

October 2008

UMN-TH-2720/08, FTPI-MINN-08/38, SLAC-PUB-13434

Confinement in Yang Mills: Elements of a Big Picture

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Abstract

This is a combined and slightly expanded version of talks delivered at 14th International QCD Conference “QCD 08,” 7-12th July 2008, Montpellier, France, the International Conference “Quark Confinement and the Hadron Spectrum,” Mainz, Germany, September 1-6, 2008 (Confinement 08), and the International Conference “Approaches to Quantum Chromodynamics,” Oberwolz, Austria, September 7-13, 2008.

Work supported in part by US Department of Energy contract DE-AC02-76SF00515

¹Unité Mixte de Recherche du CNRS, (UMR 8627).

1 Introduction

QCD is a non-Abelian gauge theory of strong interactions. It is extremely rich and describes a very wide range of natural phenomena, e.g.:

- all of nuclear physics;
- Regge behavior and Regge trajectories;
- strongly coupled quark-gluon plasma; high- T and high-density phenomena; neutron stars;
- richness of the hadronic world (chiral phenomena, light and heavy quarkonia, glueballs and exotics, exclusive and inclusive processes, interplay between strong and weak interactions, and many other issues).

At short distances QCD is weakly coupled, allowing high precision perturbative (multi-loop, multi-leg) calculations, however, analytical computations at all energy and momentum scales seem unlikely due to strong coupling nature at large distances. Let us ask ourselves: what do we want from this theory? Is it reasonable to expect high-precision predictions for the low energy observables like in QED? Can we (and should we) compute hadronic masses, matrix elements or proton magnetic moment up to, say, five digits?

The answer to the last question, as well as other similar questions, seems to be negative, at least by analytical means. Thankfully, at this front, exceedingly more precise and reliable numerical computations come from the lattice gauge theory. Lattice is the first-principles numerical framework for strongly coupled gauge theories and QCD. But, at the same time, it is a black box, “an experiment.”²

What we really need is a qualitative understanding of non-Abelian gauge dynamics in various environments and various settings, an overall “Big Picture.”

The overall picture that emerged in the last three decades – and, especially, in the last ten years or so – makes theorists (who are still active in this area) rather happy. A wealth of semi-classical techniques were developed in the past. More recently, some novel techniques allowing one to study non-perturbative gauge dynamics through “smooth transitions” were devised. These techniques work even for notoriously elusive chiral gauge theories (for

²Lattices provide non-perturbative numerical data; analytical theorists must engineer certain regimes for both QCD and lattice theories in which analytical results can be confronted with the lattice simulations.

which no practical lattice formulations exist). They provide ample opportunities for comparison of our analytical understanding of non-perturbative gauge dynamics with the lattice theory.

In the Big Picture, there are still some dark corners to be explored, which will be mentioned later. First, let us highlight general features of confinement and the elements which can be considered as well-established.

2 Color confinement: Generalities

The most salient feature of pure Yang-Mills theory is linear confinement. If one takes a heavy probe quark and an antiquark separated by a distance, the force between them does not fall off with distance, while the potential energy grows linearly.

Are there physical phenomena in which interaction energy between two interacting bodies grows with distance at large distances? What is the underlying mechanism?

The answer to this question is positive. The phenomenon was predicted by Abrikosov [1] in the superconductors of the second type which, in turn, were invented by Abrikosov [2] and discovered experimentally in the 1960s. The corresponding set up is shown in Fig. 1. In the middle of this figure we see a superconducting sample, with two very long magnets attached to it. The superconducting medium does not tolerate the magnetic field. On the other hand, the flux of the magnetic field must be conserved. Therefore, the magnetic field lines emanating from the S pole of one magnet find their way to the N pole of another magnet, through the medium, by virtue of a flux tube formation. Inside the flux tube the Cooper pair condensate vanishes and superconductivity is ruined. The flux tube has a fixed tension, implying a constant force between the magnetic poles as long as they are inside the superconducting sample. The phenomenon described above is sometimes referred to as the Meissner effect.

Of course, the Meissner effect of the Abrikosov type occurs in the Abelian theory, QED. The flux tube that forms in this case is Abelian. In Yang-Mills theories we are interested in its non-Abelian analog. Moreover, while in the Abrikosov case the flux tube is that of the magnetic field, in QCD and QCD-like theories the confined objects are the quarks; therefore, the flux tubes must be “chromoelectric” rather than chromomagnetic. In the

