October 2008
SLAC-PUB-13445

Using beam echo effect for generation of short-wavelength radiation

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Abstract

We propose to use an echo effect previously observed in hadron accelerators for up-frequency conversion of density modulation in an electron beam. We show that, for generation of high harmonics, this method is much more efficient in comparison with the currently used approach. A one dimensional model of the effect is developed which allows to optimize the amplitude of the modulation for a given harmonic number.

Submitted to the Physical Review Letters

Work supported in part by US Department of Energy contract DE-AC02-76SF00515

I. INTRODUCTION

The development of high-power short-wavelength free electron lasers (FELs) over the last years holds a promise of providing researches with a light source capable to generate coherent ultra short pulses of radiation in the spectral range from infrared to hard x-rays. The two main approaches that are currently pursued in the design of the next generation light sources are the self-amplified spontaneous emission (SASE) FELs [1, 2], and the high-gain harmonic generation (HGHG) approach [3, 4]. One of the main advantages of the HGHG approach over the SASE FEL is that, by using up-frequency conversion of the initial seed signal, HGHG allows to produce not only transversely, but also temporally coherent pulses. In contrast, the SASE radiation starts from initial shot noise in the beam, with the resulting radiation having an excellent spatial coherence, but a rather poor temporal one.

Unfortunately, the standard approach to HGHG suffers an essential drawback in that a single stage frequency conversion allows only a limited frequency multiplication factor [4]. This leads to multi-stage approach for x-ray production seeded at an ultra-violet wavelength [5], with a significant complication in the overall design. Some improvements in the cascading efficiency can be achieved by modification of the original simple approach, as demonstrated in recent paper [6]. However, generation of harmonic numbers in the range of 10 to 20, still requires a large energy modulation of the beam and deteriorates the beam properties as a lasing medium. In addition, the laser power for the seed light scales as a square of the modulation amplitude [7] and becomes expensive with an increased modulation amplitude.

The goal of this paper is to point out to a new physical mechanism that has several important advantages over the classical approach to the frequency cascading. This mechanism is based on the echo effect well known in different fields of physics, e.g., the spin echo in solids [8], the photon echo in solids and gases [9] and the echo effect in plasma [10]. A medium that exhibits the echo is characterized by excitations which decay in time due to phase mixing, or decoherence of different components of the excitation without involving an energy dissipation or diffusion.

For the beams in circular accelerators, the echo effect was first introduced in Refs. [11, 12], where it was shown how the echo can be generated and observed for betatron oscillations around an equilibrium orbit of the beam. Experimental observations, however, were initially carried out in the longitudinal direction for a coasting beam [13, 14]. In the setup for the longitudinal echo observations the beam energy is modulated by an RF cavity with the frequency ω_1 for a short period of time at some initial time $t = 0$. It is then allowed to coast in the ring until this initial perturbation gets converted to a density modulation, and then decays due to the slippage between particles of different energy. At a later time $t = \tau$, another RF system, with the frequency ω_2, modulates the beam energy. The beam signal decoheres again and for some time the beam exhibits no modulation at all. However, as it turns out, after long enough "sleep time" t_{echo}, the beam modulation recoheres for a short period at the frequency $m\omega_1 - k\omega_2$, where m and k are integer numbers [15]. The echo time actually depends on values of the values m and k, $t_{\text{echo}}(m, k)$ so that various echoes can be observed at different times with the amplitudes that depend on the initial amplitudes of the modulations.

The experiments [13, 14] were carried out using the modulation frequencies in the range from megahertz to hundreds of megahertz. As we will show below, the echo effect can also be implemented at much higher, laser frequencies. Before describing a setup for such an experiment we will quickly review a traditional approach used to modulate the beam density using a laser and an undulator tuned to the laser frequency.

II. DENSITY MODULATION CAUSED BY SINUSOIDAL BEAM ENERGY MODULATION AND DISPERSIVE SECTION

The set up for such a device is shown in Fig. 1a. An electron bunch with an average energy

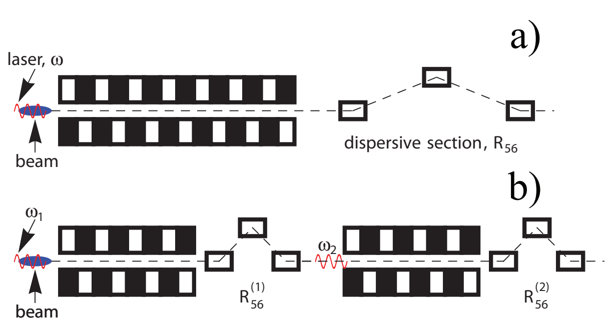

FIG. 1: A traditional system (a) consists of an undulator and a dispersive section. The proposed new scheme (b) includes two consecutive modulators.

