this first imagine $r >> R$; in this case the extra dimensions are essentially invisible and to all appearances the space looks to be 4D. Then we know that $F_{grav} \sim 1/r^2$ thanks to Newton. However, in the opposite limit $r << R$ the effects of being in a full (4+n)-dimensional space will become obvious; at such small distances

we don't even realize that the ED is compactified. Either via Gauss' Law or by recalling the nature of the solution to the inhomogeneous Laplace's equation in EDs one finds that now $F_{grav} \sim 1/r^{2+n}$. Clearly one will start to see significant deviations from conventional Newtonian gravity once $r \sim R$ so that R cannot be very large. Let's assume $n = 1$; then we can solve the equation above and obtain $R \sim 10^8$ m. This is a scale of order the Earth-Moon distance over which we know Newton's Law holds very well; thus $n = 1$ is excluded. Fortunately for us the size of R decreases rapidly as n increases; amazingly, for $n = 2$ one obtains $R \sim 100\mu$m which is close to the limit of current table top experimental searches[12] for deviations from Newton's Law of Gravity. These are summarized in Fig. 1 from the work of Adelberger *et al.*[12]. Note that these deviations from Newtonian gravity are conventionally parameterized by adding a Yukawa-type interaction of relative strength α and scale length λ to the usual Newtonian potential. In the figure the deviations expected in the $n = 2$ scenario are shown assuming $M_* = 1$ TeV; the bounds from the data tell us that $M_* >$ a few TeV in this case.

If n is further increased R becomes much too small to probe for direct deviations from the $1/r^2$ Newton's Law. It is interesting to note that for $n = 2$, which is already being constrained by table top measurements, $R^{-1} \sim 10^{-4}$ eV telling us that we have not probed gravity directly beyond energy scales of this magnitude. This ignorance is rather amazing but it is what allows the large parameter space in which the ADD model successfully functions. From this discussion it appears that the ADD model will 'work' so long as $n \geq 2$ with $n = 2$ being somewhat close to the boundary of the excluded regime. It turns out that the naive $n = 2$ case is somewhat disfavored by other measurements though larger values of n are much more weakly constrained. How large can n be? If we believe in superstring theory at high scales then we can expect that $n \leq 6$ or 7. A priori, however, there is no reason not to consider larger values in a bottom-up approach. It is curious to note that when $n \simeq 30$ one has $M_*R \sim 1$ which is perhaps what we might expect based on naturalness assumptions; for any $n \sim 15$ or less, $R^{-1} << M_* \sim$ a few TeV. This point is important for several reasons. (*i*) One may ask why we required that the SM fields remain on the brane. If the SM or any part thereof were in the bulk, those fields would have KK towers associated with them. Since the masses of these KK fields would be of order $\sim 1/R$ as discussed above *and* we have not experimentally observed any KKs of the SM particles at any collider so far, we must have $1/R \geq 100$ GeV or so. For any $n \leq 10$ it is clear that this condition cannot be satisfied. So if we believe in strings the SM fields must remain on the wall. (*ii*) Since the gravitons *are* bulk fields their KK masses are given by $m_{KK}^2 = \sum_{i=1}^{n} l_i^2/R^2$ where l_i is a integer labeling the KK momenta for the ith ED. As we noted above, for not too large values of n these masses are generally very small compared to the 1 TeV or even 100 GeV scale. This will have important phenomenological implications below.

If $n = 6$ or 7 in string theory why don't we just assume this value when discussing the ADD model? Consider a small variation on the above theme. So far we have assumed that all of the ED compactification radii are equal; this need not be the case, of course. Assume there are n EDs but let $n - p$ of them have radius R_1 and the remaining p of them radius R_2. Then from the discussion above we must have

$$\overline{M}_{Pl}^2 = (2\pi)^n R_1^{(n-p)} R_2^p M_*^{(n+2)} \ . \tag{7}$$

Now imagine that $R_2^{-1} \sim M_*$; then we'd have instead

$$\overline{M}_{Pl}^2 \sim (2\pi)^n R_1^{(n-p)} M_*^{(n-p)+2} , \qquad (8)$$

i.e., it would *appear* that we really only have $n - p$ large dimensions. The keyword here is 'large'; the p dimensions are actually 'small' of order $\sim \text{TeV}^{-1}$ and not far from the fundamental scale in size. Thus there could be, *e.g.*, 7 EDs as suggested by strings but only 4 of them are large. If any SM field lived *only* in these p small EDs that would be (at least superficially) experimentally acceptable since their KK masses would be $\sim \text{TeV}$ and as of yet these KKs would be unobserved at colliders. Since the SM fields can live in these TeV size EDs this possibility is quite popular[18] for model building purposes.

To proceed further we need to know what these KK graviton states do, *i.e.*, how they interact with the usual SM fields confined on the wall. A derivation of the Feynman rules for the ADD model is beyond the scope of the present talk but can be found in Ref.[13] with some elementary applications discussed in Ref.[14]. A glance at the Feynman rules tells us several things: (*i*) all of the states in the graviton KK tower couple to SM matter on the wall with the same strength as does the ordinary zero-mode graviton, *i.e.*, to lowest order in the coupling

$$\mathcal{L} = -\frac{1}{\overline{M}_{Pl}} \sum_n G_n^{\mu\nu} T_{\mu\nu} , \qquad (9)$$

where $G_n^{\mu\nu}$ are the KK graviton fields in the unitary gauge and $T_{\mu\nu}$ is the stress-energy tensor of the SM wall fields. (*ii*) Since there are at least 2 EDs we might expect that the vector fields (or some remaining combination of them) $G_n^{\mu i}$, where $i = 1, ..., n$, would couple to the SM particles. It turns out that such couplings are absent by symmetry arguments since the SM fields reside at **y**=0. (*iii*) Similarly we might expect that some combination of the scalar fields G_n^{ij} to couple to the SM. Here in fact one KK tower of scalars does couple to $\sim T_\mu^\mu / \overline{M}_{Pl}$. However, since $T_\mu^\mu = 0$ for massless particles (except for anomalies) this coupling is rather small for most SM fields except for top quarks and massive gauge bosons. Thus, under most circumstances, these scalar contributions to various processes are rather small. (*iv*) Though each of the $G_n^{\mu\nu}$ are rather weakly coupled there are a *lot* of them and their density of states is closely packed compared to the TeV scale. This is very important when performing sums over the graviton KK tower as we will see below.