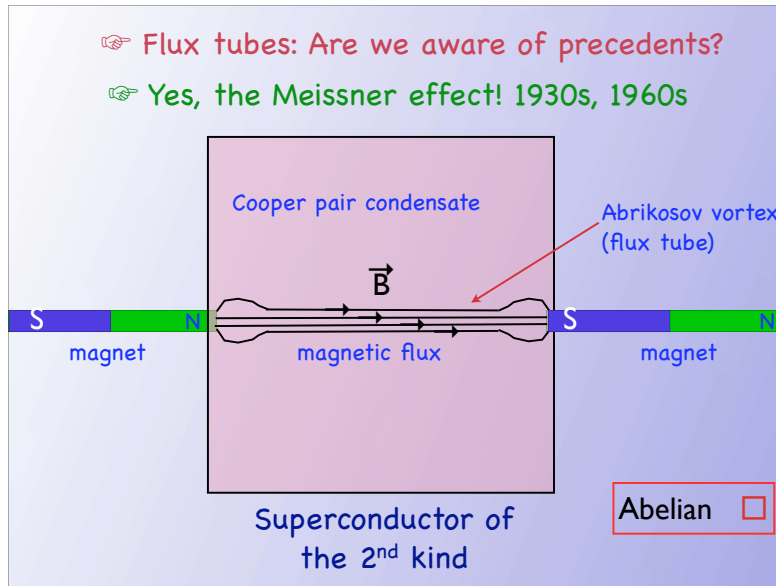


Figure 1: The Meissner effect in QED

mid-1970s Nambu, 't Hooft, and Mandelstam (independently) put forward an idea [3] of a “dual Meissner effect” as the underlying mechanism for color confinement (Fig. 2).³ Within the framework of this idea, in QCD-like theories “monopoles” condense leading to formation of “non-Abelian flux tubes” between the probe quarks. At this time the Nambu–’t Hooft–Mandelstam paradigm was not even a physical scenario, rather a dream, since people had no clue as to the main building blocks such as non-Abelian chromoelec-

³While Nambu and Mandelstam’s publications are easily accessible, it is hard to find the EPS Conference Proceedings in which ’t Hooft presented his vision. Therefore, the corresponding passage from his talk is worth quoting: “[monopoles] turn to develop a non-zero vacuum expectation value. Since they carry color-magnetic charges, the vacuum will behave like a superconductor for color-magnetic charges. What does that mean? Remember that in ordinary electric superconductors, magnetic charges are confined by magnetic vortex lines ... We now have the opposite: it is the color charges that are confined by electric flux tubes.”

tric flux tubes. In the two decades that elapsed after the Nambu–’t Hooft–Mandelstam conjecture many works had been published on this subject, with very little advancement, if at all.

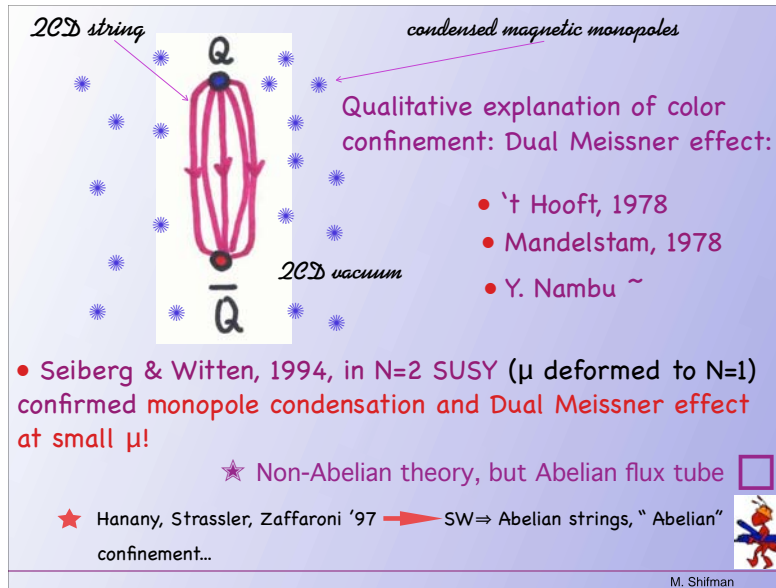


Figure 2: The Meissner effect in QED

A decisive breakthrough came in 1994, with the Seiberg–Witten solution of $\mathcal{N} = 2$ super-Yang–Mills [4] slightly deformed by a $\mu \text{Tr}\Phi^2$ term in the superpotential. The deformation term breaks $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. At $\mu = 0$ the theory has a moduli space parametrized by $\text{Tr}\Phi^2$. Assume for simplicity that the gauge symmetry of the action is $SU(2)$. On the moduli space, $SU(2)_{\text{gauge}}$ is spontaneously broken down to $U(1)$. Therefore, the theory possesses the ’t Hooft–Polyakov monopoles [5]. Two points on the moduli space were detected in [4] (the so-called monopole and dyon points) in which the monopoles (dyons) become massless. In these points the scale of the gauge symmetry breaking

$$SU(2) \rightarrow U(1) \tag{1}$$

is determined by the dynamical parameter Λ of the microscopic theory. While the neutral gauge boson (photon) remains massless, others acquire masses of the order of Λ and can be integrated out. In the low-energy limit near the monopole and dyon points one deals with electrodynamics of massless monopoles. One can formulate an effective macroscopic low-energy theory. This is a U(1) gauge theory in which the charged matter fields M, \bar{M} are those of monopoles while the gauge field is dual with respect to the photon of the microscopic theory. The superpotential has the form $\mathcal{W} = \mathcal{A}MM$, where \mathcal{A} is the $\mathcal{N} = 2$ superpartner of the dual photon/photino fields.

Now, if one switches on $\mu = 0$ (and $\mu \ll \Lambda$), the only change in the macroscopic theory is the emergence of the extra $m^2\mathcal{A}$ term in the superpotential. The dual mass parameter $m^2 \sim \mu\Lambda$. The $m^2\mathcal{A}$ term triggers the monopole condensation, $M = \bar{M} = m$, which implies, in turn, that the dual U(1) symmetry is spontaneously broken, and the dual photon acquires a mass $\sim m$. As a consequence, Abrikosov flux tubes are formed. Viewed inside the dual theory, they carry fluxes of the magnetic field. With regards to the original microscopic theory these are the electric field fluxes.

