

D-brane Instantons in Type II String Theory

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Abstract

We review recent progress in determining the effects of D-brane instantons in $\mathcal{N} = 1$ supersymmetric compactifications of Type II string theory to four dimensions. We describe the abstract D-brane instanton calculus for holomorphic couplings such as the superpotential, the gauge kinetic function and higher fermionic F-terms. This includes a discussion of multi-instanton effects and the implications of background fluxes for the instanton sector. Our presentation also highlights, but is not restricted to the computation of D-brane instanton effects in quiver gauge theories on D-branes at singularities. We then summarize the concrete consequences of stringy D-brane instantons for the construction of semi-realistic models of particle physics or SUSY-breaking in compact and non-compact geometries.

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1 INTRODUCTION

The main object of interest in any quantum field theory with a perturbative expansion is the computation of correlation functions. In general these correlation functions are already non-vanishing at tree-level and receive perturbative corrections at each loop level. If the relevant coupling constant g is small, higher loop levels are suppressed by powers of g . On top of this perturbative series, non-perturbative corrections can also arise. In the semi-classical approximation these are associated with topologically non-trivial solutions to the classical equations of motion [1, 2], and contribute terms that scale like $\exp(-1/g^2)$ to the correlation

functions. Therefore, they are more strongly suppressed than any perturbative correction.

These a priori subleading non-perturbative corrections can however become very important when all potentially larger corrections are known to be absent due to non-renormalization theorems. Such situations can be realized in supersymmetric quantum field theories. For instance, in the context of $\mathcal{N} = 1$ supersymmetric four-dimensional theories, there exist holomorphic quantities such as the superpotential W ,

$$S_W = \int d^4x d^2\theta W(\phi_i), \quad (1)$$

and the gauge kinetic function f ,

$$S_{\text{Gauge}} = \int d^4x d^2\theta f(\phi_i) \text{tr}(W^\alpha W_\alpha), \quad (2)$$

which are only integrated over half of the superspace and depend holomorphically on the chiral superfields ϕ_i . At the perturbative level, the superpotential and gauge kinetic function respectively receive only tree-level and up to one-loop level contributions [3, 4]. As a consequence, non-perturbative corrections can become very important for the dynamics of the system, in particular if for instance the tree-level superpotential coupling vanishes. Since these non-perturbative contributions are exponentially suppressed in the weak-coupling regime, when they are the leading effect they may provide a dynamical explanation of some of the hierarchy problems of fundamental physics.

In gauge theories such non-perturbative corrections arise from so-called gauge instantons. These are solutions to the Euclidean self-duality equation

$$F = *F \quad (3)$$

for the Yang-Mills gauge field. Such solutions can be explicitly constructed as local minima of the action and are classified by the instanton number $N = \int_{\mathbf{R}^4} \text{tr} F \wedge F$. Around each instanton saddle point, one can again perform perturbation theory and compute the contributions to certain correlation functions. The final result will then involve summation over all topologically non-trivial sectors. The prescription to carry out these computations is determined by the so-called instanton calculus. As a main ingredient it involves integration over the collective coordinates, also known as the moduli space of the instanton solution.

In string theory the situation is very similar. Also here one can compute perturbative corrections to tree-level correlation functions.¹ These are given by

¹Here we are speaking loosely. In asymptotically AdS solutions, one is computing correlation functions (of a dual field theory). In asymptotically Minkowski backgrounds, one computes an S-matrix, and infers an effective action indirectly. Then the corrections we discuss are really to terms in this effective action. In a non-gravitational theory, this action would give rise to meaningful off-shell correlation functions.

two-dimensional conformal field theory correlation functions on Riemannian surfaces of higher genus g . The expansion parameter is $g_s^{(2g-2)}$ with g_s the string coupling and depends on the dilaton φ via $g_s = \exp(\varphi)$. Suppose we compactify the ten-dimensional superstring on a six-dimensional background such that $\mathcal{N} = 1$ supersymmetry is preserved in four dimensions. One can then compute an effective four-dimensional supergravity action for the massless modes. The non-renormalization theorems for holomorphic couplings generalize naturally to the string case. For the holomorphic couplings W and f we expect that beyond tree- and one-loop level, respectively, non-perturbative corrections are present. Since we still lack a complete second quantized version of string theory, one must argue for the existence of these non-perturbative corrections by the analogy with field theory. Stringent tests of their presence in decoupling limits, or in cases where duality maps such effects to classical effects, provide overwhelming evidence that this analogy is correct.

From the early days of the heterotic string, effects that are non-perturbative from the worldsheet perspective have been a field of active interest. Such configurations arise as Euclidean closed strings wrapping topologically non-trivial two-cycles of the compactification manifold [5, 6]. Being localized in four dimensions, they are called worldsheet instantons, in analogy to the Euclidean topologically non-trivial solutions of Yang-Mills theory. Their contribution to the couplings is non-perturbative in the worldsheet expansion parameter α' , but not in the string coupling g_s .

However, the past one and a half decades have witnessed major progress in the understanding of objects in string theory which are non-perturbative also from the spacetime point of view. It has been shown that p-brane solutions of the supergravity equations of motions are truly non-perturbative objects in string theory. In particular for the large class of D-branes, the quantum theory around the classical solution is known to be given by an open string theory with endpoints on the D-brane [7]. These D-branes carry charge under certain Ramond-Ramond p-forms and also have tension scaling like $T_p = g_s^{-1}$. Such objects are indeed present in four-dimensional Type II string vacua preserving $\mathcal{N} = 1$ space-time supersymmetry in two different ways. Firstly, D-branes can fill four-dimensional space-time and wrap certain cycles of the internal manifold. These D-branes carry both gauge fields and chiral matter fields which are observed as physical fields on the four-dimensional effective theory localized on the D-brane. The past years have seen many attempts to realize realistic gauge and matter spectra on such intersecting D-brane models. This includes investigations of D-branes on compact manifolds as well as on non-compact geometries. Beyond the mere construction of models, a formalism has been developed to compute the resulting $\mathcal{N} = 1$ supersymmetric four-dimensional effective supergravity action.

Soon it was realized that D-branes play an important role not only as the hosts of this effective field theory, but also as actors in it: Euclidean D-branes wrapping entirely a topologically non-trivial cycle of the internal manifold appear

as truly pointlike objects in space and time and thus deserve the name D-brane instanton [8, 9, 10]. In addition, closed and open [11] worldsheet instantons lead to corrections which are non-perturbative in the string tension α' , just as in the heterotic cousin theory. The analogy with Yang-Mills theory can be made very explicit also for D-brane instantons: There the groundstate, or vacuum, of the theory is given by a trivial field configuration, and in computing correlation functions one has to sum over all non-trivial configurations, each of which is described in a perturbative saddle-point approximation. Here the vacuum corresponds to a stable configuration of spacetime-filling D-branes, and the topologically non-trivial situations include these Euclidean D_p -branes, or E_p -branes in short. Open string perturbation theory can be used to describe the fluctuations of E_p -branes and an instanton calculus can be defined in analogy to the field theory case [12, 13, 14, 15, 16, 17].

In special situations these D-brane instantons reproduce the ADHM construction of gauge instantons in the field theory limit. But D-brane instantons are not restricted to a microscopic realization of gauge instanton effects. Rather, they can generate superpotential contributions independent of the gauge degrees of freedom, as pioneered in [18]. For example the non-perturbative generation of a superpotential for some of the closed string fields is crucial in attempts to stabilize the massless moduli fields of four-dimensional compactifications [19]. The reason why non-perturbative effects become important here is precisely the absence of competing terms at the perturbative level which are forbidden due to the non-renormalization of the superpotential.

More recently it has become clear that D-brane instantons play a crucial role for the same reason also in the open string sector of intersecting brane worlds. Oftentimes the presence of global $U(1)$ symmetries forbids some of the phenomenologically desirable matter couplings such as Majorana neutrino masses, Yukawa couplings or the μ -term. It was found that D-brane instantons can contribute to these quantities by effectively breaking the global symmetries [20, 21, 22] (see also [23]). The exponential suppression by the classical instanton action $\exp(-\text{Vol}_{E_p}/g_s)$ depends on the volume (in string units) of the cycle wrapped by the instanton. As such it is in general independent of the gauge coupling in four dimensions. This property, which resulted in the name *stringy* or *exotic instantons*, explains why the non-perturbatively generated couplings can become relevant even in situations where g_s is perturbatively small. There are also obvious potential hidden sector applications of such stringy instantons. For instance, they can lead to models of dynamical supersymmetry breaking without strongly coupled field theory dynamics, and with very minimal hidden sectors [24]. These and other applications of stringy instantons have been intensely explored in the past years, and their relevance for the physics of string compactifications has revived interest also in more technical aspects of instanton calculus.

The aim of this review is both to give a pedagogical introduction to recent developments in the study of D-brane instanton effects in Type II string theory,

and to provide an overview of the various generalizations and applications which have appeared during the last couple of years. Due to lack of space we have to assume some knowledge of four-dimensional $\mathcal{N} = 1$ supersymmetric Type II orientifold compactifications. These have been an active field of research in the recent past and a number of review articles exist including [25, 26, 27, 28, 29, 30, 31, 32]. For the local string models, a certain familiarity with D-branes at singularities and the resulting quiver gauge theories is also helpful; nice reviews appear in [33, 34]. Some aspects of D-brane instantons are also covered in the recent review articles [35, 36, 37, 38].

We will begin in §2 with a general classification of D-brane instantons in Type II orientifold models. The precise couplings a given D-brane instanton can generate are determined by the zero mode content of its worldvolume theory. We then outline the rules for the computation of an instanton induced superpotential. In §3 we classify which quantities of the four-dimensional effective action are corrected by D-brane instantons of different kinds, where for the sake of brevity we have to focus on holomorphic objects. After reviewing the special case of gauge instanton effects we discuss corrections to the gauge kinetic function and higher fermionic F-terms. Consistency of the instanton calculus automatically requires the inclusion also of multi-instanton effects. Closed string background fluxes, which play a crucial role in the stabilization of massless moduli fields, modify the details of all these effects by lifting some of the fermionic zero modes. We conclude this technical section with a brief summary of known instanton contributions to D-terms. In §4, we describe how one can apply the D-instanton calculus, most easily derived in the class of free worldsheet conformal field theories, to situations which involve non-trivial geometries, e.g. branes at singularities. We find that the rules generalize in a straightforward manner, with the interactions an instanton can generate being determined entirely by data which is present in the quiver gauge theory of spacetime filling branes at the same singularity. We also describe how the powerful techniques of geometric transitions can be used, for some of these cases, to provide a dual computation of the instanton-generated superpotential, including the precise coefficients and multi-cover contributions to the superpotential. This provides a highly non-trivial check on our considerations. Finally, we describe some relations between stringy instantons and conventional Yang-Mills instantons in cascading gauge theories, that provide an alternative check on the stringy results. In §5 we very briefly discuss various possible phenomenological applications of these abstract considerations, and we close with a discussion of future directions in §6.

2 BASICS ON BPS D-BRANE INSTANTONS

2.1 Classification of D-brane instantons

Let us compactify ten-dimensional Type II string theory on a Calabi-Yau manifold \mathcal{X} to four-dimensional Minkowski space $\mathbb{R}^{1,3}$. This preserves eight supercharges corresponding to $\mathcal{N} = 2$ supersymmetry in four dimensions. To break the $\mathcal{N} = 2$ space-time supersymmetry down to a phenomenologically more appealing $\mathcal{N} = 1$ supersymmetry, one performs an orientifold projection Ω_p . We now summarize some of the main features of such orientifold models. For more details we refer the reader to the review articles [26, 27, 28]. One distinguishes three kinds of orientifolds models with very similar features.

Type I models

The starting point is the Type IIB superstring in ten dimensions. Taking the quotient by the worldsheet parity symmetry $\Omega : (\sigma, \tau) \rightarrow (-\sigma, \tau)$ one obtains the well-known Type I string. Since the orientifold acts trivially on the spacetime coordinates the theory exhibits an $O9$ -plane. The resulting tadpole can be canceled by introducing stacks of D9-branes carrying generically non-trivial stable vector bundles. Moreover, there can be D5-branes wrapping holomorphic curves of the background Calabi-Yau geometry.

The four-dimensional holomorphic superpotential receives perturbative contributions only at tree-level and depends solely on the complex structure closed string moduli, i.e. $W_0(U_i)$. In such string models, the abelian gauge anomalies are canceled by a generalized Green-Schwarz mechanism, in which the shift symmetry of the axions related to the RR-forms C_2 and C_6 is gauged. The shift symmetries of axions are generally violated by instantons. In the present case the relevant objects are the ones coupling to C_2 and C_6 . These are Euclidean E1- and E5-branes wrapping internal two-cycles of the Calabi-Yau or the whole Calabi-Yau, respectively. The classical instanton action is given by the volume of these internal cycles, which are complexified into chiral multiplets as

$$T_i = e^{-\varphi} \int_{\Gamma_2^i} J + i \int_{\Gamma_2^i} C_2, \quad S = e^{-\varphi} \int_{\mathcal{X}} J^3 + i \int_{\mathcal{X}} C_6. \quad (4)$$

One therefore expects that the superpotential can receive contributions from E1- and E5-instantons and has the following schematic form

$$W = W_0(U) + \sum_{E1_a} A_a(U) e^{-\alpha_a^i T_i} + A_S(U) e^{-S}. \quad (5)$$

Similarly, the holomorphic gauge kinetic function on a stack of D9-branes must look like

$$f_A = S - \sum_i \kappa_A^i T_i + f_A^{1\text{-loop}}(U) + \sum_{E1_a} A_a(U) e^{-\alpha_a^i T_i} + A_S(U) e^{-S}. \quad (6)$$

Here Γ_i denotes a basis of $H_4(X, \mathbb{Z})$ and $\kappa_A^i = \int_{\Gamma_i} \text{ch}_2(V_A)$ depends on the second Chern character of the vector bundle V_A defined on the D9-branes stack with gauge kinetic function f_A . Note that its one-loop correction must not depend on the Kähler moduli.

Type IIB orientifolds with O7 and O3-planes

One can generalize the orientifold projection by dressing Ω with a holomorphic involution σ . In case σ acts like

$$\sigma : J \rightarrow J, \quad \Omega_3 \rightarrow -\Omega_3 \quad (7)$$

the fixed-point set consists of $O7$ -planes wrapping holomorphic four-cycles of the Calabi-Yau and a number of $O3$ -planes localized at certain fixed points of σ on \mathcal{X} . The axion whose shift symmetry is gauged by the Green-Schwarz mechanism is the RR-form C_4 . One thus expects E3-branes wrapping four-cycles in the Calabi-Yau to contribute to the holomorphic couplings. In addition there could also be E(-1) brane instantons. The complexified Kähler moduli and the axio-dilaton field read

$$T_i = e^{-\varphi} \int_{\Gamma_4^i} J \wedge J + i \int_{\Gamma_4^i} C_4, \quad \tau = e^{-\varphi} + iC_0. \quad (8)$$

The holomorphic quantities can then have an expansion of the form

$$W = W_0(e^{-T_i}) + \sum_{E3_a} A_a(U) e^{-\alpha_a^i T_i} + A_\tau(U) e^{-\tau} \quad (9)$$

for the superpotential and

$$f_A = \sum_i \kappa_A^i T_i + f_A^{1\text{-loop}}(U) + \sum_{E3_a} A_a(U) e^{-\alpha_a^i T_i} + A_\tau(U) e^{-\tau} \quad (10)$$

for the gauge coupling on a stack of D7-branes.

Intersecting D6-brane models

Here one starts with the Type IIA superstring compactified on a Calabi-Yau and takes the quotient by the orientifold projection $\Omega\bar{\sigma}(-1)^{FL}$, where $\bar{\sigma}$ denotes an isometric anti-holomorphic involution. The fixed-point locus of such an involution is a special Lagrangian three-cycle in \mathcal{X} which gives rise to $O6$ -planes. Their tadpole can be canceled by introducing intersecting D6-branes. The axion whose shift symmetry is gauged by the Green-Schwarz mechanism is the RR-form C_3 so that one expects E2-branes wrapping three-cycles in the Calabi-Yau to contribute

to the holomorphic couplings. In this case the instanton action depends on the complex structure moduli

$$U_i = e^{-\varphi} \int_{\Gamma_3^i} \Omega_3 + i \int_{\Gamma_3^i} C_3. \quad (11)$$

However, also the complexified Kähler moduli $T_i = \int_{\Gamma_2^i} J_2 + i \int_{\Gamma_2^i} B_2$ contain as the imaginary part an axion. In combination these observations allow an expansion

$$W = W_0(e^{-T_i}) + \sum_{E_{2a}} A_a(e^{-T_i}) e^{-\alpha_a^i U_i} \quad (12)$$

and

$$f_A = \sum_i \kappa_A^i U_i + f_A^{1\text{-loop}}(e^{-T_i}) + \sum_{E_{2a}} A_a(e^{-T_i}) e^{-\alpha_a^i U_i} \quad (13)$$

for the holomorphic quantities.

Unified description of all orientifold models

It is obvious from the above that all three kinds of orientifold models are very similar in structure. This is a consequence of T-dualities (mirror symmetry) connecting the three kinds of orientifolds. In the sequel we will treat all orientifold models on the same footing by introducing a unified notation.

Let us denote the space-time filling D-branes wrapping internal cycles Γ_a as \mathcal{D}_a . For branes in Type IIB these objects also carry non-trivial holomorphic vector bundles \mathcal{V}_a . Moreover, these objects are not invariant under the orientifold projection Ω_p and are mapped to space-time filling D-branes $(\mathcal{D}'_a, \mathcal{V}'_a)$. The corresponding D-brane instantons are denoted as \mathcal{E}_i and \mathcal{E}'_i . At the intersection of two D-branes \mathcal{D}_a and \mathcal{D}_b one gets matter fields $\Phi_{a,b}$. Our convention is that an open string stretching from \mathcal{D}_a to \mathcal{D}_b yields a matter $\Phi_{a,b}$ in the bifundamental representation (\overline{N}_a, N_b) . Their multiplicity can be computed by the relevant topological cohomology groups. For D6-branes in Type IIA these are simply the positive and negative intersections between the two 3-cycles. For all three cases we use I_{ab}^+ to denote positive chirality fields and I_{ab}^- for negative chirality in the $a \rightarrow b$ sector. A positive chiral index $I_{ab} = I_{ab}^+ - I_{ab}^-$ then indicates an excess of chiral fields in the representation (\overline{N}_a, N_b) over those transforming as (N_a, \overline{N}_b) . For Type IIA orientifolds I_{ab} is simply the topological intersection number between the two internal 3-cycles. In Type IIB the chiral index is given by a unified formula in terms of the K-theoretic intersection number

$$I_{ab} = I_{ab}^+ - I_{ab}^- = \int_{\mathcal{X}} Q(\mathcal{D}_a, \mathcal{V}_a) \wedge Q(\mathcal{D}_b, \mathcal{V}_b^\vee) \quad (14)$$

with

$$Q(\mathcal{D}_a, \mathcal{V}_a) = [\Gamma_a] \wedge \text{ch}(\mathcal{V}_a) \wedge \sqrt{\frac{\hat{A}(T_{\Gamma_a})}{\hat{A}(N_{\Gamma_a})}}. \quad (15)$$

Here $[\Gamma_a]$ denotes the Poincaré dual of the cycle Γ the D-brane is wrapping and T_{Γ_a} and N_{Γ_a} are its tangent and normal bundle.

2.2 Zero modes

In the previous section we have classified which types of instantons might in principle correct the holomorphic quantities of the effective action. Our arguments were only based on non-renormalisation theorems and knowledge of the chiral superfields. The actual computation of these corrections requires precise control over the instanton zero modes. These are the massless excitations of open strings with both ends on the same instanton or at the intersection between two instantonic E-branes or between one D-brane and one E-brane. As such they can be computed with standard open string CFT methods and one can associate vertex operators with them. The only difference with respect to the more familiar case of massless modes between spacetimes filling D-branes results from the four Dirichlet-Neumann conditions of the instanton in the extended four dimensions. In particular, one cannot attribute four-dimensional momentum to the instanton modes so that only massless modes can be considered as on-shell states. With this caveat in mind one can formally compute couplings between the instanton modes among themselves and possibly involving some of the open strings in the D-brane sector.

Let us denote the collection of all instanton zero modes as \mathcal{M} . The zero mode couplings are encoded in the interaction part of the instanton effective action $S_{\mathcal{E}}^{(\text{int.})}(\mathcal{M})$. In analogy with standard wisdom for field theory instantons, the instanton contribution to the four-dimensional effective action is sketchily given by

$$S_{n.p.}^{4D} = \int d\mathcal{M} e^{-S_{\mathcal{E}}^{(0)} - S_{\mathcal{E}}^{(\text{int.})}(\mathcal{M})}, \quad (16)$$

where $S_{\mathcal{E}}^{(0)}$ denotes the classical instanton effective action given by the complexified superfields of the previous section. We will be much more precise in section 2.3.

Of special importance are the fermionic zero modes. The integral $\int d\mathcal{M}$ over the zero mode measure requires that each fermionic zero modes can be pulled down from the exponent precisely once as otherwise the Grassmannian integral vanishes. This process is called saturation of fermionic zero modes. Oftentimes knowledge of the fermionic zero mode content is therefore sufficient to decide whether or not an instanton can contribute to a certain correlation function.

After these preliminaries we classify the various kinds of zero modes of an E-brane instanton in Type II orientifolds.

Universal zero modes

The universal zero modes arise from strings starting and ending on the same E-brane. Firstly, there are four bosonic zero modes x^μ parameterizing the position of the instanton in four-dimensional spacetime. These are the Goldstone bosons associated with the breakdown of four-dimensional translational invariance due to the presence of the instanton.

Secondly, there are fermionic zero modes related to broken supersymmetries. In general the instantonic brane \mathcal{E} is not invariant under the orientifold projection and there exists an image brane \mathcal{E}' wrapping a distinct cycle. In this case, the instanton locally feels the full $\mathcal{N} = 2$ supersymmetry preserved by compactification of Type II theory on a Calabi-Yau manifold [39, 40, 41, 42]. The orientifold action preserves a specific $\mathcal{N} = 1$ subalgebra thereof with supercharges $Q^\alpha, \bar{Q}^{\dot{\alpha}}$. The orthogonal $\mathcal{N} = 1$ complement, generated by the charges $Q'^\alpha, \bar{Q}'^{\dot{\alpha}}$, is broken in four dimensions. A spacetime-filling D-brane along a 1/2-BPS cycle which is supersymmetric with respect to the orientifold preserves the supercharges $Q^\alpha, \bar{Q}^{\dot{\alpha}}$. An instanton along this same cycle likewise preserves four of the eight supersymmetries, leading to four goldstino fermionic zero modes associated with the four broken supersymmetries². However, due to its localisation in the four extended dimensions the BPS-instanton does not preserve the four supercharges $Q^\alpha, \bar{Q}^{\dot{\alpha}}$, but rather the off-diagonal combination $Q'^\alpha, \bar{Q}'^{\dot{\alpha}}$. Therefore the two chiral Goldstone modes θ^α associated with the breaking of Q^α really correspond to half of the $\mathcal{N} = 1$ superspace preserved in four dimensions, while the anti-chiral Goldstinos, denoted by $\bar{\tau}^{\dot{\alpha}}$ for distinction, are associated with its orthogonal complement in the original $\mathcal{N} = 2$ algebra [43]. It follows that such an instanton can in principle

$\mathcal{N} = 1$	$\mathcal{N} = 1'$
θ^α	τ^α
$\bar{\theta}^{\dot{\alpha}}$	$\bar{\tau}^{\dot{\alpha}}$

Table 1: Universal fermionic zero modes $\theta^\alpha, \bar{\tau}^{\dot{\alpha}}$ ($\tau^\alpha, \bar{\theta}^{\dot{\alpha}}$) of an (anti-)instanton associated with the breaking of the $\mathcal{N} = 1$ SUSY algebra preserved by the orientifold and its orthogonal complement $\mathcal{N} = 1'$.

contribute to an F-term provided some mechanism is found which saturates the extra $\bar{\tau}^{\dot{\alpha}}$ modes. We will systematically discuss various possibilities in section 3.

