

The Quirky Collider Signals of Folded Supersymmetry

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We investigate the collider signals associated with scalar quirks (“squirks”) in folded supersymmetric models. As opposed to regular superpartners in supersymmetric models these particles are uncolored, but are instead charged under a new confining group, leading to radically different collider signals. Due to the new strong dynamics, squirks that are pair produced do not hadronize separately, but rather form a highly excited bound state. The excited “squirkonium” loses energy to radiation before annihilating back into Standard Model particles. We calculate the branching fractions into various channels for this process, which is prompt on collider time-scales. The most promising annihilation channel for discovery is W+photon which dominates for squirkonium near its ground state. We demonstrate the feasibility of the LHC search, showing that the mass peak is visible above the SM continuum background and estimate the discovery reach.

I. INTRODUCTION

One of the prime motivations for physics beyond the Standard Model (SM) at scales accessible to the Large Hadron Collider (LHC) is the hierarchy problem, namely, the quadratic sensitivity of the electroweak scale to higher scales. The absence of any new physics up to very high scales would imply an unnatural fine tuning of the Higgs potential. The hierarchy problem has motivated a paradigm for model building whereby a new symmetry is introduced at the TeV scale to protect the Higgs from quadratic divergences. In such a scenario, naturalness is manifested in cancellations between those SM loop diagrams which contribute to the Higgs mass parameter and loops involving new physics. When considering the new physics that the LHC may discover it suffices to consider only the “little hierarchy” between the scales set by the LHC reach and the electroweak scale. For this purpose one loop cancellations are sufficient.

Within this paradigm the new states that cancel a given SM loop are related to the corresponding SM particles by the symmetry. The largest contribution to the Higgs mass parameter in the SM is from the top quark loop, and therefore there will be new states related to the top by a symmetry that cancel this loop. For example, if supersymmetry [1] is introduced the spectrum includes scalar top partners, \tilde{t} , to cancel the top loop. Alternatively global symmetries, or combinations of global and discrete symmetries, may be responsible for protecting the Higgs, such as in little Higgs models [2, 3] or twin Higgs models [4, 5]. In these theories new fermionic top partners, t' , are responsible for restoring naturalness.

In this class of theories the collider phenomenology crucially depends on whether or not the top partners carry SM color. While in little Higgs theories and

supersymmetric theories the top partners are indeed charged under SM color, in general this need not be the case. In particular, if the symmetry that protects the Higgs mass involves a Z_2 symmetry that interchanges SM color with a hidden color,

$$SU(3)_{\text{QCD}} \leftrightarrow SU(3)_{\text{QCD}'} \quad (1)$$

where the symmetry acts on the SM $SU(3)$ gauge fields and, more generally, on all colored matter, the cancellation can take place with top partners that are not charged under QCD, but only under QCD' . The LHC signals are drastically different in such a scenario since no new particle is strongly produced.

One possibility, realized in the mirror twin Higgs model [4], is that the top partners are not charged under any of the SM gauge groups. In this paper we focus on the alternative possibility, realized in folded supersymmetry [6], that the new quark partners are charged under QCD' , but retain SM $SU(2)_L \times U(1)_Y$ quantum numbers. The spectrum of states in this theory is similar to that of the MSSM, but with the crucial difference that the squarks are charged under $SU(3)_{\text{QCD}'}$ instead of SM QCD. The typical scale for scalar quark masses is a few hundred GeV. Since the two QCD sectors are related by the symmetry of equation (1) the scales Λ , where QCD gets strong, and Λ' , where QCD' gets strong, are close, with the difference arising from radiative effects. This implies there is a gap between the strong scale and the mass of the lightest matter field in the QCD' sector. The absence of any light states charged under both the SM gauge groups and QCD' implies this theory may be considered an example of a hidden valley model [7] in which the hidden valley particles are glueballs and in which the energy scales and particle spectrum are motivated directly by the hierarchy problem. The purpose of this paper is to study

for the first time the collider phenomenology associated with the production of heavy quark partners in this scenario. The charge assignments and strong dynamics effects will lead to signals that are very different from either supersymmetric or generic hidden valley models.