Thus, Seiberg and Witten demonstrated, for the first time ever, the existence of the dual Meissner effect in a judiciously chosen non-Abelian gauge field theory. If one “injects” a probe (very heavy) quark and antiquark in this theory, a flux tube necessarily forms between them leading to linear confinement. In the leading order in μ this flux tube is BPS saturated. Its tension is $T \sim \mu\Lambda$.

The flux tubes in the Seiberg–Witten solution were investigated in detail in 1997 by Hanany, Strassler and Zaffaroni [6]. These flux tubes are Abelian, and so is confinement caused by their formation. What does that mean? At the scale of distances at which the flux tube is formed (the inverse mass of the Higgsed U(1) photon) the gauge group that is operative is Abelian. In the Seiberg–Witten analysis this is the dual U(1). The off-diagonal (charged) gauge bosons are very heavy in this scale and play no direct role in the flux tube formation and confinement that ensues. Naturally, the spectrum of composite objects in this case turns out to be richer than that in QCD and similar theories with non-Abelian confinement.⁴ It includes not only color

⁴Anticipating further discussions let us note that by non-Abelian confinement we mean such dynamical regime in which at distances of the flux tube formation all gauge bosons are equally important.

singlets, but, in addition, a variety of states which are U(1) neutral rather than SU(2) neutral. Moreover, the string topological stability is based on $\pi_1(\text{U}(1)) = \mathbb{Z}$. Therefore, N strings do not annihilate as they should in QCD-like theories.

The double-stage symmetry breaking pattern, with (1) occurring at a high scale while the dual U(1) \rightarrow nothing at a much lower scale, disappears as we let μ approach Λ . Eventually we could send this parameter to infinity. In the limit $\mu \rightarrow \infty$ we would recover an $\mathcal{N} = 1$ rather than $\mathcal{N} = 2$ theory in which all non-Abelian gauge degrees of freedom presumably take part in the string formation, and are operative at the scale at which the strings are formed. The strings that occur (if they occur) may be called non-Abelian.

The Seiberg–Witten solution *per se* is applicable only at $\mu \ll \Lambda$. We have no quantitative (or even semi-qualitative) description of the non-Abelian confinement emerging at large μ , when the U(1) subgroup of SU(2) is no longer singled out. It is generally believed, however, that the transition from the Abelian to non-Abelian regime is smooth. It is argued that the strings occurring in the Seiberg–Witten solution belong to the same universality class as those in QCD-like theories. We will return to the issue of universality classes and non-Abelian flux tubes later, in Sect. 8. Our immediate task is to develop a similar strategy for non-supersymmetric QCD-like gauge theories which (by definition) do not possess elementary scalars in the microscopic Lagrangian.

3 QCD-like theories on a cylinder; double-trace deformation

In the Seiberg–Witten case the deformation parameter which governs the transition from Abelian to non-Abelian confinement is μ , the parameter of the $\mathcal{N} = 2$ supersymmetry breaking. In the non-supersymmetric case we have to invent another parameter which would play the same role. An appropriate set-up was suggested by Shifman and Unsal [7]. Instead of considering QCD and QCD-like cousins on R_4 one can compactify (say, a spatial) dimension, formulating the theory on a cylinder $R_3 \times S_1$ (Fig. 3). The radius of the compact dimension $r(S_1)$ is the parameter regulating dynamical regimes of the theory.

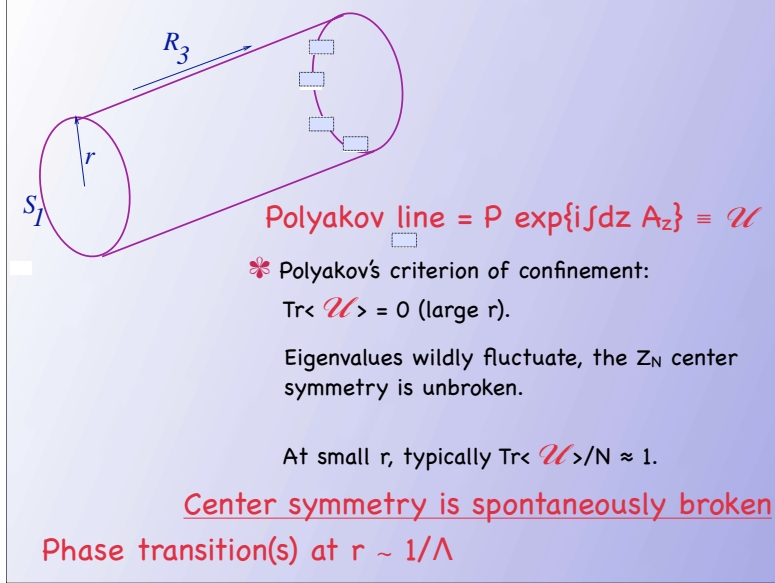


Figure 3: QCD-like theories on a cylinder.

Of course, in the past such a set-up was considered many times, for various purposes, with a number of distinct boundary conditions, with the temporal or spatial compactification. It did not bring people closer to the goal we have in mind today. The point is that the small- $r(S_1)$ domain was always separated from the decompactification limit of large $r(S_1)$ by one or more phase transitions (Fig. 4).