How would ADD EDs appear at colliders? Essentially, there are two important signatures for ADD EDs and there has been an enormous amount of work on the phenomenology of the ADD model in the literature [For a review see [16]]. The first signature is the emission of graviton KK tower states during the collision of two SM particles. Consider, *e.g.*, either the collision of $q\bar{q}$ to make a gluon or an e^+e^- pair to make a photon and during either process have the SM fields emit a tower KK graviton states. Note that since each of the graviton KKs is very weakly coupled this cross section is quite small for any given KK state. Also, once emitted, the graviton interacts so weakly it will not scatter or decay in the detector and will thus appear only as missing energy or transverse momentum. Now apart from their individual masses all graviton KK states will yield the same cross section as far as this final state is concerned, *i.e.*, a jet or photon plus missing energy; thus we should sum up all the contributions of the KK states that are kinematically

accessible. For example, at an e^+e^- collider this means we sum over the contributions of all KK states with masses less than $\sqrt{s}$. This is a *lot* of states and, since these states are closely packed, we can replace the usual the sum with an integral over the appropriate density of states. While *no one individual* KK graviton state yields a large cross section the resulting sum over so many KKs does yield a potentially large rate for either of the processes above. These resulting rates then only depend on the values of n and M_*. Both the Tevatron and LEPII have looked for such signatures with no luck and have placed bounds on the ADD model parameters[16]. Clearly it is up to searches at future colliders such as the LHC to find these signals if they exist.

Fig. 2 from Vacavant and Hinchliffe[19] shows the missing E_T spectrum at the 14 TeV LHC assuming an integrated luminosity of 100 fb^{-1} for the process $pp \rightarrow$ jet plus missing energy in the SM and the excess induced by ADD graviton emission assuming different values of $n = \delta$ and $M_* = M_D$. Once the rather large SM backgrounds are well understood this excess will be clearly visible. The more difficult question to address is whether such an excess if observed at the LHC would naturally be interpreted as arising from ADD EDs as other new physics can lead to the same apparent final state. Clearly the LHC will not be running at these energies or luminosities for some time. At the initial 7 TeV energy with a $\sim 1 fb^{-1}$ size integrated luminosity a substantial extension beyond the limits obtained from the Tevatron are certainly to be expected once the SM backgrounds become well understood. This can be seen in Fig. 3.
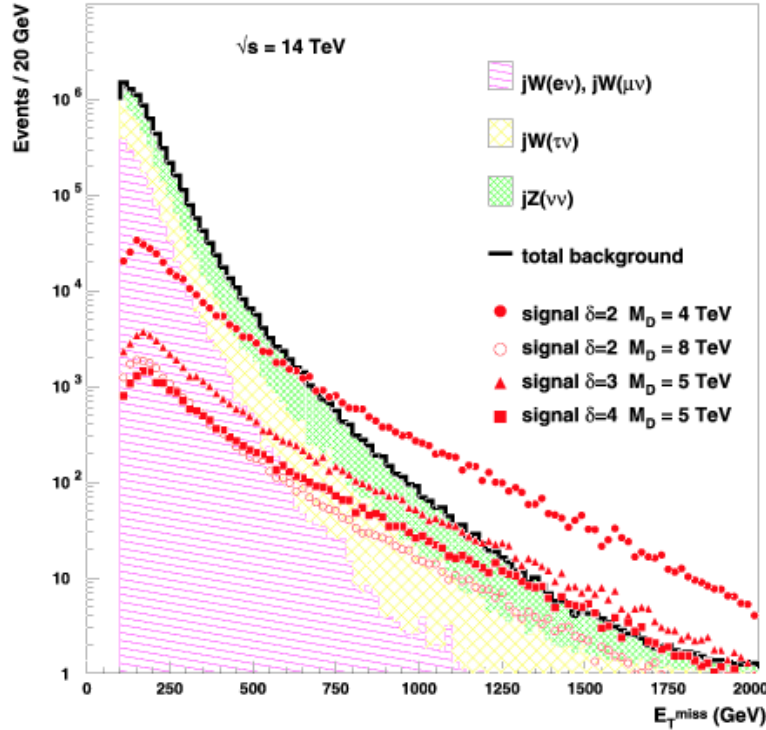
FIGURE 2. Missing transverse energy spectrum for the monojet plus missing E_T signature at the LHC assuming an integrated luminosity of 100 fb^{-1} from Ref[19]. Both the SM backgrounds and the signal excesses from graviton emission in the ADD model are shown. Here $M_D = M_*$ and $\delta = n$.

At the 0.5-1 TeV ILC, the backgrounds for the photon plus missing energy process are
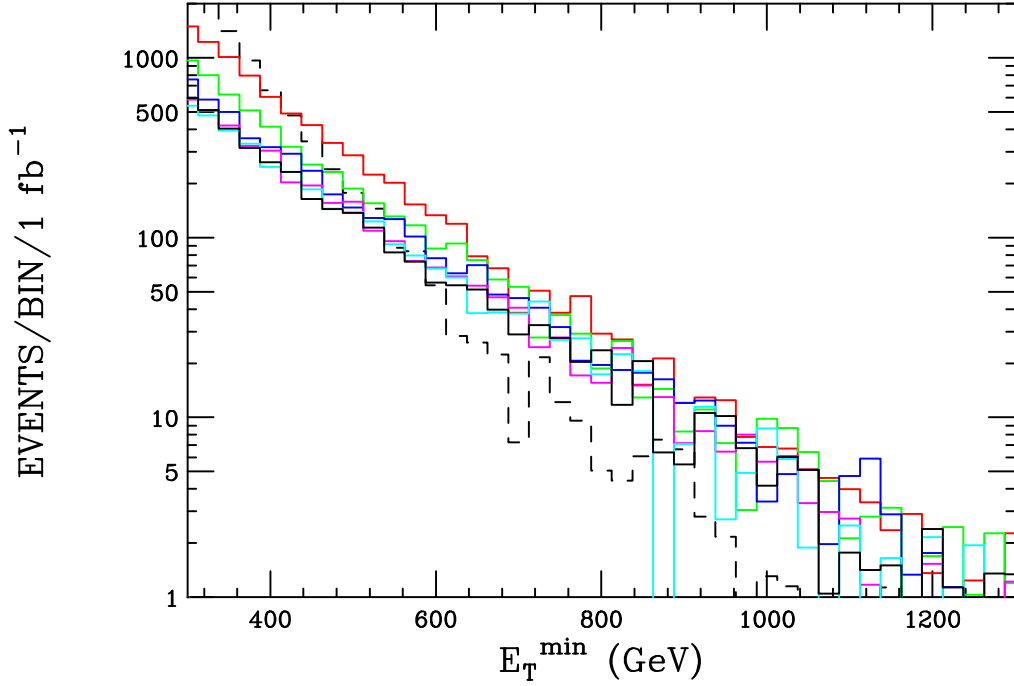
FIGURE 3. Same as the previous figure but now for the 7 TeV LHC assuming an integrated luminosity of 1 fb^{-1} with the SM background as the dotted histogram. ADD expectations are for $n = 2, 3, ..$ from top to bottom and assuming $M_D = 2$ TeV.

far simpler and better understood, essentially arising from the $\nu\bar{\nu} + \gamma$ final state. These backgrounds can be measured directly by modifying the electron (and positron) beam polarization(s) since the W^+W^- intermediate state gives the dominant contribution to this process. Measuring the excess event cross section at two different center of mass energies allows us to determine both M_* and $n = \delta$ as shown in Fig. 4 from the TESLA TDR[20]. If fitting the data taken at different center of mass energies results in a poor χ^2 using these parameters we will know that the photon plus missing energy excess is due to some other new physics source and not to ADD EDs.