Theories with a QCD' sector where there is a large hierarchy between the masses of the matter fields and the QCD' scale, $m_{q'} \gg \Lambda'$, give rise to very unusual dynamics [8][7][9][10]. For this reason the quarks (or scalar quarks) of such a sector have been dubbed quirks (or squirks) [9]. To understand this, let us first recall the dynamics of normal QCD. Consider two heavy quarks that are produced back-to-back in a hard process. As the two quarks get farther apart and their distance approaches Λ^{-1} , confining dynamics sets in and some of their energy is lost to a gluonic flux tube extending between them. When the local energy density in the flux tube is high enough it is energetically favorable to pair create a light quark anti-quark pair, ripping the tube. This mechanism of soft hadronization allows the two heavy quarks to hadronize separately.

In quirky QCD, on the other hand, such a soft hadronization mechanism is absent because there are no quarks with mass less than or comparable to Λ' . The energy density in the QCD flux tube, or more simply, the tension of the QCD string cannot exceed Λ'^2 which is far less than the $m_{q'}$ per Compton wavelength needed to create a heavy quirk anti-quirk pair. The splitting of the QCD string by a quirk anti-quirk pair is exponentially suppressed as $\exp(-m_{q'}^2/\Lambda'^2)$ [8]. In fact, one may view the entire process as single production of a highly excited bound state, squirkonium. All of the kinetic energy that the quirks possess at production, $\sqrt{\hat{s}} - 2m_{q'}$, which is typically of order $m_{q'}$, can be interpreted as squirkonium excitation energy. This energy is radiated away into glueballs of QCD' and photons. Eventually the two quirks pair-annihilate back into lighter states.

From the above discussion it is clear that the characteristic collider signatures of folded supersymmetry are determined by the final states that the squirks annihilate into. In what follows we calculate the cross-section for production of these particles at the LHC, and evaluate the branching ratios for pair-annihilation into various final states. The possibility of discovering the soft photons from the loss of excitation energy will be discussed elsewhere [11]. We then focus on the most promising annihilation channel for detection, which is $W + \text{photon}$, and demonstrate the reach for this search at the LHC. Some qualitative features of our analysis also apply to the supersymmetric model of Babu, Gogoladze and Kolda [12], which also predicts quirky behaviour at the weak scale.

II. PRODUCTION AND ENERGY LOSS

In a folded supersymmetric model, the scalar quirks have exactly the same electroweak quantum numbers as

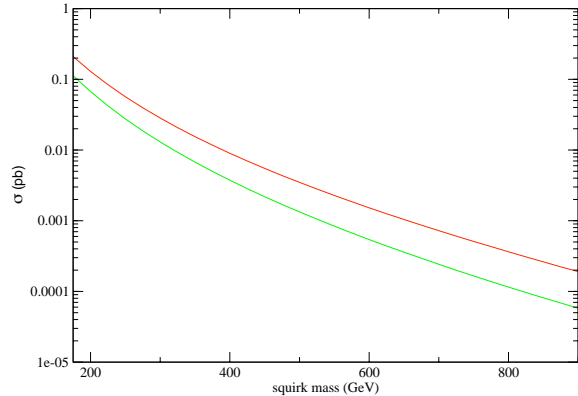


FIG. 1: The total cross-section for production of first generation squirk anti-squirk pairs via an s-channel W^+ (top curve) and W^- (bottom curve) at the LHC as a function of the squirk mass. The up and down squirks have been taken to be degenerate.

the corresponding quarks, but are charged under QCD', not under QCD. Specifically, under $SU(3)_{C'} \times SU(3)_C \times SU(2)_L \times U(1)_Y$, where $SU(3)_{C'}$ corresponds to QCD', the quantum numbers of the squirks are

$$\begin{aligned}
 \tilde{Q} & [3, 1, 2, (1/3)] \\
 \tilde{D}^c & [\bar{3}, 1, 1, (2/3)] \\
 \tilde{U}^c & [\bar{3}, 1, 1, -(4/3)]
 \end{aligned} \tag{2}$$

Therefore, at a collider squirks are only weakly produced. This could happen through the Drell-Yan process via an off-shell photon, Z , or W , or alternatively by gauge boson fusion. Production through the photon or the Z typically does not result in an observable signal, since the primary annihilation channel is to glueballs of QCD', which are invisible*. In the simple analysis of this paper we focus on production of squirks through s-channel $W^{\pm\dagger}$. In this case the conservation of electric charge implies that squirk annihilation must result in the emission of at least one charged particle.