E_0 interacts with a laser beam of frequency ω in a short undulator (called a modulator) with the resonant frequency tuned to ω. Typically, the bunch length is much larger than the laser wavelength, and one can locally consider a longitudinally uniform beam, neglecting variation of the beam current over the distance of several laser wavelength. We assume an initial Gaussian beam energy distribution with the variance σ_E and use the variable $p = (E - E_0)/\sigma_E$ for the dimensionless energy deviation of a particle. The initial distribution function of the beam, normalized by unity, is $f(p) = N(2\pi)^{-1/2}e^{-p^2/2}$, where N is the number of particles per unit length of the beam.

After passage through the undulator, the beam energy is modulated with the amplitude ΔE, so that the final dimensionless energy deviation p' is related to the initial one p by the equation $p' = p + A\sin(\kappa z)$, where $A = \Delta E/\sigma_E$, $\kappa = \omega/c$, and z is the longitudinal coordinate in the beam. The distribution function after the interaction with the laser becomes $f(z,p) = (2\pi)^{-1/2}\exp\left[-(p - A\sin\zeta)^2/2\right]$ where we now use the dimensionless variables $\zeta = \kappa z$. Sending then the beam through a dispersive system with the dispersive strength R_{56}, converts the longitudinal coordinate z into z', $z' = z + R_{56}p\,\sigma_E/E_0$, and makes the distribution function

$$f(\zeta, p) = \frac{N}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(p - A\sin(\zeta - Bp)\right)^2\right], \tag{1}$$

where $B = R_{56}\kappa\sigma_E/E_0$. Integration of f over p gives the distribution of the beam density N as a function of the coordinate ζ,

$$N(\zeta) = N_0\int_{-\infty}^{\infty} dp f(\zeta, p)\,. \tag{2}$$

Noting that this density is a periodic and even function of ζ one can expand it into Fourier series

$$\frac{N(\zeta)}{N_0} = 1 + \sum_{k=1}^{\infty} b_k\cos(k\zeta)\,, \tag{3}$$

where the coefficient b_k is the amplitude of the harmonic k. Calculations with the function (1) give an analytical expression for b_k [4]

$$|b_k| = 2\,e^{-\frac{1}{2}B^2k^2}|J_k(ABk)|\,, \tag{4}$$

where J_k is the Bessel function of order k.

It follows from this formula that if $A \lesssim 1$, then b_k decays exponentially when k increases. Indeed, for a given k, in order for the exponential factor in Eq. (4) not to suppress the result, the value of B should be such that $B \lesssim 1/k$. Taking into account that the Bessel function $J_k(x) \sim x^k/2^n n!$ if its argument is much less then k, we conclude that if the argument of the Bessel function is much smaller than one, b_k will also be strongly suppressed. Hence, we require $AB \sim 1$. Combining the two constrains for B and AB, we find that bunching factor will be small unless $A \gtrsim k$. For a numerical illustration, let us take $A = 1$; then the modulation in the forth harmonic is approximately one percent, $|b_4| = 0.01$, and $|b_5| = 1.9 \times 10^{-3}$. To obtain 10% modulation in 10-th harmonic ($|b_{10}| = 0.1$), A is required to be not less then 5.9, which would result in a six-fold increase in the energy spread of the beam after the passage through the modulator.

To overcome the low efficacy of the standard approach to modulate the beam, we propose to use a modulation scheme based on the longitudinal echo effect, in a full analogy with the experiments [13, 14], albeit extended into the region of optical frequencies. The set up is depicted in Fig. 1b.

III. USING THE ECHO EFFECT TO GENERATE HIGH HARMONICS

After passing through the system described above the beam is sent through one more energy modulator and an additional dispersive section. To distinguish between the first and the second modulators and dispersive sections we will now use indices 1 and 2, respectively (so that the distribution function (1) is now written with A and B replaced by A_1 and B_2). In general, the frequency ω_2 of the second laser beam, can differ from the frequency of the first laser ω_1.

