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# Measurement of $C P$ Asymmetries and Branching Fractions in $B^{0} \rightarrow \pi^{+} \pi^{-}, B^{0} \rightarrow K^{+} \pi^{-}, B^{0} \rightarrow \pi^{0} \pi^{0}, B^{0} \rightarrow K^{0} \pi^{0}$ and Isospin Analysis of $B \rightarrow \pi \pi$ Decays 

The BABAR Collaboration

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#### Abstract

We present preliminary results of improved measurements of the $C P$-violating asymmetries and branching fractions in the decays $B^{0} \rightarrow \pi^{+} \pi^{-}, B^{0} \rightarrow K^{+} \pi^{-}, B^{0} \rightarrow \pi^{0} \pi^{0}$, and $B^{0} \rightarrow K^{0} \pi^{0}$. This update includes all data taken at the $\Upsilon(4 S)$ resonance by the BABAR experiment at the asymmetric PEP-II $B$-meson factory at SLAC, corresponding to $467 \pm 5$ million $B \bar{B}$ pairs. We find $$
\begin{aligned} S_{\pi \pi} & =-0.68 \pm 0.10 \pm 0.03, \\ C_{\pi \pi} & =-0.25 \pm 0.08 \pm 0.02, \\ \mathcal{A}_{K \pi} & =-0.107 \pm 0.016_{-0.004}^{+0.006}, \\ C_{\pi^{0} \pi^{0}} & =-0.43 \pm 0.26 \pm 0.05, \\ \mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) & =(1.83 \pm 0.21 \pm 0.13) \times 10^{-6}, \\ \mathcal{B}\left(B^{0} \rightarrow K^{0} \pi^{0}\right) & =(10.1 \pm 0.6 \pm 0.4) \times 10^{-6}, \end{aligned}
$$ where the first error is statistical and the second is systematic. We observe $C P$ violation with a significance of $6.7 \sigma$ in $B^{0} \rightarrow \pi^{+} \pi^{-}$and $6.1 \sigma$ in $B^{0} \rightarrow K^{+} \pi^{-}$. Constraints on the Unitarity Triangle angle $\alpha$ are determined from the isospin relation between all $B \rightarrow \pi \pi$ rates and asymmetries.


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## 1 INTRODUCTION

Large $C P$-violating effects [1] in the $B$-meson system are among the most remarkable predictions of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing model [2]. These predictions have been confirmed in recent years by the $B A B A R$ and Belle collaborations, both in the interference of $B^{0}$ decays to $C P$ eigenstates with and without $B^{0}-\bar{B}^{0}$ mixing [3-5] and directly, in the interference between the decay amplitudes [6] in $B^{0} \rightarrow K^{+} \pi^{-}[5,7]$.

Effective constraints on physics beyond the Standard Model (SM) are provided by high-precision measurements of quantities whose SM predictions suffer only small theoretical uncertainties. Both experimental and theoretical uncertainties often partially cancel out in the determination of $C P$ violating asymmetries, which makes $C P$-violation measurements a sensitive probe for effects of yet-undiscovered additional interactions and heavy particles that are introduced by extensions to the SM. All measurements of $C P$ violation to date are in agreement with the indirect predictions from global SM fits $[8,9]$ that are based on measurements of the magnitudes of the elements of the CKM quark-mixing matrix; this strongly constrains [10] the flavor structure of SM extensions.

The CKM Unitarity Triangle angle $\alpha \equiv \arg \left[-V_{\mathrm{td}} V_{\mathrm{tb}}^{*} / V_{\mathrm{ud}} V_{\mathrm{ub}}^{*}\right]$ is measured through interference between decays with and without $B^{0}-\bar{B}^{0}$ mixing. Multiple measurements of $\alpha$, with different decays, further test the consistency of the CKM model. The time-dependent asymmetry in $B^{0} \rightarrow$ $\pi^{+} \pi^{-}$is proportional to $\sin 2 \alpha$ in the limit that only the $b \rightarrow u$ ("tree") quark-level amplitude contributes to this decay. In the presence of $b \rightarrow d$ ("penguin") amplitudes, the time-dependent asymmetry in $B^{0} \rightarrow \pi^{+} \pi^{-}$is modified to

$$
\begin{align*}
a(\Delta t)=\frac{|\bar{A}(\Delta t)|^{2}-|A(\Delta t)|^{2}}{|\bar{A}(\Delta t)|^{2}+|A(\Delta t)|^{2}} & =S_{\pi \pi} \sin \left(\Delta m_{d} \Delta t\right)-C_{\pi \pi} \cos \left(\Delta m_{d} \Delta t\right) \\
C_{\pi \pi} & =\frac{|A|^{2}-|\bar{A}|^{2}}{|A|^{2}+|\bar{A}|^{2}}  \tag{1}\\
S_{\pi \pi} & =\sqrt{1-C_{\pi \pi}^{2}} \sin \left(2 \alpha-2 \Delta \alpha_{\pi \pi}\right)=\sqrt{1-C_{\pi \pi}^{2}} \sin 2 \alpha_{\mathrm{eff}}
\end{align*}
$$

where $\Delta t$ is the difference between the proper decay times of the signal- and tag-side neutral $B$ mesons and $\Delta m_{d}$ is the $B^{0}$ mixing frequency. Both the phase difference $\Delta \alpha_{\pi \pi}=\alpha-\alpha_{\text {eff }}$ and the direct $C P$ asymmetry $C_{\pi \pi}$ may differ from zero due to the penguin contribution to the $B^{0} \rightarrow \pi^{+} \pi^{-}$ decay amplitude $A$.

The magnitude and relative phase of the penguin contribution to the asymmetry $S_{\pi \pi}$ may be unraveled with an analysis of isospin relations between the $B \rightarrow \pi \pi$ decay amplitudes [11]. The amplitudes $A^{i j}$ of the $B \rightarrow \pi^{i} \pi^{j}$ decays and $\bar{A}^{i j}$ of the $\bar{B} \rightarrow \pi^{i} \pi^{j}$ decays satisfy the relations

$$
\begin{align*}
& A^{+0}=\frac{1}{\sqrt{2}} A^{+-}+A^{00},  \tag{2}\\
& \bar{A}^{-0}=\frac{1}{\sqrt{2}} \bar{A}^{+-}+\bar{A}^{00} .
\end{align*}
$$

The shape of the corresponding isospin triangle is determined from measurements of the branching fractions and time-integrated $C P$ asymmetries for each of the $B \rightarrow \pi \pi$ decays. No gluonic penguin amplitudes are present in the $\Delta I=3 / 2$ decay $B^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}$, so, neglecting electroweak (EW) penguins, $A^{+0}=\bar{A}^{-0}$. We define the direct $C P$ asymmetry $C_{\pi^{0} \pi^{0}}$ in $B^{0} \rightarrow \pi^{0} \pi^{0}$ as

$$
\begin{equation*}
C_{\pi^{0} \pi^{0}}=\frac{\left|A^{00}\right|^{2}-\left|\bar{A}^{00}\right|^{2}}{\left|A^{00}\right|^{2}+\left|\bar{A}^{00}\right|^{2}} . \tag{3}
\end{equation*}
$$

From the difference in shape of these triangles for the $B$ and $\bar{B}$ decay amplitudes, a constraint on $\Delta \alpha_{\pi \pi}$ can be determined with a four-fold ambiguity.

The phenomenology of the $B \rightarrow \pi \pi$ system has been thoroughly studied in a number of theoretical frameworks and models [12]. Predictions for the relative size and phase of the penguin contribution vary considerably, so increasingly precise measurements will help distinguish among different theoretical approaches and add to our understanding of hadronic $B$ decays.