Figure 1 shows the production cross-section for first generation $SU(2)$ doublet up-down squirk anti-squirk pairs through s-channel W^+ and W^- as a function of the squirk mass. In folded supersymmetry the second generation squirks are expected to be nearly degenerate

* Although glueballs of QCD' decay back to SM states, giving rise to potentially observable signals [7], in the parameter range of interest this generally happens outside the detector [6].

† It should be pointed out that weak boson fusion may dominate over Drell-Yan production at high squirk mass [13] and may require further study.

with those of the first which will effectively double the signal rate.

The squirks are in general produced with kinetic energy much larger than $\Lambda_{\text{QCD}'}$. We expect that a significant fraction of this energy is lost to photons and hidden glueballs prior to pair-annihilation. What determines the time-scale on which this happens? In particular, should we expect displaced vertices in quirky events?

We now argue that, in the context of folded supersymmetry, the pair-annihilation of squirks is prompt on collider time-scales. Assuming the excitations of squirkonium follow a Regge behavior $E_n^2 \sim n\Lambda'^2$, the energy spacing between adjacent energy levels decreases like $1/\sqrt{n}$. In the limit of high quantum number n the energy levels may be treated as a continuum and the system may be treated semi-classically. The semi-classical description is that of two massive charged particles connected by a string with tension Λ'^2 . The string exerts a constant force on the particles, leading to an acceleration of $a = \Lambda'^2/m_{q'}$.

A system of accelerating charges must radiate soft photons as dictated by Larmor's formula

$$P = \frac{\partial E}{\partial t} = \frac{8\pi\alpha}{3} a^2. \quad (3)$$

The time scale for energy loss from photon radiation, $E/P \sim m_{q'}/P$, is thus

$$t_{\text{rad}} \sim \frac{3}{8\pi\alpha} \frac{m_{q'}^3}{\Lambda'^4} = \left(\frac{m_{q'}}{500 \text{ GeV}}\right)^3 \left(\frac{5 \text{ GeV}}{\Lambda'}\right)^4 10^{-18} \text{ sec} \quad (4)$$

which is prompt. It is plausible that, because of the glueball mass gap, radiation of soft photons contributes a substantial part of the energy loss of excited squirkonium, leading to a novel signature of this scenario [11]. In any case, other mechanisms of energy loss, such as emission of glueballs, can only make t_{rad} shorter. The rate for pair-annihilation of squirkonium from a low lying state is of order $m_{q'}\alpha_{\text{QCD}'}^3\alpha_W^2$, which is also prompt on collider time-scales. Here $\alpha_{\text{QCD}'}$ and α_W are the structure constants for QCD' and the SM weak interactions respectively. We conclude that displaced vertices are not a characteristic feature of folded supersymmetry.

III. ANNIHILATION

Since squirkonium states produced through a W carry electric charge, we expect that they will annihilate into SM particles, giving rise to promising collider signals. However, we must first verify that the charged state does not beta decay down to a neutral state prior to re-annihilation, which would result in the absence of an observable signal.

Since the first and second generation quarks have very small Yukawa couplings, the mass splittings between the corresponding up-type and down-type squirks in

an SU(2) doublet are very small. The mass splitting is dominated by radiative effects and is of order $(e^2/16\pi^2)(M_Z^2/m_{q'})$. The rate for beta decay is proportional to five powers of the mass splitting $\delta m_{q'}$, and is therefore highly phase space suppressed.

$$\Gamma = \frac{G_F^2(\delta m_{q'})^5}{15\pi^3} \quad (5)$$

Therefore the scalar partners of the first two generations will pair-annihilate before beta-decay can take place, resulting in an observable signal. For the third generation squirks, on the other hand, the mass splittings are larger, of order $m_t^2/(2mq')$ which is of order 30 GeV for squirks at 500 GeV. The lifetime of a stop squirk with this mass splitting is of order 10^{-19} seconds, which may be comparable to the time scale for energy loss by radiation. The dynamics of a squirk pair which undergoes beta decay while oscillation may be quite interesting because of the large angular momentum introduced to the system, however we will not consider this possibility here. Even if the energy loss is much faster than beta decay, which may be the case because glueball radiation, the stop-bottom quirkonium state is likely to annihilate via a charged Higgs due to the large top yukawa coupling and is thus not likely to contribute to the signal we will consider below. We therefore omit this case from further consideration, and limit our discussion to the first two generations.