The final distribution function at the exit from the second dispersion section can be easily found by applying consecutively two more transformations to (1), similar to the derivation outlined above. The first of these two transformations corresponding to the modulation of the beam energy with dimensionless amplitude A_2 is $p' = p + A_2 \sin(\kappa_2 z + \phi)$, where ϕ is a phase of the second laser beam; and the second one corresponding to the passage through the second dispersive element is $z' = z + pR_{56}^2 \sigma_E / E_0$. The resulting final distribution function is:

$$\begin{aligned} f(\zeta, p) = \frac{N}{\sqrt{2\pi}} \exp\Big[-\frac{1}{2}\big(p - A_2 \sin(K\zeta - KB_2 p + \phi) \\ - A_1 \sin(\zeta - (B_1 + B_2)p \\ + A_2 B_1 \sin(K\zeta - KB_2 p + \phi))\big)^2\Big], \end{aligned} \tag{5}$$

where we now use the notation $\zeta = \kappa_1 z$, $B_1 = R_{56}^{(1)}\kappa_1\sigma_E/E_0$, $B_2 = R_{56}^{(2)}\kappa_1\sigma_E/E_0$, and $K = \kappa_2/\kappa_1$.

To understand the main relations in the theory of echo, we first consider the limit of small energy modulations, $A_1, A_2 \ll 1$. At the same time we assume that the product $A_2 B_1$ may not be small. We then expand the distribution function (5) keeping only linear terms in A_1 and A_2. It is easy to see that the result of such an expansion gives three terms: the zero order term $N(2\pi)^{-1/2}e^{-\frac{p^2}{2}}$, the term $N(2\pi)^{-1/2}e^{-\frac{p^2}{2}}pA_2\sin(K\zeta - KB_2p+\phi)$ linear in A_2, and finally, the term, which we denote f_3,

$$f_3(\zeta,p) = \frac{N}{\sqrt{2\pi}}e^{-\frac{p^2}{2}}pA_1 \\ \times \sin(\zeta - (B_1+B_2)p + A_2B_1\sin(K\zeta - KB_2p+\phi))\,. \tag{6}$$

This last term is responsible for the echo effect, so we will focus on this term only. Using the mathematical identity

$$\sin\left(\phi_1 + x\sin\left(\phi_2\right)\right) = \sum_{k=-\infty}^{\infty} J_k(x)\sin\left(\phi_1 - k\phi_2\right)\,,$$

and integrating Eq. (6) over p gives

$$\int_{-\infty}^{\infty} f_3(\zeta,p)dp = \sum_{k=-\infty}^{\infty} c_k\cos\left(\zeta + kK\zeta + k\phi\right), \tag{7}$$

with

$$c_k = -A_1\left(B_1 + B_2(kK+1)\right) \\ \times J_k\left(B_1A_2\right)e^{-\frac{1}{2}(B_1+B_2(kK+1))^2}\,. \tag{8}$$

Each term in sum (7) corresponds to modulation with the wave number κ_{echo} equal to

$$\kappa_{\text{echo}} = k\kappa_2 + \kappa_1\,, \tag{9}$$

and we see that large values of k give up-frequency conversion of the wave number of the second laser κ_2. Note that k takes both positive and negative numbers; a negative κ_{echo} simply means a modulation with a wavelength $2\pi/|\kappa_{\text{echo}}|$.

For a given B_1, one can maximize the absolute value of $|c_k|$ by choosing

$$B_2 = -\frac{B_1 \pm 1}{kK+1}\,, \tag{10}$$

which gives

$$|c_k| = \frac{A_1}{\sqrt{e}} |J_k(A_2 B_1)| \,, \tag{11}$$

where $e = 2.71$. Note that since k takes both positive and negative values, Eq. (10) allows for solutions with $R_{56}^{(1)}$ and $R_{56}^{(2)}$ having either both the same, or opposite signs. For a given amplitude of the modulation A_1, one can now further maximize the value of the Bessel function in this expression by optimizing the value of B_1, that is by properly choosing $R_{56}^{(1)}$. For $k > 4$, with an accuracy better than a few percents, the maximum of the Bessel function J_k is approximately equal to $0.67/k^{1/3}$, and it is attained for the value of its argument equal to $k + 0.81k^{1/3}$. This gives the maximum value of $|c_k|$