The measured rates and direct $C P$-violating asymmetries in $B \rightarrow K \pi$ decays [13-18] reveal puzzling features that could indicate significant contributions from EW penguins [19, 20]. Various methods have been proposed to isolate the Standard Model contribution to this process in order to test for signs of new physics. Sum rules derived from $U$-spin symmetry relate the rates and asymmetries for the decays $B^{0}$ or $B^{+}$to $K^{+} \pi^{-}, K^{+} \pi^{0}, K^{0} \pi^{0}$, and $K^{0} \pi^{+}$[21], while $S U(3)$ symmetry can be used to make predictions for the $K \pi$ system based on hadronic parameters extracted from the $\pi \pi$ system [19].

## 2 THE BABAR DETECTOR AND DATA SET

The data used in this analysis were collected in 1999-2007 with the BABAR detector at the PEP-II asymmetric-energy $B$-meson factory at the Stanford Linear Accelerator Center. A total of $467 \pm 5$ million $B \bar{B}$ pairs were used. The preliminary results presented here supersede the results in prior publications [5, 13, 16]. Roughly $22 \%$ more $B \bar{B}$ pairs have been added to the BABAR data set, and improvements have been introduced to the analysis technique, boosting the signal significance.

In the BABAR detector [22], charged particles are detected and their momenta measured by a combination of a five-layer double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber ( DCH ) that covers $92 \%$ of the solid angle in the $\Upsilon(4 S)$ center-of-mass (c.m.) frame, both operating in a $1.5-\mathrm{T}$ solenoidal magnetic field. Discrimination between charged pions, kaons, and protons is provided by a combination of an internally reflecting ring-imaging Cherenkov detector (DIRC), which covers $84 \%$ of the c.m. solid angle in the central region of the BABAR detector and has a $91 \%$ reconstruction efficiency for pions and kaons with momenta above $1.5 \mathrm{GeV} / c$, and the ionization $(\mathrm{d} E / \mathrm{d} x)$ measurements in the DCH. Neutral-cluster (photon) positions and energies are measured with an electromagnetic calorimeter (EMC) consisting of $6580 \mathrm{CsI}(\mathrm{Tl})$ crystals. The photon energy resolution is $\sigma_{E} / E=\left\{2.3 / E(\mathrm{GeV})^{1 / 4} \oplus 1.9\right\} \%$, and the angular resolution from the interaction point is $\sigma_{\theta}=3.9^{\circ} / \sqrt{E(\mathrm{GeV})}$.

## 3 ANALYSIS METHOD

Many elements of the measurements discussed in this paper are common to the decay modes $B^{0} \rightarrow h^{+} h^{\prime-}(h=\pi$ or $K), B^{0} \rightarrow \pi^{0} \pi^{0}$, and $B^{0} \rightarrow K_{S}^{0} \pi^{0}$. The signal $B$-meson candidates ( $B_{\text {rec }}$ ) are formed by combining two particles, either tracks or $\pi^{0}$ or $K_{S}^{0}$ candidates. The event selection differs for each mode, and is described in detail below.

The number of $B$ decays and the corresponding $C P$ asymmetries are determined in extended unbinned maximum likelihood (M.L.) fits to variables described below. The likelihood is given by the expression

$$
\begin{equation*}
\mathcal{L}=\exp \left(-\sum_{i}^{M} n_{i}\right) \prod_{j}^{N}\left[\sum_{i}^{M} n_{i} \mathcal{P}_{i}\left(\vec{x}_{j} ; \vec{\alpha}_{i}\right)\right], \tag{4}
\end{equation*}
$$

where the product is over the number of events $N$, the sums are over the event categories $M, n_{i}$ is the coefficient for each category as described below, and the probability-density function (PDF) $\mathcal{P}$ describes the distribution of the variables $\vec{x}$ in terms of parameters $\vec{\alpha}$. The PDF functional forms are discussed in Sec. 3.3.1, 3.3.2, and 3.4.

### 3.1 Track and $K_{S}^{0}$ Selection

For particle identification in the $B^{0} \rightarrow h^{+} h^{--}$sample, we make use of the track's Cherenkov radiation in the DIRC as well as its ionization energy loss $\mathrm{d} E / \mathrm{d} x$ in the DCH.

For the DIRC information to be used, we require that each track have the associated Cherenkov angle $\left(\theta_{\mathrm{C}}\right)$ measured with at least six signal photons detected in the DIRC, where the value of $\theta_{\mathrm{C}}$ is required to be within 4.0 standard deviations from either the pion or kaon hypothesis, which effectively removes any candidate containing high-momentum protons. Electrons are explicitly removed based primarily on a comparison of the track momentum and the associated energy deposition in the EMC, with additional information provided by $\mathrm{DCH} \mathrm{d} E / \mathrm{d} x$ and DIRC $\theta_{\mathrm{C}}$ measurements.

The ionization energy loss $\mathrm{d} E / \mathrm{d} x$ in the DCH is used either in combination with DIRC information or alone, which enables a $35 \%$ increase in the $B^{0} \rightarrow h^{+} h^{--}$reconstruction efficiency compared to using only the tracks with good DIRC information. A detailed DCH $\mathrm{d} E / \mathrm{d} x$ calibration that we developed for the $B^{0} \rightarrow h^{+} h^{--}$analysis takes into account variations in the mean value and resolution of $\mathrm{d} E / \mathrm{d} x$ values with respect to changes in the DCH running conditions over time and the track's charge, polar and azimuthal angles, and number of ionization samples. The calibration is performed with large high-purity samples ( $>10^{6}$ events) of protons from $\Lambda \rightarrow p \pi^{-}$, pions and kaons from $D^{*+} \rightarrow D^{0} \pi^{+}\left(D^{0} \rightarrow K^{-} \pi^{+}\right)$, and $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$decays that occur in the vicinity of the interaction region.
$K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$candidates are reconstructed from pairs of oppositely charged tracks. The twotrack combinations are required to form a vertex with a $\chi^{2}$ probability greater than 0.001 and a $\pi^{+} \pi^{-}$invariant mass within $11.2 \mathrm{MeV} / c^{2}(3.7 \sigma)$ of the $K_{S}^{0}$ mass [23].

## $3.2 \quad \pi^{0}$ Selection

We form $\pi^{0} \rightarrow \gamma \gamma$ candidates from pairs of clusters in the EMC that are isolated from any charged tracks. Clusters are required to have a transverse energy deposition consistent with that of a photon and to have an energy $E_{\gamma}>30 \mathrm{MeV}$ for $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $E_{\gamma}>50 \mathrm{MeV}$ for $B^{0} \rightarrow K_{S}^{0} \pi^{0}$. We use $\pi^{0}$ candidates that fall within the invariant-mass range $110<m_{\gamma \gamma}<160 \mathrm{MeV} / c^{2}$.

For the $B^{0} \rightarrow \pi^{0} \pi^{0}$ sample, we also use $\pi^{0}$ candidates from a single EMC cluster containing two adjacent photons (a merged $\pi^{0}$ ), or one EMC cluster and two tracks from a photon conversion to an $e^{+} e^{-}$pair inside the detector. To reduce the background from random photon combinations, the angle $\theta_{\gamma}$ between the photon momentum vector in the $\pi^{0}$ rest frame and the $\pi^{0}$ momentum vector in the laboratory frame is required to satisfy $\left|\cos \theta_{\gamma}\right|<0.95$. The $\pi^{0}$ candidates are fitted kinematically with their mass constrained to the nominal $\pi^{0}$ mass [23].

Photon conversions are selected from pairs of oppositely charged tracks with an invariant mass below $30 \mathrm{MeV} / c^{2}$ whose combined momentum vector points straight away from the beam spot. The conversion point is required to lie inside the detector material. Converted photons are combined with photons from single EMC clusters to form $\pi^{0}$ candidates.