In particular, in pure Yang–Mills theory a confinement–deconfinement phase transition was identified at $r(S_1) = r_* \sim \Lambda^{-1}$. Moreover, with massless quarks included, the large- $r(S_1)$ domain, with the spontaneously broken chiral symmetry, is separated from the small- $r(S_1)$ domain, with a restored chiral symmetry, by a chiral phase transition, at (approximately) the same value of $r(S_1) \sim r_*$. The only exception is $\mathcal{N} = 1$ super–Yang–Mills endowed with the periodic boundary condition.

The reason of the unwanted phase transition(s) is the breaking of the

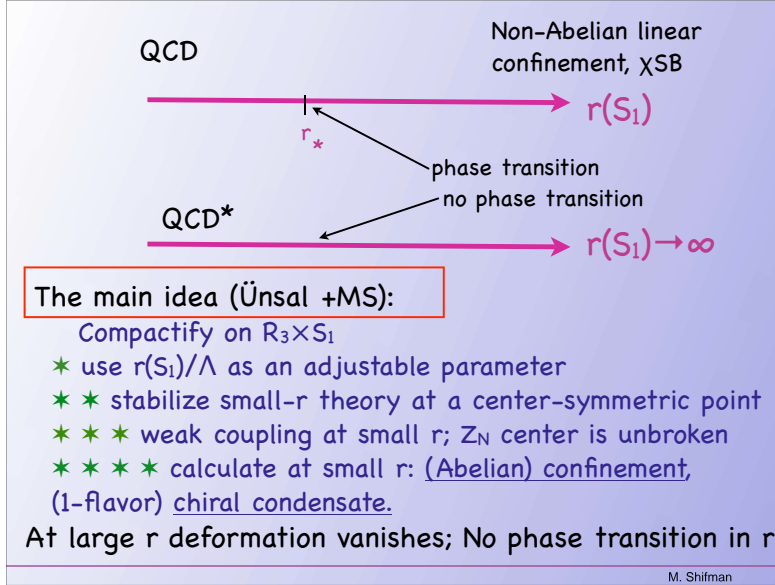


Figure 4: Proceeding from small- to large- $r(S_1)$ one usually experiences a confinement-deconfinement (and, possibly, chiral symmetry restoration) phase transition. To make the $r(S_1)$ evolution smooth we add double trace deformations.

center symmetry in the conventional (thermal) set-up at small $r(S_1)$. The famous Polyakov criterion [8, 9] tells us then that the theory is in the deconfinement phase. Moreover, the vacuum structure is such that even at small $r(S_1)$ the theory is governed by the strong-coupling regime which precludes analytic control.

The physical picture in the confinement phase is as follows. Assume that the compactified dimension is z . The Polyakov line (sometimes called the Polyakov loop) is defined as a path-ordered holonomy of the Wilson line in the compactified dimension,

$$\mathcal{U} = P \exp \left\{ i \int_0^L a_z dz \right\} \equiv V U V^\dagger \quad (2)$$

where L is the size of the compact dimension while V is a matrix diagonalizing \mathcal{U} ,

$$U = \text{diag } v_1 \ v_2 \ \dots \ v_N \quad (3)$$

According to Polyakov, non-Abelian confinement implies that the eigenvalues v_i are randomized: the phases of v_i wildly fluctuate over the entire interval $[0, 2\pi]$ so that

$$\text{Tr} U = 0 \quad (4)$$

The vanishing of $\text{Tr} U$ implies that the center symmetry is unbroken.

The double-trace deformations suggested in this context in [7] eliminate both unwanted features at once. A general design is as follows. On $R_3 \times S_1$ at small $r(S_1)$ one can deform the original theory (for the time being we mean pure Yang–Mills or one-flavor QCD) by a double-trace operator $P[U_J(\mathbf{x})]$ where

$$P[U(\mathbf{x})] = \frac{2}{\pi^2 L^4} \prod_{n=1}^{\lfloor \frac{N}{2} \rfloor} d_n \left(\text{Tr} U^n(\mathbf{x}) \right)^2 \quad (5)$$

Here d_n are numerical parameters of order one, and $\lfloor \cdot \rfloor$ denotes the integer part of the argument in the brackets. The deformed action is

$$S^* = S + \int_{R_3 \times S_1} P[U(\mathbf{x})] \quad (6)$$

We label the deformed theories with an asterisk (e.g. YM*). For judiciously chosen d_n , the center symmetry remains unbroken in the vacuum. This is due to the fact that the gauge symmetry $\text{SU}(N)$ spontaneously breaks

$$\text{SU}(N) \rightarrow \text{U}(1)^{N-1} \quad (7)$$

with the pattern of the eigenvalues of U depicted in Fig. 5. This means, the long-distance dynamics is effectively Abelian.

At strong coupling confinement takes place because the eigenvalues of U wildly fluctuate implying vanishing of the Polyakov line. These wild fluctuations are in one-to-one correspondence with formation of confining flux tubes. At weak coupling the Polyakov line vanishes for a different reason, but this is still a confinement.