These is another way to see, at least indirectly, the effect of graviton KKs in the ADD model: gravitons can be exchanged between colliding SM particles. This means that processes such as $q\bar{q} \rightarrow gg$ or $e^+e^- \rightarrow \mu^+\mu^-$ can proceed through graviton KK tower exchange as well as through the usual SM fields. As before, the amplitude for one KK intermediate state is quite tiny but we must again sum over all their exchanges (of which there are very many) thus obtaining a potentially large result. Unlike the case of graviton emission where the KK sum was cut off by the kinematics here there is no obvious cutoff and, in principle, the KK sum should include all the tower states. Furthermore, note that here the KK sum occurs at the amplitude level, *i.e.*, it is a coherent sum. One problem with this is that this KK sum is divergent once $n > 1$ as is the case here. (In fact the sum is log divergent for $n = 2$ and power law divergent for larger n.) The conventional approach to this problem is to remember that once we pass the mass scale $\sim M_*$ the gravitons in the ADD model become strongly coupled and we can no longer rely on
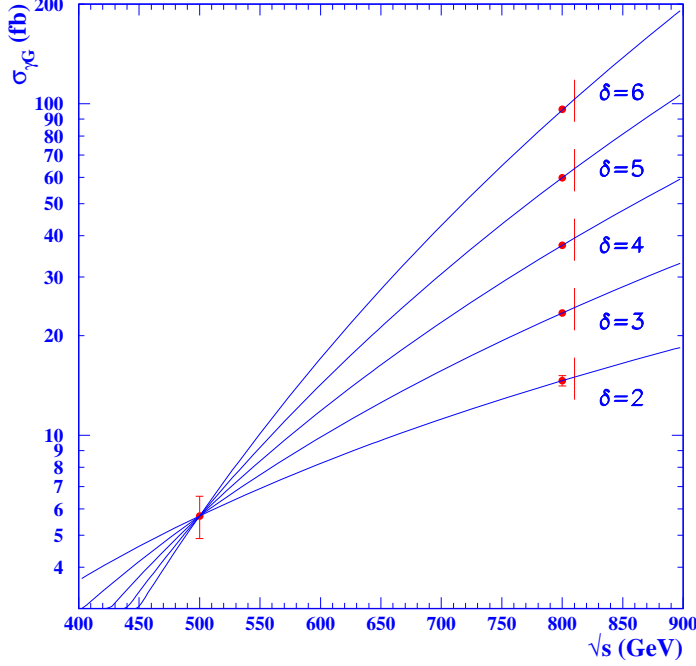
FIGURE 4. Signal cross section for the γ plus missing energy final state at the ILC in the ADD model as a function of $\sqrt{s}$ for various $\delta = n$ from Ref.[20] normalized to a common value at $\sqrt{s} = 500$ GeV. Combining measurements at two distinct values of $\sqrt{s}$ one can extract both the values of n and M_* for the ADD model.

perturbation theory so perhaps we should cut off the sum near M_* since the theory is not well-defined above that scale. There are several ways to implement this or even circumvent this entire problem[21] described in the literature[13, 14]. In all cases the effect of graviton exchange is to produce a set of dimension-8 operators containing SM fields, *e.g.*, in the notation of Hewett[14]

$$\mathscr{L} = \frac{4\lambda}{\Lambda_H^4} T_{\mu\nu}^i T_f^{\mu\nu}, \tag{10}$$

where $\Lambda_H \sim M_*$ is the cutoff scale, $\lambda = \pm 1$ and $T_{i,f}^{\mu\nu}$ are the stress energy tensors for the SM fields in the initial and final state, respectively. This is just a contact interaction albeit of dimension-8 and with an unconventional tensor structure owing to the spin-2 nature of the gravitons being exchanged. Graviton exchange contributions to SM processes can lead to substantial deviations from conventional expectations; Fig 5 shows the effects of graviton KK exchange on the process $e^+e^- \to b\bar{b}$ at the ILC. Note that the differential cross section as well as the left-right polarization asymmetry, A_{LR}, are both altered from the usual SM predictions.

Fig. 6 shows the corresponding expectations for these ADD-induced contact interactions at the 7 TeV LHC assuming an integrated luminosity of 1 fb^{-1} in the Drell-
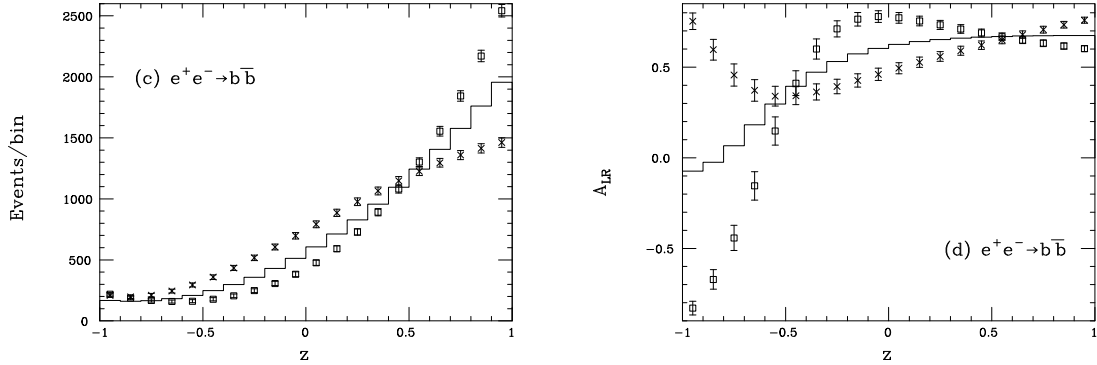
FIGURE 5. Deviations in the process $e^+e^- \to b\bar{b}$ at the ILC due to graviton KK tower exchange in the ADD model from Hewett[14]. The left panel is the angular distribution while the right panel is the left-right polarization asymmetry. Here $\sqrt{s} = 500$ GeV and $\Lambda_H = 1.5$ TeV. The histograms are the SM predictions while the 'data' points are for the ADD model with $\lambda = \pm 1$. An integrated luminosity of 75 fb^{-1} has been assumed.

Yan channel. The present limit from the Tevatron and LEPII are roughly given by $\Lambda_H \sim 1 - 1.5$ TeV.