Single EMC clusters containing two photons are selected with the transverse second moment, $S=\sum_{i} E_{i} \times\left(\Delta \alpha_{i}\right)^{2} / E$, where $E_{i}$ is the energy in each $\operatorname{CsI}(\mathrm{Tl})$ crystal and $\Delta \alpha_{i}$ is the angle
between the cluster centroid and the crystal. The second moment is used to distinguish merged $\pi^{0}$ candidates from both single photons and neutral hadrons.

### 3.3 Event Selection in $B^{0} \rightarrow \pi^{+} \pi^{-}, B^{0} \rightarrow K^{+} \pi^{-}$, and $B^{0} \rightarrow \pi^{0} \pi^{0}$

Two kinematic variables are used in the $B^{0} \rightarrow h^{+} h^{--}$and $B^{0} \rightarrow \pi^{0} \pi^{0}$ analyses to separate $B$ meson decays from the large $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ combinatoric background [22]. One is the beam-energy-substituted mass $m_{\mathrm{ES}}=\sqrt{\left(s / 2+\mathbf{p}_{i} \cdot \mathbf{p}_{B}\right)^{2} / E_{i}^{2}-\mathbf{p}_{B}^{2}}$, where $\sqrt{s}$ is the total $e^{+} e^{-}$ c.m. energy, $\left(E_{i}, \mathbf{p}_{i}\right)$ is the four-momentum of the initial $e^{+} e^{-}$system in the laboratory frame, and $\mathbf{p}_{B}$ is the laboratory momentum of the $B$ candidate. The other is $\Delta E=E_{B}^{*}-\sqrt{s} / 2$, where $E_{B}^{*}$ is the $B$ candidate's energy in the c.m. frame.

Two additional quantities take advantage of the event topology to further separate $B$ decays from the $q \bar{q}$ background. The absolute value of the cosine of the angle $\theta_{S}$ between the sphericity axes [24] of the $B$ candidate's decay products and that of the remaining tracks and neutral clusters in the event, computed in the c.m. frame, is peaked at 1.0 for the jet-like $q \bar{q}$ events but has a flat distribution for $B$ decays. We require $\left|\cos \theta_{S}\right|<0.7$ for $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $\left|\cos \theta_{S}\right|<0.91$ for $B^{0} \rightarrow h^{+} h^{\prime-}$. For the $B^{0} \rightarrow h^{+} h^{\prime-}$ sample, we further require that the second Fox-Wolfram moment [26] satisfy $R_{2}<0.7$ to remove a small remaining background from $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-}$events.

To improve the discrimination against $q \bar{q}$ events, a Fisher discriminant $\mathcal{F}$ is formed as a linear combination of the sums $L_{0} \equiv \sum_{i}\left|\mathbf{p}_{i}^{*}\right|$ and $L_{2} \equiv \sum_{i}\left|\mathbf{p}_{i}^{*}\right| \cos ^{2} \theta_{i}^{*}$, where $\mathbf{p}_{i}^{*}$ are the momenta and $\theta_{i}^{*}$ are the angles with respect to the thrust axis [25] of the $B$ candidate, both in the c.m. frame, of all tracks and clusters not used to reconstruct the signal $B$-meson candidate. In the case of $B^{0} \rightarrow \pi^{0} \pi^{0}$, we improve the sensitivity of the signal by combining $\mathcal{F}$ with three other quantities in a neural network. These are the $\left|\cos \theta_{S}\right|$ described above, $\left|\cos \theta_{B}\right|$, where $\theta_{B}$ is the angle between the center-of-mass momentum vector of the signal $B$ and the beam axis, and $\left|\cos \theta_{T}\right|$, where $\theta_{T}$ is the angle between the thrust axis of the signal $B$-meson's daughters and the beam axis.

### 3.3.1 $\quad B^{0} \rightarrow \pi^{+} \pi^{-}$and $B^{0} \rightarrow K^{+} \pi^{-}$

We reconstruct candidate decays $B_{\text {rec }} \rightarrow h^{+} h^{--}$from pairs of oppositely charged tracks in the polar-angle range $0.35<\theta_{\text {lab }}<2.40$ that are consistent with originating from a common decay point with a $\chi^{2}$ probability of at least 0.001 . The remaining particles are examined to infer whether the other $B$ meson in the event ( $B_{\mathrm{tag}}$ ) decayed as a $B^{0}$ or $\bar{B}^{0}$ (flavor tag). We perform an unbinned extended M.L. fit to separate $B^{0} \rightarrow \pi^{+} \pi^{-}$and $B^{0} \rightarrow K^{+} \pi^{-}$decays and determine simultaneously their $C P$-violating asymmetries $S_{\pi \pi}, C_{\pi \pi}$, and $\mathcal{A}_{K \pi}$ and the signal and background yields and PDF parameters. The fit uses particle-identification, kinematic, event-shape, $B_{\text {tag }}$ flavor, and $\Delta t$ information.

The variables $m_{\mathrm{ES}}$ and $\Delta E$ are calculated assuming that both tracks are charged pions. The $B^{0} \rightarrow \pi^{+} \pi^{-}$events are described by a Gaussian distribution for both $m_{\mathrm{ES}}$ and $\Delta E$, where the resolutions are found to be $2.6 \mathrm{MeV} / c^{2}$ and 29 MeV , respectively. For each kaon in the final state, the $\Delta E$ peak position is shifted from zero by an amount that depends on the kaon momentum, with an average shift of -45 MeV . We require $m_{\mathrm{ES}}>5.20 \mathrm{GeV} / c^{2}$ and $|\Delta E|<0.150 \mathrm{GeV}$. The large region below the signal in $m_{\mathrm{ES}}$ effectively determines the background shape parameters, while the wide range in $\Delta E$ allows us to separate $B^{0}$ decays to all four final states $\left(\pi^{+} \pi^{-}, K^{+} \pi^{-}, \pi^{+} K^{-}\right.$, and $K^{+} K^{-}$) in a single fit.

We construct $\theta_{\mathrm{C}}$ PDFs for the pion and kaon hypotheses, and $\mathrm{d} E / \mathrm{d} x$ PDFs for the pion, kaon,


Figure 1: The average difference between the expected values of DIRC $\theta_{\mathrm{C}}$ and $\mathrm{DCH} \mathrm{d} E / \mathrm{d} x$ for pions and kaons at $0.35<\theta_{\text {lab }}<2.40$, divided by the uncertainty, as a function of laboratory momentum in $B^{0} \rightarrow K^{+} \pi^{-}$decays in BABAR.

Table 1: Average tagging efficiency $\epsilon$, average mistag fraction $w$, mistag fraction difference $\Delta w=w\left(B^{0}\right)-$ $w\left(\bar{B}^{0}\right)$, and effective tagging efficiency $Q$ for signal events in each tagging category. The quantities are measured in the large-statistics $B_{\text {flav }}$ sample of fully reconstructed neutral $B$-meson decays.

| Category | $\epsilon(\%)$ | $w(\%)$ | $\Delta w(\%)$ | $Q(\%)$ |
| :---: | ---: | ---: | ---: | :---: |
| Lepton | $8.96 \pm 0.07$ | $2.9 \pm 0.3$ | $0.2 \pm 0.5$ | $7.95 \pm 0.11$ |
| Kaon I | $10.81 \pm 0.07$ | $5.3 \pm 0.3$ | $0.0 \pm 0.6$ | $8.64 \pm 0.14$ |
| Kaon II | $17.18 \pm 0.09$ | $14.5 \pm 0.3$ | $0.4 \pm 0.6$ | $8.64 \pm 0.17$ |
| Kaon Pion | $13.67 \pm 0.08$ | $23.3 \pm 0.4$ | $-0.6 \pm 0.7$ | $3.91 \pm 0.12$ |
| Pion | $14.19 \pm 0.08$ | $32.6 \pm 0.4$ | $5.1 \pm 0.7$ | $1.73 \pm 0.09$ |
| Other | $9.55 \pm 0.07$ | $41.5 \pm 0.5$ | $3.8 \pm 0.8$ | $0.28 \pm 0.04$ |
| Total |  |  |  | $31.1 \pm 0.3$ |

and proton hypotheses, separately for each charge. The $K-\pi$ separations provided by $\theta_{\mathrm{C}}$ and $\mathrm{d} E / \mathrm{d} x$ are complementary: for $\theta_{\mathrm{C}}$, the separation varies from $2.5 \sigma$ at $4.5 \mathrm{GeV} / c$ to $13 \sigma$ at $1.5 \mathrm{GeV} / c$, while for $\mathrm{d} E / \mathrm{d} x$ it varies from less than $1.0 \sigma$ at $1.5 \mathrm{GeV} / c$ to $1.9 \sigma$ at $4.5 \mathrm{GeV} / c$ (Fig. 1). For more details, please see Ref. [5].