What are the possible final states? One possibility is that the squirk and anti-squirk go back into two SM fermions through an s-channel W. In the limit that the masses of the W and the final state fermions are neglected, the total cross-section for a squirk anti-squirk pair into all possible SM fermion-antifermion pairs is given by

$$\sigma = N_{\text{QCD}'} N_f v_{\text{rel}} \frac{\pi\alpha_W^2}{48E^2} \quad (6)$$

Here E is the energy of each squirk, v_{rel} is the relative velocity between the squirks and $N_f = 12$ is the total number of possible SM final states. Other possible annihilation channels include W + photon and W + Z. Consider first the W + photon final state. In the limit that v_{rel} is very small, we obtain

$$\sigma v_{\text{rel}} = N_{\text{QCD}'} \frac{\pi\alpha_W\alpha}{18E^2} \quad (7)$$

In this expression we have neglected the mass of the W. Finally, for the W + Z final state, in the limit of very small relative velocity, we have

$$\sigma v_{\text{rel}} = N_{\text{QCD}'} \tan^2\theta_W \frac{\pi\alpha_W\alpha}{18E^2} \quad (8)$$

Here θ_W is the weak mixing angle, and we have again neglected the masses of the W and the Z. We see from these expressions that for slow-moving squirks which have lost most of their energy or were produced close to

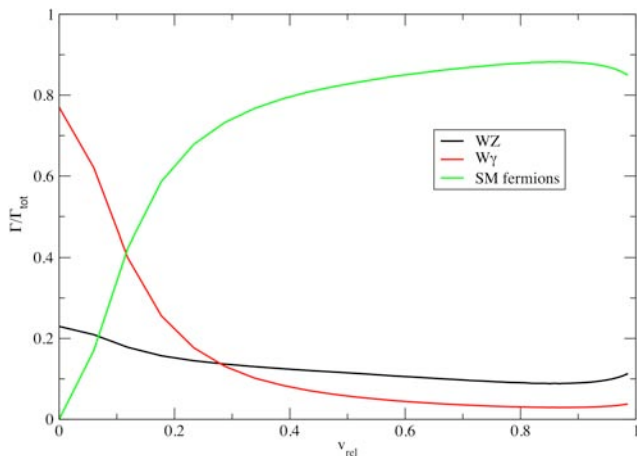


FIG. 2: Branching ratios for a charged squirk-antisquirk pair into various final states, as a function of the relative velocity of the pair.

threshold the most important channel is to $W + \text{photon}$, with annihilation to two fermions becoming significant only for larger values of the relative velocity.

Figure 2 shows the branching ratios for pair-annihilation of squirks into various final states, as a function of the relative velocity of the incoming squirks. The relevant cross-sections have been evaluated using CalcHEP [14]. We see that for slow-moving squirks the dominant annihilation channel is to $W + \text{photon}$, in agreement with our analytic expressions above. This is promising for the LHC. The $W + Z$ channel is also large enough to play a role. For more energetic squirks annihilation through an off-shell W into two SM fermions dominates. In this case the charged lepton + missing energy and top + bottom final states are most likely to prove useful.

In order to devise a search strategy it is therefore important to understand whether annihilation typically occurs at high or low relative velocities. As described in more detail in [9, 11], annihilation will occur dominantly at low values of angular momentum. The angular momentum in the squirkonium system will grow in a random walk as the excited state radiates quanta. Following the formalism developed in [9] we estimate that the probability for squirk annihilation in a single period at low angular momentum and high velocity is approximately 10^{-4} ‡. This small probability implies that the squirk system will typically gain significant angular momentum before annihilation occurs, and the squirkonium pair will reach near its ground state before annihilating. With this motivation in mind we will consider a search for squirks in

‡ This is significantly less than the annihilation probability for fermionic quarks.

the $W + \text{photon}$ annihilation channel in the next section.