²Note that if the instanton did not preserve any supersymmetry, i.e. if it were non-BPS, one would get eight fermionic zero modes, so that such an object cannot contribute to any supersymmetric quantity.

It is important to appreciate, though, that the generation of a D-term requires instead the four Goldstone modes $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$. We will comment on appropriate configurations in section 3.6.

The simplest configuration leading to an F-term is given by an instanton that already locally feels only the $\mathcal{N} = 1$ supersymmetry. Since the breaking of $\mathcal{N} = 2$ to $\mathcal{N} = 1$ occurs on the orientifold planes and on the space-time filling D-branes, we must either place the instanton on top of a D-brane or place it in an Ω_p invariant position.

The first case, $\mathcal{E} \subset \mathcal{D}$, is nothing else than the stringy description of a gauge instanton for the Chan-Paton gauge theory on the D-brane \mathcal{D} . This can be made very precise in that one can derive the celebrated ADHM constraints by evaluating open string disc diagrams and is the subject of §3.1. As will be detailed there, such instantons contribute to the superpotential even in presence of just a single parallel brane \mathcal{D} even though they cannot directly be interpreted as gauge instantons of the associated $U(1)$ theory.

For the second case with $\mathcal{E} = \mathcal{E}'$ one has to distinguish the possibilities that the Ω_p projection either symmetrizes or anti-symmetrizes the CP-gauge group [39, 40, 41, 42]. Special care has to be taken due the four Dirichlet-Neumann (DN) boundary conditions between \mathcal{E} and an auxiliary D-brane \mathcal{D} wrapping the same internal cycle. If the Chan-Paton gauge group on \mathcal{D} is $(S)O(N)/SP(N)$, then the CP-gauge group on \mathcal{E} is switched to $SP(N)/(S)O(N)$. This is because the action on the Chan-Paton factors switches from symmetrization to anti-symmetrization and vice versa. In addition, the orientifold acts with an extra minus sign on chiral spinors as well as on bosonic excitations along the four extended directions. More details can be found in [39, 40, 41, 42, 43]. The result can be phrased as follows:

- $SP(N)$ instanton: In this case one needs an even number of branes \mathcal{E} . As a result one gets $\frac{N(N-1)}{2}$ zero modes θ^α and $\frac{N(N+1)}{2}$ zero modes $\bar{\tau}^{\dot{\alpha}}$. For $N = 2$ one therefore has one chiral and three anti-chiral Weyl-spinors.
- $O(N)$ instanton: Here one can also have an odd number of \mathcal{E} branes. One gets $\frac{N(N+1)}{2}$ zero modes θ^α and $\frac{N(N-1)}{2}$ zero modes $\bar{\tau}^{\dot{\alpha}}$. For $N = 1$ one therefore ends up with two universal zero modes θ^α .

On the open string worldsheet, these universal zero modes are described by the vertex operator (in the $(-1/2)$ -ghost picture)

$$V_\theta^{-\frac{1}{2}}(z) = \theta_\alpha e^{-\frac{\varphi(z)}{2}} S^\alpha(z) \Sigma_{\frac{3}{8}, \frac{3}{2}}^{\mathcal{E}, \mathcal{E}}(z), \quad (17)$$

where θ_α , $\alpha = 1, 2$ is the polarization and S^α denotes the 4D spin field of $SO(1, 3)$. This is a Weyl spinor of conformal dimension $h = 1/4$. The twist field $\Sigma_{\frac{3}{8}, \frac{3}{2}}^{\mathcal{E}, \mathcal{E}}$ is essentially the spectral flow operator of the $\mathcal{N} = 2$ superconformal field theory. The subscripts denote its conformal dimension $\frac{3}{8}$ and $U(1)$ worldsheet charge

$\frac{3}{2}$. A pedagogical introduction to the basics of the $\mathcal{N} = 2$ superconformal field theory can be found e.g. in [44]. Since the instantonic brane \mathcal{E} wraps a cycle on the CY manifold, the four-dimensional spacetime is transversal so that there appears no four-dimensional momentum factor in the vertex operator.

Finally let us introduce some nomenclature: D-brane instantons along a cycle not populated by a D-brane, which do therefore not directly have a gauge instanton description, have been called *stringy* or *exotic* instantons in the literature. Stringy instantons on top of an orientifold leading to a universal zero mode measure $\int d^4x d^2\theta$ are known as $O(1)$ instantons, as opposed to so-called $U(1)$ instantons \mathcal{E} in a non-invariant position $\mathcal{E} \neq \mathcal{E}'$.

Deformation zero modes There can be further zero modes from the $\mathcal{E} - \mathcal{E}$ sector due to possible deformations of the instantonic brane. For a $U(1)$ instanton, each complex valued deformation leads to one complex bosonic zero mode as well as one chiral and one anti-chiral Weyl spinor, making a total of four fermionic degrees of freedom. If the instanton is however of type $O(1)$, then the orientifold projection acts also on these deformation zero modes as shown in Table 2.

zero modes	E1	E3	E2
γ^α	$H^{(1,0)}(E1)$	$H^{(1,0)}(E3)$	$b^1(E2)_-$
$(c, \bar{\chi}^\alpha)$	$H^0(E1, N)$	$H^{(2,0)}(E3)$	$b^1(E2)_+$

Table 2: Deformation zero modes for three classes of Type II orientifolds.

There the cohomology classes of type $H^{(1,0)}(E)$ count the Wilson-line moduli and $H^0(E, N)$ the transversal deformations of the cycle N . For E2 instantons $b^1(E2)_\pm$ count the moduli even and odd under the orientifold projection. As anticipated an instanton with deformation moduli can only contribute to a correlation function if the fermionic zero modes can be soaked up or lifted by flux, as will be discussed in §3.2, §3.3 and §3.5. An instanton which does not have these extra zero modes from the $\mathcal{E}_i - \mathcal{E}_i$ sector is also called *rigid*.

Charged zero modes

Finally, there will generically be zero modes which arise at the intersection of the instanton \mathcal{E} with the D-branes \mathcal{D}_a [20, 21, 22]. These zero modes are called *charged zero modes* as they are charged under the four-dimensional gauge symmetry localized on the \mathcal{D}_a branes. They were first discussed in the context of F-theory compactifications in [45]. Let us focus here on the case of a stringy instanton with a non-trivial intersection with a D-brane along a different cycles. Due to the four Neumann-Dirichlet boundary conditions between \mathcal{E} and \mathcal{D}_a along the four space-time directions, the zero point energy in the NS-sector is already shifted by $L_0 = 1/2$ so that for internally intersecting branes, there can only be

fermionic zero modes from the R-sector. The GSO-projection only allows for *chiral* zero modes from the worldsheet point of view [46] corresponding to a single Grassmannian degree of freedom. The total number of such charged fermionic zero modes is displayed for a $U(1)$ instanton in table 3. If the instanton wraps the same cycle as a spacetime filling D-brane \mathcal{D} there exist also bosonic modes in the $\mathcal{E} - \mathcal{D}$ sector. These will be discussed in §3.1.

zero mode	Reps.	number
$\lambda_a \equiv \lambda_{\mathcal{E}a}$	$(-1_{\mathcal{E}}, \square_a)$	$I_{\mathcal{E}, \mathcal{D}_a}^+$
$\bar{\lambda}_a \equiv \lambda_{a\mathcal{E}}$	$(1_{\mathcal{E}}, \bar{\square}_a)$	$I_{\mathcal{E}, \mathcal{D}_a}^-$
$\lambda'_a \equiv \lambda_{\mathcal{E}'a}$	$(1_{\mathcal{E}}, \square_a)$	$I_{\mathcal{E}', \mathcal{D}_a}^+$
$\bar{\lambda}'_a \equiv \lambda_{a\mathcal{E}'}$	$(-1_{\mathcal{E}}, \bar{\square}_a)$	$I_{\mathcal{E}', \mathcal{D}_a}^-$

Table 3: Charged fermionic zero modes from $\mathcal{E} - \mathcal{D}$ intersections.

From table 3 it is clear that the total $U(1)_a$ charge of all the fermionic zero modes on the intersection of $\mathcal{E} + \mathcal{E}'$ and $\mathcal{D}_a + \mathcal{D}'_a$ is $Q_a(\mathcal{E}) = N_a (I_{\mathcal{E}, \mathcal{D}_a} - I_{\mathcal{E}', \mathcal{D}'_a})$. For the case of an $O(1)$ instanton the table simplifies as $\mathcal{E} = \mathcal{E}'$ and only the first two lines in Table 3 give independent zero modes.

In order for such an instanton to contribute to a coupling, the charged zero modes have to be soaked up. This is possible because the part $S_{inst.}(\mathcal{M})$ in equ. (16) contains couplings of the schematic form $\lambda_{\mathcal{E}a_i} \Phi_{a_i b_i} \lambda_{b_i \mathcal{E}}$. The saturation of the λ modes thus pulls down charged matter fields $\Phi_{a_i b_i}$. This happens in such a way that for each absorption diagram the (global) $U(1)_a$ charges are preserved. Therefore for the superpotential, only terms like

$$W = \prod_{i=1}^M \Phi_{a_i b_i} \exp(-S_{\mathcal{E}}) \quad (18)$$

can be generated for which the $U(1)_a$ charges of the product of matter fields $\prod_i \Phi_i$ are canceled by the sum of the $U(1)_a$ charges of the zero modes, i.e.

$$\sum_{i=1}^M Q_a(\Phi_{a_i b_i}) = -N_a (I_{\mathcal{E}, \mathcal{D}_a} - I_{\mathcal{E}', \mathcal{D}'_a}) . \quad (19)$$

It was shown in [20, 21] that this relation can also be deduced by using the gauging of the axionic shift symmetries of the RR-forms C_p , $p = 2, 4, 3$ due to the generalized Green-Schwarz mechanism. Therefore, such instantons at non-trivial intersection with D-branes can generate charged matter couplings which per se violate the global $U(1)_a$ symmetries.

Let us also give the worldsheet description of these matter field zero modes. The corresponding Ramond sector open string vertex operators are of the form

$$V_{\lambda_a^i}^{-\frac{1}{2}}(z) = \lambda_a^i e^{-\frac{\varphi(z)}{2}} \Sigma_{\frac{3}{8}, -\frac{1}{2}}^{\mathcal{D}_a, \mathcal{E}}(z) \sigma_{h=1/4}(z), \quad (20)$$

Here $\Sigma_{\frac{3}{8}, -\frac{1}{2}}^{\mathcal{D}_a, \mathcal{E}}$ denotes a spin field with conformal dimension $h = \frac{3}{8}$ and $U(1)$ worldsheet charge $-\frac{1}{2}$ in the R-sector for the internal SCFT and $\sigma_{h=1/4}$ the 4D spin field arising from the twisted 4D worldsheet bosons carrying half-integer modes. Note that also these zero modes carry no momentum along the flat 4D directions.

Multi-instanton zero modes

So far we have focused on the zero modes associated with single instantons, but a general configuration may feature several instantonic branes at once. In this case there appear new modes in the sector between two different instantons. In fact, such multi-instanton configurations are almost inevitable in the context of Type II orientifolds. Recall that if an instanton \mathcal{E} wraps a cycle not invariant under the geometric orientifold action we have to consider in addition its orientifold image \mathcal{E}' . This leads, in the upstairs geometry prior to orientifolding, to a two-instanton configuration consisting of \mathcal{E} and \mathcal{E}' .

As in the case of spacetime-filling D-branes the intersection locus of two instantons \mathcal{E}_1 and \mathcal{E}_2 hosts massless zero modes in form of one chiral multiplet together with its CPT conjugate. These are counted by the same intersection numbers as in the corresponding $\mathcal{D}_1 - \mathcal{D}_2$ case. Note that this is in contrast with the charged zero modes in the $\mathcal{E} - \mathcal{D}$ sector, where the boson is projected out due to the four DN boundary conditions in the extended spacetime dimensions and only a single Grassmann degree of freedom λ survives.

Special care has to be taken, though, of the orientifold action for zero modes between an instanton and its image on top of the orientifold plane. As encountered before, the orientifold action on the CP factors changes in the instanton sector due to the localization of the instanton in four dimensions and the orientifold acts with an extra minus sign on the chiral Weyl spinors. Together with the non-projected sector away from the orientifold locus this gives rise to the zero mode content for $\mathcal{E} - \mathcal{E}'$ instantons displayed in table 4 [43].

zero mode	$U(1)_{\mathcal{E}}$ charge	Multiplicity
$(m, \bar{\mu}^{\dot{\alpha}})$	$(2, -2)$	$\frac{1}{2}(I_{\mathcal{E}', \mathcal{E}} + pI_{O, \mathcal{E}})^+$
μ^{α}	2	$\frac{1}{2}(I_{\mathcal{E}', \mathcal{E}} - pI_{O, \mathcal{E}})^+$
$(n, \bar{\nu}^{\dot{\alpha}})$	$(-2, 2)$	$\frac{1}{2}(I_{\mathcal{E}', \mathcal{E}} + pI_{O, \mathcal{E}})^-$
ν^{α}	-2	$\frac{1}{2}(I_{\mathcal{E}', \mathcal{E}} - pI_{O, \mathcal{E}})^-$

Table 4: Charged zero modes at an $\mathcal{E} - \mathcal{E}'$ intersection, with $p=1,2$ for E2-instantons in Type IIA and E3-instantons in Type IIB, respectively.

2.3 Superpotential calculus

In the previous section we have seen that a rigid $O(1)$ instanton has the appropriate universal zero mode structure $d^4x d^2\theta$ to yield a contribution to the holomorphic F-terms in the effective supergravity action. In this section we will review the explicit computation of such corrections to the superpotential. As mentioned in the introduction, in lack of a second quantized version of string theory we cannot derive the instanton calculus from first principles, but have to define it from analogy considerations to field theory.

The starting point is to find the interactions of the instanton zero modes appearing in $S_{\mathcal{E}}^{(\text{int.})}(\mathcal{M})$ in equ. (16). They can be computed in terms of correlators in the boundary conformal field theory describing the interactions of the D-branes and the instanton. One then integrates out the zero modes by pulling down appropriate interaction terms from the exponent.

Equivalently one can view the computation entirely from the CFT perspective: To detect a contribution like (18) to the superpotential one computes an M-point correlator in an instanton background $\langle \Phi_{a_1, b_1} \cdot \dots \cdot \Phi_{a_M, b_M} \rangle_{\mathcal{E}}$. For canonically normalized conformal fields this yields the physical correlator. In terms of the quantities in the effective supergravity action, it involves a combination of the superpotential coefficient Y , the Kähler potential \mathcal{K} and the matter field Kähler metrics K_{a_i, b_i} ,

$$\langle \Phi_{a_1, b_1} \cdot \dots \cdot \Phi_{a_M, b_M} \rangle_{\mathcal{E}} = \frac{e^{\frac{\mathcal{K}}{2}} Y_{\Phi_{a_1, b_1}, \dots, \Phi_{a_M, b_M}}}{\sqrt{K_{a_1, b_1} \cdot \dots \cdot K_{a_M, b_M}}}. \quad (21)$$

Now focus on the superpotential contribution. As for the quantum fluctuations around the classical instanton solution, only terms proportional to g_s^0 are relevant as these translate into a constant dependence on the holomorphic superfields T_i and U_i in eqs. (4), (8) and, respectively, in eq. (11). Any other dependence on these axions and therefore on g_s can be ruled out due to the axionic shift symmetries. In conclusion, the counting of factors of g_s together with the need to insert all fermionic zero modes gives the terms which can appear in the superpotential.

Now, each disc diagram carries an overall normalization factor proportional to g_s^{-1} . Each annulus or Möbius diagram comes with an additional factor of g_s . In [20, 46] it was argued, in analogy with the ADHM construction by D-brane instantons [17], that one should assign to each charged fermionic zero mode λ_a an extra factor of $\sqrt{g_s}$. From the counting of g_s it is therefore clear that only worldsheets with the topology of a disc or of an annulus or Möbius strip can contribute to the superpotential. Furthermore, charged fermionic zero modes can only contribute to the disc amplitudes and in such a way that precisely two of them are inserted, whereas the 1-loop amplitudes have to be uncharged. We will provide more evidence for this picture momentarily.

Summarizing all these observation, in [20] the following formula for computing

the correlation function in the semi-classical approximation³ was proposed

$$\begin{aligned}
\langle \Phi_{a_1, b_1} \cdots \Phi_{a_M, b_M} \rangle_{\mathcal{E}} &\simeq \int d^4x d^2\theta \sum_{\text{conf.}} \prod_a (\prod_{i=1}^{I_{\mathcal{E}, \mathcal{D}_a}^+} d\lambda_a^i) (\prod_{i=1}^{I_{\mathcal{E}, \mathcal{D}_a}^-} d\bar{\lambda}_a^i) \\
&\exp(-S_{\mathcal{E}}^{(0)}) \exp\left(\sum_b \left(\text{disc diagram}^* + \text{one-loop diagram}^* \right)\right) \quad (22) \\
&\langle \widehat{\Phi}_{a_1, b_1}[\vec{x}_1] \rangle_{\lambda_{a_1}, \bar{\lambda}_{b_1}} \cdots \langle \widehat{\Phi}_{a_L, b_L}[\vec{x}_L] \rangle_{\lambda_{a_L}, \bar{\lambda}_{b_L}} .
\end{aligned}$$

For more details on the overall normalization see [46]. The amplitude (22) involves an integration over all instanton zero modes and a sum over all configurations of distributing the vertex operators for the charged matter fields Φ_{a_i, b_i} on disc diagrams, on each of which two charged zero modes are inserted.⁴ The abbreviation $\widehat{\Phi}_{a_k, b_k}[\vec{x}_k]$ denotes a chain-product of vertex operators

$$\widehat{\Phi}_{a_k, b_k}[\vec{x}_k] = \Phi_{a_k, x_{k,1}} \cdot \Phi_{x_{k,1}, x_{k,2}} \cdot \Phi_{x_{k,2}, x_{k,3}} \cdots \Phi_{x_{k,n-1}, x_{k,n}} \cdot \Phi_{x_{k,n}, b_k} \quad , \quad (23)$$

while $\langle \widehat{\Phi}_{a_1, b_1}[\vec{x}_1] \rangle_{\lambda_{a_1}, \bar{\lambda}_{b_1}}$ is a CFT disc correlator with the vertex operators for $\widehat{\Phi}_{a_1, b_1}[\vec{x}_1]$ and those for the charged zero modes λ_{a_1} and $\bar{\lambda}_{b_1}$ inserted on the boundary. Therefore, the charged matter zero modes are soaked up by boundary changing CFT disc correlators as shown in figure 1. They were computed explicitly in [46].

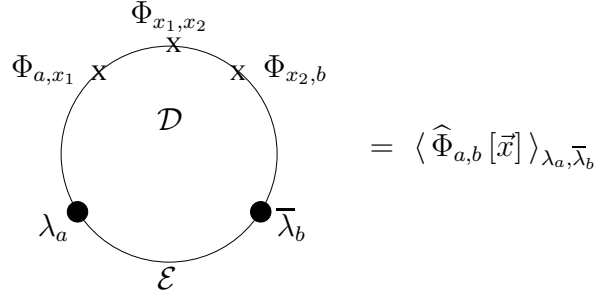


Figure 1: Disc diagram for charged zero mode absorption.

Note that the respective arguments of the two exponential functions in (22) are the disc vacuum diagram and the one-loop vacuum diagram with at least one boundary on the instanton \mathcal{E} . The vacuum disc amplitude for an Ep-instanton is given by [12]

$$S_{\mathcal{E}}^{(0)} = -\langle 1 \rangle^{\text{disc}} = \frac{1}{g_s} \frac{V_{\mathcal{E}}}{\ell_s^{p+1}} = \frac{8\pi^2}{g_{\text{YM}, \mathcal{E}}^2} . \quad (24)$$

³Higher loop corrections are certainly not vanishing, but come from corrections to the Kähler potential resp. Kähler metrics in (21).

⁴Note that for simplicity the possibility of including annuli with charged *matter* fields inserted is neglected.

Here $g_{\text{YM},\mathcal{E}}$ is the gauge coupling on an auxiliary spacetime-filling D-brane $\mathcal{D}_{\mathcal{E}}$ that would be wrapping the same internal cycles as the instanton \mathcal{E} . In particular, this quantity is not identical to the gauge couplings on the branes $\mathcal{D}_{a,b}$.

For the one-loop diagrams $\frac{\bigcirc\bigcirc}{\mathcal{E}\mathcal{E}} = 0$ and $\frac{\bigcirc\bigcirc}{\mathcal{D}_a\mathcal{D}_b} = 0$ by supersymmetry. Thus only the annulus and Möbius strip amplitudes with precisely one boundary on \mathcal{E} really contribute. (The upper index $*$ in (22) will be explained momentarily.) In [47, 48] it was shown that these vacuum diagrams are related to the gauge threshold corrections associated with the auxiliary D-brane $\mathcal{D}_{\mathcal{E}}$. In stringy Feynman diagrams we therefore arrive at the relation shown in figure 2.

$$F^* \left(\begin{array}{c} \bigcirc \\ \mathcal{D}_{\mathcal{E}} \\ \bigcirc \end{array} \right) \begin{array}{c} \bigcirc \\ \mathcal{D}_b \\ \bigcirc \end{array} = \Re \left[\begin{array}{c} \bigcirc \\ \mathcal{E} \\ \bigcirc \end{array} \right] \begin{array}{c} \bigcirc \\ \mathcal{D}_b \\ \bigcirc \end{array}$$

Figure 2: Annulus 1-loop vacuum diagram.

Analogously, one finds a relation for the instanton vacuum Möbius strip amplitudes $\frac{\bigcirc\otimes}{\mathcal{D}_{\mathcal{E}}\mathcal{O}} = \Re \left[\frac{\bigcirc\otimes}{\mathcal{E}\mathcal{O}} \right]$.