We use a multivariate technique [27] to determine the flavor of $B_{\text {tag }}$. Separate neural networks are trained to identify leptons and kaons from $B$ and $D$ decays and soft pions from $D^{*}$ decays. Events are assigned to one of seven mutually exclusive tagging categories (including untagged events) based on the estimated average mistag probability and the source of the tagging information. The quality of tagging is expressed in terms of the effective efficiency $Q=\sum_{k} \epsilon_{k}\left(1-2 w_{k}\right)^{2}$, where $\epsilon_{k}$ and $w_{k}$ are the efficiencies and mistag probabilities, respectively, for events tagged in category $k$. The difference in mistag probabilities is given by $\Delta w=w_{B^{0}}-w_{\bar{B}^{0}}$. Table 1 summarizes the tagging performance measured in a large data sample of fully reconstructed neutral $B_{\text {flav }}$ decays to $D^{(*)-}\left(\pi^{+}, \rho^{+}, a_{1}^{+}\right)$.

The time difference $\Delta t=\Delta z / \beta \gamma c$ is obtained from the known boost of the $e^{+} e^{-}$system $(\beta \gamma=0.56)$ and the measured distance $\Delta z$ along the beam $(z)$ axis between the $B_{\text {rec }}$ and $B_{\text {tag }}$ decay vertices. A description of the inclusive reconstruction of the $B_{\text {tag }}$ vertex is given in [28]. We require $|\Delta t|<20 \mathrm{ps}$ and $\sigma_{\Delta t}<2.5 \mathrm{ps}$, where $\sigma_{\Delta t}$ is the error on $\Delta t$ determined separately for each event. The signal $\Delta t \mathrm{PDF}$ for $B^{0} \rightarrow \pi^{+} \pi^{-}$is given by

$$
\begin{align*}
f_{k}^{ \pm}\left(\Delta t_{\text {meas }}\right)=\frac{e^{-|\Delta t| / \tau}}{4 \tau}\{ & (1 \mp \Delta w) \\
& \left. \pm\left(1-2 w_{k}\right)\left[S_{\pi \pi} \sin \left(\Delta m_{d} \Delta t\right)-C_{\pi \pi} \cos \left(\Delta m_{d} \Delta t\right)\right]\right\} \otimes R\left(\Delta t_{\text {meas }}-\Delta t\right) \tag{5}
\end{align*}
$$

where $f_{k}^{+}\left(f_{k}^{-}\right)$indicates a $B^{0}\left(\bar{B}^{0}\right)$ flavor tag and the index $k$ indicates the tagging category. The resolution function $R\left(\Delta t_{\text {meas }}-\Delta t\right)$ for signal candidates is a sum of three Gaussian functions, identical to the one described in Ref. [28], with parameters determined from a fit to the $B_{\text {flav }}$ sample (including events in all seven tagging categories). The background $\Delta t$ distribution is also modeled as the sum of three Gaussians, where the common parameters used to describe the background shape for all tagging categories are determined simultaneously with the $C P$ parameters in the maximum likelihood fit.

The M.L. fit includes 28 components: $B^{0}$ signal decays and background with the final states $\pi^{+} \pi^{-}, K^{+} \pi^{-}, K^{-} \pi^{+}$, and $K^{+} K^{-}$where either the positively charged or the negatively charged track, or both, have good DIRC information $\left(2 \times 4 \times 3=24\right.$ components) plus the $p \pi^{-}, p K^{-}, \pi^{+} \bar{p}$ and $K^{+} \bar{p}$ background components where the (anti)proton has no DIRC information. The $K^{ \pm} \pi^{\mp}$ event yields are parameterized as $n_{K^{ \pm} \pi^{\mp}}=n_{K \pi}\left(1 \mp \mathcal{A}_{K \pi}^{\text {raw }}\right) / 2$. All other coefficients are products of the fraction of events in each tagging category, taken from $B_{\text {flav }}$ events, and the event yield. The background PDFs are a threshold function [29] for $m_{\mathrm{ES}}$ and a second-order polynomial for $\Delta E$. The $\mathcal{F}$ PDF is a sum of two asymmetric Gaussians for both the signal and background. We used large samples of simulated $B$ decays to investigate the effects of backgrounds from other $B$ decays on the determination of the $C P$-violating asymmetries in $B^{0} \rightarrow \pi^{+} \pi^{-}$and $B^{0} \rightarrow K^{+} \pi^{-}$ and determined them to be negligible.

### 3.3.2 $\quad B^{0} \rightarrow \pi^{0} \pi^{0}$

$B^{0} \rightarrow \pi^{0} \pi^{0}$ events are identified with an M.L. fit to the variables $m_{\mathrm{ES}}, \Delta E$, and $N N$, the output of the event-shape neural network. We require $m_{\mathrm{ES}}>5.20 \mathrm{GeV} / c^{2}$ and $|\Delta E|<0.2 \mathrm{GeV}$. Tails in the EMC response produce a correlation between $m_{\mathrm{ES}}$ and $\Delta E$, so a two-dimensional PDF, derived from detailed Monte Carlo (MC) simulation, is used to describe signal. The $N N$ distribution is binned in ten bins (equally populated for signal) and described by a parametric step-function PDF with 9 height parameters taken from the MC and fixed in the fit. $B_{\text {flav }}$ data are used to verify that the MC accurately reproduces the $N N$ distribution. The $q \bar{q}$ background PDFs are a threshold function [29] for $m_{\mathrm{ES}}$, a second-order polynomial for $\Delta E$, and a parametric step function for $N N$. For $q \bar{q}$ events, $N N$ is not distributed uniformly across the bins but rises sharply toward the highest bins. We see a small linear correlation between the shape parameter of the $m_{\mathrm{ES}}$ threshold function and the $N N$ bin number, and this linear relation is taken into account in the fit. All $q \bar{q}$ background PDF parameters are allowed to float in the M.L. fit.

The decays $B^{+} \rightarrow \rho^{+} \pi^{0}$ and $B^{0} \rightarrow K_{S}^{0} \pi^{0}\left(K_{S}^{0} \rightarrow \pi^{0} \pi^{0}\right)$ add $71 \pm 10$ background events to $B^{0} \rightarrow \pi^{0} \pi^{0}$ and are included as an additional fixed component in the M.L. fit. We model these $B-$
decay backgrounds with a two-dimensional PDF to describe $m_{\mathrm{ES}}$ and $\Delta E$, and with a step function for $N N$, all taken from MC simulation.