Before moving on to collider searches we should briefly comment on the possibility of annihilating to $W + \text{glueball}$. The direct annihilation to $W + \text{gluon}$ naively vanishes because any such amplitude is proportional to the trace of a single $SU(3)$ generator. Annihilating to $W + \text{glueball}$ thus requires emitting at least two gluons, and is therefore phase space suppressed. However, there may be another possibility for this annihilation to occur. It is believed that hybrid states exist in QCD [17]. These exotic states may be viewed as bound states consisting of a meson in the color octet state and a gluon. Lattice studies show that QCD hybrid states carry a variety of spin and parity quantum numbers, including some states that mix directly with conventional mesons, and some that do not. In regular QCD the states that do not mix with mesons can decay rapidly to, for example, pions. However in quirky QCD', these states may be semi-stable. In particular, lattice studies show that the mass splitting between some hybrid states and their corresponding (same flavor) conventional meson state is smaller than the estimated QCD glueball mass [17]. If this is indeed the case, some quirky hybrid state will be stable to glueball emission due to kinematics and will thus annihilate directly. Since the $q'q'^*$ state is in the color octet, the amplitude for single gluon emission is no longer a trace of a single generator and will not vanish. These states are thus most likely to annihilate to $W + \text{glueball}$. The dynamical question is then, how often does the radiative energy loss of excited squirkonium lead to one of the exotic hybrid state rather than the conventional ground state? Keeping the $W + \text{glueball}$ channel in mind we leave this question for further work.

IV. LHC SEARCH

In this final section we will demonstrate the feasibility of searching for the annihilation products of squirk bound states at the LHC and estimate the signal to background ratios after employing various kinematic cuts. We will focus on the $W + \text{photon}$ annihilation signal. As discussed above, this final state dominates when the squirkonium bound state has lost most of its energy by radiation and the annihilation takes place at or near the ground state. The expected signal is a *peak* in the invariant mass of $W + \text{photon}$. The concentration of the signal at a mass peak will be crucial for the signal to stand above background.

We focus on events in which the W boson decays leptonicly. Despite the inability to measure the neutrino energy and longitudinal momentum the invariant mass of the squirkonium may be reconstructed if one *assumes* the lepton and missing E_T come from an on shell W , and that the neutrino is the only source of missing transverse energy. If this is the case, the width of the mass peak arises predominantly from detector effects and from squirkonium decays from low lying excited states,

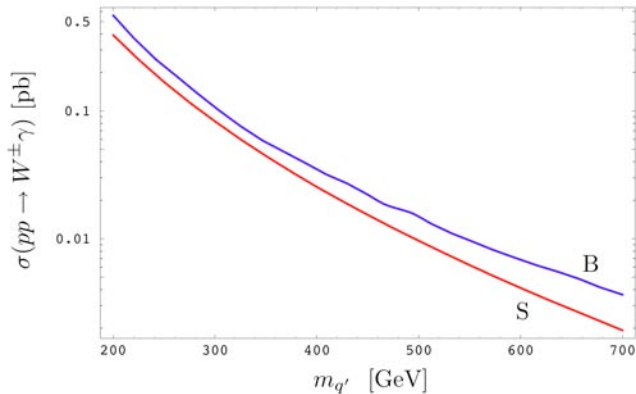


FIG. 3: An estimate of the signal versus the background as a function of the squirk mass in GeV. The bottom curve is the squirk pair production cross section in the charged channel. The annihilation of these squirk pair will dominantly produce W^{\pm} +photon with an invariant mass of $\sim 2m_{q'}$. The top curve is the SM W^{\pm} +photon with $|m_{W\gamma} - 2m_{q'}| < \sqrt{\Lambda m_{q'}}$ for $\Lambda = 15$ GeV.

rather than true ground states (of order $\Lambda' \sim 10$ GeV).

This mass peak is likely to be smeared due to additional transverse missing energy from the radiative decay. The energy loss due to radiation is expected to be distributed in a particular pattern which is symmetric under $\vec{x} \rightarrow -\vec{x}$, that is, for every glueball or photon emitted in one direction, a similar photon will be emitted in the opposite direction. In the classical limit (an infinite number of soft quanta) such energy loss does not introduce new transverse momentum. However, for glueball radiation the classical limit is not appropriate because the glueball mass cannot be neglected when compared to the total amount of energy radiated. The additional missing transverse energy is inversely proportional to the square root of the number of glueball quanta emitted, in the limit that the total energy radiated is held fixed. We will (conservatively) assume that most of the energy is lost by glueball emission giving a missing E_T of order $\sqrt{m_{q'}\Lambda'}$, which is of order 100 GeV or less in most of our parameter space.