All disc and 1-loop CFT amplitudes in the instanton correlator (22) factorize into holomorphic and non-holomorphic parts. For disc amplitudes a formula completely analogous to (21) holds and for the annulus and Möbius strip amplitudes one employs the Kaplunovsky-Louis formula

$$\begin{aligned} \sum_b \Re \left[\frac{\bigcirc\bigcirc}{\mathcal{E}\mathcal{D}_b} \right] + \Re \left[\frac{\bigcirc\otimes}{\mathcal{E}\mathcal{O}} \right] &= -8\pi^2 \text{Re}(f_{\mathcal{E}}^{(1)}) - \frac{\beta}{2} \log \left(\frac{M_p^2}{\mu^2} \right) - \frac{\gamma}{2} \mathcal{K}_{\text{tree}} \quad (25) \\ &\quad - \log \left(\frac{V_{\mathcal{E}}}{g_s} \right)_{\text{tree}} + \sum_b \frac{|I_{\mathcal{E},\mathcal{D}_b} N_b|}{2} \log [\det K^{\mathcal{E}\mathcal{D}_b}]_{\text{tree}} . \end{aligned}$$

Here $f_{\mathcal{E}}^{(1)}$ denotes the holomorphic piece from the annulus diagram (Wilsonian one-loop threshold correction). For the brane and instanton configuration in question the coefficients are

$$\beta = \sum_b \frac{|I_{\mathcal{E},\mathcal{D}_b} N_b|}{2} - 3, \quad \gamma = \sum_b \frac{|I_{\mathcal{E},\mathcal{D}_b} N_b|}{2} - 1. \quad (26)$$

The sum over the \mathcal{D}_b branes include also the Ω_p image branes. The coefficient β is nothing else than the one-loop β -function coefficient for the gauge theory of an auxiliary \mathcal{D} brane wrapping the same cycle as \mathcal{E} . It involves the one-loop correction of massless modes. However, since in (22) the integral over these zero modes is carried out explicitly, to avoid double counting we have to remove their contribution from (26). This is the definition of the $*$ in (22). Consistently, the

zero mode measure leads precisely to a divergence

$$\mu^{\frac{N_f}{2} - N_b} = \mu^\beta \quad (27)$$

with N_f denoting the total number of fermionic zero modes and N_b the number of bosonic zero modes.

It was shown in [49] that a number of cancellations appear, which indeed allow one to express the holomorphic superpotential coupling entirely in terms of the holomorphic couplings in the CFT amplitudes

$$Y_{\Phi_{a_1, b_1}, \dots, \Phi_{a_M, b_M}} = \sum_{\text{conf.}} \exp(-S_{\mathcal{E}}^{(0)}) \exp(-f_{\mathcal{E}}^{(1)}) \quad (28)$$

$$Y_{\lambda_{a_1} \hat{\Phi}_{a_1, b_1}[\vec{x}_1] \bar{\lambda}_{b_1}} \cdot \dots \cdot Y_{\lambda_{a_L} \hat{\Phi}_{a_L, b_L}[\vec{x}_L] \bar{\lambda}_{b_L}}.$$

This can be considered a non-trivial consistency check of the instanton calculus. In particular the one-loop vacuum amplitudes and the rule that only two charged zero modes are attached to each disc play a crucial role. The latter prescription is also a consequence of the following observation: If one replaces the instanton by a spacetime-filling D-brane the vertex operators of the charged zero modes will correspond to fermionic fields, and disc amplitudes with more than two fermions do not give rise to holomorphic contact terms. While these arguments make it clear that discs with more than two charged zero mode insertions do not yield superpotential terms, it would be interesting to further investigate their role.

In the sequel we will often denote by

$$S_{\mathcal{E}} = S_{\mathcal{E}}^{(0)} + f_{\mathcal{E}}^{(1)} \quad (29)$$

the tree-level plus one-loop holomorphic piece of the instanton suppression factor.

Let us mention that the Kaplunovsky-Louis formula (26) can also be applied to extract information on the non-holomorphic quantities, i.e. in particular on the matter field Kähler metrics. This was carried out for intersecting D6-branes in various toroidal orbifolds in [35, 50, 51, 52, 53].

3 GENERATION OF F-TERMS: OVERVIEW

Based on this instanton calculus, a lot of recent work has been devoted to its applications and generalizations. This section aims at providing a guide through and a logical ordering of the extensive literature. Since we will elaborate on phenomenological applications to string model building with Type II orientifolds in §5, here we discuss other interesting, but slightly more formal developments. A summary of the various effects associated with BPS instantons of different types is given in table 5.

instanton gauge group	parallel branes	universal zero modes	extra fermionic zero modes	effective contribution
$\mathcal{O}(1)$	-	x^μ, θ^α	- γ^α $\bar{\chi}^{\dot{\alpha}}$	superpotential gauge kin. function multi-fermion for vectors
$U(1)$	- - 1 $N_c > 1$	$x^\mu, \theta^\alpha, \bar{\tau}^{\dot{\alpha}}$	- $\bar{\mu}^{\dot{\alpha}}, \bar{\nu}^{\dot{\alpha}}$ ADHM ADHM	multi-fermion for hypers multi-fermion for vectors/ superpotential superpotential gauge instanton

Table 5: Overview of F-term generation by BPS instantons in absence of background flux.

3.1 Gauge instanton effects

ADS superpotential for $SU(N_c)$ SQCD

Historically the string theoretic derivation [9, 10] of the famous ADHM construction of gauge instantons [54] was among the first appearances of Euclidean D-branes in the context of instanton computations and has been the subject of many early investigations including [15, 16, 17] (for reviews and more references see [55, 56, 57]).

Here we will focus on well-known non-perturbative effects in $\mathcal{N} = 1$ supersymmetric gauge theories [58]. It is interesting to verify that indeed the D-brane instanton calculus contains these effects. In this section, we provide some of the details of how this works for the prototype example of $SU(N_c)$ $\mathcal{N} = 1$ supersymmetric QCD with $N_f = N_c - 1$ flavors. In this supersymmetric field theory a gauge instanton generates the so-called Affleck-Dine-Seiberg (ADS) superpotential [59]

$$S_W = \int d^4x d^2\theta \frac{\Lambda^{3N_c - N_f}}{\det[M_{ff'}]}, \quad (30)$$

where $M_{ff'}$ is the meson matrix and Λ the dynamically generated scale. The way to proceed is to engineer a local D-brane set-up realizing this situation and to describe the gauge instanton as a D-brane instanton. One then computes the instanton amplitude following the rules explained in §2.3 and takes the field theory limit.

To engineer $\mathcal{N} = 1$ SQCD we wrap a stack of N_c D-branes on a rigid cycle and N_f D-branes on another cycle such that the intersection realizes precisely one vector-like pair of matter fields Q and \tilde{Q} transforming in the bifundamental representation (N_c, N_f) .⁵ The gauge instanton (in the zero size limit) is described

⁵Such configurations can also be engineered in quiver gauge theories arising from fractional

in string theory by a Euclidean D-brane \mathcal{E} wrapping the same internal cycle as the color brane \mathcal{D}_c . The final local $\mathcal{D} - \mathcal{E}$ brane configuration and the resulting zero modes are shown in figure 3.

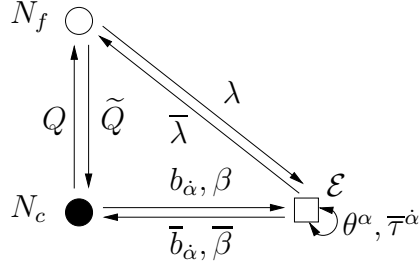


Figure 3: Extended quiver diagram showing the matter fields and the instanton zero modes for $SU(N_c)$ SQCD with N_f flavors.

Let us discuss the appearing zero modes in some more detail:

\mathcal{E} - \mathcal{E} : Since the \mathcal{E} -instanton wraps the same rigid cycle as the \mathcal{D}_c brane it is a $U(1)$ instanton. Its universal \mathcal{E} - \mathcal{E} zero modes are the four positions in Minkowski space x^μ with $\mu = 0, \dots, 3$ and the four fermionic zero modes $\theta^\alpha, \bar{\tau}^{\dot{\alpha}}$.

\mathcal{E} - \mathcal{D}_f : In this sector, one only gets the N_f pairs of non-chiral $\lambda_f, \bar{\lambda}_f$ zero modes from Table 3.

\mathcal{E} - \mathcal{D}_c : This sector, which has not been discussed in §2.2, is characteristic of gauge instantons. Since the \mathcal{E} -instanton and the \mathcal{D}_c branes wrap the same cycle, open strings between them are subject to six NN, DD boundary conditions and four DN boundary conditions. Therefore, the ground state energy in both the NS- and the R-sector vanishes and one finds $4N_c$ bosonic zero modes $b_\alpha^u, \bar{b}_{\dot{\alpha}, u}$ and $2N_c$ fermionic ones $\beta^u, \bar{\beta}_u$ with $u = 1, \dots, N_c$.

However, not all of these zero modes are independent. In fact, as shown in [17] the effective action on the \mathcal{E} contains the two terms

$$S_1 = i \bar{\tau}^{\dot{\alpha}} (b_\alpha^u \bar{\beta}_u + \bar{b}_{\dot{\alpha}, u} \beta^u) - i D^c \left(\bar{b}_u^{\dot{\alpha}} (\tau_c)_{\dot{\alpha}}^\beta b_\beta^u \right). \quad (31)$$

Here D_c with $c = 1, 2, 3$ are auxiliary fields which, together with the extra Goldstinos $\bar{\tau}^{\dot{\alpha}}$, appear as Lagrangian multipliers implementing the D- and F-term constraints in the effective action on the instanton. Integrating out $\bar{\tau}^{\dot{\alpha}}$ and D^c yields precisely the fermionic and, respectively, bosonic ADHM constraints for the case of a single instanton,

$$b_\alpha \bar{\beta} + \bar{b}_{\dot{\alpha}} \beta = 0, \quad \bar{b}^{\dot{\alpha}} (\tau_c)_{\dot{\alpha}}^\beta b_\beta = 0. \quad (32)$$

Note that the universal zero modes $\bar{\tau}^{\dot{\alpha}}$ are soaked up in this process. This is the microscopic reason why an \mathcal{E} brane instanton on top of a \mathcal{D} brane can contribute

D-branes on singularities. More on that in section 4.

to the holomorphic superpotential

$$S = \int d^4x d^2\theta W \quad (33)$$

with

$$W = \mathcal{C} \int d\{b_{\dot{\alpha}}^u, \bar{b}_{\dot{\alpha},u}, \beta^u, \bar{\beta}_u, \lambda_f, \bar{\lambda}_f\} \delta(b_{\dot{\alpha}} \bar{\beta} + \bar{b}_{\dot{\alpha}} \beta) \delta(\bar{b}^{\dot{\alpha}} (\tau_c)_{\dot{\alpha}}^{\dot{\beta}} b_{\dot{\beta}}) e^{-S_{\mathcal{E}} - S_2}. \quad (34)$$

Here in the two fermionic and three bosonic ADHM constraints summation over the color index is understood.

One now has to soak up the remaining fermionic zero modes in such a way that the bosonic integral converges. The computation of the appropriate three-point and four-point disc amplitudes yields couplings [48]

$$S_2 = \beta^u (Q^\dagger)_u^f \bar{\lambda}_f + \lambda^f (\tilde{Q}^\dagger)_f^u \bar{\beta}_u + \frac{1}{2} \bar{b}_{\dot{\alpha},u} \left(Q_f^u (Q^\dagger)_v^f + (\tilde{Q}^\dagger)_f^u \tilde{Q}_v^f \right) b^{\dot{\alpha},v}. \quad (35)$$

Note that it is really the anti-holomorphic fields Q^\dagger and \tilde{Q}^\dagger which enter into these couplings. Inserting this action into eq. (34), one can now compute the resulting integrals. In view of the two fermionic ADHM constraints, a simple counting argument yields that only for $N_f = N_c - 1$ the fermionic zero mode integral is non-vanishing.⁶ After first integrating over the fermionic zero modes, one is left with a Gaussian integration over the bosonic ones. These integrals can be carried out as detailed for instance in [48]. The D-terms for the $SU(N_c)$ gauge theory constrain the vevs of the quark fields such that $Q Q^\dagger = \tilde{Q}^\dagger \tilde{Q}$. This indeed leads to a cancellation of the anti-holomorphic terms. One eventually arrives at the ADS superpotential

$$W = \frac{M_s^{2N_c+1}}{\det(\tilde{Q} Q)} \exp\left(-\frac{8\pi^2}{g_c^2(M_s)}\right) = \frac{\Lambda^{3N_c-N_f}}{\det[M_{ff'}]}, \quad (36)$$

where we have introduced the correct dimensionfull scale and, in the field theory limit, have neglected all contributions from massive modes in the vacuum one-loop diagrams. The dynamically generated scale Λ is defined as

$$\left(\frac{\Lambda}{\mu}\right)^{3N_c-N_f} = \exp\left(-\frac{8\pi^2}{g_c^2(\mu)}\right). \quad (37)$$

Very similar computations can be performed for $\mathcal{N} = 1$ SQCD like theories with gauge groups $SO(N_c)$ and $SP(2N_c)$ with $N_f = N_c - 3$ and $N_f = N_c$ flavors, respectively. Here the engineering of the gauge theory requires the introduction of orientifold planes [48]. Various generalizations of such local $\mathcal{N} = 1$ quiver type gauge theories and the respective modelling of gauge instanton effects by Euclidean D-branes have been discussed in the literature [60, 40, 41, 61, 62, 63]. A prototype of such geometries will be explained in §4.

⁶For $N_f \geq N_c$ the remaining zero modes have to be absorbed by additional interactions which will be discussed in §3.3 and lead to higher fermion F-terms instead of a superpotential.

Stringy instantons for the special case $N_c = 1$ and generalizations

The ADHM computation performed around equ. (34) admits an interesting application beyond proper gauge instantons. In fact, the absorption of the $\bar{\tau}^\alpha$ modes with the help of the bosonic and fermionic zero modes in the $\mathcal{D}_c - \mathcal{E}$ sector works even in the special case $N_c = 1$ [64]. This describes an instanton along a cycle not-invariant under the orientifold action that is wrapped by a single spacetime-filling D-brane. From the point of view of the abelian gauge theory along this cycle the instanton effect should not be interpreted as a gauge instanton since a $U(1)$ theory does not lead to any strong gauge dynamics. Still, after absorption of the $\bar{\tau}^\alpha$ modes and performing the bosonic moduli integral the instanton can generate a superpotential term [64, 63] - provided a contribution is not prohibited by additional modes such as those arising in the $\mathcal{E} - \mathcal{E}'$ sector. This effect had been anticipated by quite different methods in [65] and will be analysed more closely in §4.2 and §4.3.

In [66, 67] it was proposed that this reasoning holds true even in a more general situation. These papers consider a single E3-instanton wrapping the same cycle as a D7-brane, but carrying in addition non-trivial gauge flux $\mathcal{F}_{E3} \neq \mathcal{F}_{D7}$. In this case the bosonic and fermionic modes b_α, β in the E3-D7 sector are massive due to the twisting by the relative gauge flux. However, it was argued that couplings of the form (31) involving their Kaluza-Klein partners, i.e. $\bar{\tau}^\alpha (b_\alpha^{KK} \bar{\beta}^{KK} + \bar{b}_\alpha^{KK} \beta^{KK})$, can still be used to saturate the $\bar{\tau}$ -modes as to generate a superpotential. Note that this is in deviation from our previous policy to include the massive modes, which are off-shell, only in the one-loop factors and not in the instanton effective action. It will be interesting to further verify if such configurations, which were also used in the phenomenological applications of [68, 69], really contribute to the superpotential.

3.2 Corrections to the gauge kinetic function

Up to now we have discussed instanton induced corrections to the superpotential. In $\mathcal{N} = 1$ supergravity there exists another holomorphic quantity, namely the gauge kinetic function appearing in

$$S_{\text{Gauge}} = \int d^4x d^2\theta f(T, U) \text{tr} W^\alpha W_\alpha. \quad (38)$$

Here we consider the gauge fields to come from space-time filling intersecting \mathcal{D}_a branes. In §2.1 we have already discussed that $f(T, U)$ only receives perturbative corrections up to one-loop order beyond which only D-brane instanton corrections are allowed. Moreover, we discussed the possible dependence on the complex structure U and Kähler moduli T .

Again $O(1)$ instantons have the correct universal zero mode structure to yield a non-vanishing contribution to this F-term. But the two zero modes θ^α alone

are not sufficient to generate a non-vanishing instanton amplitude. It turns out that one needs precisely one pair of deformation zero modes of the type γ^α [35] as listed in the first line of table 2. In addition there must be no charged zero modes from intersections with other D-branes. For a discussion of analogous corrections in heterotic string theory by worldsheet instantons wrapping higher genus curves see [70].

The four fermionic zero modes $\theta^\alpha, \gamma^\alpha$ of the instanton can be absorbed by an annulus diagram as shown in figure 4.

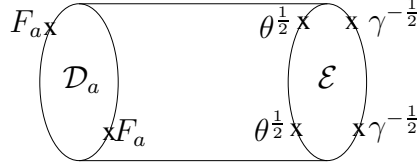


Figure 4: Annulus diagram for an \mathcal{E} -instanton correction to the gauge coupling of a stack of \mathcal{D}_a branes. The upper indices give the ghost number of the vertex operators.

The total amplitude that computes an \mathcal{E} -instanton correction to the gauge kinetic function f_a has the form

$$f_a^\mathcal{E} = \int d^2\theta d^2\gamma \left(\text{diagram of } \mathcal{D}_a \text{ and } \mathcal{E} \text{ with } \theta \text{ and } \gamma \text{ insertions} \right) e^{-S_\mathcal{E}^{(0)}} \exp \left(\sum_b \left(\text{diagram of } \mathcal{E} \text{ and } \mathcal{D}_b \text{ with } \theta \text{ and } \gamma \text{ insertions} \right) + \text{diagram of } \mathcal{E} \text{ and } \mathcal{O} \text{ with } \theta \text{ and } \gamma \text{ insertions} \right). \quad (39)$$

The last factor represents the exponentiated disc and one-loop vacuum diagrams with at least one boundary on the \mathcal{E} instanton, as is by now familiar from the superpotential calculus.

Next we need to know the zero mode absorption amplitude between the \mathcal{D}_a -brane and an \mathcal{E} -instanton. It was shown in [71] that this at first sight highly complicated six-point function can be related to the derivative of a much easier two-point function. In fact it is the derivation of a one-loop gauge threshold correction with respect to the deformation moduli m

$$\Re \left[\int d^2\theta d^2\gamma \left(\text{diagram of } \mathcal{D}_a \text{ and } \mathcal{E} \text{ with } \theta \text{ and } \gamma \text{ insertions} \right) \right] = \frac{\partial^2}{\partial m^2} \left(\text{diagram of } \mathcal{D} \text{ and } \mathcal{D}_\mathcal{E} \text{ with } \theta \text{ and } \gamma \text{ insertions} \right) \Big|_{m=m_0}, \quad (40)$$

where again $\mathcal{D}_\mathcal{E}$ denotes an auxiliary space-time filling \mathcal{D} brane wrapping the same internal cycle as the instanton brane \mathcal{E} .

Such single instanton corrections to the gauge kinetic function were evaluated for a simple toroidal orientifold model in [71]. These results were compared to worldsheet instanton corrections in an S-dual heterotic string model [72] (see also [73]) and complete agreement for the single instanton contributions were found. It was pointed out in [71] that the existence of instanton corrections to the gauge kinetic function of D-branes leads to an iterative structure. This can be interpreted

as multi-D-brane instanton corrections to f_a , where the additional zero modes are absorbed among the instantons themselves. More on that will be presented in §3.4 on multi-instanton corrections. For a different recent test of *six-dimensional* Type I - Heterotic duality with the help of stringy instantons see [74].

3.3 Beasley-Witten F-terms

There exists yet another class of supersymmetric F-terms in addition to the familiar superpotential and gauge kinetic function. At the fermionic level these interactions involve a product of $2n$ anti-chiral Weyl fermions beyond the chiral fermion bilinear characteristic of a superpotential. They are therefore often called multi-fermion or higher derivative F-terms. Such interactions were first studied systematically by Beasley and Witten, originally in connection with $\mathcal{N} = 1$ supersymmetric QCD with gauge group $SU(N_c)$ and $N_f \geq N_c$ (in the special case $N_c = 2$ in [75]) and more generally in the context of heterotic worldsheet instantons in [70].

In superspace notation, multi-fermion F-terms can be written as

$$S = \int d^4x d^2\theta w_{\bar{i}_1, \dots, \bar{i}_n, \bar{j}_1, \dots, \bar{j}_n}(\Psi) \left(\bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi}^{\bar{i}_1} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^{\bar{j}_1} \right) \dots \left(\bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi}^{\bar{i}_n} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^{\bar{j}_n} \right). \quad (41)$$

Here the degrees of freedom assembled in the chiral superfield $\Phi = \varphi + \theta^\alpha \psi_\alpha$ appear in the combination

$$\bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi} = \bar{\psi}^{\dot{\alpha}} + \theta_\alpha (\sigma^\mu)^{\dot{\alpha}\alpha} \partial_\mu \bar{\varphi}. \quad (42)$$

Even though this is not manifest in equ. (41), multi-fermion F-terms are supersymmetric if ω depends holomorphically on some chiral superfields Ψ and is antisymmetric in the \bar{i} and \bar{j} indices separately and symmetric under their exchange. In addition ω is subject to a certain equivalence relation discussed in detail in [70] which ensures that (41) cannot be written globally as a D-term. More background on the geometric interpretation of higher F-terms can be found in [75, 70]. For example the case $n = 1$, corresponding to a four-fermi interaction, is of the form that describes quantum deformations of the moduli space of $\mathcal{N} = 1$ supersymmetric QCD [76] with $N_f = N_c$ [75]. This concept of encoding deformations of the moduli space of $\mathcal{N} = 1$ supersymmetric theories in higher F-terms is more general [70].

At a technical level, multi-fermion F-terms of degree n are generated by a BPS-instanton whose zero modes comprise n extra anti-chiral Weyl spinors, denoted collectively as $\bar{\mu}_{\bar{i}}^{\dot{\alpha}}$, $\bar{i} = 1, \dots, n$, which couple in the instanton effective action as

$$S_{B.W.} = (\bar{\mu}_{\bar{i}}^{\dot{\alpha}})^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^{\bar{i}}. \quad (43)$$

Integrating out these extra anti-chiral zero modes pulls down corresponding powers of $\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}$. Depending on the nature of the spinors $\bar{\mu}_{\bar{i}}^{\dot{\alpha}}$ one obtains

- deformations of the moduli space of SQCD with gauge group $SU(N_c)$ and $N_f \geq N_c$ (and generalizations thereof) from gauge instantons; here the $(\bar{\mu}_{\tilde{t}})^{\dot{\alpha}}$ are the bosonic modes $b^{\dot{\alpha}}, \bar{b}^{\dot{\alpha}}$ in the $\mathcal{E} - D_c$ sector,
- deformations of the vector multiplet moduli space from stringy $\mathcal{O}(1)$ instantons with n deformation modes $(c, \bar{\chi}^{\dot{\alpha}})$, or
- deformations of the hypermultiplet moduli space from stringy isolated $U(1)$ instantons due to the universal zero modes $\bar{\tau}^{\dot{\alpha}}$.

We now discuss the different cases in turn.

Gauge instantons

Consider again the microscopic realization of a gauge instanton in $\mathcal{N} = 1$ SQCD with gauge group $SU(N_c)$ and N_f flavors as introduced in §3.1. For $N_f = N_c - 1$ such an instanton reproduces the ADS superpotential (29) upon absorbing the fermionic zero modes via the terms (31) and (34) in the instanton effective action. As argued in [77] the instanton effective action contains in addition the couplings

$$S_3 = \bar{b}_{\dot{\alpha},u} (\bar{\mathcal{D}}^{\dot{\alpha}} \tilde{Q})_f^u \lambda^f + b_{\dot{\alpha},u} (\bar{\mathcal{D}}^{\dot{\alpha}} \bar{Q})_f^u \bar{\lambda}^f. \quad (44)$$

The existence of these terms was verified in [62] via an explicit CFT computation for a local $D3 - E(-1)$ system on $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$.