Can the effects of graviton exchange be uniquely identified, *i.e.*, separated from other new physics which induces contact interaction-like effects, such as Z' exchange? This has been addressed by several groups of authors[15]. For example, by taking moments of the $e^+e^- \to f\bar{f}, W^+W^-$ angular distributions and employing polarized beams it is possible to uniquely identify the spin-2 nature of the graviton KK exchange up to ~ 6 TeV at a $\sqrt{s} = 1$ TeV ILC with an integrated luminosity of 1 ab^{-1}. This is about half of the discovery reach at the ILC for ADD EDs: $\Lambda_H \simeq 10 - 11\sqrt{s}$ for reasonable luminosities. If both beams could be polarized this could be improved somewhat by also employing transverse polarization asymmetries.

It is possible to constrain the ADD model in other ways, *e.g.*, the emission of ADD KK gravitons can be constrained by astrophysical processes as reviewed in Ref.[16]. These essentially disfavor values of M_* less than several hundred TeV for $n = 2$ but yield significantly weaker bounds as n increases.

Before turning to a different model let us briefly discuss the dirty little secret of the ADD model. The purpose of this model was to eliminate the hierarchy problem, *i.e.*, remove the large ratio between the weak scale and the true fundamental scale, hence the requirement that $M_* \sim$ a few TeV. However, if we look carefully we see that this large ratio has been eliminated in terms of another large ratio, *i.e.*, $RM_* \sim (\overline{M}_{Pl}^2/M_*^2)^{1/n}$, which for smallish n is a very large number–as large as the hierarchy we wanted to avoid. Thus we see that ADD really only trades one large ratio for another and does not really eliminate the hierarchy problem. The next model we will discuss does a much better job in that regard.
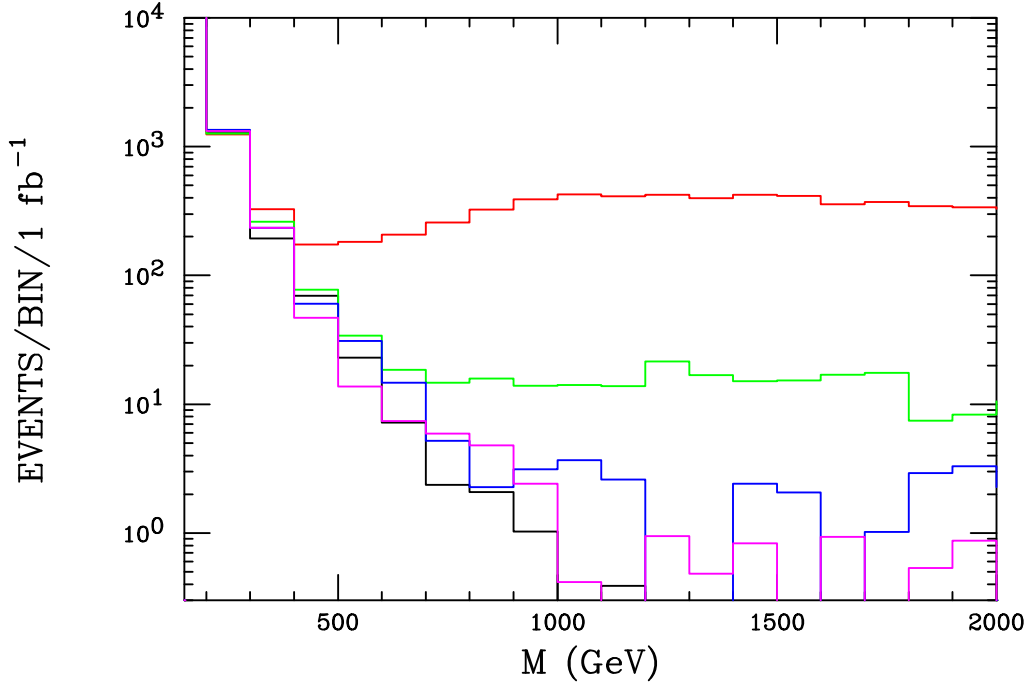
FIGURE 6. Deviations from the SM (solid) expectations for the invariant mass distribution of the dilepton pair for the Drell-Yan process, $pp \rightarrow e^+ e^- + X$, at the 7 TeV LHC assuming an integrated luminosity of 1 fb^{-1}. The colored histograms, from top to bottom, correspond to $\Lambda_H = 1$, 1.5, 2 and 2.5 TeV, respectively.

WARPED EXTRA DIMENSIONS

The Warped Extra Dimensions scenario was created by Randall and Sundrum(RS)[2] and is quite different and more flexible than the ADD model. The RS model assumes the existence of only one ED which is compactified on the now-familiar S^1/Z_2 orbifold discussed above. In this setup there are two branes, one at $y = 0$ (called the Planck brane) while the other is at $y = \pi r_c$ (called the TeV or SM brane) which are the two orbifold fixed points. What makes this model special is the metric:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \,, \tag{11}$$

where $\eta_{\mu\nu} = diag(1, -1, -1, -1)$ is the usual Minkowski metric and $\sigma(y)$ is some a priori unknown function. This type of geometry is called 'non-factorizable' because the metric of the 4D subspace is $y-$dependent. In the simplest version of the RS model (*i.e.*, the original RS I) it is assumed, like in the ADD case, that the SM fields live on the so-called TeV brane while gravity lives everywhere. Unlike in the ADD case, however, there is a 'cosmological' constant in the 5D bulk and both branes have distinct tensions. [2] Solving the 5D Einstein's equations provides a unique solution for these quantities and

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[2] More complex versions of this model are possible with SM bulk matter fields but we will limit our discussion here to this more simple case.

also determines that $\sigma = k|y|$, where k is a dimensionful parameter. A basic assumption of this model is that there are no large mass hierarchies present so that very roughly we expect that $k \sim M_*$, the 5D fundamental or Planck scale. In fact, once we solve Einstein's Equations and plug the solutions back into the original action and integrate over y we find that

$$\overline{M}_{Pl}^2 = \frac{M_*^3}{k}\left(1 - e^{-2\pi k r_c}\right). \tag{12}$$

As we will see below the *warp factor* $e^{-\pi k r_c}$ will be a very small quantity which implies that $\overline{M}_{Pl}, M_*$ and k have essentially comparable magnitudes following from the assumption that no hierarchies exist. If we calculate the Ricci curvature invariant for the 5D bulk space we find it is a constant, *i.e.*, $R_5 = -20k^2$ and thus k is a measure of the constant curvature of this space. A space with constant negative curvature is called an Anti-DeSitter space and so this 5D version is called AdS_5. Due to the presence of the exponential warp factor this space is also called a *warped* space. On the other hand, a space with a constant metric is called 'flat'; the ADD model is an example of flat EDs when compactified on T^n as has been assumed here. (The T^n spaces are flat since one can make a conformal transformation to a metric with only constant coefficients.)