In the case where the W decays leptonically, the leading background is continuum SM production of W+photon. For simplicity we will estimate the signal-to-background ratio by comparing the W+photon signal cross section to the invariant mass distributions of background W+photon in the mass window $|m_{W\gamma} - 2m_{q'}| < \sqrt{\Lambda m_{q'}}$ GeV. For the purpose of this estimate we have conservatively chosen a rather high value of $\Lambda = 15\text{GeV}$, about a factor of 10 above our QCD glueball mass calculated on the lattice. Background events were generated using MadEvent [16]. As shown in Figure 3, the signal to background ratio is of order a half for all of the mass range within LHC reach. Taking the leptonic branching fraction of the W boson (to electrons or muons

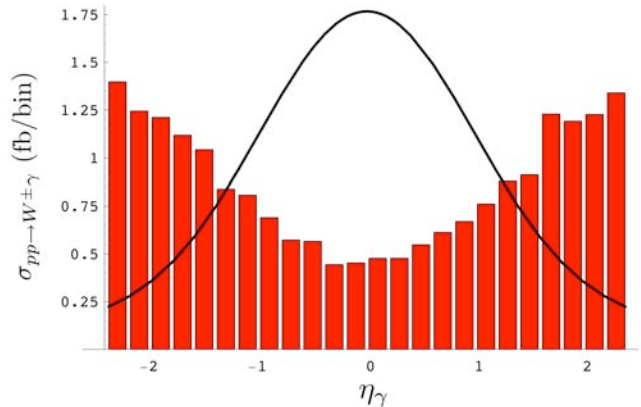


FIG. 4: The photon η distribution for SM W+photon events with center of mass energy of 1 TeV or more (red bars). The estimated signal distribution is shown as a solid curve. Employing a cut of $\eta < 1.5$ increases the signal to background ratio by more than a factor of two.

only) and a low branching fraction of squirkonium to W+photon of 0.6, a 5 sigma discovery of squirks with a mass of 400 GeV is possible with $\sim 11 \text{ fb}^{-1}$ of data. With 100 fb^{-1} the discovery reach is approximately 620 GeV.

One can improve the discovery reach by employing a pseudorapidity cut on the final state photon. The signal events involve the production and decay of heavy squirkonium. Even after losing its excitation energy, the squirkonium will be nearly at rest in the lab frame and is expected to decay isotropically. In Figure 4 we show the distribution of the photon's pseudorapidity, η , for SM W+photon events with invariant mass of 1 TeV or more. The shape of the distribution is similar for different invariant masses. This distribution is compared with the expected signal distribution (shown as solid curve). This was estimated by convoluting the η distribution of isotropic decay with the longitudinal boost of the squirkonium system (which is simply the longitudinal boost of the CoM frame in squirk production, generated by MadEvent).

Placing a cut on η of the photon of 1.5 reduces the background by a factor of ~ 2.2 while the signal is only reduced by $\sim 15\%$. Including the η cut, 5 sigma discovery of 400 GeV squirks may be reached with approximately 8.5 fb^{-1} . Taking this estimate at face value, with 100 fb^{-1} the LHC will be able to discover one generation of squirks up to a mass of 500 GeV and two degenerate generations of squirks (as is the case in folded SUSY) up to a mass of 650 GeV. However, one could imagine improving this analysis, for example, by optimization of the η cut. Alternatively, we could perform additional cuts on lepton rapidity or compare transverse mass distributions instead of reconstructed

invariant masses [15]. Observation of soft photons from the radiative decay of squirkonium could also reduce the background and thereby increase the reach [11].

In summary, we have performed the first detailed study of the collider phenomenology associated with squirks in folded supersymmetric models, and identified several promising discovery channels for the LHC.

Acknowledgments It is a pleasure to thank Johan Alwall, Elliott Cheu, Markus Luty, Shmuel Nussinov and Michael Peskin for useful discussions. GB acknowledges the support of the State of São Paulo Research Founda-

tion (FAPESP), as well as the Brazilian National Council for Technological and Scientific Development (CNPq). The work of ZC and CAK was partially supported by the NSF under award number PHY-0408954. The work of HSG was supported in part by the NSF grant PHY-04-57315 and by DOE under contract DE-AC02-05CH11231. The work of RH was supported by DOE grant DE-AC02-76SF00515. RH and HSG also wish to thank the Aspen Center for physics where some of this work was conducted.

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