The point is now that while $N_c - 1$ of the N_f pairs of zero modes λ^f and $\bar{\lambda}^f$ are already saturated by the interaction terms (31) and (34), for $N_f \geq N_c$ the remaining $n = N_f - (N_c - 1)$ pairs have to be absorbed via the couplings (44). This pulls down a term $(\bar{\mathcal{D}}^{\dot{\alpha}} \bar{Q} \mathcal{D}_{\dot{\alpha}} \tilde{Q})^n$.

Integrating out the ADHM moduli is more complicated than for $N_f = N_c - 1$ due to the appearance of extra factors of bosonic modes $b^{\dot{\alpha}}, \bar{b}^{\dot{\alpha}}$ and we refrain from performing this computation here. E.g. for simplest case $N_f = N_c = 2$ the final result is proportional to [77, 62]

$$S = \text{tr}(\bar{M}M)^{-3/2} \epsilon^{ijkl} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{M}_{ij} \mathcal{D}^{\dot{\alpha}} \bar{M}_{kl} \quad (45)$$

in terms of the meson field $\bar{M}_{ij} = \epsilon_{uv} Q_i^u Q_j^v$, in agreement with the field theoretic derivation of [75]. Generalizations to $N_f > N_c$ and to SQCD with gauge group $SP(2N_c)$ and $N_f \geq N_c + 1$ can be found in [77, 62].

Deformations of the vector multiplet moduli space and open string terms

Let us move on to the generation of higher F-terms by stringy instantons with additional zero modes other than $d^4x d^2\theta$. The first example of such modes are

the subclass of deformation moduli $(c, \bar{\chi}^\alpha)$ displayed in the second line of table 2. For heterotic worldsheet instantons analyzed in [70] these moduli correspond to instantons along Riemann spheres moving in a family.

In general the anti-chiral deformation moduli couple to the closed string moduli sitting in those $\mathcal{N} = 1$ chiral multiplets which descend from vector multiplets of the underlying $\mathcal{N} = 2$ compactification [43]. The respective closed string fields in Type I/Type IIB orientifolds and Type IIA orientifolds are the complex structure and Kähler moduli.

To be explicit consider an E2-instanton in Type IIA orientifolds of $\mathcal{O}(1)$ type with $b_1(E2) = b_1(E2)_+ = 1$ and corresponding deformation moduli $(c, \bar{\chi}^\alpha)$. If we schematically denote by $\mathcal{T} = T + \theta^\alpha t_\alpha$ the $\mathcal{N} = 1$ chiral superfields associated with the Kähler moduli, then the $\bar{\chi}^\alpha$ -moduli couple in the instanton effective action as

$$S_{B.W.} = \bar{\chi}^\alpha \bar{\mathcal{D}}_\alpha \bar{\mathcal{T}}. \quad (46)$$

This was demonstrated in [43] by verifying that both the fermionic open-closed disc amplitude $\langle \bar{\chi} \bar{t} \rangle$ and its superpartner $\langle \theta^\alpha \bar{\chi}^\alpha \bar{T} \rangle$ are allowed by $U(1)$ worldsheet charge selection rules. Note that the latter coupling leads to the interaction term $\theta \sigma^\mu \bar{\chi} \partial_\mu \bar{T}$, which indeed combines with $\bar{\chi}^\alpha \bar{t}_\alpha$ into the coupling (46).

Integration over the deformation modulus ⁷ yields an F-term of the form (41),

$$S = \int d^4x d^2\theta e^{-S_\mathcal{E}} f_{\bar{i}, \bar{j}} \bar{\mathcal{D}}^{\bar{i}} \bar{\mathcal{T}}^{\bar{i}} \bar{\mathcal{D}}_{\bar{\alpha}} \bar{\mathcal{T}}^{\bar{j}}. \quad (47)$$

Here $S_\mathcal{E} = S_\mathcal{E}^{(0)} + f_\mathcal{E}^{(1)}$ denotes again the tree-level plus one-loop corrected action of \mathcal{E} as in the case of an ordinary superpotential. The information on the concrete vector multiplet moduli appearing in the Beasley-Witten term is encoded in the tensor $f_{\bar{i}, \bar{j}}$ (for the case with one deformation modulus) and depends on the geometric details of the setup. For an explicit example on $K_3 \times T_2$ see [78].

Finally, in the presence of suitable charged zero modes the CFT selection rules also allow Beasley-Witten terms involving open string fields in the $\mathcal{D}_i - \mathcal{D}_i$ or $\mathcal{D}_i - \mathcal{D}_j$ sector of other D-branes [43]. This requires instanton couplings of the schematic type

$$S_{B.W.} = \lambda_a \bar{\chi}^\alpha \bar{\mathcal{D}}_\alpha \bar{\Phi}_{ab} \bar{\lambda}_b \quad (48)$$

and charged zero modes $\lambda_a, \bar{\lambda}_b$ in the $\mathcal{E} - \mathcal{D}_a$ and $\mathcal{D}_b - \mathcal{E}$ sector, respectively.

Deformations of the hypermultiplet moduli space

A new phenomenon arises for stringy $U(1)$ instantons along a non-invariant supersymmetric cycle due the presence of the extra anti-chiral Goldstinos $\bar{\tau}^\alpha$. As

⁷The bosonic moduli c decouple from the computation and merely lead to powers of moduli space volume.

discussed in [43] if these modes are not lifted by any other mechanism, such $U(1)$ instantons generate four-fermi interactions of Beasley-Witten type which involve the chiral fields descending from the hypermultiplets of the underlying $\mathcal{N} = 2$ compactification. This is what happens e.g. for an isolated $U(1)$ instanton not intersecting its orientifold image. The relevant terms lifting the $\bar{\tau}$ -modes in the instanton effective are completely analogous to the ones described for the deformation modes except that they involve the hypermultiplets. For the example of E2-instantons in Type IIA orientifolds with complex structure moduli $\mathcal{U} = U + \theta^\alpha u_\alpha$, the instanton effective action schematically contains the coupling [43]

$$S_{B.W.} = \bar{\tau}^\alpha \bar{\mathcal{D}}_\alpha \bar{\mathcal{U}}, \quad (49)$$

as follows again from general $U(1)$ worldsheet charge selection rules of the open-closed CFT. In view of the role of the θ^α and $\bar{\tau}^\alpha$ modes as Goldstinos associated with the two different $\mathcal{N} = 1$ subalgebras, cf. table 1, it was further argued in [79] that the combination of hypermultiplet moduli appearing in (49) is precisely the superfield Σ whose bosonic vev controls the Fayet-Iliopoulos term of the instanton cycle and therefore vanishes for an instanton on its BPS locus. Note that Σ appears only in the combination $\bar{\mathcal{D}}\Sigma$ so that only the derivatives of its bosonic components enter, see equ. (42). For a rigid E2-instanton the resulting Beasley-Witten term is then of the form

$$S = \int d^4x d^2\theta e^{-S_\varepsilon} \bar{\mathcal{D}}^\alpha \bar{\Sigma} \bar{\mathcal{D}}_\alpha \bar{\Sigma}. \quad (50)$$

3.4 Multi-Instanton processes

Our presentation has so far focused on single-instanton processes. As anticipated already in §2.2 the general situation involves multiple instantons at the same time. There are at least two ways in which such multi-instanton corrections are almost forced upon us. First, in many cases what is described as a single instanton effect in some regions of moduli space becomes a multi-instanton process for other values of the closed string moduli [43, 80, 81, 79]. Second, there also exist configurations where the multi-instanton contribution is not related to the decay of stable BPS instantons into several BPS constituents [71] but arises due to the iterated effect that stringy instantons can correct string instanton actions.

Multi-instantons and Instanton recombination

This is due to the appearance of lines of marginal or threshold stability in closed string moduli space where supersymmetric cycles decay into several cycles. This transforms the single instanton associated with the original cycle into a multi-instanton. The closed string moduli governing this behaviour are the ones descending from the $\mathcal{N} = 2$ hypermultiplets, i.e. the Kähler and complex structure

moduli for Type IIB and Type IIA orientifolds, respectively. There also exist configurations where the multi-instanton contribution is not related to the decay of stable BPS instantons into several BPS constituents [71].

Instantons across lines of threshold stability

The simplest and possibly most abundant type of configurations of the first kind is that of an $O(1)$ instanton decaying into a $U(1)$ instanton and its orientifold image. The latter system is really a two-instanton configuration, at least in the upstairs geometry prior to orientifolding. This process was first analysed in [43]. Other configurations including such beyond instanton-image instanton systems were considered in [80]. $O(1)$ instantons that undergo such a decay are sometimes referred to as decomposable, in contrast to isolated $U(1)$ instantons which never merge with their orientifold image into a single invariant instanton.

Following [43] let us consider this process of instanton recombination of an $\mathcal{E} - \mathcal{E}'$ system starting from the locus in hypermultiplet space where the cycle and its image are split. The recombination moduli are given by the $\mathcal{E} - \mathcal{E}'$ modes of table 4. To illustrate the point let us consider the situation where we have only one vector-like pair of such zero modes $(m, \bar{m}, \bar{\mu}^{\dot{\alpha}})$ and $(n, \bar{n}, \bar{\nu}^{\dot{\alpha}})$. The universal moduli of the two sectors are identified, and if we assume that the instanton has no further deformation or charged modes we have to cope with the measure

$$\int d\mathcal{M} = \int d^4x d^2\theta d^2\bar{\tau} dm d\bar{m} d^2\bar{\mu} dn d\bar{n} d^2\bar{\nu}. \quad (51)$$

At first sight it seems hopeless to ever generate a superpotential term. In particular the extra Goldstone modes $\bar{\tau}^{\dot{\alpha}}$ appear as an obstruction. The crucial point is, though, that there exist new interaction terms in the instanton effective action that allow us to absorb in particular the extra fermionic modes. First to mention is the interaction

$$S_{\bar{\tau}} = \bar{\tau}_{\dot{\alpha}} (m \bar{\mu}^{\dot{\alpha}} - n \bar{\nu}^{\dot{\alpha}}), \quad (52)$$

whose generic presence was shown in [43] with elementary CFT methods. It follows that the $\bar{\tau}^{\dot{\alpha}}$ -modes absorb one linear combination of the fermionic zero modes $\bar{\mu}^{\dot{\alpha}}, \bar{\nu}^{\dot{\alpha}}$, bringing down in addition two powers of bosonic modes from the exponents. The resulting bosonic integral is damped due to the D-term

$$S_D = (2m\bar{m} - 2n\bar{n} - \xi(\mathcal{U}))^2, \quad (53)$$

see also [80]. This D-term is in complete analogy with the situation for spacetime-filling \mathcal{D} branes. In particular the Fayet-Iliopoulos term ξ depends, as always, on the hypermultiplet moduli and measures the misalignment of the \mathcal{E} (and \mathcal{E}') brane with respect to the orientifold. It vanishes for supersymmetric configurations.

In absence of any other interaction terms, we are left with one extra pair of zero modes given by the linear combination of μ and ν which do not enter (52). In agreement with what we learnt so far, such an $\mathcal{E} - \mathcal{E}'$ configuration generates Beasley-Witten terms involving the vector multiplets. There may however be other interaction terms that lift also this remaining combination of modes. In this case the two-instanton does contribute to the superpotential. A particular way to lift these extra modes was proposed in [43] and involves extra charged zero modes. In [80] it is argued that there also exist configurations with couplings whose analogue for the D-branes would be derivable from a quartic superpotential term of the form

$$W = (MN)^2, \quad (54)$$

where M and N denote the chiral superfields corresponding to the bosonic and fermionic $\mathcal{E} - \mathcal{E}'$ modes. Taking into account that only the anti-chiral Weyl spinors $\bar{\mu}^{\dot{\alpha}}, \bar{\nu}^{\dot{\alpha}}$ survive the orientifold, this leads to bosonic and fermionic terms of the form

$$\begin{aligned} S_{quart} &= \bar{\mu} \bar{\mu} \bar{n} \bar{n} + \bar{\nu} \bar{\nu} \bar{m} \bar{m} + 2 \bar{\mu} \bar{\nu} \bar{m} \bar{n}, \\ S_F &= |m n m|^2 + |n m m|^2. \end{aligned} \quad (55)$$

In this case, S_{quart} allows also for the absorption of the remaining linear combination of fermionic $\mathcal{E} - \mathcal{E}'$ modes, and the configuration generates a superpotential term.

These considerations fit with the aforementioned picture of instanton recombination as follows. In absence of the F-term (55) for a supersymmetric configuration with vanishing Fayet-Iliopoulos terms the $\mathcal{E} - \mathcal{E}'$ system is at threshold with the bound state formed by condensing the bosonic moduli m, n in a D-flat manner. As is familiar from the context of D-branes the bound state corresponding to $|m| = |n|$ has one deformation modulus if the \mathcal{E} (and \mathcal{E}') brane is rigid. Similarly, if one hypothetically moves in hypermultiplet moduli space the Fayet-Iliopoulos parameter ξ becomes non-zero. Depending on its sign m or n acquires a vev in a D-flat manner. This is the recombination of instantons in different regions of moduli space referred to at the beginning of this subsection. Either way the recombined object is an $O(1)$ instanton with one deformation modulus and therefore just of the right kind to generate Beasley-Witten F-terms. By contrast, situations with a quartic superpotential of the form (54) describe an $\mathcal{E} - \mathcal{E}'$ state at threshold whose bound state, in regions of moduli space with $\xi \neq 0$, is indeed rigid and thus has every right to contribute to the superpotential. In particular, the computation on the two-instanton locus agrees with the expectations for the corresponding bound state. As argued in [80] this is as it has to be for the superpotential to be a holomorphic function in particular of the hypermultiplet moduli governing the instanton decay/recombination. Indeed, the line of threshold $\xi = 0$ describes a real codimension one surface in hypermultiplet space, and

a holomorphic function cannot jump across such a real surface. This analysis can be generalised to other multi-instanton configurations at threshold where the individual components are not related to one another by the orientifold action [80]. Further concrete examples along these lines appear in [82, 83].

Instantons across lines of marginal stability

In all these examples continuity of the superpotential across lines of threshold stability is guaranteed by the fact that for either sign of the Fayet-Iliopoulos term there does exist a supersymmetric multi-instanton configuration. More generally, however, supersymmetric cycles can actually decay across proper lines of marginal stability with no BPS object of the same charge existing on the other side. The simplest such situation occurs, in the present context, again for an $\mathcal{E} - \mathcal{E}'$ system with, however, just a single set of extra zero modes $m, \bar{m}, \bar{\mu}^\alpha$. As is apparent from the D-term

$$S_D = (2m\bar{m} - \xi(\mathcal{U}))^2, \quad (56)$$

a supersymmetric $O(1)$ instanton exists only in regions of moduli space where $\xi > 0$, while for $\xi < 0$ the $\mathcal{E} - \mathcal{E}'$ configuration ceases to be supersymmetric. Consistently, in this case there exists a microscopic obstruction for the $\mathcal{E} - \mathcal{E}'$ system to yield F-terms in the effective action [43]. The point is that in a globally supersymmetric configuration of this type there necessarily exist extra charged zero modes λ^i in the sector between \mathcal{E} and some of the D-branes present in the configuration. This follows from the net $U(1)_\mathcal{E}$ charge in the $\mathcal{E} - \mathcal{D}$ sector as

$$\begin{aligned} \sum_i Q_\mathcal{E}(\lambda^i) &= \sum_a N_a \left(-I_{\mathcal{E}, \mathcal{D}_a}^+ + I_{\mathcal{E}, \mathcal{D}_a}^- - I_{\mathcal{E}, \mathcal{D}_{a'}}^+ + I_{\mathcal{E}, \mathcal{D}_{a'}}^- \right) \\ &= - \sum_a N_a (I_{\mathcal{E}, \mathcal{D}_a} + I_{\mathcal{E}, \mathcal{D}_{a'}}). \end{aligned} \quad (57)$$

With the help of the tadpole cancellation condition the latter expression can be seen to be proportional to the chiral intersection number $I_{\mathcal{E}, O_6}$ [43]. According to table 4, for an $\mathcal{E} - \mathcal{E}'$ system with modes $m, \bar{m}, \bar{\mu}$ this is non-vanishing. Closer inspection reveals that these extra zero modes required for $U(1)_\mathcal{E}$ invariance of the zero mode measure cannot be lifted perturbatively in the instanton effective action. Their presence thus annihilates the contribution of the instanton to the superpotential. Again, this microscopic picture fits nicely with the arguments of [80] that an instanton undergoing actual decay should not generate holomorphic couplings. Having said this, there do exist more sophisticated multi-instanton setups where the role of the additional instantons is to lift the excess λ^i modes [81]. In agreement with the general philosophy the line of marginal stability is thereby transformed into a line of threshold stability, and again no instanton relevant for a superpotential can actually disappear from the BPS spectrum.

To summarize, we have seen that in favourable circumstances even $U(1)$ instantons in type II orientifolds can contribute to the superpotential despite the appearance of two extra Goldstone modes $\bar{\tau}^{\dot{\alpha}}$. For this to be the case it must be possible to interpret the configuration as a two-instanton configuration $\mathcal{E} - \mathcal{E}'$ in the geometry before orientifolding. The interactions in the $\mathcal{E} - \mathcal{E}'$ sector can then lift the extra Goldstones. Contributions are possible whenever there exists some region in hypermultiplet moduli space where this two-instanton configuration forms a bound state of $O(1)$ type, and the results in different patches of moduli space agree. By contrast, there do exist isolated $U(1)$ instantons which can never form a bound state with their image. Consistently, such instantons only yield Beasley-Witten type terms of the form (50), at least in absence of fluxes or other extra ingredients to lift the $\bar{\tau}^{\dot{\alpha}}$ Goldstones.

Our discussion has focused on the microscopic description of (multi-)instantons in Type II orientifolds preserving $\mathcal{N} = 1$ supersymmetry. Eventually one will want to find closed expressions by performing the sum over all multi-instanton contributions. In the context of $\mathcal{N} = 2$ supersymmetric Calabi-Yau compactifications of Type II theory powerful techniques have been developed to capture instanton corrections to the moduli space metric. Recent progress in determining instanton corrections to the hypermultiplet moduli space includes [84] (for Type IIB) and [85, 86, 87, 88, 89] (for Type IIA). Some supergravity techniques even carry over to $\mathcal{N} = 1$ orientifolds, see [90] for a recent example.

Finally, the continuity of physical quantities despite jumps in the BPS spectrum across lines of marginal stability can be made very concrete in field theoretic settings [91] and put in precise connection with the recent mathematical insights of [92].

Power towers of multi-instantons

Another interesting aspect of the multi-instanton configurations just described was put forward in [80]. The effect of a, say, two-instanton configuration system involving \mathcal{E}_1 and \mathcal{E}_2 can also be interpreted as the single instanton contribution from \mathcal{E}_1 after including non-perturbative corrections $\Delta S_{\mathcal{E}_1}^{n.p.}$ to the effective action of \mathcal{E}_1 due to \mathcal{E}_2 (or vice versa). Already in the simple example of the vectorlike $\mathcal{E} - \mathcal{E}'$ system introduced previously we can interpret \mathcal{E}' as generating an effective mass term for the $\bar{\tau}$ -modes of the form

$$\Delta S_{\mathcal{E}}^{n.p.} = \int d^2\bar{\mu} d^2\bar{\nu} dm d\bar{m} dn d\bar{n} \exp(-S_{\mathcal{E}'} - S_{\text{int}}(m, n, \bar{\mu}, \bar{\nu}, \bar{\tau})) = \bar{\tau}\bar{\tau} e^{-S_{\mathcal{E}'}}. \quad (58)$$

Here $S_{\mathcal{E}'} = S_{\mathcal{E}'}^{(0)} + f_{\mathcal{E}'}^{(1)}$ denotes the tree-level plus one-loop corrected action of \mathcal{E}' and $S_{\text{int}}(m, n, \bar{\mu}, \bar{\nu}, \bar{\tau})$ is the sum of the interaction terms (52), (53) and (55). In

this spirit the contribution of the instanton \mathcal{E} is schematically [80]

$$\begin{aligned} \int d^4x d^2\theta d^2\bar{\tau} \exp(-S_{\mathcal{E}} - \Delta S_{\mathcal{E}}^{n.p.}) &= \int d^4x d^2\theta d^2\bar{\tau} \exp(-S_{\mathcal{E}} - e^{-S_{\mathcal{E}'}} \bar{\tau}\bar{\tau}) \\ &= \int d^4x d^2\theta e^{-S_{\mathcal{E}} - S_{\mathcal{E}'}}. \end{aligned} \quad (59)$$

The final result follows once we take the square root due to the orientifold identification.

This picture was further generalised and extended in [71]. The starting point is the observation that the exponential suppression factor in an F-term generated by an instanton \mathcal{E} is given by the Wilsonian gauge kinetic function $f_{\mathcal{E}}$ of the hypothetical spacetime-filling D-brane $\mathcal{D}_{\mathcal{E}}$ wrapping the same cycle,

$$\exp(-S_{\mathcal{E}}) = \exp(-f_{\mathcal{E}}). \quad (60)$$

As discussed in §3.2, this gauge kinetic function $f_{\mathcal{E}}$ is itself corrected, beyond the one-loop thresholds $f_{\mathcal{E}}^{(1)}$, by suitable D-brane instantons,

$$f_{\mathcal{E}} = f_{\mathcal{E}}^{(0)} + f_{\mathcal{E}}^{(1)} + \sum_r f_{\mathcal{E}}^{\mathcal{E}_r}. \quad (61)$$

The non-perturbative correction $f_{\mathcal{E}}^{\mathcal{E}_r}$ due to another instanton \mathcal{E}_r is given in equ. (39). Recall that \mathcal{E}_r has to be an $O(1)$ instanton with precisely two Wilson line moduli γ_r^α . While the threshold $f_{\mathcal{E}}^{(1)}$ is already included in the instanton calculus outlined in §2.3, it is natural to conjecture that also the non-perturbative corrections $\sum_r f_{\mathcal{E}}^{\mathcal{E}_r}$ appear in the full answer. This means that the full exponential suppression factor of a non-perturbative holomorphic F-term is given by

$$\exp(-S_{\mathcal{E}}) = \exp\left(-S_{\mathcal{E}}^{(0)} - f_{\mathcal{E}}^{(1)} - \sum_r \int d^4x_r d^2\theta_r d^2\gamma_r \underbrace{\bigcirc}_{\mathcal{E}} \underbrace{\bigcirc}_{\mathcal{E}_r} e^{-S_{\mathcal{E}_r}^{(0)} - f_{\mathcal{E}_r}^{(1)} - \dots}\right). \quad (62)$$

Here x_r are the zero modes associated with the relative position of the instanton \mathcal{E} and \mathcal{E}_r , and $\theta_r^\alpha, \gamma_r^\alpha$ denote the universal and Wilson line fermionic zero modes of \mathcal{E}_r . Similar to equ. (40), it was shown in [71] that the holomorphic piece in the four-zero mode absorption amplitude $\underbrace{\bigcirc}_{\mathcal{E}} \underbrace{\bigcirc}_{\mathcal{E}_r}$ can be computed by the second derivative with respect to the Wilson line moduli of the threshold correction of the corresponding hypothetical space-time filling D-branes

$$\Re \left[\int d^4x_r d^2\theta_r d^2\gamma_r \underbrace{\bigcirc}_{\mathcal{E}} \underbrace{\bigcirc}_{\mathcal{E}_r} \right] = \frac{\partial^2}{\partial m^2} \underbrace{\bigcirc}_{\mathcal{D}_{\mathcal{E}}} \underbrace{\bigcirc}_{\mathcal{D}_{\mathcal{E}_r}} \Big|_{m=m_0}. \quad (63)$$

Let us assume for simplicity that $f_{\mathcal{E}}$ receives corrections from just one single instanton \mathcal{E}_r . In a somewhat reverse spirit as around eqs. (58) and (59) we can expand the exponent in terms of \mathcal{E}_r and interpret equ. (62) as the sum over

various multi-instanton configurations involving \mathcal{E} and n copies of the instanton \mathcal{E}_r , with $n = 0, 1, \dots$,

$$\begin{aligned} \int d\mathcal{M} e^{-S_{\mathcal{E}}} &= \int d\mathcal{M} e^{-S_{\mathcal{E}}^{(0)} - f_{\mathcal{E}}^{(1)}} + \\ &+ \int d\mathcal{M} \int d\mathcal{M}_r \left(\text{Diagram} \right) e^{-S_{\mathcal{E}_r}^{(0)} - f_{\mathcal{E}_r}^{(1)}} e^{-S_{\mathcal{E}}^{(0)} - f_{\mathcal{E}}^{(1)}} + \dots \end{aligned} \quad (64)$$

Here we abbreviated the respective zero mode measures by $d\mathcal{M}$ and $d\mathcal{M}_r$. To distinguish these stringy multi-instantons effects from ordinary field theory multi-instanton corrections, they were called poly-instantons in [71]. In general the world-volumes of the participating string instantons wrap different cycles of the underlying geometry.