The time-integrated $C P$ asymmetry is measured by the $B$-flavor tagging algorithm described previously. The fraction of events in each tagging category is also constrained to the corresponding fraction determined from MC simulation. The PDF coefficient for the $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal is given by the expression

$$
\begin{equation*}
n_{\pi^{0} \pi^{0}, k}=\frac{1}{2} f_{k} N_{\pi^{0} \pi^{0}}\left\{1-s_{j}(1-2 \chi)\left(1-2 w_{k}\right) C_{\pi^{0} \pi^{0}}\right\}, \tag{6}
\end{equation*}
$$

where $f_{k}$ is the fraction of events in the tagging category $k, N_{\pi^{0} \pi^{0}}$ is the number of $B^{0} \rightarrow \pi^{0} \pi^{0}$ decays, $\chi=0.188 \pm 0.003$ [23] is the time-integrated $B^{0}$ mixing probability, and $s_{j}=+1(-1)$ when the $B_{\text {tag }}$ is a $B^{0}\left(\bar{B}^{0}\right)$.

## $3.4 \quad B^{0} \rightarrow K^{0} \pi^{0}$

For each $B^{0} \rightarrow K_{S}^{0} \pi^{0}$ candidate, two independent kinematic variables are computed. The first one is $m_{B}$, the invariant mass of the reconstructed $B$ meson, $B_{\text {rec }}$. The second one is $m_{\text {miss }}$, the invariant mass of the other $B, B_{\mathrm{tag}}$, computed from the known beam energy, by applying a mass constraint to $B_{\text {rec }}[30]$. For signal decays, $m_{B}$ and $m_{\text {miss }}$ peak near the $B^{0}$ mass with resolutions of $\sim 36 \mathrm{MeV} / c^{2}$ and $\sim 5.3 \mathrm{MeV} / c^{2}$, respectively. Both the $m_{\text {miss }}$ and $m_{B}$ distributions exhibit a low-side tail due to the leakage of energy deposits out of the EMC. We select candidates within the ranges $5.11<m_{\text {miss }}<5.31 \mathrm{GeV} / c^{2}$ and $5.13<m_{B}<5.43 \mathrm{GeV} / c^{2}$, which include a signal peak and a "sideband" region for background characterization. In the events with more than one reconstructed candidate ( $0.8 \%$ of the total), we select the candidate with the smallest $\chi^{2}=\sum_{i=\pi^{0}, K_{S}^{0}}\left(m_{i}-m_{i}^{\prime}\right)^{2} / \sigma_{m_{i}}^{2}$, where $m_{i}\left(m_{i}^{\prime}\right)$ is the measured (nominal) mass and $\sigma_{m_{i}}$ is the estimated uncertainty on the measured mass of particle $i$.

We exploit topological observables, computed in the c.m. frame, to discriminate jet-like $e^{+} e^{-} \rightarrow$ $q \bar{q}$ events ( $q=u, d, s, c$ ) from the nearly spherical $B \bar{B}$ events. In order to reduce the number of background events, we require $L_{2} / L_{0}<0.55$, where $L_{j} \equiv \sum_{i}\left|\mathbf{p}_{i}^{*}\right| \cos ^{j} \theta_{i}^{*}$ and $\theta_{i}^{*}$ are computed with respect to the sphericity axis [24] of the $B_{\text {rec }}$ candidate. We compute $\cos \theta_{B}^{*}$, the cosine of the angle between the direction of the $B$ meson and the nominal direction of the magnetic field ( $z$ axis). This variable is distributed as $1-\cos ^{2} \theta_{B}^{*}$ for signal events and is nearly flat for background events. We select events with $\left|\cos \theta_{B}^{*}\right|<0.9$. We also use the distributions of $L_{2} / L_{0}$ and $\cos \theta_{B}^{*}$ to discriminate the signal from the residual background in a M.L. fit. Using a full detector simulation, we estimate that our selection retains ( $34.2 \pm 1.2$ )\% of the signal events, where the error includes both statistical and systematic contributions. The selected sample of $B^{0} \rightarrow K_{S}^{0} \pi^{0}$ candidates is dominated by random $K_{S}^{0} \pi^{0}$ combinations from $e^{+} e^{-} \rightarrow q \bar{q}$ fragmentation. Using large samples of simulated $B \bar{B}$ events, we find that backgrounds from other $B$-meson decays are small, $\mathcal{O}(0.1 \%)$; however, we study in detail the effect of a number of specific $B$ decay channels. The dominant ones are $B^{+} \rightarrow \rho^{+} K_{S}^{0}, B^{+} \rightarrow K^{*+} \pi^{0}$, and $B^{+} \rightarrow K_{S}^{0} \pi^{0} \pi^{+}$, and we include this effect in our study of the systematic errors.

For the $B^{0} \rightarrow K_{S}^{0} \pi^{0}$ decay, where no charged particles are present at the decay vertex, we compute the decay point of the $B_{\text {rec }}$ using the knowledge of the $K_{S}^{0}$ trajectory from the measurement of $\pi^{+} \pi^{-}$momenta and the knowledge of the average interaction point [31].

We extract the signal yield from an extended unbinned M.L. fit to kinematic, event-shape, flavor-tag, and decay-time quantities. The use of tagging and decay-time information in the M.L. fit further improves discrimination between signal and background. We have verified that all
correlations are negligible, and so construct the likelihood function as a product of one-dimensional PDFs. Residual correlations are taken into account in the systematic uncertainty, as explained below.

The PDFs for signal events are parameterized based on a large sample of fully reconstructed $B$ decays in data and from simulated events. For background PDFs, we select the functional form from the background-dominated sideband regions in the data.

The likelihood function is defined as:

$$
\begin{align*}
& \mathcal{L}\left(S_{f}, C_{f}, N_{\text {sig }}, N_{\mathrm{bkg}}, f_{\mathrm{sig}}, f_{\mathrm{bkg}}, \vec{\alpha}\right)=\frac{e^{-\left(N_{\mathrm{sig}}+N_{\mathrm{bkg}}\right)}}{N!}  \tag{7}\\
& \quad \times \prod_{i \in g}\left[N_{\mathrm{sig}} f_{\mathrm{sig}} \epsilon_{\mathrm{sig}}^{c} \mathcal{P}_{\mathrm{sig}}\left(\vec{x}_{i}, \vec{y}_{i} ; S_{f}, C_{f}\right)+N_{\mathrm{bkg}} f_{\mathrm{bkg}} \epsilon_{\mathrm{bkg}}^{c} \mathcal{P}_{\mathrm{bkg}}\left(\vec{x}_{i}, \vec{y}_{i} ; \vec{\alpha}\right)\right] \\
& \quad \times \prod_{i \in b}\left[N_{\mathrm{sig}}\left(1-f_{\mathrm{sig}}\right) \epsilon_{\mathrm{sig}}^{c} \mathcal{P}_{\text {sig }}^{\prime}\left(\vec{x}_{i} ; C_{f}\right)+N_{\mathrm{bkg}}\left(1-f_{\mathrm{bkg}}\right) \epsilon_{\mathrm{bkg}}^{c} \mathcal{P}_{\mathrm{bkg}}^{\prime}\left(\vec{x}_{i} ; \vec{\alpha}\right)\right],
\end{align*}
$$

where the $N$ selected events are partitioned into two subsets: $i \in g$ events have $\Delta t$ information, while $i \in b$ events do not. Here, $f_{\text {sig }}\left(f_{\mathrm{bkg}}\right)$ is the fraction of signal (background) events $\in g$, and $1-f_{\text {sig }}\left(f_{\text {bkg }}\right)$ is the fraction of events $\in b$. The probabilities $\mathcal{P}_{\text {sig }}$ and $\mathcal{P}_{\text {bkg }}$ are products of PDFs for signal (sig) and background (bkg) hypotheses evaluated for the measurements $\vec{x}_{i}=$ $\left\{m_{B}, m_{\text {miss }}, L_{2} / L_{0}, \cos \theta_{B}^{*}\right.$, flavor tag, tagging category $\}$ and $\vec{y}_{i}=\left\{\Delta t, \sigma_{\Delta t}\right\} . \mathcal{P}_{\text {sig }}^{\prime}$ and $\mathcal{P}_{\text {bkg }}^{\prime}$ are the corresponding probabilities for events without $\Delta t$ information. In the formula, $\vec{\alpha}$ represents the set of parameters that define the shape of the PDFs. Along with the $C P$ asymmetries $S_{f}$ and $C_{f}$, the fit extracts the yields $N_{\text {sig }}$ and $N_{\mathrm{bkg}}$, the fraction of events $f_{\mathrm{sig}}$ and $f_{\mathrm{bkg}}$, and the parameters $\vec{\alpha}$ that describe the background PDFs.