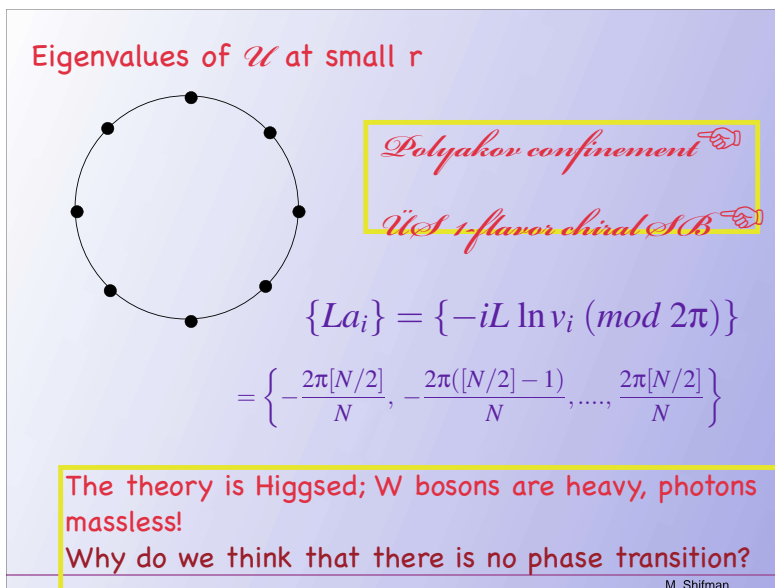


Figure 5: Center symmetry stabilization by double trace deformations.

4 Linear confinement at small $r(S_1)$

Consider the deformed Yang–Mills theory. The $SU(N)$ gauge group is Higgsed down to an Abelian $U(1)^{N-1}$ Cartan subgroup at the scale of compactification (remember, $r(S_1)$ is small for the time being). At low energies, keeping only the zero modes, we end up with $N - 1$ “photons” in three dimensions. Perturbatively, they are massless. Correspondingly, there is no linear confinement to all orders in perturbation theory.

The picture drastically changes at the non-perturbative level. In Yang–Mills-(adjoint)Higgs system this is known from 1977, when Polyakov demonstrated that three-dimensional instantons generate a mass for the dual photons [10]. Technically, these instantons look exactly as monopoles in four dimensions; hence we will refer to them as instanton-monopoles.

It is obvious that in 2+1 dimensions the photon field has only one physical

(transverse) polarization. This means that the photon field must have a dual description in terms of one scalar field ϕ , namely [10]

$$F_{\mu\nu} = k_{\mu\nu\rho}(\partial^\rho \phi) \quad (8)$$

where $k = g_{3D}^2 / 4\pi$ and $g_{3D}^2 = g_{4D}^2 L^{-1}$. The circumference of S_1 is $L = 2\pi r(S_1)$. Equation (8) defines the field ϕ in terms of $F_{\mu\nu}$ in a non-local way. At the same time, $F_{\mu\nu}$ is related to ϕ locally.

Consider a probe heavy charge $\pm 1/2$ at the origin. The electric field induced by the probe particle is radially oriented. A brief inspection of Eq. (8) shows that the radial orientation of E requires the scalar function ϕ to be r -independent. Moreover, it should depend on the polar angle α as

$$\phi = \alpha \quad (9)$$

Needless to say, Eq. (9) implies that the scalar field ϕ is compact and defined mod 2π . The points

$$\alpha \pm 2\pi \quad \alpha \pm 4\pi \quad (10)$$

are identified. Thus, Polyakov's observation that in 2+1 dimensions the photon field is dual to a real scalar field needs an additional specification: the real scalar field at hand is compact; it is defined on a circle of circumference 2π .⁵ The minimal electric charge in the original formulation is equal to the minimal vortex in the dual formulation, and vice versa.

As was noted by Polyakov, the instanton-monopole contribution generates a mass term for the field ϕ ,

$$\mathcal{L}_{\text{dual}} = \frac{\kappa^2}{2} (\partial_\mu \phi)^2 + \mu^3 \cos \phi \quad (11)$$

where $\kappa = g_{3D} / 4\pi$ and

$$\mu^3 = e^{-S_{\text{inst}}} \quad (12)$$

where

$$S_{\text{inst}} = \frac{4\pi m_W}{g_{3D}^2} \quad (13)$$

⁵The non-compact (free) Maxwell theory is dual to a free scalar theory with a continuous shift symmetry. Compact electrodynamics has instantons; this explicitly breaks the continuous shift symmetry down to a 2π shift symmetry. The latter can no longer protect masslessness of the dual scalar.

Hence, μ^3 is exponentially small at small $r(S_1)$. The dual photon mass $m = \mu^3 \kappa^{-1}$ is exponentially small too.

In the dual language it is quite obvious that if $m = 0$, there exist domain “lines” in 1+2 dimensions, a.k.a. strings, separating the (physically equivalent) domains $\phi = 0$ and $\phi = 2\pi$ (or, more generally, $\phi_{\text{vac}} = 2\pi k$ and $\phi_{\text{vac}} = 2\pi(k+1)$ where k is an arbitrary integer), see Fig. 6. The endpoints of the “line” are vortices. The line thickness is $\sim 1/m$.

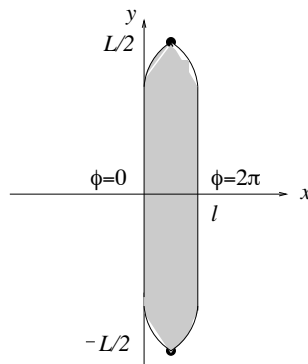


Figure 6: “Domain wall” in 2+1 dimensions. The circles at the endpoints represent probe charges $\pm 1/2$. These are vortex-anti-vortex pairs in the dual formulation. The picture assumes that $L \gg \lambda$. The transitional domain which is a domain line and a string simultaneously, is shaded.