Before going further we note that if the scale of curvature is too small, *e.g.*, if the inverse radius of curvature becomes larger than the 5D Planck scale, then higher curvature/quantum gravity effects can dominate our discussion and the whole RS scenario may break down since we are studying the model in its 'classical', *i.e.*, non-quantum limit. This essentially means that we must require $|R_5| \leq M_*^2$ which further implies a bound that $k/\overline{M}_{Pl} \leq 0.1$ or so, which is not much of a hierarchy. It is, of course, possible to formulate a version of the RS model including higher curvature terms where this assumption can be dropped.

Now for the magic of the RS model. In fitting in with the RS philosophy it will be assumed that all dimensionful parameters in the action will have their mass scale set by $M_* \sim \overline{M}_{Pl} \sim k$ so that there is no fine-tuning. However, the warp factor rescales them as one moves about in y so that, in particular, all masses will appear to be of order the TeV scale on the SM brane, *i.e.*, to us. This means that if there is some mass parameter, m, in the action which is order $\overline{M}_{Pl}$, we on the TeV brane will measure it to be reduced by the warp factor, *i.e.*, $me^{-\pi k r_c}$. Note that if $k r_c \sim 11$ (adain, a small hierarchy) this exponential suppression reduces a mass of order 10^{18} GeV to only 1 TeV. Thus the ratio of the weak scale to $\overline{M}_{Pl}$ is explained through an exponential factor and no large ratios appear anywhere else in the model. It has been shown by Goldberger and Wise[11] that values of $k r_c \sim 11$ are indeed natural and can be provided by a stable configuration. Hence we have obtained a true solution to the hierarchy problem.

How does this 'warping' really work? Let's discuss a simple example by considering the action for the SM Higgs field on the TeV brane:

$$S = \int d^4x dy \, \sqrt{-g}\left(g^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - \lambda\left(H^2 - v_0^2\right)^2\right)\delta(y - \pi r_c), \tag{13}$$

where g is the determinant of the metric tensor, λ is the usual quartic coupling and v_0 is the Higgs vev, which, keeping with the RS philosophy, is assumed to be of order $\overline{M}_{Pl}$ and *not* at the TeV scale. Now $\sqrt{-g} = e^{-4k|y|}$ and $g^{\mu\nu} = e^{2k|y|}\eta^{\mu\nu}$ so that we can trivially
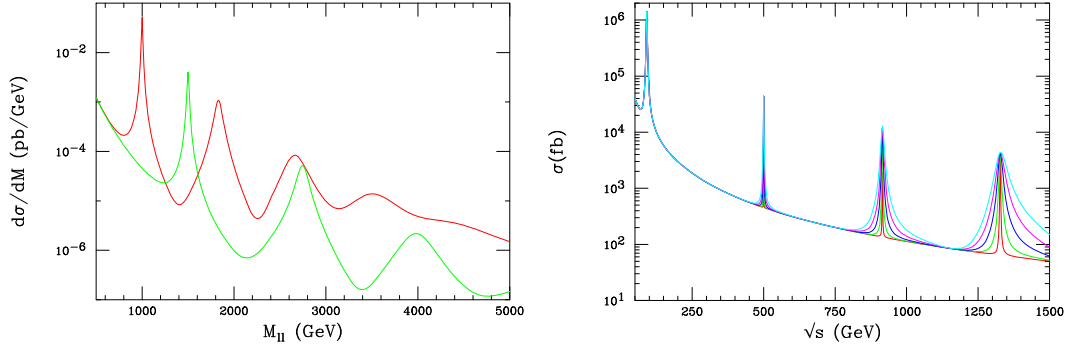
FIGURE 7. Graviton resonance production in Drell-Yan at the LHC(top) and at the ILC in $\mu^+\mu^-$ in the original RS I model from Ref[23]. The different curves correspond to various choices of $k/\overline{M}_{Pl}$ and m_1 as described in the text. The ever widening resonances correspond to increasing the value of $k/\overline{M}_{Pl}$.

integrate over y due to the delta-function. This yields

$$S = \int d^4x \left(e^{-2\pi kr_c} \eta^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - \lambda e^{-4\pi kr_c} (H^2 - v_0^2)^2 \right). \tag{14}$$

Now to get a canonically normalized Higgs field (one with no extra constants in front of the kinetic term) we rescale the field by letting $H \to e^{\pi kr_c} h$ which now gives us

$$S = \int d^4x \left(\partial^\mu h^\dagger \partial_\mu h - \lambda (h^2 - v_0^2 e^{-2\pi kr_c})^2 \right), \tag{15}$$

where we have contracted the indices using $\eta^{\mu\nu}$. Here we see that the vev that we observe on the SM brane is *not* v_0 but the warped down quantity $v_0 e^{-\pi kr_c}$ which is of order the TeV scale. Thus, in the end, the Higgs gets a TeV scale vev even though the parameters we started with in the action are all of order $\overline{M}_{Pl}$! The Hierarchy Problem is solved. This warping effect is a general result of the RS model.

This result can be a curse as well as a blessing. Since this warping effects all scales and no scale is larger than M_* it is difficult to suppress dangerous higher dimensional operators that can induce proton decay or flavor changing neutral currents in the original RS model. This motivates taking the SM fermions and gauge fields into the bulk while keeping the Higgs on or near the TeV brane. A discussion of such possibilities is, however, beyond the scope of the current introductory notes.

What do the KK gravitons look like in this model? Even though gravitons are spin-2 it turns out that their masses and wave functions are identical to the case of a scalar field in the RS bulk[22] which is far simpler to analyze. Let us return to the Klein-Gordon equation above but now in the case of curved space; one obtains

$$(\sqrt{-g})^{-1} \partial_A \left(\sqrt{-g} g^{AB} \partial_B \Phi \right) = 0. \tag{16}$$