Let us finally mention that the existence of these poly-instanton corrections is still puzzling. In [72] a heterotic freely acting $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with gauge group $SO(32)$ was considered and via standard worldsheet techniques the one-loop (in g_s) gauge threshold corrections were computed. This model has a proposed S-dual Type I description, for which the poly-instanton calculus was explicitly applied in [71]. It is expected that worldsheet instanton corrections for the heterotic string are mapped to E1-brane instanton corrections in the Type I string. Indeed, the heterotic result exhibits the usual sum over single worldsheet instantons multiply covering toroidal curves of the internal geometry. These contributions can precisely be matched on the Type I side. However, on the Type I side there are also non-vanishing contributions from poly-instantons, i.e. from E1-instantons not lying on top of each other in the sense that they carry different Wilson lines. These corrections are absent on the heterotic side. The resolution of this puzzle is still an open issue. If S-duality holds, two options seem conceivable. First, the naive (tree-level) S-duality map might receive instanton corrections. Second, since the starting point for computing the heterotic threshold corrections involves just the partition function of a single heterotic string, one might be missing these multiple (poly) worldsheet instanton corrections from the very beginning. More data seems to be necessary to settle this issue.⁸

3.5 Instantons and background fluxes

The details of the instanton calculus are modified in the presence of non-trivial closed string background fluxes. In general such fluxes can introduce couplings in the instanton effective moduli action which provide new ways to integrate out some of the fermionic zero modes. In this manner extra zero modes may acquire flux dependent effective mass terms and are thus lifted without inducing higher derivative F-terms.

⁸The heterotic worldsheet instanton corrections to the gauge kinetic function for a Gimon-Polchinski like model have been computed in [73]. Here the Type I S-dual computation still remains to be worked out.

Suppose for simplicity that an instanton \mathcal{E} possesses, in addition to the universal modes x^μ and θ^α , two extra zero modes $\bar{\psi}^{\dot{\alpha}}$. At this stage we are not specifying whether these correspond to deformation, Wilson line or extra universal modes. The situation we are interested in occurs when some background flux G induces a mass term of the form

$$S_G = \int \mathcal{O}_G \bar{\psi} \bar{\psi} \quad (65)$$

with a non-vanishing flux-dependent operator \mathcal{O}_G . Integrating out the $\bar{\psi}$ modes leads to a contribution of the form

$$\int d^4x d^2\theta d^2\bar{\psi} e^{-S_{\mathcal{E}} - \mathcal{O}_G \bar{\psi} \bar{\psi}} = \int d^4x d^2\theta \mathcal{O}_G e^{-S_{\mathcal{E}}}. \quad (66)$$

In principle the mechanism of equ. (66) can work for all types of D-brane instantons in orientifolds. Determining the non-vanishing operators of the form (65) requires precise knowledge of the couplings of the background fluxes to the instanton zero modes. These can be derived either within a supergravity approach starting from the superembedding of the worldvolume of the instanton or by a direct CFT computation of the relevant open-closed couplings. Most efforts in the literature have focused on the flux-induced lifting of deformation zero modes of M5-brane instantons in M/F-theory [93, 94, 95, 96] and of E3-brane instantons in Type IIB orientifolds [97, 98, 99, 100, 101, 102, 103]. More subtle is the effect of fluxes on the extra Goldstinos $\bar{\tau}^{\dot{\alpha}}$. It has been analysed for E3-brane instantons in [43, 104] and for fractional E(-1) instantons in [104, 62]. The general compatibility of fluxes and instanton contributions to the superpotential is analysed in the abstract and in concrete backgrounds in [105] and [60].

E3-instantons in Type IIB orientifolds with 3-form flux

The probably best understood class of flux compactifications is that of Type IIB Calabi-Yau orientifolds with O3/O7-branes and background 3-form flux. We will only consider compactifications of the type [106] with a non-trivial background value of $G_3 = F_3 - \tau H_3$ of RR- and NS-flux $F_3 = dC_2$ and $H_3 = dB_2$ and constant dilaton $\tau = C_0 + ie^{-\phi}$.

To set the stage recall from [106] that for compactifications with D3-, anti-D3 and D7-branes as well as O3/O7-planes as local sources, the global consistency conditions of the supergravity equations force the 3-form flux to be imaginary self-dual (ISD). Such ISD flux can be of Hodge type (0,3), (2,1) primitive or (1,2) non-primitive. Primitivity, which amounts to the condition $J \wedge G = 0$, is automatically satisfied for 3-form flux on Calabi-Yau spaces, which do not have non-trivial 5-cycles, but is a non-vacuous constraint on T^6 or $K3 \times T^2/\mathbb{Z}_2$. As shown in [107] in absence of any non-perturbative effects only the (2,1) primitive part of the 3-form flux satisfies $\mathcal{N} = 1$ supersymmetry. These results were generalised in [108,

101, 102]: In the presence of non-perturbative contributions to the superpotential beyond the flux-induced superpotential⁹, supersymmetric vacua exist also for flux of Hodge type (3,0), (1,2) and (0,3). This is because the F-terms induced by these fluxes can be cancelled against the F-terms of the dilaton, the complex structure and the Kähler moduli, respectively, such that $D_i W_{flux} + D_i W_{non-pert.} = 0$.

Our aim is to determine the flux induced mass terms of the form (65) for an E3-brane instanton wrapping a holomorphic divisor Γ of the Calabi-Yau. The result will be given schematically in eqs. (73) and (74). For explicitness we focus on the case of an isolated $U(1)$ instanton.¹⁰ In general such instantons can carry non-vanishing worldvolume flux $\mathcal{F} = F - B|_\Gamma$, where F is the gauge flux associated with the $U(1)$ gauge group of the instanton. For the time being, however, let us set $\mathcal{F} = 0$. As derived in [109, 110, 99] with supergravity methods and confirmed by the CFT analysis of [104] the 3-form flux couplings in absence of gauge flux take the form

$$S = \int_\Gamma d^4 \zeta \sqrt{\det g} \omega \left(e^{-\phi} \Gamma^{\tilde{m}} \nabla_{\tilde{m}} + \frac{1}{8} \tilde{G}_{\tilde{m}\tilde{n}p} \Gamma^{\tilde{m}\tilde{n}p} \right) \omega. \quad (67)$$

Here the 3-form flux appears in the combination

$$\tilde{G}_{\tilde{m}\tilde{n}p} = e^{-\phi} H_{\tilde{m}\tilde{n}p} + i F'_{\tilde{m}\tilde{n}p} \gamma_5 \quad (68)$$

in terms of $F'_{\tilde{m}\tilde{n}p} = F_{\tilde{m}\tilde{n}p} - C_0 H_{\tilde{m}\tilde{n}p}$ and the four-dimensional matrix γ_5 . The indices \tilde{m}, \tilde{n} are along the four-cycle Γ and p is transverse to it.

The above action uses a ten-dimensional notation for the fermionic degrees of freedom encoded in the object ω . Locally ω can be decomposed into a four-dimensional chiral (anti-chiral) Weyl-spinor times an internal part ϵ_+ (ϵ_-) given by

$$\begin{aligned} \epsilon_+ &= \phi |\Omega\rangle + \phi_{\bar{a}} \Gamma^{\bar{a}} |\Omega\rangle + \phi_{\bar{a}\bar{b}} \Gamma^{\bar{a}\bar{b}} |\Omega\rangle, \\ \epsilon_- &= \phi_{\bar{z}} \Gamma^{\bar{z}} |\Omega\rangle + \phi_{\bar{a}\bar{z}} \Gamma^{\bar{a}\bar{z}} |\Omega\rangle + \phi_{\bar{a}\bar{b}\bar{z}} \Gamma^{\bar{a}\bar{b}\bar{z}} |\Omega\rangle. \end{aligned} \quad (69)$$

This decomposition makes use of the local choice of complex coordinates $a, b = 1, 2$ along Γ and z, \bar{z} for the transverse direction as well as the standard definition of the Clifford vacuum $|\Omega\rangle$,

$$\Gamma^z |\Omega\rangle = 0, \quad \Gamma^a |\Omega\rangle = 0. \quad (70)$$

Consider now the flux-induced lifting of what would be a zero mode in the absence of any three-form flux.¹¹ For $G = 0$ the zero modes, i.e. the solutions to

⁹Here we mean superpotential contributions by instantons which would exist already without taking into account the flux induced lifting of zero modes.

¹⁰Recall that isolated $U(1)$ instantons are instantons not invariant under the orientifold action and which do not intersect their orientifold image. In particular there are no $\mathcal{E} - \mathcal{E}'$ zero modes.

¹¹A more general treatment in terms of the full Dirac equation involving a flux-induced torsion piece can be found in [95, 99]

the ordinary Dirac equation, are given by the harmonic piece of the modes (69). The universal fermionic zero modes with four-dimensional polarisation θ^α and $\bar{\tau}^\alpha$ can be identified with

$$\omega_0^{(1)} = \theta^\alpha \otimes \phi|\Omega\rangle, \quad \omega_0^{(2)} = \bar{\tau}^\alpha \otimes \phi_{\bar{a}\bar{b}\bar{z}}\Gamma^{\bar{a}\bar{b}\bar{z}}|\Omega\rangle. \quad (71)$$

Recall furthermore from table 2 that the Wilson line and deformation modes are counted by $H^{(0,1)}(\Gamma)$ and $H^{(0,2)}(\Gamma)$, respectively. It follows that the Wilson line modulini correspond to $\gamma^\alpha \otimes \phi_{\bar{a}}\Gamma^{\bar{a}}|\Omega\rangle$ and their conjugates $\bar{\gamma}^\alpha \otimes \phi_{\bar{a}z}\Gamma^{\bar{a}z}|\Omega\rangle$, while the deformation modulini are given by $\chi^\alpha \otimes \phi_{\bar{a}\bar{b}}\Gamma^{\bar{a}\bar{b}}|\Omega\rangle$ (plus their conjugates $\bar{\chi}^\alpha \otimes \phi_{\bar{z}}\Gamma^{\bar{z}}|\Omega\rangle$).¹²

Given a particular combination of 3-form flux G_3 one can now evaluate its induced couplings to the fermionic zero modes using the action (67) and the decomposition (69). The result is that

- (2,1) flux can in principle couple to the anti-chiral deformation and to the anti-chiral Wilson line modulini of unmagnetised E3-brane instantons; under appropriate circumstances these can therefore be lifted [98, 99]. Lifting their chiral counterparts requires (1,2) flux.
- primitive (2,1) flux does not couple to the extra Goldstinos $\bar{\tau}^\alpha$ of unmagnetised E3-brane instantons [43]; only (3,0) and (2,1) non-primitive flux couples to these modes. Corresponding statements for the θ^α modes hold by conjugation.

To illustrate this latter point for (2,1) primitive flux we compute the action e.g. of $\tilde{G}_{\bar{a}bz}\Gamma^{\bar{a}bz}$ on the internal part of the extra Goldstinos $\omega_0^{(2)}$, $\phi_{\bar{a}\bar{b}\bar{c}}\Gamma^{\bar{a}\bar{b}\bar{c}}|\Omega\rangle$. Elementary gamma-matrix algebra reveals that

$$\tilde{G}_{\bar{a}bz}\Gamma^{\bar{a}bz}\Gamma^{\bar{1}}\Gamma^{\bar{2}}\Gamma^{\bar{3}}|\Omega\rangle = \tilde{G}_{\bar{a}bz}g^{b\bar{a}}\left(g^{z\bar{1}}\Gamma^{\bar{2}}\Gamma^{\bar{3}}|\Omega\rangle - g^{z\bar{2}}\Gamma^{\bar{1}}\Gamma^{\bar{3}}|\Omega\rangle + g^{z\bar{3}}\Gamma^{\bar{1}}\Gamma^{\bar{2}}|\Omega\rangle\right) = 0. \quad (72)$$

The last equation follows from the identity [99] $\tilde{G}|\Omega\rangle = iG|\Omega\rangle$ together with primitivity of G , $g^{c\bar{c}}G_{bc\bar{c}} = 0$ [43].¹³ By the same token one finds that (3,0) flux can lift the $\bar{\tau}$ modes, while (0,3) and (1,2) flux does not couple to them. These results were confirmed by CFT methods in [104].

To summarize, 3-form flux on a Calabi-Yau manifold¹⁴ induces mass terms for unmagnetized E3-instantons of the schematic form

$$\begin{aligned} S_G^{E3} &= \int G_{(0,3)}\theta\theta + G_{(3,0)}\bar{\tau}\bar{\tau} \\ &+ \int G_{(2,1)}^{\text{prim.}}\bar{\chi}\bar{\chi} + G_{(2,1)}^{\text{prim.}}\bar{\gamma}\bar{\gamma} + G_{(1,2)}^{\text{prim.}}\chi\chi + G_{(1,2)}^{\text{prim.}}\gamma\gamma. \end{aligned} \quad (73)$$

¹²Since we are considering here a $U(1)$ instanton away from the orientifold plane the deformation and Wilson line modulini are not subject to the projections of table 2.

¹³Note for completeness that for non-primitive (2,1) flux the right-hand side of equ. (72) is non-zero and leads to a non-diagonal coupling to some deformation modes $\bar{\chi}$ of the schematic form $G_{(2,1)\text{n.p.}}\bar{\chi}\bar{\tau}$.

¹⁴Note that in this case no non-primitive (2,1) or (1,2) flux exists.

The lifting of the Goldstinos in vacua with (2,1) flux requires the interplay of 3-form flux and non-trivial supersymmetric worldvolume flux \mathcal{F} [43]. Indeed, for $\mathcal{F} \neq 0$ new interaction terms appear [98]. The BPS condition on the gauge flux amounts to primitivity of \mathcal{F} . The part in the instanton effective action relevant for the lifting of the universal modes can be written as [43]

$$S_G \simeq \int_{\Gamma} d^4\zeta \sqrt{\det g} \omega \mathcal{O}(G_{(2,1)}^{\text{prim.}}, \mathcal{F}) \omega \quad \text{with} \quad \mathcal{O}(G, \mathcal{F}) = \mathcal{F}_{\tilde{ij}} \Gamma^{\tilde{ipq}} g^{\tilde{jk}} G_{\tilde{kpq}}. \quad (74)$$

Here a tilde denotes indices parallel to the worldvolume, whereas p, q are general internal indices.

In the presence of suitable three-form flux the interaction term (74) leads to a coupling of the zero mode $\omega_0^{(2)}$ proportional to

$$G_{\tilde{abz}} \mathcal{F}^{\tilde{b}\tilde{a}} g^{\tilde{z}\tilde{3}} \Gamma^{\tilde{1}} \Gamma^{\tilde{2}} |\Omega\rangle. \quad (75)$$

Note that unlike the coupling (72) this need not vanish by primitivity of \mathcal{F} and G . In fact, a simple local configuration of a magnetised E3-instanton on a fluxed T^6/\mathbb{Z}_2 was found in [43] where the lifting of the τ -modes via this mechanism can indeed be achieved.

Let us pause a second to interpret these results. The fact that $\mathcal{N} = 1$ supersymmetric background flux alone does not lift the extra Goldstone moduli is surprising, but not inconsistent. After all, their appearance is rooted in the local enhancement of the $\mathcal{N} = 1$ supersymmetry preserved by the orientifold projection to the full $\mathcal{N} = 2$ supersymmetry of the Calabi-Yau compactification away from the orientifold locus. In the presence of background flux this $\mathcal{N} = 2$ supersymmetry is reduced to $\mathcal{N} = 1$ even away from the orientifold plane, and the $\bar{\tau}$ -modes are no longer protected as the Goldstone modes associated with the breakdown of a global symmetry by the instanton. Consistently, new interactions can be found which give them a mass term, even though the mere absence of the supersymmetry enhancement is not sufficient for the former Goldstinos to be lifted.

Second, one might wonder if the contribution of an isolated $U(1)$ instanton to the superpotential can be consistent with holomorphicity of the superpotential. This question was analysed in [79], see also [78]. In fact, the isolated instanton considered in the toroidal example of [43] can become non-supersymmetric across a line of marginal stability upon deforming the Kähler moduli as to depart from the primitivity condition $J \wedge \mathcal{F} = 0$. Being an isolated instanton it cannot compensate for the deviation from the BPS condition by recombination with its orientifold image. In this case it should not contribute to the superpotential any more. What resolves the paradox in the toroidal example is that this deformation of J automatically renders the 3-form flux non-primitive as well. Since in this region of moduli space supersymmetry is broken completely, the non-BPS instanton need not exhibit additional zero modes which would forbid the generation of

a superpotential-like contribution. As of this writing it remains to be seen if the lifting of $\bar{\tau}$ -modes via the coupling (74) is available also on genuine Calabi-Yau manifolds.

Fractional E(-1)-instantons in Type IIB orientifolds with 3-form flux

A similar analysis was carried out in [104, 62] for fractional E(-1)-instantons at singularities in Type IIB compactifications. These E(-1) instantons can be viewed as E1-brane instantons wrapping a vanishing holomorphic two-cycle. Unlike in the case discussed above, the relevant flux interactions were analysed here entirely with the help of conformal field theory methods. For brevity we stick to the case of stringy E(-1) instantons and refer the reader to [62] for a discussion of gauge instantons in the presence of background flux. Schematically, the 3-form flux couples via [104, 62]

$$S_G^{E(-1)} \simeq G_{3,0} \theta \theta - G_{0,3} \bar{\tau} \bar{\tau}. \quad (76)$$

Note that for E(-1) instantons (0,3) flux does indeed couple to the $\bar{\tau}$ -modes. This is in contrast to E3-brane instantons where, as summarised in equ. (73), even (0,3) flux does not couple to the extra Goldstones. As a result, it was proposed in [62] (see also [63]) that E(-1) instantons of $U(1)$ type can generate superpotential terms in the presence of (0,3) flux. Of course such instantons are trivially supersymmetric and there arises no paradox from a potential crossing of a line of marginal stability.

Interpretation and Outlook

Some more comments are in order concerning the general philosophy of lifting zero modes by flux-induced terms as in equ. (65).

Consider first the case of (2,1) primitive flux, which is supersymmetric already by itself. Suppose the flux lifts some deformation modes of type $(c, \bar{\chi}^\alpha)$. As described previously, in absence of flux these extra zero modes lead to Beasley-Witten multi-fermion interactions.¹⁵ The effect of the flux induced mass term is therefore to turn the multi-fermion F-terms into contributions to the superpotential.

The resulting generation of a superpotential term can also be understood from a purely four-dimensional effective field theoretic approach. As argued in [78] background flux can induce mass terms of the form (65) precisely when it

¹⁵As for the deformation modes we are having the anti-chiral deformation modes $(c, \bar{\chi}^\alpha)$ in mind. Their chiral counterparts, if not projected out, can only be lifted by fluxes other than of (2,1) type, see equ. (73). The same applies in principle to the chiral Wilson line modulini γ^α , which are involved in the generation of corrections to the gauge kinetic function. Only the $\bar{\gamma}^\alpha$ can be lifted by (2,1) flux, but they are not relevant for the generation of interesting F-terms.

also lifts those closed string moduli which would be involved in the Beasley-Witten terms generated by the instanton for vanishing flux. Integrating out these massive closed string moduli indeed transforms the original multi-fermion interactions into a superpotential interaction below the mass scale of the fixed closed string moduli.

Less clear is the lifting of moduli by fluxes of other Hodge type for which a supersymmetric vacuum exists only in presence of a non-perturbative superpotential even before taking the effect of the fluxes into account [108, 102]. In [101] it is argued that the naive couplings of this flux to the moduli are cancelled against additional terms which can be understood as the backreaction of the instanton on the setup. It will be interesting to see how the lifting of, say, the $\bar{\tau}$ -modes of E(-1) instantons by (0,3) flux is affected by these considerations.

3.6 D-terms from non-BPS instantons

While a thorough discussion of instanton induced D-terms is beyond the scope of this review we would like to briefly point out some pertinent developments in the recent literature. Through the lack of holomorphicity instanton corrections to D-terms such as the Fayet-Iliopoulos term or the Kähler potential are currently under comparatively poor computational control. The generation of a D-term requires an instanton whose universal fermionic zero modes span the full $\mathcal{N} = 1$ supersymmetry algebra preserved in four dimensions. In the notation of table 1 these are the modes $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$. From the general arguments in §2.2 we therefore need a non-BPS instanton whose presence indeed breaks all supersymmetries in such a way that the four zero modes $\tau^\alpha, \bar{\tau}^{\dot{\alpha}}$ are lifted appropriately.

In [35] it was argued that a Fayet-Iliopoulos D-term can be generated by $\mathcal{O}(1)$ instantons (with two chiral and two anti-chiral deformation moduli) which are non-supersymmetric due to a non-zero pull-back of RR potentials to its world-volume. Another natural system of non-BPS instantons is given by isolated $U(1)$ instantons which become non-supersymmetric away from the line of marginal stability in hypermultiplet moduli space [79]. In particular this reference discusses how the multi-fermion F-terms generated on the BPS locus pick up proper D-term contributions once the instanton becomes non-supersymmetric. Finally, non-perturbative corrections to the Kähler potential by instanton-anti instanton pairs were considered in [111].

4 INSTANTONS IN QUIVER THEORIES

In the discussion to this point, we have implicitly used free worldsheet conformal field theory techniques in quantizing the open strings that stretch between our stringy instanton and other branes that are present in the background. However, many of the most interesting geometries for string compactification are highly

curved, giving rise to strongly interacting worldsheet conformal field theories. In this section, we describe the existing techniques to infer stringy instanton contributions to holomorphic couplings even in these situations. In order to avoid introducing cumbersome notation while still making the major points clear, we focus on one particular class of geometries; the generalization of these ideas to other geometries should however be transparent, and we indicate how it proceeds at various points. We will mostly follow the references that have focused on local conifold geometries and their close relatives, but other related works with many related results appear in [60, 40, 41, 112, 82, 61, 113].