$$|c_k| \approx \frac{0.41 A_1}{|k|^{1/3}} \,, \tag{12}$$

for

$$|B_1| \approx \frac{|k|}{A_2} (1 + 0.81 |k|^{-2/3}) \,. \tag{13}$$

Since we assume that $A_2 \ll 1$, it follows from the above equation that $|B_1|$ should be much larger than unity for the validity of our approximation. Note that although our definitions of $A_{1,2}$ and $B_{1,2}$ involve the energy spread in the beam σ_E, the optimal settings for the dimensional factors $R_{56}^{(1)}$ and $R_{56}^{(2)}$, as follows from Eqs. (13) and (11), do not depend on σ_E. They do depend however on the amplitude of the energy modulation $A_2 \sigma_E$ in the second modulator.

Eq. (12) demonstrates a remarkable feature of the echo modulation—a very slow decay $\propto |k|^{-1/3}$ with the harmonic number k. This kind of dependence is in sharp contrast with the exponential decrease of the bunching factor with k demonstrated by Eq. (4) for $A_1 \lesssim 1$. Note also that the amplitude of the second modulation of the beam A_2 did not come in the final result (12). Of course, a vanishingly small A_2 is not practical because, according to Eq. (13), it would require an extremely large value of $R_{56}^{(1)}$.

One can show that in the general case of arbitrary values of parameters A_1 and A_2, the wavenumber for the echo signal is

$$\kappa_{\text{echo}} = k\kappa_2 + m\kappa_1 \,, \tag{14}$$

where k and m are arbitrary (positive or negative) integer numbers.

IV. THE CASE OF EQUAL SEED FREQUENCIES

From a practical point of view, a particularly interesting case is represented by selection of equal wavelengths for both modulators, $\kappa_1 = \kappa_2$. This case can be realized with a single laser system by splitting the laser light and sending it to both undulators. Remarkably, this case allows an analytical solution for arbitrary values of parameters A_1 and A_2. As it follows from Eq. (14), the echo wavenumber in this case is equal to an integer numbers of κ_1, and hence the dimensionless beam density can be expanded in Fourier series

$$\frac{N(\zeta)}{N_0} = 1 + \sum_{k=1}^{\infty} b_k \cos(k\zeta + \psi_k)\,. \tag{15}$$

Omitting a lengthy derivation we present here the final result for the amplitudes of the modulation in this case:

$$\begin{aligned} b_k = 2\Big| \sum_{m=-\infty}^{\infty} & e^{im\phi} J_{-m-k}\left(A_1((m+k)B_1 + kB_2)\right) \\ & \times J_m\left(kA_2B_2\right) e^{-\frac{1}{2}((m+k)B_1+kB_2)^2} \Big|\,. \end{aligned} \tag{16}$$

Eq. (16) is too complicated for analytical maximization of b_k. However, if A_1 and A_2 are not very large, one can use the results of the small amplitude analysis Eq. (10) and (13) as a first approximation, with the following numerical optimization of Eq. (16).

We will demonstrate this approach for the case $A_1 = A_2 = 1$, that is when the energy modulation in both modulators is equal to the original energy spread in the beam. We will first try to maximize the amplitude of the 10th harmonic using Eq. (10) (13) for B_1 and B_2. Assuming that they are of the same sign (which means a negative $k = -11$ in Eq. (9)), we find $B_1 = 12.8$ and $B_2 = 1.18$. Eq. (12) predicts the amplitude $b_{10} = 0.18$. This prediction is then corrected by scanning the values of B_1 and B_2 in the vicinity of the above values. The plot of b_{10} as a function of B_1 obtained this way with the help of Eq. (16) is shown in Fig. 2 for several different values of B_2. As it follows from this plot, the maximal value of b_{10} is actually attained when $B_1 = 12.1$ and $B_2 = 1.3$, and is equal to 0.16, in a reasonably good agreement with the small A approximation. Note also a weaker echo effect in the region from 5 to 7 of values of B_1. It is important to emphasize here that, as an additional numerical analysis shows, the maximum value of b_{10} is insensitive to the value of the phase ϕ.