'Separation of variables' via the KK decomposition then yields

$$-e^{2ky} \partial_y \left(e^{-2ky} \partial_y \chi_n \right) = m_n^2 \chi_n, \tag{17}$$
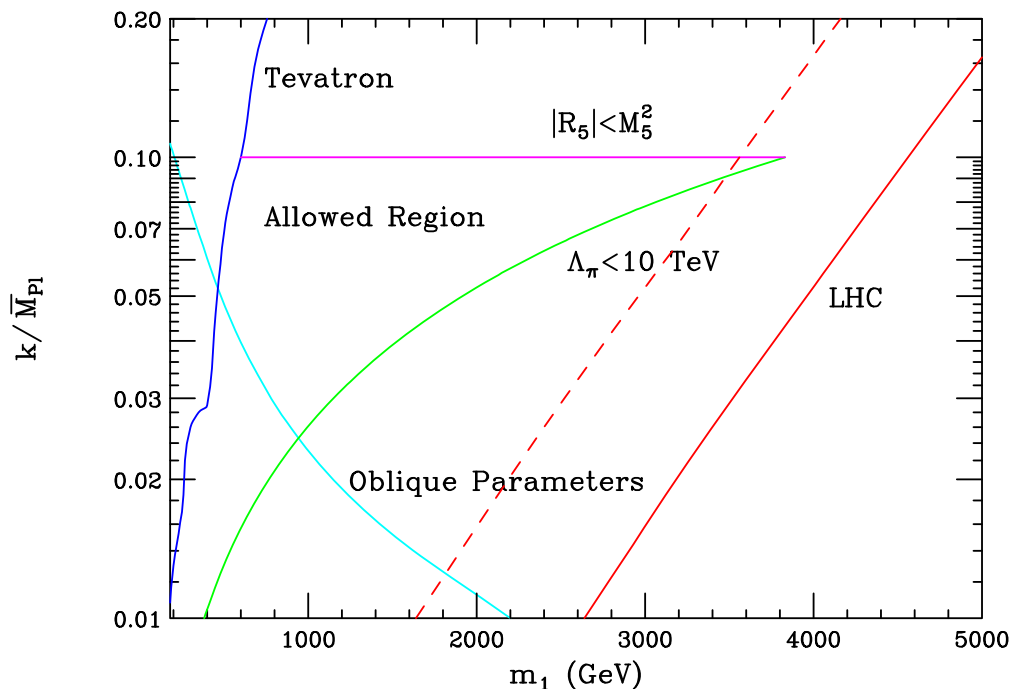
FIGURE 8. Allowed region in the RS model parameter space implied by various theoretical and experimental constraints from Ref.[23]. The regions to the left of the horizontal lines are excluded by direct searches at colliders. The dashed(solid) line for the 14 TeV LHC corresponds to an integrated luminosity of $10(100)\,\text{fb}^{-1}$. The present anticipated parameter space is inside the triangular shaped region.

which reduces to the result above for a space of zero constant curvature, *i.e.*, when $k \to 0$. The solutions to this equation for the χ_n wave functions yield linear combinations of the J_2, Y_2 Bessel functions and not sines and cosines as in the flat space case and the masses of the KK states are given by

$$m_n = x_n k e^{-\pi k r_c}, \tag{18}$$

where the x_n are roots of $J_1(x_n) = 0$. Here $x_n = 0,\ 3.8317..,\ 7.0155..,\ 10.173..,..\ etc.$ Since $k e^{-\pi k r_c}$ is maybe $\sim$ a few hundred GeV, we see that the KK graviton masses are of a similar magnitude with comparable, but unequal, spacing, *i.e.*, the KK gravitons have approximately TeV scale masses. This is quite different than in the ADD model.

Returning to the 5D Einstein action we can insert the wavefunctions for the KK states and determine how they couple to SM fields on the TeV brane; one finds that

$$\mathscr{L} = -\left(\frac{G_0^{\mu\nu}}{\overline{M}_{Pl}} + \sum_{n>0} \frac{G_n^{\mu\nu}}{\Lambda_\pi}\right) T_{\mu\nu}, \tag{19}$$

where $\Lambda_\pi = \overline{M}_{Pl} e^{-\pi k r_c}$ is of order TeV. Here we see that the ordinary graviton zero mode couples as it does in the ADD model as it should but all the higher KK modes have couplings that are exponentially larger due to the common warp factor. We thus have weak scale graviton KKs with weak scale couplings that should be produced as spin-2 *resonances* at colliders. Due to the universality of gravity these KK graviton resonances should be observable in many processes. There are no table top or astrophysical

constraints on this scenario unlike in the ADD model. It is interesting to note that this model also has only 2 free parameters which we can conveniently take to be the mass of the lightest KK excitation, m_1, and the ratio $k/\overline{M}_{Pl}$; given these parameters as input all other masses and couplings can be determined. As we will see $k/\overline{M}_{Pl}$ essentially controls the KK resonance width for a fixed value of the resonance mass. The RS model in its simplest form is thus highly predictive.
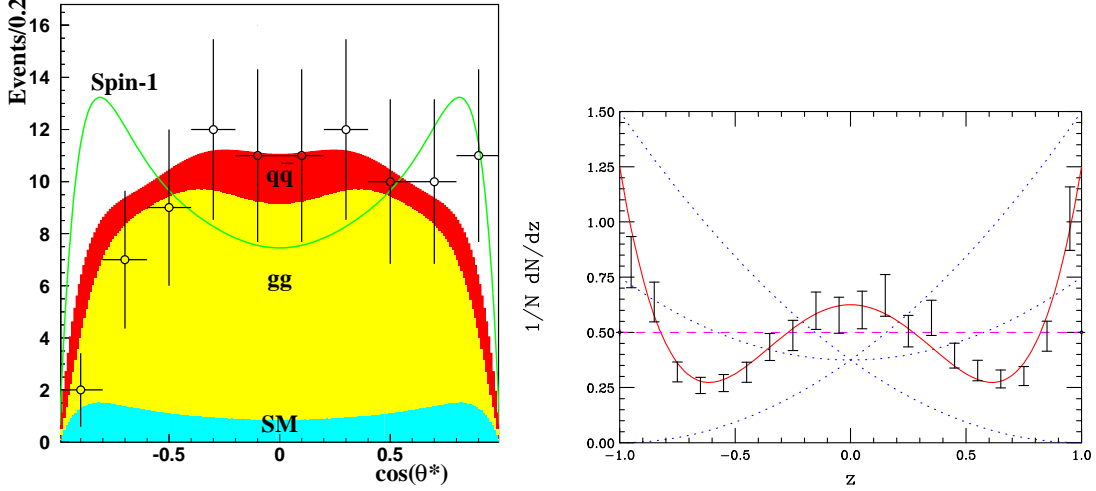

FIGURE 9. Results from Refs.[24, 23] showing that the spin of the KK graviton in the RS model can be determined at either the LHC(left) or ILC(right) from the angular distribution of final state dilepton pairs. Fitting the dilepton data to different spin hypotheses is relatively straightforward.

At this point one may wonder why in the RS model the zero-mode coupling is so weak while the couplings of all the other KK tower states are so much stronger. The strength of the graviton KK coupling to any SM state on the TeV brane is proportional to the value of its 5D wavefunction at $y = \pi r_c$. In the flat space cases discussed above the typical wavefunctions for KK gravitons were $\sim \cos ny/R$ and so took on essentially the same O(1) value at the location of the SM fields for all n. Here the relevant combinations of the J_2, Y_2 Bessel functions behave quite differently when $x_n = 0$, *i.e.*, for the zero mode, versus the case when x_n takes on a non-zero value as it does for the KK excitations. For the zero mode the 5D wavefunction is highly peaked near the Planck brane and so its value is very small near the TeV brane where we are; the opposite is true for the other KK states. Thus it is the strong peaking of the graviton wavefunctions that determine the strength of the gravitational interactions of the KK states with us.