4.1 Rules for rigid stringy instantons at singularities

Starting with the seminal work of Douglas and Moore [114], it has been realized that the field theories arising on D-branes at Calabi-Yau singularities can be represented in terms of quiver diagrams. Here, we assume the reader is familiar with the basic notions of such diagrams. Discussions of how to understand the quiver associated to a given geometry can be found in the reviews [33, 34] and in the many references therein.

Here, we focus on the quivers arising for D-branes in IIB string theory probing the singular geometries defined by the constraint

$$(xy)^n = zw \tag{77}$$

in \mathbb{C}^4 . These are just \mathbb{Z}_n orbifolds of the conifold; the resulting gauge theories are described in detail in [115, 116]. While the standard conifold quiver theory has two nodes with bi-fundamentals $A_{1,2}$ and $B_{1,2}$ running between them in opposite directions [117], the \mathbb{Z}_n quotient gives rise to $2n$ nodes. The content for $n = 2$ appears in figure 5; the general case is the obvious extension to a larger number of nodes. The superpotential governing the matter fields (with the notation that X_{12} is a bifundamental between nodes 1 and 2, and X_{21} is a bifundamental in the conjugate representation) is

$$W = h (X_{12}X_{23}X_{32}X_{21} - X_{23}X_{34}X_{43}X_{32} + X_{34}X_{41}X_{14}X_{43} - X_{41}X_{12}X_{21}X_{14}) . \tag{78}$$

Because the quiver is completely non-chiral, we are free to occupy the nodes with arbitrary numbers of spacetime filling branes without inducing any anomalies in the field theory. Our greatest interest is in stringy instantons, so we focus on the case where some node is unoccupied. What is the spectrum of zero modes on such an instanton?

We can follow the general classification of §2.1. First of all, there are universal zero modes. Because the geometry breaks the SUSY to $\mathcal{N} = 2$, while further spacetime filling branes break it to $\mathcal{N} = 1$, the stringy instanton on an unoccupied node will see *four* fermionic zero modes – two from the 2 broken $\mathcal{N} = 1$ supercharges, and two more from the orthogonal $\mathcal{N} = 1'$. So a priori, one

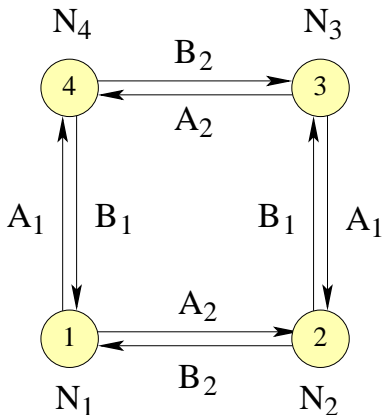


Figure 5: Quiver diagram for the \mathbb{Z}_2 orbifold of the conifold, for arbitrary numbers of fractional and regular D3-branes. We have labeled bifundamentals according to the parent field in the un-orbifolded conifold theory.

does not expect any contribution to the superpotential. This is easily fixed by introducing suitable orientifold planes into the geometry as in [39, 40, 41, 42, 43]. When we proceed we will therefore assume that the instanton wraps a node of the quiver which has an SO orientifold projection on the instanton (and would have an SP projection on a corresponding spacetime filling D-brane at the same node). The simplest orientifold action on the $2n$ node quiver above truncates the number of nodes to $n + 1$ (exchanging $2n - 2$ nodes pairwise and fixing two of them), and places an orientifold (with an SO projection for the instantons) at the two fixed quiver nodes. It changes the “circular” $2n$ -node quiver into a chain with $n + 1$ nodes, with the orientifold projections on the first and last node. For a more detailed discussion, see [118].

What about deformation zero modes? In the quiver gauge theory on spacetime filling branes, the deformation zero modes shown in table 2 give rise to adjoint matter fields. Since the quiver has no adjoints, our stringy instanton will be *rigid*.

Finally, we must classify the charged zero modes. The arrows in our quiver represent precisely the charged fermionic zero modes tabulated in table 3. For each arrow entering/leaving the instanton node, one finds a single Grassmann degree of freedom in the appropriate bi-fundamental representation of the instanton and spacetime filling gauge group. That is, the chiral arrows representing chiral matter fields in the quiver geometry for spacetime filling branes, turn into single Grassmann variables on the instanton world-volume for an instanton occupying the same node.

We therefore see that the instanton zero modes are simply derived from the quiver governing the spacetime filling gauge theory at the same singularity. These rules have been discussed in more detail in [119, 61]. In fact, at the level of the worldsheet conformal field theory, it is also easy to prove that the interactions of

the charged instanton zero modes with the rest of the quiver fields may be summarized as follows. One obtains an interaction of the instanton zero modes with the charged matter for each superpotential coupling which a spacetime filling brane at the same node would have possessed. One simply replaces the charged chiral fields entering the instanton node with the appropriate instanton Grassmann zero modes.

So for instance, in our orientifold above, the charged chiral fields Q, \tilde{Q} leaving the end-points of the linear quiver would be replaced by charged instanton zero modes $\lambda, \bar{\lambda}$. And they would have an effective action

$$S_{eff} = t + \lambda X_{23} X_{32} \bar{\lambda}, \quad (79)$$

where t is the Kähler parameter controlling the size of the node the instanton wraps. In the particular case that one has $N_2 = 1, N_3 = N$, for instance, from the integral

$$\int d\lambda d\bar{\lambda} e^{-S_{eff}} = e^{-t} X_{23} X_{32} \quad (80)$$

one infers that one can generate an exponentially small non-perturbative mass term for the fields X_{23}, X_{32} , i.e. for a flavor of the $SU(N)$ theory at node 3 [39]. For $N_2 > 1$, one would instead find a higher dimension (irrelevant) operator in that SUSY QCD theory.

In summary, the general rules for adding stringy instantons to quiver gauge theories arising at Calabi-Yau singularities are quite simple. One gets a charged Grassmann zero mode λ for each charged chiral field Q emanating from the quiver node for the related spacetime filling gauge theory. And, one gets interaction terms between these Grassmann fields and the matter fields of the space-time gauge theory, for each superpotential term that would have coupled Q to matter at the other nodes.

4.2 Geometric transitions and stringy instantons

In some cases, the non-perturbative dynamics on D-branes at a singular geometry X can be determined by performing a “geometric transition” to a different geometry X' , replacing the branes with fluxes. In the IIB theory, for instance, there are three-form fluxes H_3, F_3 from the NS and RR sector. Turning on background values of these fluxes generates a superpotential for the Calabi-Yau complex moduli [120]

$$W = \int (F - \tau H_3) \wedge \Omega. \quad (81)$$

By appropriate mapping of the D-brane quanta on X to fluxes on X' , and the parameters (like the dynamical scale) of the brane field theory to the moduli of X' , one can determine the non-perturbative dynamics of the QFT by computing (81).

The most famous example of such a correspondence again involves the conifold geometry

$$\sum_{i=1}^4 z_i^2 = \epsilon^2 . \quad (82)$$

In the limit $\epsilon \rightarrow 0$ with real ϵ , one can see that a three-sphere (the real slice of the defining equation above) is collapsing to zero size. One can repair this singularity either by deforming the geometry with finite ϵ , or by performing a small resolution which introduces a finite-sized \mathbb{P}^1 at the tip. The quiver gauge theory representing the conifold [117] has two nodes. Equal occupation of the nodes by $U(N)$ gauge groups describes N D3 branes on the conifold geometry, while any difference in the numbers $N_2 - N_1 = M$ maps to adding M additional D5 branes wrapping the small \mathbb{P}^1 .

The canonical example of a geometric transition occurs if one has e.g. $N_2 = M, N_1 = 0$ [121, 122]. The low-energy gauge theory on the D5-branes is an $SU(M)$ $\mathcal{N} = 1$ pure Yang-Mills theory. It is expected to have confinement and chiral symmetry breaking at an exponentially small scale $\Lambda \sim e^{-1/g_s^2 M}$, with M vacua resulting from a spontaneous breaking of a non-anomalous \mathbb{Z}_{2M} R-symmetry down to \mathbb{Z}_2 .

In this geometric transition, X is the resolved conifold and X' is the deformed conifold. The M D5-branes map to M units of F_3 flux through the small sphere A in X' , while the NS 3-form flux through the non-compact dual cycle to this S^3 , B , is chosen to be t , the \mathbb{P}^1 volume in X (controlling the gauge theoretic coupling on the D5s). The end result is

$$\int_A F_3 = M, \quad \int_B H_3 = t, \quad W = -i \frac{t}{g_s} z + M \left(\frac{z}{2\pi i} \right) \log(z), \quad (83)$$

where z is the size of the S^3 (the complex modulus determining ϵ). Here, in computing W in terms of z , we have used basic facts about the periods of Ω in the conifold geometry [123]. Minimizing the flux potential on X' , we in fact discover M vacua with

$$|z| \sim e^{-2\pi t/g_s M}, \quad (84)$$

precisely in accord with the gauge theory expectations. The parameter z in the X' geometry represents the dynamical scale Λ in the dual gauge theory.

It is interesting to ask whether one can similarly perform transitions to sum up purely stringy non-perturbative effects, like the stringy instantons. Here, following [65] (see also [124]), we give an example where in fact a stringy instanton effect (and an infinite series of multicovers) are reproduced as expected by a geometric transition. This provides an alternative check on our computations.

The geometries X for us will be non-compact Calabi-Yau threefolds which are A_r ALE spaces fibered over the complex x -plane. These are described by hypersurfaces in \mathbb{C}^4 via a defining equation

$$uv = \prod_{i=1}^{r+1} (z - z_i(x)) . \quad (85)$$

This geometry is singular at the points where $u, v = 0$ and $z_i(x) = z_j(x) = z$. At these points there are vanishing \mathbb{P}^1 s, which can be blown up by deforming the Kähler parameters of the Calabi-Yau (analogous to the small resolution of the conifold). There are r such two-cycle classes, which we will denote by S_i^2 . These correspond to the blow-ups of the singularities at $z_i = z_{i+1}$ for $i = 1, \dots, r$. By wrapping D5-branes on these spheres, we can engineer the gauge theories of interest. The study of transitions in such geometries was pioneered in the paper [125].

If the z_i were independent of x , then this geometry would become the product of an A_r ALE space with the x -plane. In this circumstance, D5-branes wrapped on the small \mathbb{P}^1 s would have a moduli space; they would have adjoint fields whose vev parameterizes their location on the x -plane. The superpotential which the non-trivial fibration induces for these adjoints can be computed as follows [126, 127]. For the i th brane stack, introduce a 3-chain \mathcal{C} whose boundary is S_i^2 . Then

$$W = \int_{\mathcal{C}} \Omega . \quad (86)$$

For this particular geometry, one can show that it simplifies to

$$W_i = \int_{\mathcal{C}} (z_i(x) - z_{i+1}(x)) dx \quad (87)$$

for the i th adjoint field.

In addition to the adjoints, there are quarks stretching between D5-branes wrapped on adjacent \mathbb{P}^1 s, and they have $\mathcal{N} = 2$ - like couplings to the adjoints. So the full superpotential is

$$W_{\text{total}} = \sum_i W_i(\Phi_i) + Tr(Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i+1,i}\Phi_iQ_{i,i+1}) . \quad (88)$$

Note that if one chooses the z_i so that the adjoints are all massive with equal vevs, then after integrating out the adjoints, one gets precisely the quiver geometries studied in the previous subsection. In that sense, this is the simplest generalization of those orbifold geometries.

In fact, we will generalize the previous subsection in another way as well. While in §4.1 we focused on instantons wrapping empty (orientifolded) nodes, here we will instead sum up instantons on $U(1)$ nodes. These are still “stringy,” since the $U(1)$ gauge theory does not have smooth Yang-Mills instantons. And as described in §3.1, the zero mode selection rules are expected to allow contributions even in this case. We will perform a geometric transition on such a $U(1)$ node, and use the dual geometry to show that one gets non-perturbative generation of an exponentially small mass term (as also in the previous subsection).

We consider the A_3 case of (85), choosing the $z_i(x)$ so that

$$uv = (z - mx)(z + mx)(z - mx)(z + m(x - 2a)) . \quad (89)$$

After blowing up, in our geometry X (pre-transition) we wrap M branes each on S_1^2 at $z_1(x) = z_2(x)$ and S_2^2 at $z_2(x) = z_3(x)$, and a single brane on S_3^2 at $z_3(x) = z_4(x)$. The tree level superpotential is then

$$W = \sum_{i=1}^3 W_i(\Phi_i) + Tr(Q_{12}\Phi_2Q_{21} - Q_{21}\Phi_1Q_{12}) + Tr(Q_{23}\Phi_3Q_{32} - Q_{32}\Phi_2Q_{23}), \quad (90)$$

with the $W_i(\Phi_i)$ taking the values

$$W_1 = m\Phi_1^2, \quad W_2 = -m\Phi_2^2, \quad W_3 = m(\Phi_3 - a)^2. \quad (91)$$

The adjoint superpotentials have localized brane stacks 1 and 2 at $x = 0$. So they intersect, and the intervening quark flavors remain massless after inserting the adjoint vevs. However, the third node is localized at $x = a$; both its adjoint and its quark matter are massive, and hence it is a fully massive node (ignoring the free abelian gauge field). Correspondingly, we expect that we should be able to perform a geometric transition on this node.

The result deforms the (formerly resolved) singularity after shrinking S_3^2 , changing the complex structure to that of a new manifold X'

$$uv = (z - mx)(z + mx)((z - mx)(z + m(x - 2a)) - s). \quad (92)$$

The size of the new “deformed” S^3 which replaces S_3^2 is

$$\int_{S^3} \Omega = S = \frac{s}{m}. \quad (93)$$

Since the third D5-brane is gone, so are the fields Q_{23}, Q_{32} and Φ_3 . We replace them in the effective superpotential instead by the flux superpotential for S , and by the deformed superpotential for Φ_2 (which has had its potential changed since we have integrated out fields that it couples to). The result is

$$W_{eff} = W_1(\Phi_1) + \tilde{W}_2(\Phi_2, S) + Tr(Q_{12}\Phi_2Q_{21} - Q_{21}\Phi_1Q_{12}) + W_{flux}(S). \quad (94)$$

In this geometry, the exact flux superpotential is as in the case of the conifold that we previously discussed; the geometric transition is *locally* identical. The new superpotential term \tilde{W}_2 is

$$\tilde{W}_2(x) = \int (z_2(x) - \tilde{z}_3(x)) dx \quad (95)$$

where we define $\tilde{z}_3(x)$ via the relation

$$(z - \tilde{z}_3(x))(z - \tilde{z}_4(x)) = (z - z_3(x))(z - z_4(x)) - s \quad (96)$$

with \tilde{z}_3 being chosen on the branch which asymptotes to $z_3(x)$ at large x . Concretely, this implies

$$\tilde{W}_2(x) = \int_{\Delta}^x (-m(x' + a) - \sqrt{m^2(x' - a)^2 + s}) dx' . \quad (97)$$

Here, Δ is an IR cut-off in the geometry (which maps to a UV cutoff in the field theory).

One can now integrate out S and the remaining adjoints. It is intuitively quite clear that the superpotential W_{flux} stabilizes S at an exponentially small value $S \sim e^{-t/g_s}$. Then, plugging the vev of S into \tilde{W}_2 , and expanding in powers of $s = mS$, one finds an infinite series of instanton contributions. Their leading effect is to shift the vacuum for Φ_2 a bit away from its old location by an amount of order e^{-t/g_s} ; so one obtains from the term

$$W_{eff} = \dots + Q_{21}\Phi_2Q_{12} + \dots \rightarrow e^{-t/g_s}Q_{12}Q_{21} . \quad (98)$$

In other words, one obtains an exponentially small mass for the quarks stretching between the remaining nodes. This is of course reminiscent of the phenomena described in §4.1. The precise formulae, including small corrections to the above scalings and coefficients for all terms in the multi-instanton series, can be found in [65].

4.3 Another check: stringy instantons in RG cascades

One of the most interesting phenomena discovered in quiver gauge theories is RG cascades, where as one moves towards the infrared, the effective gauge theory description changes by a self-similar sequence of Seiberg dualities. The simplest example occurs in the conifold quiver with unequal ranks [121]; generalizations to the orbifolded conifold geometries of §4.1 are also easy to exhibit [116].

We argued in §4.1 that if one orientifolds the geometry $(xy)^n = zw$, one can obtain a quiver with $n + 1$ nodes. The gauge theory realized by spacetime filling branes in this quiver is $Sp(N_1) \times U(N_2) \times \dots \times U(N_n) \times Sp(N_{n+1})$ – i.e., there is an SO projection for Euclidean branes on the first and last nodes.

We also argued that in the special case with e.g. $N_1 = 0, N_2 = 1, N_3 = N, \dots$, one obtains from a stringy instanton on the first node a non-perturbatively generated mass for the quarks X_{23}, X_{32} stretching between nodes 2 and 3.

The existence of the RG cascade offers us another possibility to check this claim [118]. Suppose we start with large occupation numbers at the nodes, chosen so that at the final step of the RG cascade, we end up with the configuration above. Then, one should be able to derive the effective low-energy theory in two different ways:

1) One can do the path integral over D-instanton zero modes at node 1, with occupation numbers in the quiver gauge theory describing the final cascade step. This is the class of techniques we have been describing in this review.

2) One could also try to derive the *same* effective low-energy theory by analyzing the gauge theory at higher steps in the cascade, where the relevant node is occupied by spacetime filling branes. In this case, one should be able to reproduce the “stringy instanton” effect by using standard techniques and results in $\mathcal{N} = 1$ supersymmetric gauge theory.

While going through the details of the renormalization group cascades for orientifolded orbifolded conifolds is beyond the scope of this brief review, we simply state here the results. The gauge theory analysis is completely consistent with the microscopic expectation from instanton calculus: in the case that one has the orientifolded quiver, with configuration $N_1 = 0, N_2 = 1, N_3 = N, \dots$ at the final cascade step, one can prove from gauge theory analysis that gauge dynamics generates an exponentially small mass for the quarks X_{23}, X_{32} . In the case that one studies the cascade with only $U(N)$ nodes and no orientifold, one does not generate such a mass via gauge dynamics in the cascade. It is interesting that in the case that the non-trivial effect occurs, the stringy instanton effect turns into a strong coupling effect (and not an instanton effect) in the cascading gauge theory [118]. Extensions of these results, giving more cases where alternative UV completions involving gauge theory can be used to derive stringy instanton effects, have also been noted in [80, 128, 129].¹⁶

We emphasize here, however, that the UV completions involving small numbers of D-branes in a Calabi-Yau geometry, or larger rank non-Abelian theories which produce strong dynamics, are *different*. These results should not be interpreted as indicating that the effects are not stringy; rather that in cases where a duality relates the stringy configuration to a (UV distinct but IR equivalent) field theory, the results of computations are in accord as expected.

5 PHENOMENOLOGICAL IMPLICATIONS

In this section we highlight specific phenomenological implications of D-brane instanton generated couplings for string model building. Our main focus will be on superpotential corrections in the charged matter sector. At the end of this section we will also comment on some implications of other types of coupling corrections. A general overview of the status of F-term corrections due to (multi-) instantons has been provided in §3.

As discussed in §2 and §3, an instanton configuration can contribute to the superpotential provided all uncharged fermionic zero modes other than the two universal θ^α are lifted or saturated without inducing higher derivative terms. In particular rigid $O(1)$ instantons are natural candidates to generate superpotential corrections. Recall from §2.2 that such an object has only two uncharged

¹⁶An interesting relation between stringy instantons and matrix models is observed in [130] (see also [63]). A possible interpretation of some stringy instantons as octonionic field theory instantons is considered in [131].

fermionic zero modes θ^α to begin with, which, along with four bosonic ones x^μ , constitute the universal superpotential zero mode measure $\int d^4x d^2\theta$.

In the presence of charged zero modes $\lambda_{\mathcal{E}a_i}$ (see table 3) the superpotential involves charged matter fields, and can thus have drastic effects for string model building. The form of the matter couplings is such that the total $U(1)_a$ charges of the zero modes are canceled by those of the charged matter superpotential terms, see equ. (19).

A detailed analysis of the superpotential calculus is spelled out in §2.3 where the holomorphic superpotential couplings (18) are determined explicitly in terms of the classical instanton action, disc diagram couplings of matter fields with two charged zero modes and the holomorphic part of the annulus diagram (see figure 2). Such a superpotential coupling of mass-dimension D is thus of the form

$$\mu^D = x M_s^D \exp(-S_{\mathcal{E}}^{(0)}), \quad (99)$$

where we have introduced the string mass scale $M_s = \ell_s^{-1}$. The tree-level string action $S_{\mathcal{E}}^{(0)}$ is determined by the volume in string mass units of the cycle wrapped by the instanton as in equ. (24). The pre-factor $x = \mathcal{O}(1)$ can be determined precisely by carrying out the disc and annulus diagram calculations when a conformal field theory description of the string model is available.

It is of utmost importance that the instanton suppression factor does in general not coincide with $8\pi/g_{YM}^2$ as would be the case for a gauge instanton, which is given by a Euclidean brane wrapped along the same cycle as a matter brane. For example in the Type IIA framework of intersecting D6-branes the exponent of the classical E2-instanton action can be cast into the form

$$S_{\mathcal{E}}^{(0)} = \frac{8\pi^2}{g_a^2} \frac{\text{Vol}_{E2}}{\text{Vol}_{D6_a}}, \quad (100)$$

where Vol_{E2} and Vol_{D6_a} are the respective volumes of the three-cycles wrapped by the E2-instanton and D6_a-branes in the internal space, and g_a is the gauge coupling of the $U(N_a)$ gauge theory on the D6_a-branes. These couplings introduce a new hierarchy: in appropriate circumstances the ratios of the volumes of the instanton and D-brane can be just right to generate the desired magnitude of specific non-perturbatively induced couplings. In this context we stress that given a particular string vacuum the instanton effects cannot be turned on or off at will, but are determined by the internal geometry. In particular the modulus describing the ratio of cycle volumes as in equ. (100) is constrained by the standard D-term supersymmetry conditions for the $U(1)$ gauge fields living on the spacetime-filling D-branes. See e.g. the reviews [26, 27, 28, 29, 30, 32] for more information.

In the sequel we shall highlight some phenomenological implications of these superpotential couplings. We focus on mass-dimension three, two, one and zero couplings, where effects could be significant, and mention some potential effects for non-renormalizable operators.

5.1 Mass dimension three couplings

In absence of any charged zero modes the contribution to the superpotential of an $\mathcal{O}(1)$ D-brane instanton is a function of the closed string moduli via the dependence (24) of the tree-level suppression factor $S_{\mathcal{E}}^{(0)}$ on the instanton volume. For example in Type IIB compactifications this yields an exponential dependence of the superpotential on the Kähler moduli and the dilaton. As the perturbative flux-induced superpotential only involves the complex structure, but not the Kähler moduli, E3-instantons are therefore vital in attempts to stabilize the moduli in Type IIB orientifolds with fluxes. This was pioneered in [19] and demonstrated in very explicit examples e.g. in [132, 133, 108, 102]. A modified scenario where these E3-instanton contributions to the superpotential are balanced against perturbative corrections to the Kähler potential was developed in [134], while its mirror dual Type IIA construction was worked out in [135]. On the other hand, the complex structure dependent one-loop Pfaffian $f_{\mathcal{E}}^{(1)}$ of E3-brane instantons is quite difficult to extract in concrete settings. In general it depends also on the open string moduli of other D-branes and can become relevant for their stabilization, as e.g. for D3-branes [136]. Of particular use in generating hierarchies in the closed string moduli sector can be the double suppression by the poly-instanton effects described around equ. (62) [137]. In all these setups it is crucial that the instanton does not intersect any other D-brane to avoid chiral charged zero modes. In particular the volume modulus associated with cycles wrapped by chirally intersecting D-branes cannot be stabilized in this manner [138].