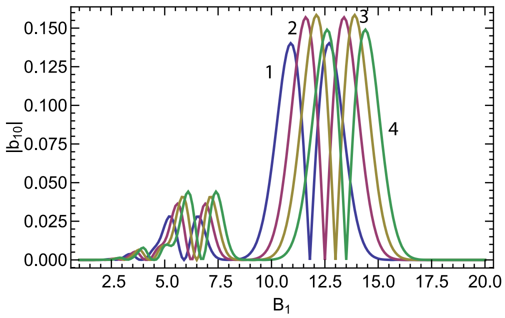

FIG. 2: The bunching factor for $k = 10$ as a function of parameter B_1 for four different values of B_2: *1* – $B_2 = 1.18$, *2* – $B_2 = 1.25$, *3* – $B_2 = 1.3$, *4* – $B_2 = 1.35$.

Fig. 3 shows the evolution of the phase space of the beam as it travels through the system, for parameters discussed above. These pictures demonstrate a simple physical mechanism

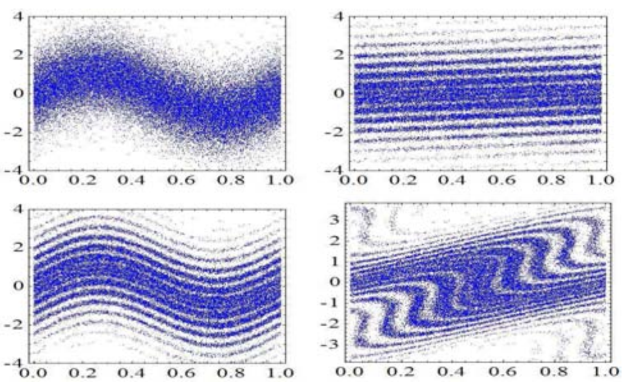

FIG. 3: The phase space of the beam after the first undulator (top left), the first dispersive element (top right), the second undulator (bottom left), and the second dispersive element (bottom right), for the parameters of the example in the text. Horizontal axes on the plots are ζ, and the vertical axes are p.

behind the echo effect. A large value of $R_{56}^{(1)}$ in the first modulator leads to "shredding" of the beam phase space in the longitudinal direction and generation of multiple "beamlets" in the phase space. Each beamlet is imaged as a stripe at the top right picture of Fig. 3. It has an almost uniform density distribution in the z direction and an energy spread much smaller than that of the original beam. The role of the second modulator consists in a simultaneous compression of all beamlets with a relatively modest value of $R_{56}^{(2)}$.

As an example of practical parameters for a possible application of the proposed scheme, we will estimate here the required strengths of the dispersion elements using the beam and the seed laser parameters of the ELETTRA@FERMI project in Trieste [16]. The relevant parameters are: the beam energy $E_0 = 1.2$ GeV, the beam energy spread $\sigma_E = 150$ keV, and, the laser wavelength 0.26 microns. For the example of an optimized 10th harmonic setup discussed above, one finds that the attaining the maximum amplitude of the modulation 0.16 requires $R_{56}^{(1)} = 4$ mm and $R_{56}^{(2)} = 0.37$ mm. As an another example, increasing three times the modulation in the first undulator, $A_1 = 3$ (and leaving $A_2 = 1$), allows to reach the relative amplitude of the 24 harmonics (corresponding to the wavelength of 10 nm) of 0.2. The required dispersive elements for this case are: $R_{56}^{(1)} = 7.9$ mm and $R_{56}^{(2)} = 0.22$ mm.

There are several practical physical effects that are left behind our simplified one-dimensional analysis of the echo effect. They include coupling between transverse and longitudinal degrees of freedom; coherent and incoherent radiation of the bunched beam in dispersion elements and additional energy spread introduced by the radiation; radial inhomogeneity of the laser beam in the undulator and its effect on the amplitude of the echo effect. These an other effects should be taken in the account in the design and optimization of the echo experiment setup. It is likely that they will determine the ultimate shortest wavelength of modulation achievable with the proposed approach.

V. ACKNOWLEDGEMENTS

The author would like to thank M. Zolotorev for a useful discussion, and Dao Xiang for help in calculations and critical remarks.

This work was supported by US DOE contracts DE-AC03-76SF00515.

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