What will these graviton KK states look like at a collider? Fig. 7 shows the production of graviton resonances at the LHC in the Drell-Yan channel and in μ-pairs at the ILC for different values of m_1 and $k/\overline{M}_{Pl}$[22, 23]. Note that the width of the resonance grows as $\sim (k/\overline{M}_{Pl})^2$ so that the resonance appears rather like a spike when this ratio is small. Also note that due to the nonrenormalizable coupling of the graviton KK states to the SM fields the resonance width also grows as $\Gamma \sim m_n^3$ as we go up the KK tower. Hence heavier states are rather wide; for any reasonable fixed value of $k/\overline{M}_{Pl}$ at some point as one goes up the KK tower one reaches states which are quite wide with $\Gamma \simeq m_n$ signaling the existence of the strongly interaction sector of the theory. Examining the $k/\overline{M}_{Pl} - m_1$ parameter space and making some simple assumptions one sees that the 14

TeV LHC has a very good chance of covering all of it once $100\,\text{fb}^{-1}$ or so of integrated luminosity have been accumulated. Part of the present constraints on RS follow from the requirement that $k/\overline{M}_{Pl} \leq 0.1$, as discussed above, and how large we are willing to let Λ_π be before we start worrying about fine tuning again. Given these considerations we see that the 14 TeV LHC has excellent RS parameter space coverage as seen in Fig. 8.

Once we discover a new resonance at the LHC or ILC we'd like to know whether or not it is a graviton KK state. The first thing to do is to determine the spin of the state; Fig. 9 shows that differentiating spin-2 from other possibilities is relatively straightforward[24, 23] at either machine. To truly identify these spin-2 resonances as gravitons, however, we need to demonstrate that they couple universally as expected from General Relativity. The only way to do this is to measure the various final state branching fractions and this is most easily done at the ILC. Fig. 10 shows the expected branching fractions for a graviton KK as a function of its mass assuming only decays to SM particles with a Higgs mass of 120 GeV. One unique test [23] is based on the fact that $\Gamma(G_n \to \gamma\gamma) = 2\Gamma(G_n \to \ell^+\ell^-)$ both of which can be easily measured at either collider.
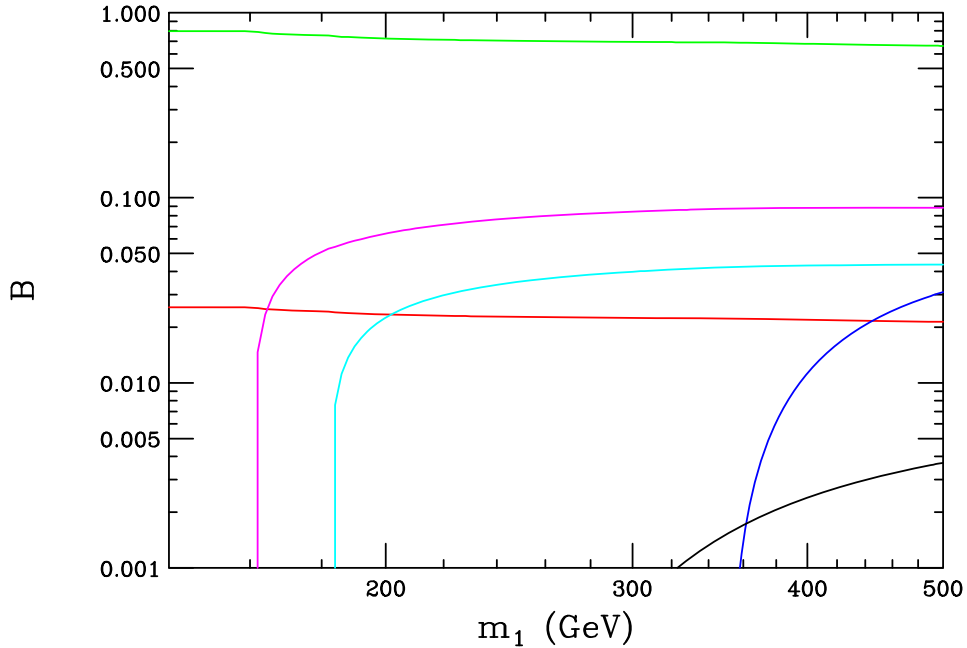

FIGURE 10. Branching fractions for the RS graviton KK state as a function of its mass from Ref.[23]. From top to bottom on the right hand side of the figure the curves correspond to the following final states: jj, W^+W^-, ZZ, $t\bar{t}$, $\ell^+\ell^-$, and hh, respectively.

Before concluding this section we should note that this simple RS model scenario is barely the tip of the iceberg and has been extended in many ways to help with various model building efforts. A few possibilities that have been considered (with limited references!) are

- Extend to 3 or more branes[25]
- Extend to 6 or more dimensions[26]

- Put the SM gauge fields and fermions in the bulk[27] with or without localized brane term interactions[28]. This is very active are of current research.

TEV-SCALE BLACK HOLES

Since gravity becomes strong at the M_* scale it is natural to imagine that black holes may form in TeV collisions at the LHC[8, 31]. We then imagine that in the collision of any 2 partons, above some mass threshold, λM_*, some large fraction of their total energy, $\varepsilon\sqrt{\hat{s}}$, will go into the formation of a BH with the rest being lost as gravitational radiation. Thus, at the parton level we expect a cross section of the approximate form

$$\hat{\sigma} = F_n \pi R_s^2(n, M = \varepsilon\sqrt{\hat{s}}) \, \Theta(\sqrt{\hat{s}} - \lambda M_*), \tag{20}$$

where n is the number of extra dimensions, F_n are an O(1) geometric factors to account for possible geometric and angular momentum effects in the collision process and R_s is the Schwarzschild radius for a BH of mass M given by

$$\frac{M}{M_*} = c(M_* R_s)^{n+1}, \tag{21}$$

with

$$c = \frac{(n+2)\pi^{(n+3)/2}}{\Gamma(\frac{n+3}{2})}. \tag{22}$$

Note that in the above cross section expression it is assumed that the mass threshold is a simple step function which is unlikely to be realistic in a complete model. Furthmore it has been assumed that the SM fields are localized as in ADD or the original RS model and that the size of the BH, R_s, is much smaller than the compactification radius, R_c, of any of the extra dimensions. In order to obtain the predicted cross section for the LHC we must multiply $\hat{\sigma}$ by the appropriate parton densities and then perform the necessary integrations. Fig. 11 shows a sample result of this calculation for the 7 TeV LHC assuming for purposes of demonstration that $M_* = 1$ TeV, $\varepsilon = 0.7$ and taking the F_n as given in Ref.[32]. Here we see that these cross sections may be potentially quite large even if we assume a minimum BH mass of 3 TeV. We note that we can expect 'refinements' to the above cross section estimate if any of our assumptions that we made above prove not to be valid. Of course, to have a complete picture of this BH production possibility we require a quantum theory of gravity.