## 4 RESULTS AND SYSTEMATIC UNCERTAINTIES

## $4.1 \quad B^{0} \rightarrow \pi^{0} \pi^{0}$ Results

Results from the M.L. fit for the $B^{0} \rightarrow \pi^{0} \pi^{0}$ decay mode are summarized in Table 2. Distributions of $m_{\mathrm{ES}}, \Delta E$, and $N N$ for $B^{0} \rightarrow \pi^{0} \pi^{0}$ are shown in Fig. 2, where a weighting and backgroundsubtraction technique, ${ }_{s} \mathcal{P}$ lots [32], is used to display the signal events. The same technique is used to display the $q \bar{q}$ background as well, shown in the insets.

The uncertainty in the efficiency for the $B^{0} \rightarrow \pi^{0} \pi^{0}$ decay mode is dominated by a $3 \%$ systematic uncertainty per $\pi^{0}$, estimated from a study of $\tau \rightarrow \pi \pi^{0} \nu_{\tau}$ decays. There is an additional $1.0 \%$ uncertainty in the resolution of the signal shape and a $0.45 \%$ uncertainty due to the limited knowlegde of the $m_{\mathrm{ES}}$ and $\Delta E$ peak positions in data, estimated by shifting the $m_{\mathrm{ES}}$ and $\Delta E$

Table 2: Results for the $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{0} \rightarrow K^{0} \pi^{0}$ decay modes: signal yields $N_{\text {sig }}$, efficiencies, branching fractions, and time-integrated $C P$ asymmetries. When two uncertainties are given, the first is statistical and the second systematic.

|  | $N_{\text {sig }}$ | Efficiency | Branching fraction | Asymmetry |
| :---: | :---: | :---: | :---: | :---: |
| $B^{0} \rightarrow \pi^{0} \pi^{0}$ | $247 \pm 29$ | $(28.8 \pm 1.8) \%$ | $(1.83 \pm 0.21 \pm 0.13) \times 10^{-6}$ | $-0.43 \pm 0.26 \pm 0.05$ |
| $B^{0} \rightarrow K_{S}^{0} \pi^{0}$ | $556 \pm 32$ | $(34.2 \pm 1.2) \%$ | $(10.1 \pm 0.6 \pm 0.4) \times 10^{-6}$ | $[33]$ |



Figure 2: ${ }_{s} \mathcal{P}$ lots for $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal (background shown in the inset plots): (top left) $m_{\mathrm{ES}}$, (top right) $\Delta E$, (bottom) the binned $N N$. The line in each plot shows the corresponding PDF.
means and resolutions by amounts determined from MC-data comparison in a control sample of $B^{+} \rightarrow \pi^{+} \pi^{0}$ events. We also take an uncertainty of $1.5 \%$, determined from the $B_{\text {flav }}$ sample, due to the $\left|\cos \theta_{S}\right|$ requirement. Systematic uncertainties involving the M.L. fit are evaluated by varying the PDF parameters and refitting the data. These contribute an uncertainty of 8.3 events to the branching-fraction measurement and an uncertainty of 0.05 to $C_{\pi^{0}} \pi^{0}$. The various systematics sources are tabulated in Table 3.

## $4.2 \quad B^{0} \rightarrow \pi^{+} \pi^{-}$and $B^{0} \rightarrow K^{+} \boldsymbol{\pi}^{-}$Results

Results for the $B^{0} \rightarrow h^{+} h^{--}$decay modes are listed in Table 4. The correlation coefficient between $S_{\pi \pi}$ and $C_{\pi \pi}$ is found to be -0.056 , and the correlation between $C_{\pi \pi}$ and $\mathcal{A}_{K \pi}$ is 0.019. In Fig. 3, we show ${ }_{s} \mathcal{P l o t s}$ for $m_{\mathrm{ES}}, \Delta E$, and $\mathcal{F}$ for the $B^{0} \rightarrow h^{+} h^{\prime-}$ signal and background. The direct $C P$ asymmetry in $B^{0} \rightarrow K^{+} \pi^{-}$is apparent in the distribution of $\Delta E$ plotted separately for $B^{0}$ and $\bar{B}^{0}$ decays, shown in Fig. 4. We show the distributions of $\Delta t$ for $B^{0} \rightarrow K^{ \pm} \pi^{\mp}$ signal and background decays in Fig. 5. In Fig. 6, we show the distribution of $\Delta t$ separately for $B^{0} \rightarrow \pi^{+} \pi^{-}$events tagged as $B^{0}$ or $\bar{B}^{0}$, and the asymmetry $a(\Delta t)$. The central values and errors for $S_{\pi \pi}$ and $C_{\pi \pi}$ are shown in Fig. 7, along with confidence-level contours corresponding to statistical significances ranging from 1 to 7 standard deviations. Our measurement excludes the absence of $C P$ violation in $B^{0} \rightarrow \pi^{+} \pi^{-}$ $\left(S_{\pi \pi}=0, C_{\pi \pi}=0\right)$ at a confidence level of $2 \times 10^{-11}$, or $6.7 \sigma$ (where systematic errors are taken into account).

Table 3: Systematic uncertainties in the determination of the $B^{0} \rightarrow \pi^{0} \pi^{0}$ signal yield $\left(N_{\pi^{0}} \pi^{0}\right)$ and branching fraction, and the direct $C P$ asymmetry $C_{\pi^{0} \pi^{0}}$. The total branching-fraction systematic is the sum in quadrature of the uncertainties on the signal yield, the signal efficiency, and the $B$-meson counting.

| Source | $N_{\pi^{0} \pi^{0}}$ | $\sigma_{\text {syst }}(\mathcal{B}) / \mathcal{B}$ | $C_{\pi^{0} \pi^{0}}$ |
| :--- | :---: | :---: | :---: |
| Peaking background | $\pm 4.9$ |  | $\pm 0.030$ |
| Tagging | $\pm 0.35$ |  | $\pm 0.034$ |
| Background shape | $\pm 5.5$ |  | $\pm 0.023$ |
| Signal shape | $\pm 3.8$ |  | $\pm 0.020$ |
| Total fit systematics | $\pm 8.3$ | $3.4 \%$ | $\pm 0.055$ |
|  |  |  |  |
| $\pi^{0}$ efficiency |  | $6.0 \%$ |  |
| $\left\|\cos \theta_{S}\right\|$ selection | $1.5 \%$ |  |  |
| neutrals resolution |  | $1.0 \%$ |  |
| $m_{\text {ES }}$ and $\Delta E$ shape |  | $0.5 \%$ |  |
| Total efficiency systematics |  | $6.3 \%$ |  |
|  |  | $1.1 \%$ |  |
| Number of $B \bar{B}$ pairs |  | $7.2 \%$ | $\pm 0.055$ |
| Total systematic error |  |  |  |

Table 4: Results for the $B^{0} \rightarrow h^{+} h^{--}$decay modes. For each mode, the number of signal events $N_{\text {sig }}$ and $C P$ asymmetries are shown. Statistical, followed by systematic, uncertainties are given for the asymmetries.