The Polyakov string tension is

$$T = 8\mu^3 \kappa = 8m \kappa^2 = 2km \quad (14)$$

Note that this tension is much larger than λ^{-2} .

For this string to develop between two probe charges the distance L between the charges must be $L \gg \lambda$. At distances $\lesssim \lambda$ each charge is surrounded by essentially a (two-dimensional) Coulomb field, with the force lines spreading homogeneously in all directions. At distances $\sim \lambda$ the “flux tube” starts forming. If the probe charges have opposite signs, and $L \gg \lambda$, they are connected by the “flux tube,” and the energy of the configuration grows as TL . At huge distances $L \gg \lambda$ the linear confinement sets in.

The YM* theory at small $r(S_1)$ can be viewed as a very similar three-dimensional YM–Higgs system, in which a compact adjoint Higgs field – the

holonomy – replaces the non-compact field of the Polyakov model. This “compactness” introduces an extra topological excitation (see e.g. [11]). Some technicalities of the analysis change; but, in essence, we have the same dynamical system. However, introduction of massless quarks drastically changes the picture.

5 Massless quarks

Let us speak for definiteness of a single massless quark in the $SU(N)$ gauge theory: either in the fundamental representation (for QCD) or in the two-index representations (for QCD-like theories). If such a massless quark is introduced in the Polyakov three-dimensional model *per se* the model ceases to be confining! This is due to the fact that the instanton-monopoles have zero fermion modes and no longer generate the dual photon mass. Rather, they give rise to a chiral condensate which may break a discrete chiral symmetry.

However, in QCD* on $R_3 \times S_1$ the situation is different. In this theory there are other, composite, topological excitations (different from the instanton-monopoles) which carry a net magnetic charge, but no topological charge and, hence, no fermion zero modes. Such excitations do generate a potential and produce a mass term for dual photons, implying linear confinement.

In this aspect there is a crucial difference between the Polyakov construction which is genuinely three-dimensional and the $R_3 \times S_1$ reduction of QCD and QCD-like theories. For example, in the compactified four-dimensional $SU(2)$ Yang–Mills theory there are two types of monopoles: one is the standard ’t Hooft–Polyakov (tHP) monopole; another can be called the Kaluza–Klein (KK) monopole [11]; its existence is due to the fact that $\pi_1(S_1) = \mathbb{Z}$. In QCD-like theories with the two-index representation fermions, the bound state of the tHP monopole and the KK antimonopole carries no topological charge; hence, no fermion zero modes. It does carry a magnetic charge and, therefore, generates a mass term for the dual photon. The above pair was shown to be stable [12]; it was termed a magnetic bion. In $SU(N)$ theories the set of bions responsible for nonperturbative physics on $R_3 \times S_1$ is quite varied; they contribute to the dual photon effective potential (i.e. to the dual photon mass squared) at the level $\exp(-2S_{\text{inst}})$.

Summarizing the situation in one-flavor QCD*, the instanton-monopoles generate the chiral condensate at the level $\exp(-S_{\text{inst}})$. Bions are responsible for the dual photon masses which lead, in turn, to (Abelian) linear confinement through domain lines [7]. Four-dimensional instantons produce effects which are suppressed at the level $\exp(-NS_{\text{inst}})$.

The multiflavor QCD* theories exhibit an interplay of different dynamics; it is currently under consideration.

6 Transition to large $r(S_1)$

Much in the same way as passing from small to large μ in the Seiberg–Witten constructions is believed to take us smoothly from the Abelian confinement to the non-Abelian one, we would like to argue that the transition to large $r(S_1)$ does the same job in YM* and one-flavor QCD*, and is smooth too.

Looking from four dimensions we see the following order parameters: the Polyakov line (confinement/deconfinement) and the chiral condensate. The first vanishes both at small and large $r(S_1)$. Hence, it gives no signal as to a possible confinement-deconfinement phase transition. On the contrary, the second does not vanish both at small and large $r(S_1)$, indicating spontaneous breaking of a discrete chiral symmetry. Therefore, there is no reason to expect a chiral phase transition either (for quarks belonging to higher representations).

At small $r(S_1)$ confinement is obviously Abelian. At large $r(S_1)$ we expect non-Abelian confinement, much in the same way as in the Seiberg–Witten problem at large μ . There is no obvious obstacle to a smooth transition from the Abelian confinement to that of the non-Abelian type. From the four-dimensional standpoint we can indicate no appropriate order parameter.

Let us ask ourselves what quantitative parameters could guide us in the description of evolution from the Abelian to non-Abelian confinement? The source of quantitative information might be the effective theory on the string world sheet. At small $r(S_1)$ the string is Abelian and the effective Lagrangian is that describing transverse displacements of the string position. If, in addition, there are hidden light degrees of freedom, which belong to this Lagrangian, and make the vacuum structure on the string world sheet nontrivial, then the phase transition on the string world sheet could separate the small and large- $r(S_1)$ domains.

Of course, in two dimensions only discrete symmetries (e.g. Z_N) can be spontaneously broken. There are no obvious reasons for the occurrence of Z_N symmetric extra moduli on the string world sheet. That's why we believe that the domain of the Abelian confinement at small $r(S_1)$ is not separated from the decompactification limit (non-Abelian confinement) by phase transition(s). The center-symmetric construction we develop is smooth.