5.2 Mass dimension two couplings

Linear couplings can be generated non-perturbatively for matter fields Φ_{ab} which are charged under (massive) Abelian factors $U(1)_a \times U(1)_b$ only. Namely, the charge condition (19) for fermionic zero modes in this case requires $N_a = N_b = 1$, along with one $\lambda_{\mathcal{E}a}$ and one $\lambda_{b\mathcal{E}}$ mode. For Φ_{ab} in representation $(-1_a, 1_b)$, the $\mathcal{O}(1)$ instanton must have the following non-zero topological intersection numbers,

$$I_{\mathcal{E},\mathcal{D}_a}^+ = 1, \quad I_{\mathcal{E},\mathcal{D}_b}^- = 1. \quad (101)$$

A single disc diagram generates the effective instanton action term $\lambda_{\mathcal{E}a}\Phi_{ab}\lambda_{b\mathcal{E}}$. After absorption of the fermionic zero modes this generates a superpotential term linear in Φ_{ab} . This is illustrated in figure 6 for an E2-instanton along the cycle Ξ in the framework of Type IIA models with intersecting D6-branes. A particular challenge in implementing this and similar effects in phenomenologically appealing string models is associated with the requirement that the instanton may have intersections only with those D-branes which host the respective matter fields in the bifundamental sector. Any additional charged zero mode beyond (101) will lead to higher dimensional couplings at best and thus annihilate the effect.

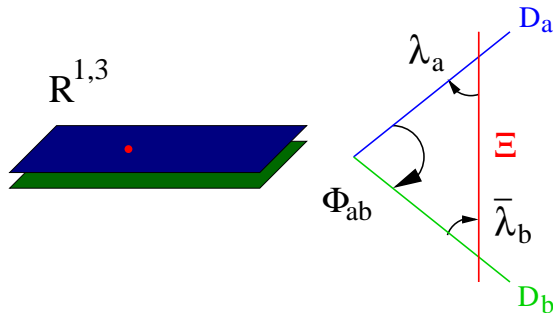


Figure 6: The disc diagram for a Polonyi-type coupling, represented in the Type IIA framework.

Supersymmetry breaking

These mass-dimension two couplings $W = \mu^2 \Phi$ can trigger F-term supersymmetry breaking à la Polonyi at the scale μ^2 . Originally [24] this scenario was envisaged in the special case where the Polonyi field Φ arises from the $\mathcal{D}_a - \mathcal{D}_a$ sector of a single D-brane rather than at the intersection of two branes $\mathcal{D}_a, \mathcal{D}_b$. Such a setup requires a vectorlike pair of zero modes $\lambda_{a\mathcal{E}}, \lambda_{\mathcal{E}a}$. In both variants D-brane instantons can account not only for the presence of such a Polonyi term [22, 24] but also give an appealing explanation of the hierarchical suppression of its scale μ . This was demonstrated even in globally consistent examples in [139, 140] based on chiral $SU(5)$ GUT constructions of Type I theory and in [141] on the orbifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z})$ with torsion.

This Polonyi-type supersymmetry breaking can in principle be embedded into a scenario of gauge mediation [142] via perturbative Yukawa couplings of the type $\Phi_{ab} M_{bc} M_{ca}$. Here it is understood that the Standard Model gauge symmetry is part of the gauge group factor $U(N_c)$. In this case the fields M_{bc}, M_{ca} play the role of messenger fields. This scenario has been investigated in various contexts in [39, 111, 140, 66, 68, 67, 143] (see also [144]).

Next to the Minimal Supersymmetric Standard Model

The field Φ_{ab} can also play the role of a Standard Model singlet field in the next to the minimal supersymmetric Standard Model (NMSSM). Its perturbative coupling to the Standard Model Higgs doublets H_{bc} and H_{ca} can induce the μ -parameter after Φ_{ab} acquires a non-zero vacuum expectation value in the desired regime, triggered by the D-instanton induced linear couplings for Φ_{ab} and supersymmetry breaking in a separate, “hidden” sector. Further investigation of these types of models is underway [145] (see also [143]).

5.3 Mass dimension one couplings

Neutrino Majorana masses

Perhaps the most prominent example of non-perturbatively generated mass terms are Majorana masses for right-handed neutrinos [20, 21]. The prototype example involves fields Φ_{ab} which are singlets under the Standard Model gauge symmetry, and are charged under additional massive Abelian gauge group factors $U(1)_a \times U(1)_b$, say in the $(-1_a, 1_b)$ representation. The condition on the topological instanton intersection numbers

$$I_{\mathcal{E}, \mathcal{D}_a}^+ = 2, \quad I_{\mathcal{E}, \mathcal{D}_b}^- = 2 \quad (102)$$

then ensures that two disc diagrams (depicted schematically in the Type IIA framework in figure 7) generate superpotential terms quadratic in Φ_{ab} . For the desired magnitude of Majorana masses in the range $\mu_1 = \mathcal{O}(10^{10} - 10^{15})$ GeV, the volume of the instanton cycle has to lie in the corresponding regime. Furthermore, the constructions should allow for perturbative Dirac neutrino Yukawa couplings m_D of the order of the charged sector of the Standard Model $m_D = \mathcal{O}(0.1 - 10)$ GeV. This results in a see-saw mechanism with physical neutrino masses $\sim m_D^2/\mu_1$ of the order of $(10^{-2} - 10^{-3})$ eV.

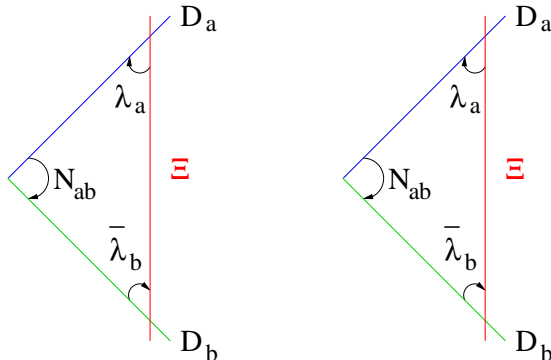


Figure 7: Two disc diagrams contributing to the Majorana mass term.

This effect was realized within a locally defined chiral GUT theory with gauge symmetry $U(5)_{GUT} \times U(1)_a \times U(1)_b$ in the Type IIA framework of intersecting D6-branes on an orientifold of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$. In [46] a class of $\mathcal{O}(1)$ instantons with precisely the instanton numbers (102) is identified that can produce the desired hierarchy for Majorana masses. The explicit conformal field theory on such orientifolds also allows for an explicit calculation of both the disc [46] and annulus [35] contributions to μ_1 . A systematic search for globally consistent three-family Standard models with D-instanton induced Majorana masses was performed in [42] for models based on Type IIA rational conformal field theories. The first globally consistent examples with the desired hierarchy for Majorana

masses were presented in the context of chiral Type I models on elliptically fibered Calabi-Yau spaces in [139]. Further phenomenological implications of D-brane instanton induced neutrino masses including the possible generation of a realistic family structure were studied in [146].

μ -parameter

A potential explanation of the hierarchically small μ -term $\mu H^u H^d$ can involve D-brane instantons in situations where a perturbative μ -term is forbidden explicitly by the global $U(1)$ charges of the H^u, H^d fields [20, 21, 147]. For typical Standard Model constructions Higgs doublets arise from a chiral sector, say H_{ac} and H_{cb} in respective representations $(-1_a, \mathbf{2}_c)$ and $(\bar{\mathbf{2}}_c, 1_b)$ under $U(1)_a \times U(1)_b \times U(2)_c$ ($SU(2)_L \in U(2)_c$). An instanton with the non-zero topological intersection numbers (101) has the correct zero mode structure to generate the μ -term due to the quartic disc diagram coupling $\lambda_{\mathcal{E}a} H_{ac} H_{cb} \lambda_{b\mathcal{E}}$ in the effective instanton action. In this case the classical instanton action requires a stronger suppression than for Majorana neutrino masses in order to achieve $\mu = \mathcal{O}(\text{TeV})$. A concrete realisation appears e.g. in [82].

Decoupling of non-chiral exotics and further effects

Many explicit string models with intersecting D-branes are plagued by the appearance of unwanted exotic matter fields. For example non-chiral matter exotics in the (anti-)symmetric representation of $U(N_a)$ arise due to non-zero topological intersection numbers $I_{\mathcal{D}_a, \mathcal{D}_{a'}}$. It turns out that rigid $O(1)$ instantons with appropriate intersection numbers can ensure the correct number of charge instanton zero modes and generate mass terms for these non-chiral exotics. This mechanism has been demonstrated within globally consistent models both on the Type I [60] and the Type IIA [148] side.

As yet another interesting implication of instanton generated mass terms, [141] investigates the so-induced breakdown of perturbatively realised conformal invariance in a globally consistent toroidal orbifold.

5.4 Mass dimension zero couplings

Both $SU(5)$ GUT and multi-stack Standard Model constructions in the Type II framework generically suffer from the absence of certain desired Yukawa couplings. D-brane instantons are natural candidates to generate such perturbatively absent terms. On the other hand, in typical multi-stack Standard-Model like constructions D-instantons could in principle generate R-parity violating couplings; these in turn would induce experimentally excluded lepton and baryon violating processes. The absence of such dangerous interactions even at a non-perturbative level might thus further constrain the model building possibilities.

In the sequel we exemplify the implications for different tri-linear couplings. We also refer the reader to [149] for a systematic analysis of $O(1)$ instanton generated tri-linear couplings (as well as mass terms) for specific four-stack quiver Standard Model constructions. One should point out that a given instanton can induce more than one couplings. Since in this case all associated non-perturbative couplings are suppressed by the same classical instanton action, it can happen that some couplings turn out to be just of the desired magnitude while others may then be too large. Examples of this phenomenon were also encountered in [149].

Top quark Yukawa couplings in $SU(5)$ GUT's

Perhaps the most glaring deficiency of $SU(5)$ GUT constructions in Type II orientifolds is the absence of a perturbative top quark Yukawa coupling. This coupling is of the type $\mathbf{10}^{(2,0)} \mathbf{10}^{(2,0)} \mathbf{5}_H^{(1,1)}$. Here the superscripts denote the $U(1)_a \times U(1)_b$ charges of the respective fields in a minimal two-stack $SU(5)$ setup based on gauge groups $U(5)_a \times U(1)_b$ (where the diagonal Abelian factors are assumed to acquire Stückelberg masses via the Green-Schwarz mechanism). Evidently this coupling is not invariant under the $U(1)$ charges.¹⁷ It was shown in [148] that a rigid $O(1)$ instanton can generate this coupling provided its non-zero intersection numbers are

$$I_{\mathcal{E}, \mathcal{D}_a}^- = I_{\mathcal{E}, \mathcal{D}_b}^- = 1. \quad (103)$$

In this case the three disc diagrams illustrated in figure 8 generate the top Yukawa coupling. The desired hierarchy for this Yukawa coupling requires an appropriately small volume of the instanton cycle. This was achieved explicitly in the globally consistent $SU(5)$ GUT models constructed in [152] on Type IIB orientifolds with D3/D7-branes.¹⁸ Note, however, that the instanton cycle has to be of string scale size. While one might be worried that one therefore has to sum up the infinite series of all multiply-wrapped instanton corrections this is actually not the case: N -fold wrapped instantons along the small cycle cannot contribute to this Yukawa coupling but only generate string-scale suppressed higher dimensional operators as the charged zero mode sector comprises N times more modes. By contrast, in flipped $SU(5)$ models the $\mathbf{10}^{(2,0)} \mathbf{10}^{(2,0)} \mathbf{5}_H^{(1,1)}$ coupling accounts for the mass of the down-type quark and can more naturally produce the associated hierarchy.

¹⁷Note that within non-perturbative F-theory constructions [150, 151] these couplings are in principle allowed.

¹⁸An explicit globally consistent realisation of the coupling $\mathbf{15} \mathbf{15} \mathbf{15}$ based on gauge group $U(6)$, from which the $\mathbf{10} \mathbf{10} \mathbf{5}$ emanates upon breaking $U(6) \rightarrow U(5) \times U(1)$, appears in [82].

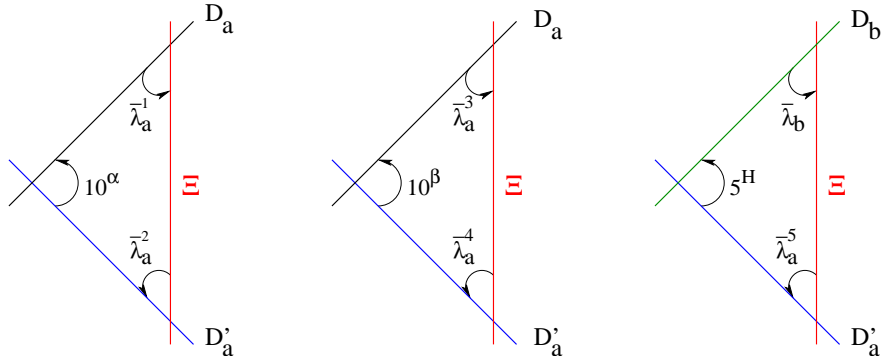


Figure 8: Three disc diagrams contributing to the top quark Yukawa coupling.

Dirac neutrino masses

Another interesting framework to explain the smallness of neutrino masses is given by models where the Dirac neutrino masses are absent perturbatively. This can occur if the anomalous $U(1)$ charges of the right-handed neutrino do not allow for such couplings at a perturbative level. Appropriate $O(1)$ instanton intersection numbers may in turn ensure the non-perturbative appearance of such couplings. For details and a concrete local Type IIA construction based on a chiral $SU(5)$ GUT see [153]. In this case the hierarchical coupling emerges naturally without further tuning of the volume of the instanton cycles. Namely, for $S_E \sim 8\pi^2/g_{GUT}^2$ the Dirac neutrino masses are of the order of 10^{-3} eV.

R-parity violating couplings

In multi-stack constructions R-parity violating couplings are absent perturbatively. However, it was shown that $O(1)$ instantons can generate both baryon number violating tri-linear couplings [20, 42] as well as lepton number violating ones [42]. There exist strong experimental limits on such couplings; in particular strong bounds from proton decay basically exclude the existence of both types of couplings at the same time. This example shows how important detailed knowledge of the non-perturbative sector of a vacuum can be.

5.5 Negative mass dimension couplings

Non-renormalizable terms induced by instantons are typically subleading since they are not only exponentially suppressed by the instanton effective action, but in addition by powers of the inverse string mass scale. Nevertheless, such terms could in principle introduce left-handed neutrino masses via Weinberg operators of the type $(LH_u)(LH_u)/\mu$ [42]. This term can compete with the see-saw induced left-handed neutrino masses. However, as the Weinberg operator coupling μ^{-1}

is suppressed by the instanton action, its contribution is generically subleading relative to the see-saw mass.

D-brane instantons can also induce dimension-five proton decay operators [42] which are however sufficiently small for models with large enough string scale. D-brane instanton generated non-renormalizable operators can also serve as an interesting stringy mediation mechanism by connecting the visible and the hidden sector [111]. For an attempt to generate top quark couplings of $SU(5)$ GUTs via certain non-renormalizable terms, see [154], though such couplings are usually too small to reproduce a desired top quark mass.

5.6 Other corrections

As discussed in §3 D-brane instantons can contribute to other terms beyond superpotential corrections such as higher order F-terms and threshold corrections. While one might naively expect higher fermionic F-terms to be only of minor phenomenological interest, it was shown in [78] that they become important in the context of moduli stabilization. Once the moduli appearing at a derivative level in the F-terms receive a mass, say the complex structure moduli in the presence of Type IIB 3-form fluxes, they can be integrated out, thus transforming the higher F-term into an effective superpotential at energies below the mass of the moduli.

While our discussion has focused on the generation of otherwise forbidden couplings, instanton corrections can also modify existing physical Yukawa couplings of charged matter fields [47]. These corrections can help improve some phenomenological properties of the Standard Model fermion mass matrix. For example, in certain toroidal constructions the mass matrix has only rank one at the perturbative level and instanton effects constitute the leading contribution to family mixing [47].

6 CONCLUSIONS AND OUTLOOK

We have reviewed a number of recent developments pertinent to D-brane instanton effects in $\mathcal{N} = 1$ Type II compactifications to four dimensions. This admittedly includes only a small portion of all the strenuous work on instanton effects in both field and string theory. The main motivation behind these recent efforts was to gain a better understanding of the implications of D-brane instantons for string phenomenology. Here the exponentially suppressed instanton contributions can become important if by some selection rule the perturbative contributions are vanishing. These might allow to solve by a *genuinely stringy mechanism* some of the small and large hierarchy problems present in supersymmetric extensions of the Standard Model. In addition instantons can yield non-trivial contributions to the closed string moduli potential and as such are essential ingredients for moduli

stabilisation, and very consequential for cosmic inflation.

We have tried to summarize many recent papers on the development of a D-brane instanton calculus for the computation of correlation functions in a D-brane instanton background. A rather coherent picture has emerged for the computation of correlators corresponding to holomorphic couplings in the four-dimensional $\mathcal{N} = 1$ supersymmetries effective action. We have outlined both the (boundary) conformal field theory based calculus as well as summarized the rules for local quiver type models, which are defined on highly curved background geometries. All results are in perfect agreement with each other and with expectations from well-known non-perturbative dynamics in field theory.

The most obvious types of instantons contributing to a superpotential are of rigid $\mathcal{O}(1)$ type, but we have described various more complicated instances where extra zero modes can be lifted or saturated in a way compatible with the generation of a superpotential. We have focused on situations with background fluxes, $U(1)$ instantons on top of single D-branes or configurations with non-trivial instanton-instanton couplings. While the covered results represent the state of the art as of this writing, a deeper understanding of the lifting of fermion zero modes is expected to generalize this picture in the future.

We have also reviewed recent efforts to better understand the physics of multi-instantons. Multi-instantons are important to account for the microscopic behavior of holomorphic four-dimensional couplings across lines of marginal stability. Taking the so extracted multi-instanton calculus seriously one is driven to the conclusion that the multi-instanton calculus is much richer and more involved than its field theoretic counterpart. This can be traced back to the existence of many stringy (exotic) D-brane instantons in string theory, which can induce mutual corrections to their instanton actions. More work is required to complete this picture. Not unrelatedly, it is also desirable to improve our understanding of instanton effects on D-terms in the four-dimensional effective action.

Finally, coming back to our main motivation, we have summarized some of the implications of D-brane instantons for phenomenologically important quantities such as the scalar potential, neutrino masses, Yukawa couplings and supersymmetry breaking linear couplings of Polonyi type. Needless to say that in a top-down approach, once a concrete string background has been fixed, so are all the non-perturbative effects. Our list thus only shows which terms can be generated in principle by what type of instantons; it still remains to define concrete models where instantons with precisely the right zero mode structure are indeed present. Typically this is particularly sensitive to global constraints such as the tadpole cancellation condition as a hidden sector might introduce extra charged zero modes at its intersections with the instanton. Another technical challenge is the exact computation and summation of all D-brane instanton effects in a given compactification. While we have outlined some existing technology relating this formidable task to a classical dual via geometric transitions, a long way remains to go until the same degree of sophistication is achieved as in the mirror

symmetric computation of worldsheet instanton effects.

Acknowledgements:

We thank our collaborators M. Aganagic, O. Aharony, N. Akerblom, R. Argurio, C. Beem, M. Bertolini, B. Florea, S. Franco, P. Langacker, D. Lüüst, J. McGreevy, S. Moster, E. Plauschinn, R. Richter, N. Saulina, M. Schmidt-Sommerfeld, E. Silverstein and D. Simic for sharing with us the excitement of stringy instantons over the past couple of years. We are indebted to R. Richter for valuable comments on the manuscript and help with the figures. This work was supported in part by the US Department of Energy under contract DE-AC02-76SF00515, the NSF under grant PHY-0244728, and the Stanford Institute for Theoretical Physics.

References

- [1] G. 't Hooft, “Computation of the quantum effects due to a four-dimensional pseudoparticle,” *Phys. Rev.* **D14** (1976) 3432–3450.
- [2] A. A. Belavin, A. M. Polyakov, A. S. Shvarts, and Y. S. Tyupkin, “Pseudoparticle solutions of the Yang-Mills equations,” *Phys. Lett.* **B59** (1975) 85–87.
- [3] M. T. Grisaru, W. Siegel, and M. Rocek, “Improved Methods for Supergraphs,” *Nucl. Phys.* **B159** (1979) 429.
- [4] N. Seiberg, “Naturalness Versus Supersymmetric Non-renormalization Theorems,” *Phys. Lett.* **B318** (1993) 469–475, [hep-ph/9309335](#).
- [5] M. Dine, N. Seiberg, X. G. Wen, and E. Witten, “Nonperturbative Effects on the String World Sheet,” *Nucl. Phys.* **B278** (1986) 769.
- [6] M. Dine, N. Seiberg, X. G. Wen, and E. Witten, “Nonperturbative Effects on the String World Sheet. 2,” *Nucl. Phys.* **B289** (1987) 319.
- [7] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges,” *Phys. Rev. Lett.* **75** (1995) 4724–4727, [hep-th/9510017](#).
- [8] E. Witten, “Bound states of strings and p-branes,” *Nucl. Phys.* **B460** (1996) 335–350, [hep-th/9510135](#).
- [9] E. Witten, “Small Instantons in String Theory,” *Nucl. Phys.* **B460** (1996) 541–559, [hep-th/9511030](#).
- [10] M. R. Douglas, “Branes within branes,” [hep-th/9512077](#).
- [11] S. Kachru, S. H. Katz, A. E. Lawrence, and J. McGreevy, “Open string instantons and superpotentials,” *Phys. Rev.* **D62** (2000) 026001, [hep-th/9912151](#).
- [12] J. Polchinski, “Combinatorics of boundaries in string theory,” *Phys. Rev.* **D50** (1994) 6041–6045, [hep-th/9407031](#).
- [13] K. Becker, M. Becker, and A. Strominger, “Five-branes, membranes and nonperturbative string theory,” *Nucl. Phys.* **B456** (1995) 130–152, [hep-th/9507158](#).
- [14] J. A. Harvey and G. W. Moore, “Superpotentials and membrane instantons,” [hep-th/9907026](#).
- [15] M. B. Green and M. Gutperle, “Effects of D-instantons,” *Nucl. Phys.* **B498** (1997) 195–227, [hep-th/9701093](#).