| Mode | $N_{\text {sig }}$ | Asymmetry |
| :--- | :---: | :---: |
| $B^{0} \rightarrow \pi^{+} \pi^{-}$ | $1394 \pm 54$ | $S_{\pi \pi}=-0.68 \pm 0.10 \pm 0.03 ; C_{\pi \pi}=-0.25 \pm 0.08 \pm 0.02$ |
| $B^{0} \rightarrow K^{+} \pi^{-}$ | $5410 \pm 91$ | $\mathcal{A}_{K \pi}=-0.107 \pm 0.016_{-0.004}^{+0.006}$ |

Table 5: Summary of absolute systematic errors on $\mathcal{A}_{K \pi}$. The total is calculated as the quadrature sum of each contribution. To address the $\mathcal{A}_{K \pi}$ bias due to hadronic interactions of charged kaons with the detector material, we shift the $\mathcal{A}_{K \pi}$ value obtained in the fit by +0.0050 .

| Source | Uncertainty |
| :---: | :---: |
| Material interactions | $+0.0053-0.0025$ |
| $\theta_{\mathrm{C}}$ and $\mathrm{d} E / \mathrm{d} x$ PDFs | 0.0020 |
| Potential MC bias | 0.0011 |
| Alternative DIRC parameterization | 0.0016 |
| Total | $+0.0060-0.0037$ |



Figure 3: The distributions of (left) $m_{\mathrm{ES}}$, (middle) $\Delta E$, and (right) Fisher discriminant $\mathcal{F}$ : (top) background-subtracted for $B^{0} \rightarrow \pi^{+} \pi^{-}$signal, (middle) background-subtracted for $B^{0} \rightarrow K^{+} \pi^{-}$signal, (bottom) signal-subtracted for all $h^{+} h^{--}$background candidates in the data. The curves represent the PDFs used in the fit and reflect the fit result. The structure to the left of the signal $\Delta E$ peak for $B^{0} \rightarrow \pi^{+} \pi^{-}$is consistent with the expected background from other charmless modes, which is negligible above -0.10 GeV .


Figure 4: The background-subtracted distribution of $\Delta E$ for signal $K^{ \pm} \pi^{\mp}$ events, comparing (solid) $B^{0}$ and (dashed) $\bar{B}^{0}$ decays.


Figure 5: (Left) the background-subtracted distribution of $\Delta t$ for signal $K^{ \pm} \pi^{\mp}$ and (right) the signalsubtracted $\Delta t$ distribution for background candidates in the data. The curves represent the PDFs used in the fit and reflect the fit result.

Table 6: Summary of systematic uncertainties on $S_{\pi \pi}$ and $C_{\pi \pi}$.

| Source | $S_{\pi \pi}$ | $C_{\pi \pi}$ |
| :--- | :---: | :---: |
| DIRC $\theta_{\mathrm{C}}$ | 0.0064 | 0.0050 |
| DCH $\mathrm{d} E / \mathrm{d} x$ | 0.0032 | 0.0037 |
| Signal $\Delta t$ | 0.0199 | 0.0055 |
| SVT local alignment | 0.0004 | 0.0002 |
| Boost $/ z$ scale | 0.0021 | 0.0013 |
| PEP-II beam spot | 0.0028 | 0.0014 |
| $B$ flavor tagging | 0.0146 | 0.0138 |
| $\Delta m_{d}, \tau_{B^{0}}[23]$ | 0.0004 | 0.0017 |
| Potential bias | 0.0041 | 0.0043 |
| Doubly Cabibbo-suppressed decays $[35]$ | 0.007 | 0.016 |
| Total | 0.027 | 0.023 |



Figure 6: The background-subtracted distributions of $\Delta t$ for signal $\pi^{+} \pi^{-}$events tagged as (top) $B^{0}$ or (middle) $\bar{B}^{0}$, and (bottom) their asymmetry $a(\Delta t)$ (Eq. 1). The curves represent the PDFs used in the fit and reflect the fit result.


Figure 7: $S_{\pi \pi}$ and $C_{\pi \pi}$ in $B^{0} \rightarrow \pi^{+} \pi^{-}$: the central values, errors, and confidence-level (C.L.) contours for $1-$ C.L. $=0.317(1 \sigma), 4.55 \times 10^{-2}(2 \sigma), 2.70 \times 10^{-3}(3 \sigma), 6.33 \times 10^{-5}(4 \sigma), 5.73 \times 10^{-7}$ $(5 \sigma), 1.97 \times 10^{-9}(6 \sigma)$ and $2.56 \times 10^{-12}(7 \sigma)$, calculated from the square root of the change in the value of $-2 \ln \mathcal{L}$ compared with its value at the minimum. The systematic errors are included. The measured value is $6.7 \sigma$ from the point of no $C P$ violation ( $S_{\pi \pi}=0, C_{\pi \pi}=0$ ).

Systematic uncertainties for the direct $C P$ asymmetry $\mathcal{A}_{K \pi}$ are listed in Table 5. Here, $\mathcal{A}_{K \pi}$ is the fitted value of the $K^{\mp} \pi^{ \pm}$event-yield asymmetry $\mathcal{A}_{K \pi}^{\text {raw }}$ shifted by $+0.005_{-0.003}^{+0.005}$ to account for a bias that arises from the difference between the cross sections of $K^{+}$and $K^{-}$hadronic interactions within the BABAR detector. We determine this bias from a detailed MC simulation based on GEANT4 [34] version 7.1; it is independently verified with a calculation based on the known material composition of the BABAR detector [22] and the cross sections and material properties tabulated in Ref. [23]. The corrected $K^{\mp} \pi^{ \pm}$event-yield asymmetry in the background, where no observable $C P$ violation is expected, is consistent with zero: $-0.005 \pm 0.004$ (stat) ${ }_{-0.003}^{+0.005}$ (syst). Systematic uncertainties for the $C P$ asymmetries $S_{\pi \pi}$ and $C_{\pi \pi}$ are listed in Table 6. They are dominated by uncertainties in the parameterization of $B$-flavor tagging and vertexing, and (for $C_{\pi \pi}$ ) in the effect of $C P$ violation in $B_{\operatorname{tag}}$ [35].

## $4.3 \quad B^{0} \rightarrow K^{0} \pi^{0}$ Results

Results for the $B^{0} \rightarrow K^{0} \pi^{0}$ decay mode are summarized in Table 2. In Fig. 8, we show ${ }_{s} \mathcal{P}$ lots for $m_{\text {miss }}, m_{B}, L_{2} / L_{0}$, and $\left|\cos \theta_{B}^{*}\right|$ for signal events, with background distributions shown in the insets.

To compute the systematic error associated with the statistical precision on the parameters of the likelihood function, we shift each parameter by its associated uncertainty and repeat the fit. For $\Delta t$ and the tagging parameters, the uncertainty is obtained from the fit to the $B_{\text {flav }}$ sample, while for the other parameters it is obtained from MC; the total error is obtained by summing the individual contributions in quadrature. This fit systematic also accounts for the limited statistics available to determine the shape of the likelihood function in Eq. 7. We find a systematic error of 1.2 events on the $K_{S}^{0} \pi^{0}$ yield. As an additional systematic error associated with the data-MC agreement of the shape of the signal PDFs, we also quote the largest deviation observed when the parameters of the individual signal PDFs for $m_{\text {miss }}, m_{B}, L_{2} / L_{0}$, and $\cos \theta_{B}^{*}$ are floated in the fit. This gives a systematic error on the yield of 2.5 events. The output values of the PDF parameters are also used to assign a systematic error to the selection efficiency of the cuts on the likelihood variables. Comparing the efficiency in data to that in the MC, we obtain a relative systematic error of $1.5 \%$. We do not assign a systematic uncertainty on the scale of $m_{\text {miss }}$ and $m_{B}$ because we float these variables in the fit. We evaluate the systematic error due to the neglected correlations among fit variables using a set of MC experiments, in which we embed signal events from a full detector simulation with events generated from the background PDFs. Since the shifts are small and only marginally significant, we use the average shift in the yield ( +2.2 events) as the associated systematic uncertainty.