7 A few words on chiral theories

Strongly coupled chiral Yang–Mills theories did not receive sufficient attention that they deserve. This is not because they are dynamically uninteresting (on the contrary, they are interesting and may even be relevant for TeV scale physics) but, rather, because it is notoriously difficult to come up with a reliable quantitative framework for their analysis. They are certainly the most mysterious ones among non-Abelian gauge theories. That we were unable to say anything at all about their non-perturbative dynamics was very disappointing.

We will split the chiral Yang–Mills theories in two classes. A more traditional one, which was to an extent addressed in the 1980s [13] is represented, e.g. by the $SU(N)$ gauge theory with one two-index antisymmetric or symmetric (AS and S, for short) Weyl fermion plus $N-4$ or $N+4$ antifundamental Weyl fermions,

$${}_{ab} (N+4) \quad {}^a \quad \text{and} \quad {}_{[ab]} (N-4) \quad {}^a$$

A novel class of chiral (quiver) gauge theories are the Z_K orbifold projections of $SU(KN)$ supersymmetric gluodynamics with $K \geq 3$. These are perturbatively planar equivalent (PPE) to supersymmetric gluodynamics. Two other types of new chiral theories are $SU(N)$ gauge theories with the following matter content:

$${}_{ab} \quad {}^{[ab]} \quad 8 \quad {}^a \quad \text{and} \quad {}_{ab} \quad {}_{[ab]} \quad 2N \quad {}^a$$

For the both classes, internal gauge anomalies cancel. For example, in the first case, $d_S + d_{\overline{AS}} + 8d_{\overline{F}} = N + 4 - (N - 4) - 8 = 0$, and these theories are consistent perturbatively. These theories arise in the context of planar

orientifold equivalence. Note that these are essentially the two independent combinations of the chiral theories mentioned in the previous paragraph.

Again, much in the same way as in the non-chiral theories, we can compactify the space-time onto $R_3 \times S_1$ and perform double-trace deformations to stabilize the center-symmetric vacuum at small- $r(S_1)$. This allows one to smoothly connect small- $r(S_1)$ to large- $r(S_1)$ physics where the double-trace deformations are switched off. In this way bion-induced linear confinement was analytically demonstrated [14]. A remarkably rich pattern of the chiral symmetry realizations which depends on the structure of the ring operators, a novel class of topological excitations, was revealed in [14] on the way! Perhaps, one of the most interesting aspect of these theories is that the simplest monopole-instanton operators drop out. Another surprising aspect of the $[\text{SU}(N)]^K$ chiral quiver theories is that, although they possess a Z_{2N} axial chiral symmetry – just like pure supersymmetric gluodynamics – this symmetry may or may not be broken depending on the relation between N and K . What does this imply for PPE?

PPE extends to a full non-perturbative equivalence provided: (i) the orbifold projection symmetry is not spontaneously broken in the parent theory; and (ii) the Z_K cyclic symmetry is not spontaneously broken in the daughter quiver theory [15]. In our case with $K \geq 3$, the necessary and sufficient conditions are violated in the supersymmetric parent rather than in the non-supersymmetric daughter. The Z_K symmetry used in the orbifold projection is part of the discrete chiral symmetry which breaks down spontaneously to Z_2 . Consequently, the non-perturbative equivalence fails in the chiral case $K \geq 3$, while it is valid in the vector-like case $K = 2$ [7]. Our microscopic description of the chiral quiver theories also confirms this result.

The chiral gauge dynamics is very different from vector-like or supersymmetric gluodynamics. We refer the reader to [14] for a very detailed description.

8 Prototypes for the non-Abelian strings

Now we will briefly discuss a prototype non-Abelian flux tube. Note that the flux it carries is that of the magnetic rather than electric field. Dualization of this particular model is not yet performed. The (non-supersymmetric) gauge theory supporting such strings [16] is at weak coupling, while the theory on

the string world sheet is at strong coupling (Fig. 7).

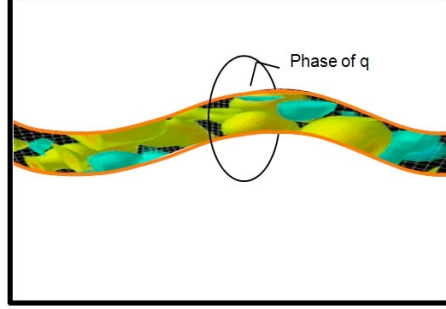


Figure 7: A prototype non-Abelian string in the model of Ref. [16]. This figure is borrowed from [17].

A prototype model has the gauge group $U(N)$. Besides $SU(N)$ and $U(1)$ gauge bosons the model contains N scalar fields charged with respect to $U(1)$ which form N fundamental representations of $SU(N)$. It is convenient to write these fields in the form of $N \times N$ matrix $\Phi = \Phi^{kA}$ where k is the $SU(N)$ gauge index while A is the flavor index,

$$\Phi = \begin{pmatrix} 11 & 12 & \dots & 1N \\ 21 & 22 & \dots & 2N \\ \vdots & \vdots & \ddots & \vdots \\ N1 & N2 & \dots & NN \end{pmatrix} \quad (15)$$