- [16] M. B. Green and M. Gutperle, “D-particle bound states and the D-instanton measure,” *JHEP* **01** (1998) 005, [hep-th/9711107](#).
- [17] M. Billo *et al.*, “Classical gauge instantons from open strings,” *JHEP* **02** (2003) 045, [hep-th/0211250](#).
- [18] E. Witten, “Non-Perturbative Superpotentials In String Theory,” *Nucl. Phys.* **B474** (1996) 343–360, [hep-th/9604030](#).
- [19] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev.* **D68** (2003) 046005, [hep-th/0301240](#).
- [20] R. Blumenhagen, M. Cvetič, and T. Weigand, “Spacetime instanton corrections in 4D string vacua - the seesaw mechanism for D-brane models,” *Nucl. Phys.* **B771** (2007) 113–142, [hep-th/0609191](#).
- [21] L. E. Ibanez and A. M. Uranga, “Neutrino Majorana masses from string theory instanton effects,” *JHEP* **03** (2007) 052, [hep-th/0609213](#).
- [22] B. Florea, S. Kachru, J. McGreevy, and N. Saulina, “Stringy instantons and quiver gauge theories,” *JHEP* **05** (2007) 024, [hep-th/0610003](#).
- [23] M. Haack, D. Krefl, D. Lüst, A. Van Proeyen, and M. Zagermann, “Gaugino condensates and D-terms from D7-branes,” *JHEP* **01** (2007) 078, [hep-th/0609211](#).
- [24] O. Aharony, S. Kachru, and E. Silverstein, “Simple Stringy Dynamical SUSY Breaking,” *Phys. Rev.* **D76** (2007) 126009, [0708.0493](#).
- [25] C. Angelantonj and A. Sagnotti, “Open strings,” *Phys. Rept.* **371** (2002) 1–150, [hep-th/0204089](#).
- [26] R. Blumenhagen, M. Cvetič, P. Langacker, and G. Shiu, “Toward realistic intersecting D-brane models,” *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71–139, [hep-th/0502005](#).
- [27] M. R. Douglas and S. Kachru, “Flux compactification,” *Rev. Mod. Phys.* **79** (2007) 733–796, [hep-th/0610102](#).
- [28] R. Blumenhagen, B. Kors, D. Lüst, and S. Stieberger, “Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,” *Phys. Rept.* **445** (2007) 1–193, [hep-th/0610327](#).
- [29] A. M. Uranga, “The standard model in string theory from D-branes,” *Nucl. Phys. Proc. Suppl.* **171** (2007) 119–138.
- [30] F. Marchesano, “Progress in D-brane model building,” *Fortsch. Phys.* **55** (2007) 491–518, [hep-th/0702094](#).

- [31] F. Denef, “Les Houches Lectures on Constructing String Vacua,” 0803.1194.
- [32] D. Lüst, “The landscape of string theory (orientifolds and their statistics, D-brane instantons, AdS(4) domain walls and black holes),” *Fortsch. Phys.* **56** (2008) 694–722.
- [33] D. Malyshev and H. Verlinde, “D-branes at Singularities and String Phenomenology,” *Nucl. Phys. Proc. Suppl.* **171** (2007) 139–163, 0711.2451.
- [34] M. Wijnholt, “Geometry of Particle Physics,” hep-th/0703047.
- [35] N. Akerblom, R. Blumenhagen, D. Lüst, and M. Schmidt-Sommerfeld, “D-brane Instantons in 4D Supersymmetric String Vacua,” *Fortsch. Phys.* **56** (2008) 313–323, 0712.1793.
- [36] M. Cvetič, R. Richter, and T. Weigand, “D-brane instanton effects in Type II orientifolds: local and global issues,” 0712.2845.
- [37] N. Akerblom, “D-instantons and effective couplings in intersecting D-brane models,” *Fortsch. Phys.* **56** (2008) 1065–1142.
- [38] M. Billo, “(D)-instanton effects in magnetized brane worlds,” *Fortsch. Phys.* **56** (2008) 735–743.
- [39] R. Argurio, M. Bertolini, S. Franco, and S. Kachru, “Metastable vacua and D-branes at the conifold,” *JHEP* **06** (2007) 017, hep-th/0703236.
- [40] R. Argurio, M. Bertolini, G. Ferretti, A. Lerda, and C. Petersson, “Stringy Instantons at Orbifold Singularities,” *JHEP* **06** (2007) 067, 0704.0262.
- [41] M. Bianchi, F. Fucito, and J. F. Morales, “D-brane Instantons on the T^6/Z_3 orientifold,” *JHEP* **07** (2007) 038, 0704.0784.
- [42] L. E. Ibanez, A. N. Schellekens, and A. M. Uranga, “Instanton Induced Neutrino Majorana Masses in CFT Orientifolds with MSSM-like spectra,” *JHEP* **06** (2007) 011, 0704.1079.
- [43] R. Blumenhagen, M. Cvetič, R. Richter, and T. Weigand, “Lifting D-Instanton Zero Modes by Recombination and Background Fluxes,” *JHEP* **10** (2007) 098, 0708.0403.
- [44] B. R. Greene, “String theory on Calabi-Yau manifolds,” hep-th/9702155.
- [45] O. J. Ganor, “A note on zeroes of superpotentials in F-theory,” *Nucl. Phys.* **B499** (1997) 55–66, hep-th/9612077.

- [46] M. Cvetič, R. Richter, and T. Weigand, “Computation of D-brane instanton induced superpotential couplings - Majorana masses from string theory,” *Phys. Rev.* **D76** (2007) 086002, [hep-th/0703028](#).
- [47] S. A. Abel and M. D. Goodsell, “Realistic Yukawa couplings through instantons in intersecting brane worlds,” *JHEP* **10** (2007) 034, [hep-th/0612110](#).
- [48] N. Akerblom, R. Blumenhagen, D. Lüst, E. Plauschinn, and M. Schmidt-Sommerfeld, “Non-perturbative SQCD Superpotentials from String Instantons,” *JHEP* **04** (2007) 076, [hep-th/0612132](#).
- [49] N. Akerblom, R. Blumenhagen, D. Lust, and M. Schmidt-Sommerfeld, “Instantons and Holomorphic Couplings in Intersecting D- brane Models,” *JHEP* **08** (2007) 044, [0705.2366](#).
- [50] N. Akerblom, R. Blumenhagen, D. Lüst, and M. Schmidt-Sommerfeld, “Thresholds for intersecting D-branes revisited,” *Phys. Lett.* **B652** (2007) 53–59, [0705.2150](#).
- [51] M. Billo *et al.*, “Instantons in N=2 magnetized D-brane worlds,” *JHEP* **10** (2007) 091, [0708.3806](#).
- [52] M. Billo *et al.*, “Instanton effects in N=1 brane models and the Kahler metric of twisted matter,” *JHEP* **12** (2007) 051, [0709.0245](#).
- [53] R. Blumenhagen and M. Schmidt-Sommerfeld, “Gauge Thresholds and Kaehler Metrics for Rigid Intersecting D-brane Models,” *JHEP* **12** (2007) 072, [0711.0866](#).
- [54] M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld, and Y. I. Manin, “Construction of instantons,” *Phys. Lett.* **A65** (1978) 185–187.
- [55] A. V. Belitsky, S. Vandoren, and P. van Nieuwenhuizen, “Yang-Mills and D-instantons,” *Class. Quant. Grav.* **17** (2000) 3521–3570, [hep-th/0004186](#).
- [56] N. Dorey, T. J. Hollowood, V. V. Khoze, and M. P. Mattis, “The calculus of many instantons,” *Phys. Rept.* **371** (2002) 231–459, [hep-th/0206063](#).
- [57] M. Bianchi, S. Kovacs, and G. Rossi, “Instantons and supersymmetry,” *Lect. Notes Phys.* **737** (2008) 303–470, [hep-th/0703142](#).
- [58] K. A. Intriligator and N. Seiberg, “Lectures on supersymmetric gauge theories and electric- magnetic duality,” *Nucl. Phys. Proc. Suppl.* **45BC** (1996) 1–28, [hep-th/9509066](#).

- [59] I. Affleck, M. Dine, and N. Seiberg, “Dynamical Supersymmetry Breaking in Supersymmetric QCD,” *Nucl. Phys.* **B241** (1984) 493–534.
- [60] M. Bianchi and E. Kiritsis, “Non-perturbative and Flux superpotentials for Type I strings on the Z_3 orbifold,” *Nucl. Phys.* **B782** (2007) 26–50, [hep-th/0702015](#).
- [61] R. Argurio, G. Ferretti, and C. Petersson, “Instantons and Toric Quiver Gauge Theories,” *JHEP* **07** (2008) 123, [0803.2041](#).
- [62] M. Billo’ *et al.*, “Non-perturbative effective interactions from fluxes,” [0807.4098](#).
- [63] G. Ferretti and C. Petersson, “Non-Perturbative Effects on a Fractional D3-Brane,” [0901.1182](#).
- [64] C. Petersson, “Superpotentials From Stringy Instantons Without Orientifolds,” *JHEP* **05** (2008) 078, [0711.1837](#).
- [65] M. Aganagic, C. Beem, and S. Kachru, “Geometric Transitions and Dynamical SUSY Breaking,” *Nucl. Phys.* **B796** (2008) 1–24, [0709.4277](#).
- [66] J. J. Heckman, J. Marsano, N. Saulina, S. Schafer-Nameki, and C. Vafa, “Instantons and SUSY breaking in F-theory,” [0808.1286](#).
- [67] J. Marsano, N. Saulina, and S. Schafer-Nameki, “An Instanton Toolbox for F-Theory Model Building,” [0808.2450](#).
- [68] J. Marsano, N. Saulina, and S. Schafer-Nameki, “Gauge Mediation in F-Theory GUT Models,” [0808.1571](#).
- [69] J. J. Heckman and C. Vafa, “F-theory, GUTs, and the Weak Scale,” [0809.1098](#).
- [70] C. Beasley and E. Witten, “New instanton effects in string theory,” *JHEP* **02** (2006) 060, [hep-th/0512039](#).
- [71] R. Blumenhagen and M. Schmidt-Sommerfeld, “Power Towers of String Instantons for N=1 Vacua,” *JHEP* **07** (2008) 027, [0803.1562](#).
- [72] P. G. Camara, E. Dudas, T. Maillard, and G. Pradisi, “String instantons, fluxes and moduli stabilization,” *Nucl. Phys.* **B795** (2008) 453–489, [0710.3080](#).
- [73] P. G. Camara and E. Dudas, “Multi-instanton and string loop corrections in toroidal orbifold models,” *JHEP* **08** (2008) 069, [0806.3102](#).

- [74] M. Bianchi and J. F. Morales, “Unoriented D-brane Instantons vs Heterotic worldsheet Instantons,” *JHEP* **02** (2008) 073, 0712.1895.
- [75] C. Beasley and E. Witten, “New instanton effects in supersymmetric QCD,” *JHEP* **01** (2005) 056, hep-th/0409149.
- [76] N. Seiberg, “Exact results on the space of vacua of four-dimensional SUSY gauge theories,” *Phys. Rev.* **D49** (1994) 6857–6863, hep-th/9402044.
- [77] Y. Matsuo, J. Park, C. Ryou, and M. Yamamoto, “D-instanton derivation of multi-fermion F-terms in supersymmetric QCD,” 0803.0798.
- [78] A. M. Uranga, “D-brane instantons and the effective field theory of flux compactifications,” 0808.2918.
- [79] I. Garcia-Etxebarria, F. Marchesano, and A. M. Uranga, “Non-perturbative F-terms across lines of BPS stability,” *JHEP* **07** (2008) 028, 0805.0713.
- [80] I. Garcia-Etxebarria and A. M. Uranga, “Non-perturbative superpotentials across lines of marginal stability,” *JHEP* **01** (2008) 033, 0711.1430.
- [81] M. Cvetič, R. Richter, and T. Weigand, “(Non-)BPS bound states and D-brane instantons,” *JHEP* **07** (2008) 012, 0803.2513.
- [82] L. E. Ibanez and A. M. Uranga, “Instanton Induced Open String Superpotentials and Branes at Singularities,” *JHEP* **02** (2008) 103, 0711.1316.
- [83] T. Jelinski and J. Pawelczyk, “Multi-Instanton Corrections to Superpotentials in Type II Compactifications,” 0810.4369.
- [84] D. Robles-Llana, M. Rocek, F. Saueressig, U. Theis, and S. Vandoren, “Nonperturbative corrections to 4D string theory effective actions from $SL(2,Z)$ duality and supersymmetry,” *Phys. Rev. Lett.* **98** (2007) 211602, hep-th/0612027.
- [85] F. Saueressig and S. Vandoren, “Conifold singularities, resumming instantons and non-perturbative mirror symmetry,” *JHEP* **07** (2007) 018, 0704.2229.
- [86] N. Halmagyi, I. V. Melnikov, and S. Sethi, “Instantons, Hypermultiplets and the Heterotic String,” *JHEP* **07** (2007) 086, 0704.3308.
- [87] D. Robles-Llana, F. Saueressig, U. Theis, and S. Vandoren, “Membrane instantons from mirror symmetry,” *Commun. Num. Theor. Phys.* **1** (2007) 681, 0707.0838.

- [88] S. Alexandrov, B. Pioline, F. Saueressig, and S. Vandoren, “D-instantons and twistors,” 0812.4219.
- [89] S. Alexandrov, “D-instantons and twistors: some exact results,” 0902.2761.
- [90] T. W. Grimm, “Non-Perturbative Corrections and Modularity in N=1 Type IIB Compactifications,” *JHEP* **10** (2007) 004, 0705.3253.
- [91] D. Gaiotto, G. W. Moore, and A. Neitzke, “Four-dimensional wall-crossing via three-dimensional field theory,” 0807.4723.
- [92] M. Kontsevich and Y. Soibelman, “Stability structures, motivic Donaldson-Thomas invariants and cluster transformations,” 0811.2435.
- [93] R. Kallosh and D. Sorokin, “Dirac action on M5 and M2 branes with bulk fluxes,” *JHEP* **05** (2005) 005, hep-th/0501081.
- [94] N. Saulina, “Topological constraints on stabilized flux vacua,” *Nucl. Phys.* **B720** (2005) 203–210, hep-th/0503125.
- [95] R. Kallosh, A.-K. Kashani-Poor, and A. Tomasiello, “Counting fermionic zero modes on M5 with fluxes,” *JHEP* **06** (2005) 069, hep-th/0503138.
- [96] D. Tsimpis, “Fivebrane instantons and Calabi-Yau fourfolds with flux,” *JHEP* **03** (2007) 099, hep-th/0701287.
- [97] L. Gorlich, S. Kachru, P. K. Tripathy, and S. P. Trivedi, “Gaugino condensation and nonperturbative superpotentials in flux compactifications,” *JHEP* **12** (2004) 074, hep-th/0407130.
- [98] P. K. Tripathy and S. P. Trivedi, “D3 Brane Action and Fermion Zero Modes in Presence of Background Flux,” *JHEP* **06** (2005) 066, hep-th/0503072.
- [99] E. Bergshoeff, R. Kallosh, A.-K. Kashani-Poor, D. Sorokin, and A. Tomasiello, “An index for the Dirac operator on D3 branes with background fluxes,” *JHEP* **10** (2005) 102, hep-th/0507069.
- [100] J. Park, “D3 instantons in Calabi-Yau orientifolds with(out) fluxes,” hep-th/0507091.
- [101] D. Lüst, S. Reffert, W. Schulgin, and P. K. Tripathy, “Fermion Zero Modes in the Presence of Fluxes and a Non-perturbative Superpotential,” *JHEP* **08** (2006) 071, hep-th/0509082.

- [102] D. Lüster, S. Reffert, E. Scheidegger, W. Schulgin, and S. Stieberger, “Moduli stabilization in type IIB orientifolds. II,” *Nucl. Phys.* **B766** (2007) 178–231, [hep-th/0609013](#).
- [103] W. Schulgin, “Zero mode counting in the presence of background fluxes,” *Nucl. Phys. Proc. Suppl.* **171** (2007) 316–318.
- [104] M. Billo’ *et al.*, “Flux interactions on D-branes and instantons,” *JHEP* **10** (2008) 112, [0807.1666](#).
- [105] A.-K. Kashani-Poor and A. Tomasiello, “A stringy test of flux-induced isometry gauging,” *Nucl. Phys.* **B728** (2005) 135–147, [hep-th/0505208](#).
- [106] S. B. Giddings, S. Kachru, and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev.* **D66** (2002) 106006, [hep-th/0105097](#).
- [107] M. Grana and J. Polchinski, “Supersymmetric three-form flux perturbations on AdS(5),” *Phys. Rev.* **D63** (2001) 026001, [hep-th/0009211](#).
- [108] D. Lüster, S. Reffert, W. Schulgin, and S. Stieberger, “Moduli stabilization in type IIB orientifolds. I: Orbifold limits,” *Nucl. Phys.* **B766** (2007) 68–149, [hep-th/0506090](#).
- [109] D. Marolf, L. Martucci, and P. J. Silva, “Actions and fermionic symmetries for D-branes in bosonic backgrounds,” *JHEP* **07** (2003) 019, [hep-th/0306066](#).
- [110] L. Martucci, J. Rosseel, D. Van den Bleeken, and A. Van Proeyen, “Dirac actions for D-branes on backgrounds with fluxes,” *Class. Quant. Grav.* **22** (2005) 2745–2764, [hep-th/0504041](#).
- [111] M. Buican and S. Franco, “SUSY breaking mediation by D-brane instantons,” [0806.1964](#).
- [112] S. Franco *et al.*, “Dimers and Orientifolds,” *JHEP* **09** (2007) 075, [0707.0298](#).
- [113] D. Forcella, I. Garcia-Etxebarria, and A. Uranga, “E3-brane instantons and baryonic operators for D3-branes on toric singularities,” [0806.2291](#).
- [114] M. R. Douglas and G. W. Moore, “D-branes, Quivers, and ALE Instantons,” [hep-th/9603167](#).
- [115] A. M. Uranga, “Brane Configurations for Branes at Conifolds,” *JHEP* **01** (1999) 022, [hep-th/9811004](#).

- [116] R. Argurio, M. Bertolini, S. Franco, and S. Kachru, “Gauge/gravity duality and meta-stable dynamical supersymmetry breaking,” *JHEP* **01** (2007) 083, [hep-th/0610212](#).
- [117] I. R. Klebanov and E. Witten, “Superconformal field theory on threebranes at a Calabi-Yau singularity,” *Nucl. Phys.* **B536** (1998) 199–218, [hep-th/9807080](#).
- [118] O. Aharony and S. Kachru, “Stringy Instantons and Cascading Quivers,” *JHEP* **09** (2007) 060, [0707.3126](#).
- [119] S. Kachru and D. Simic, “Stringy Instantons in IIB Brane Systems,” [0803.2514](#).
- [120] S. Gukov, C. Vafa, and E. Witten, “CFT’s from Calabi-Yau four-folds,” *Nucl. Phys.* **B584** (2000) 69–108, [hep-th/9906070](#).
- [121] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities,” *JHEP* **08** (2000) 052, [hep-th/0007191](#).
- [122] C. Vafa, “Superstrings and topological strings at large N,” *J. Math. Phys.* **42** (2001) 2798–2817, [hep-th/0008142](#).
- [123] P. Candelas, X. C. De La Ossa, P. S. Green, and L. Parkes, “A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory,” *Nucl. Phys.* **B359** (1991) 21–74.
- [124] M. Buican, D. Malyshev, and H. Verlinde, “On the Geometry of Metastable Supersymmetry Breaking,” *JHEP* **06** (2008) 108, [0710.5519](#).
- [125] F. Cachazo, S. Katz, and C. Vafa, “Geometric transitions and $N = 1$ quiver theories,” [hep-th/0108120](#).
- [126] E. Witten, “Branes and the dynamics of QCD,” *Nucl. Phys.* **B507** (1997) 658–690, [hep-th/9706109](#).
- [127] M. Aganagic and C. Vafa, “Mirror symmetry, D-branes and counting holomorphic discs,” [hep-th/0012041](#).
- [128] D. Krefl, “A gauge theory analog of some ‘stringy’ D-instantons,” [0803.2829](#).
- [129] A. Amariti, L. Girardello, and A. Mariotti, “Stringy Instantons as Strong Dynamics,” *JHEP* **11** (2008) 041, [0809.3432](#).
- [130] I. Garcia-Etxebarria, “D-brane instantons and matrix models,” [0810.1482](#).

- [131] M. Billo', M. Frau, L. Gallot, A. Lerda, and I. Pesando, "Classical solutions for exotic instantons?," 0901.1666.
- [132] F. Denef, M. R. Douglas, and B. Florea, "Building a better racetrack," *JHEP* **06** (2004) 034, hep-th/0404257.
- [133] F. Denef, M. R. Douglas, B. Florea, A. Grassi, and S. Kachru, "Fixing all moduli in a simple F-theory compactification," *Adv. Theor. Math. Phys.* **9** (2005) 861–929, hep-th/0503124.
- [134] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, "Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications," *JHEP* **03** (2005) 007, hep-th/0502058.
- [135] E. Palti, G. Tasinato, and J. Ward, "WEAKLY-coupled IIA Flux Compactifications," *JHEP* **06** (2008) 084, 0804.1248.
- [136] D. Baumann *et al.*, "On D3-brane potentials in compactifications with fluxes and wrapped D-branes," *JHEP* **11** (2006) 031, hep-th/0607050.
- [137] R. Blumenhagen, S. Moster, and E. Plauschinn, "String GUT Scenarios with Stabilised Moduli," 0806.2667.
- [138] R. Blumenhagen, S. Moster, and E. Plauschinn, "Moduli Stabilisation versus Chirality for MSSM like Type IIB Orientifolds," *JHEP* **01** (2008) 058, 0711.3389.
- [139] M. Cvetič and T. Weigand, "Hierarchies from D-brane instantons in globally defined Calabi-Yau Orientifolds," *Phys. Rev. Lett.* **100** (2008) 251601, 0711.0209.
- [140] M. Cvetič and T. Weigand, "A string theoretic model of gauge mediated supersymmetry beaking," 0807.3953.
- [141] C. Angelantonj, C. Condeescu, E. Dudas, and M. Lennek, "Stringy Instanton Effects in Models with Rigid Magnetised D-branes," 0902.1694.
- [142] G. F. Giudice and R. Rattazzi, "Theories with gauge-mediated supersymmetry breaking," *Phys. Rept.* **322** (1999) 419–499, hep-ph/9801271.
- [143] J. J. Heckman and C. Vafa, "F-theory, GUTs, and the Weak Scale," 0809.1098.
- [144] J. Kumar, "Dynamical SUSY Breaking in Intersecting Brane Models," *Phys. Rev.* **D77** (2008) 046010, 0708.4116.

- [145] M. Cvetič and P. Langacker, “work in progress,”.
- [146] S. Antusch, L. E. Ibanez, and T. Macri, “Neutrino Masses and Mixings from String Theory Instantons,” *JHEP* **09** (2007) 087, 0706.2132.
- [147] M. Buican, D. Malyshev, D. R. Morrison, H. Verlinde, and M. Wijnholt, “D-branes at singularities, compactification, and hypercharge,” *JHEP* **01** (2007) 107, hep-th/0610007.
- [148] R. Blumenhagen, M. Cvetič, D. Lüst, R. Richter, and T. Weigand, “Non-perturbative Yukawa Couplings from String Instantons,” *Phys. Rev. Lett.* **100** (2008) 061602, 0707.1871.
- [149] L. E. Ibanez and R. Richter, “Stringy Instantons and Yukawa Couplings in MSSM-like Orientifold Models,” 0811.1583.
- [150] C. Beasley, J. J. Heckman, and C. Vafa, “GUTs and Exceptional Branes in F-theory - II: Experimental Predictions,” 0806.0102.
- [151] R. Donagi and M. Wijnholt, “Model Building with F-Theory,” 0802.2969.
- [152] R. Blumenhagen, V. Braun, T. W. Grimm, and T. Weigand, “GUTs in Type IIB Orientifold Compactifications,” 0811.2936.
- [153] M. Cvetič and P. Langacker, “D-Instanton Generated Dirac Neutrino Masses,” 0803.2876.
- [154] C. Kokorelis, “On the (Non) Perturbative Origin of Quark Masses in D-brane GUT Models,” 0812.4804.