We estimate the background from other $B$ decays to be small in the nominal fit. We account for a systematic shift induced on the signal yield by this neglected component by embedding simulated $B$ background events in the data set and evaluating the average shift in the fit result: +5.2 events on the signal yield. We adjust the signal yield accordingly and use half of the shift as a systematic uncertainty.

For the branching fraction, additional systematic errors come from the uncertainty in the selection efficiency, the counting of $B \bar{B}$ pairs in the data sample ( $1.1 \%$ ), and the branching fractions in the $B^{0}$ decay chain, $\mathcal{B}\left(K_{S}^{0} \rightarrow \pi^{+} \pi^{-}\right)=0.6920 \pm 0.0005$ and $\mathcal{B}\left(\pi^{0} \rightarrow \gamma \gamma\right)=0.98798 \pm 0.00032$ [23]. The systematic uncertainties are summarized in Table 7.


Figure 8: Distributions of (a) $m_{\text {miss }}$, (b) $m_{B}$, (c) $L_{2} / L_{0}$, (d) $\cos \theta_{B}^{*}$ for background-subtracted events in the $B^{0} \rightarrow K_{S}^{0} \pi^{0}$ sample. The solid curves represent the shapes of signal PDFs as obtained from the M.L. fit. The insets show the distributions and PDFs for signal-subtracted data.

Table 7: Summary of dominant contributions to the systematic error on the measurement of $\mathcal{B}\left(B^{0} \rightarrow K^{0} \pi^{0}\right)$

|  |  | $\sigma_{\text {syst }}(\mathcal{B}) / \mathcal{B}(\%)$ |
| :---: | :---: | :---: |
| Efficiencies | $\pi^{0}$ efficiency | 3.0 |
|  | $K_{s}^{0}$ efficiency | 0.5 |
|  | Cut on likelihood variables | 1.5 |
| Yield | stat. precision on PDF parameters | 0.22 |
|  | Shape of signal PDFs | 0.45 |
|  | $B \bar{B}$ background | 0.47 |
|  | Correlations among likelihood variables | 0.40 |
|  | Resolution function | 0.49 |
| Normalization | Number of $B \bar{B}$ pairs | 1.1 |
| Total |  | 3.7 |

## 5 CONCLUSIONS

The CP-asymmetry and branching-fraction results described in this paper are:

$$
\begin{aligned}
S_{\pi \pi} & =-0.68 \pm 0.10 \pm 0.03, \\
C_{\pi \pi} & =-0.25 \pm 0.08 \pm 0.02, \\
\mathcal{A}_{K \pi} & =-0.107 \pm 0.016_{-0.004}^{+0.006}, \\
C_{\pi^{0} \pi^{0}} & =-0.43 \pm 0.26 \pm 0.05, \\
\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) & =(1.83 \pm 0.21 \pm 0.13) \times 10^{-6}, \\
\mathcal{B}\left(B^{0} \rightarrow K^{0} \pi^{0}\right) & =(10.1 \pm 0.6 \pm 0.4) \times 10^{-6} .
\end{aligned}
$$

We combine $\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)$ with the branching fractions $\mathcal{B}\left(B^{0} \rightarrow \pi^{+} \pi^{-}\right)=(5.5 \pm 0.4 \pm 0.3) \times 10^{-6}$ and $\mathcal{B}\left(B^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}\right)=(5.02 \pm 0.46 \pm 0.29) \times 10^{-6}$ previously measured by BABAR $[13,14]$ to evaluate the constraints on both the penguin contribution to $\alpha$ and on the CKM angle $\alpha$ itself. Constraints are evaluated by scanning the parameters of interest, $\left|\Delta \alpha_{\pi \pi}\right|=\left|\alpha-\alpha_{\text {eff }}\right|$ and $\alpha$, and then calculating the $\chi^{2}$ for the five amplitudes $\left(A^{+0}, A^{+-}, A^{00}, \bar{A}^{+-}, \bar{A}^{00}\right)$ from our measurements and the isospintriangle relations [8]. The $\chi^{2}$ is converted to a confidence level (C.L.) as shown in Fig. 9. The upper bound on $\left|\Delta \alpha_{\pi \pi}\right|$ is $43^{\circ}$ at the $90 \%$ C.L., and the range $\left[23^{\circ}, 67^{\circ}\right]$ in $\alpha$ is excluded at the $90 \%$ C.L. If we consider only the solution preferred in the SM [37], $\alpha$ is in the range [ $71^{\circ}, 109^{\circ}$ ] at the $68 \%$ C.L. Somewhat more restrictive new constraints on $\alpha$ have been found in the measurements of $B \rightarrow \rho \rho$ and $B^{0} \rightarrow(\rho \pi)^{0}$ decays [38].

We have also presented an improved measurement of the $C P$-violating charge asymmetry $\mathcal{A}_{K \pi}$ in the $B^{0} \rightarrow K^{+} \pi^{-}$decay. We observe direct $C P$ violation in $B^{0} \rightarrow K^{+} \pi^{-}$with a significance of $6.1 \sigma$. Ignoring color-suppressed tree amplitudes, the charge asymmetries in $K^{+} \pi^{-}$and $K^{+} \pi^{0}$ should be equal (see Gronau and Rosner in Ref. [21]), which has not been supported by recent $B A B A R$ and Belle data $[5,7,13]$. These results might indicate a large color-suppressed amplitude, an enhanced electroweak penguin, or possibly new-physics effects [39].

Finally, we have presented an improved measurement of $\mathcal{B}\left(B^{0} \rightarrow K^{0} \pi^{0}\right)$. From the rate sum-rule prediction [21] $2 \mathcal{B}\left(K^{0} \pi^{0}\right)^{\text {sr }}=\mathcal{B}\left(K^{+} \pi^{-}\right)+\frac{\tau_{0}}{\tau_{+}}\left[\mathcal{B}\left(K^{0} \pi^{+}\right)-2 \mathcal{B}\left(K^{+} \pi^{0}\right)\right]$ and the currently published results for the other three $B \rightarrow K \pi$ modes, we find the sum-rule prediction to be $\mathcal{B}\left(B^{0} \rightarrow K^{0} \pi^{0}\right)^{\text {sr }}=$ $(8.4 \pm 0.8) \times 10^{-6}$, which is consistent with our new experimental result.

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Figure 9: (Top) constraint on the angle $\Delta \alpha_{\pi \pi}=\alpha-\alpha_{\text {eff }}$, expressed as one minus the confidence level (C.L.), as a function of $\left|\Delta \alpha_{\pi \pi}\right|$. We find an upper bound on $\left|\Delta \alpha_{\pi \pi}\right|$ of $43^{\circ}$ at the $90 \%$ C.L. (Bottom) constraint on the CKM angle $\alpha$ expressed as 1-C.L. There are eight peaks, two of them nearly merged, corresponding to an eight-fold ambiguity in the extraction of $\alpha$; four solutions are from the value and sign of $\Delta \alpha_{\pi \pi}$, which is doubled due to the trigonometric reflections between $\alpha_{\text {eff }}$ and $\pi / 2-\alpha_{\text {eff }}$. We exclude the range $\left[23^{\circ}, 67^{\circ}\right]$ in $\alpha$ at the $90 \%$ C.L. Only the isospin-triangle relations and the expressions in Eq. 1 are used in this constraint. The point $\alpha=0$, which corresponds to no $C P$ violation, and the values of $\alpha$ near 0 or $\pi$ can be excluded with additional physics input [13,36].
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