The action of the model is

$$S = \int d^4x \left\{ \frac{1}{4g_2^2} (F_{\mu\nu}^a)^2 + \frac{1}{4g_1^2} (F_{\mu\nu})^2 + \text{Tr} (D_\mu \Phi)^\dagger (D^\mu \Phi) \right. \\ \left. + \frac{g_2^2}{2} [\text{Tr} (\Phi^\dagger T^a \Phi)]^2 + \frac{g_1^2}{8} [\text{Tr} (\Phi^\dagger \Phi) - N\xi]^2 \right\} \quad (16)$$

where N stands for the number of colors and the parameter ξ which has dimension m^2 is assumed to be large which guarantees weak coupling. The last term in the second line forces Φ to develop a vacuum expectation value (VEV) while the last but one term forces the VEV to be diagonal,

$$\Phi_{\text{vac}} = \bar{\xi} \text{diag} (1 \ 1 \ \dots \ 1) \quad (17)$$

Thus, the model is fully Higgsed in the bulk; there are no massless excitations. A diagonal global $SU(N)$ survives, however, namely,

$$U(N)_{\text{gauge}} \times SU(N)_{\text{flavor}} \rightarrow SU(N)_{\text{diag}} \quad (18)$$

Thus, color-flavor locking takes place in the vacuum.

The string solution can be obtained by winding one of the elements of the field matrix (17). Its topological stability is due to the fact that

$$\pi_1(SU(N) \times U(1) \times Z_N) = 0 \quad (19)$$

The string solution breaks the symmetry (18) down⁶ to $U(1) \times SU(N-1)$ (at $N > 2$). This means that the world-sheet (two-dimensional) theory of the elementary string moduli is the $SU(N)/(U(1) \times SU(N-1))$ sigma model. This is the famous $CP(N-1)$ model. It is strongly coupled in the infrared and develops its own dynamical scale Λ .

This string is formed at the distances of the order of the inverse gauge boson masses (which are all of the same order if we do not consider N to be parametrically large). The occurrence of the orientational moduli (dynamical fields of the $CP(N-1)$ model) is a clear-cut indication of the non-Abelian nature of the string. The tension of the above non-Abelian string is $T = 2\pi\xi$, up to small corrections. This tension is lower than that of the Abrikosov string by $1/N$.

Classically, the $CP(N-1)$ model has a moduli field of vacua and gapless excitations. The vacuum structure is drastically changed by nonperturbative infrared effects. As well-known, the vacuum of the model is unique, the $U(1) \times SU(N-1)$ is restored, and the gapless excitations disappear. From the four-dimensional point of view this means that while geometrically the flux is oriented along the string, fluctuations in the color space are gigantic. The magnetic field inside the non-Abelian flux tube presented above has no specific orientation in the $SU(N)$ space.

9 What remains to be done

Contours of the Big Picture are rather clear at present. Abelian confinement or its absence in non-Abelian gauge theories (in which the corresponding low-energy theory effectively becomes Abelian) seems possible to establish. The

⁶At $N = 2$ the string solution breaks $SU(2)$ down to $U(1)$.

instructive examples are provided by the Polyakov model with or without massless fermions, supersymmetric gauge theories, and deformed QCD-like theories, including chiral theories. In each case, the detailed mechanisms explaining confinement are different; they are tied up, however, with certain topological excitations (see Table 1). The Abelian duality (either in three or four dimensions) helps us in each one of the above applications. Looking optimistically we can say that in a few years a much more crisp classification will arise along these lines.

Still, there are dark corners. We essentially do not know how the non-Abelian duality works. Dualizing non-Abelian Yang–Mills theories with strings and linear confinement, and without any long-distance Abelian regime, is a challenge, regardless of whether the underlying theory is supersymmetric or not. A better understanding/description of the (presumably smooth) transition from Abelian to non-Abelian confinement is badly needed.

| | Seiberg–Witten | One-flavor QCD* |
|---|---|---|
| Control parameter | $\mu \Lambda \equiv \epsilon$ | $L\Lambda \equiv \epsilon$ |
| $\epsilon = 0$ limit | no mass gap, no confinement | no mass gap, no confinement |
| (First) gauge symmetry breaking | $SU(N) \rightarrow U(1)^{N-1}$ | $SU(N) \rightarrow U(1)^{N-1}$ |
| $\epsilon \ll 1$ regime | mass gap, confinement | mass gap, confinement |
| Mechanism | Monopole condensation | Bion mechanism or instanton-monopoles |
| (Second) gauge symmetry breaking | $U(1)^{N-1} \rightarrow \text{nothing}$ | $U(1)^{N-1} \rightarrow \text{nothing}$ (or discrete) |
| Expectation for $\epsilon \gtrsim 1$ regime | non-Abelian confinement | non-Abelian confinement |

Table 1: Main features of the SW and QCD* solutions (N is not assumed to be large).

On a more practical side, it seems necessary to design a quasiclassical “string theory,” with strings resembling those in the old Veneziano model, but modernized to somehow include quarks at the endpoints (it is highly

desirable to have chiral symmetry of quarks built in, perhaps by somehow adding pions separately), for the description of highly excited mesons. This string should be responsible not only for rotational excitations, but for a mixture of rotational and oscillational (radial) excitations. This is in addition to purely stringy excitations. Then a description of highly excited mesons could be obtained.

Acknowledgments

We are grateful to A. Armoni and A. Yung for multiple and very useful discussions. M.S. is supported in part by DOE Grant DE-FG02-94ER-40823, and by *Chaire Internationale de Recherche Blaise Pascal* de l'Etat et de la Région d'Ile-de-France, gérée par la Fondation de l'Ecole Normale Supérieure. The work of M.U. is supported by the U.S. Department of Energy Grant DE-AC02-76SF00515.

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