Once the BH is produced it, approximately, decays as a blackbody with a Hawking temperature (in the absence of angular momentum) given by $T_H = (n+1)/4\pi R_s$ as can be seen in Fig. 12. However, there are many corrections to this approximation including (*i*) spin effects, (*ii*) so called 'grey body' factors (to account for quantum effects associated with the wave functions of the emitted particles), (*iii*) non-spherical emission of particles, (*iv*) the cooling of the BH as particles are emitted (*i.e.*, canonical vs. microcanonical statistical treatment), (*v*) the emission of gravitons into the bulk, (*vi*) the recoil of the BH during the emission process and (*vii*) the possibility that a Planck scale mass remnant would be the final result of the evaporation process. Many of these
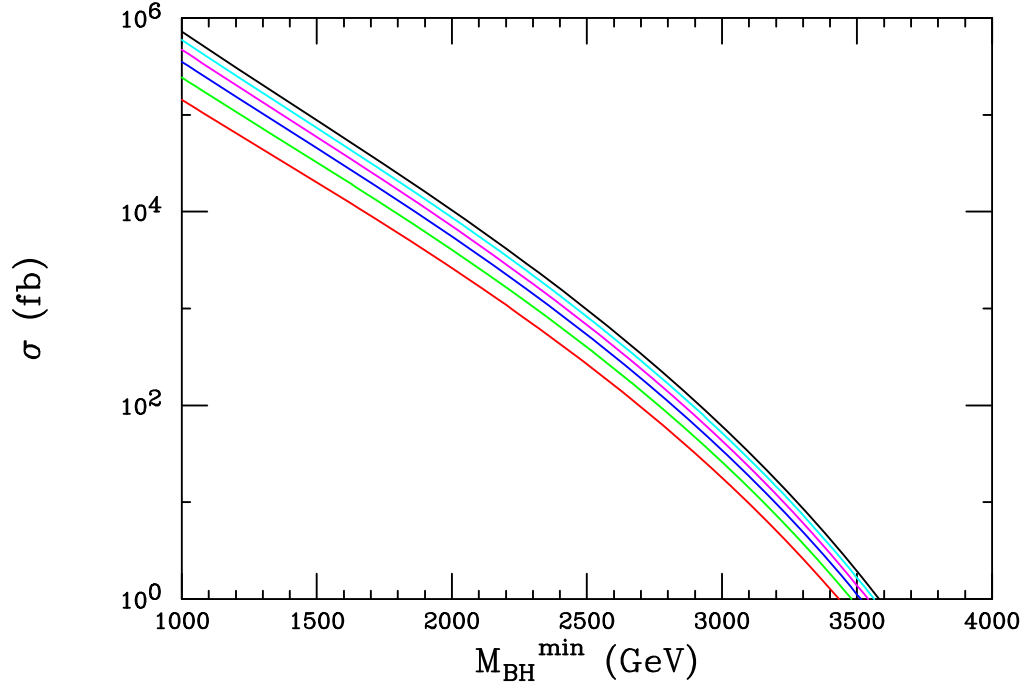
FIGURE 11. Black hole production cross section as a function of the minimum BH mass at the 7 TeV LHC for (from bottom to top) $n = 2$ to 7 assuming $M_* = 1$ TeV with $\varepsilon = 0.7$ as discussed in the text.
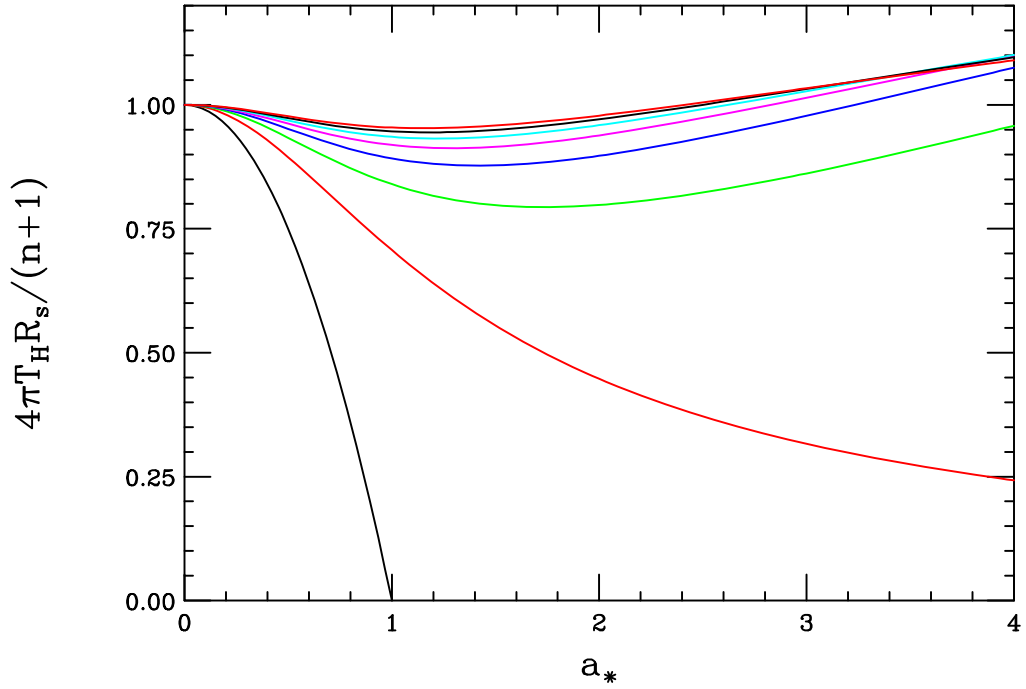

FIGURE 12. Scaled BH Hawking temperature for n=0,1,2,... (from bottom to top) extra dimensions as a function of the angular momentum parameter $a_* = (n+2)J/2MR_h$ with $R-h$ being the horizon radius.

effects combine to significantly lengthen the BH lifetime in comparison to the most naive calculations and not all of these effects have been considered in a single simultaneous treatment. Crudely, the BH final state is one consisting of a high multiplicity of various SM states that are emitted in a roughly spherical pattern around the collision point, a signal which is very hard to fake in the SM. Again, a complete picture describing this decay process will require a quantum theory of gravity.

SUMMARY AND CONCLUSION

The subject of EDs has become a huge research area over the last dozen years and we have hardly scratched the surface in the present discussion. As one can see there are at present an immense number of ideas and models floating around connected to EDs and we certainly can expect there to be many more in the future. EDs can lead to a wide range of new phenomena (Dark Matter, collider signatures, BH, *etc*) that will be sought over the coming decade. Of course, only experiment can tell us if EDs have anything to do with reality and, if they do exist, what their nature may be. The discovery of EDs will certainly radically alter our view of the universe on the very small and very large scales.

ACKNOWLEDGMENTS

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