

Extraction of $|V_{ub}|$ with Reduced Dependence on Shape Functions

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Using *BABAR* measurements of the inclusive electron spectrum in $B \rightarrow X_u e \nu$ decays and the inclusive photon spectrum in $B \rightarrow X_s \gamma$ decays, we extract the magnitude of the CKM matrix element V_{ub} . The extraction is based on theoretical calculations designed to reduce the theoretical uncertainties by exploiting the assumption that the leading shape functions are the same for all $b \rightarrow q$ transitions (q is a light quark). The results agree well with the previous analysis, have indeed smaller theoretical errors, but are presently limited by the knowledge of the photon spectrum and the experimental errors on the lepton spectrum.

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I. INTRODUCTION

The determination of the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element V_{ub} from charmless semileptonic B decays is complicated by the fact that over most of the phase space $B \rightarrow X_c \ell \nu$ decays dominate and are very difficult to distinguish from the signal $B \rightarrow X_u \ell \nu$ decays. Here X_c and X_u refer to hadrons, mostly mesons, with and without charm. To reduce impact of the dominant $B \rightarrow X_c \ell \nu$ background, partial rates for $B \rightarrow X_u \ell \nu$ decays are measured in regions of phase space where these background decays are forbidden or highly suppressed, for instance, near the endpoint of the energy spectrum of the charged lepton ℓ . The partial decay rates are extrapolated to the total rate by comparing the experimentally measured rate with a theoretical prediction.

Theoretically, the most precise predictions can be made for the total $B \rightarrow X_u \ell \nu$ decay rate. Accounting for restriction in phase space is difficult because decay spectra close to the kinematic limit are susceptible to non-perturbative strong-interaction effects. Theoretical tools for the calculations of the partial inclusive decay rates are QCD factorization and local operator product expansions (OPE). These calculations separate non-perturbative from perturbative quantities, and use expansions in inverse powers of the b -quark mass, m_b , and in powers of the strong coupling α_s [1]. At leading order in Λ_{QCD}/m_b , the non-perturbative bound-state effects are accounted for by a shape function describing the ‘‘Fermi-motion’’ of the b quark inside the B meson. These shape functions cannot be calculated. Different shapes have been proposed, and parameters defining these shapes have to be extracted from data. This

introduces significant additional hadronic uncertainties. At leading order, these shape functions are assumed to be universal functions for $b \rightarrow q$ transitions, where q is a light quark, either s or u .

In the past, we have extracted the non-perturbative parameters of these shape functions, the b -quark mass m_b and its mean kinetic energy squared, $\mu_\pi^2(\mu)$, from the moments of the inclusive photon spectrum in $B \rightarrow X_s \gamma$ decays, as well as from moments of the hadron mass and the lepton energy distributions in $B \rightarrow X_c \ell \nu$ decay. These parameters depend on the choice of the renormalization scale, μ .

It was suggested many years ago [2] that $|V_{ub}|$ can be extracted with smaller theoretical uncertainties by combining integrals over the lepton spectrum in $B \rightarrow X_u \ell \nu$ decays with weighted integrals over the photon spectrum from $B \rightarrow X_s \gamma$ decays, each above a cut-off energy E_0 . The underlying assumption is that the QCD interactions affecting these two processes are the same and thus will cancel to first order in the appropriate ratio of weighted decay rates. The advantage of this approach is that it reduces the sensitivity to the choice of the shape-function parameterization and thus avoids uncertainties that are difficult to quantify.

In the following, we extract $|V_{ub}|$ using two different prescriptions, one proposed by Leibovich, Low, and Rothstein (LLR) [3–5], and the other based on more recent calculations by Lange, Neubert, and Paz (LNP) [6, 7]. The two prescriptions use different calculations of the weight functions, and thus result in different estimates of the theoretical uncertainties.

The *BABAR* Collaboration was the first to apply the LLR prescription [3, 4] to extract $|V_{ub}|$, based on measurements of the hadron mass spectrum in charmless

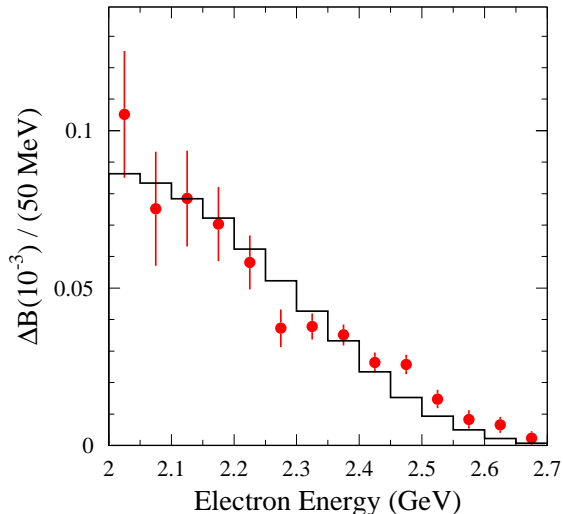


FIG. 1: The differential branching fraction for $B \rightarrow X_u e \nu$ decays [9] as a function of the charged lepton momentum, measured in the $\Upsilon(4S)$ rest frame. The error bars indicate the statistical errors; the histogram represents the theoretical prediction [11].

semileptonic B decays and the inclusive photon spectrum [8].

II. EXPERIMENTAL INPUT

The experimental inputs for this analysis are the published *BABAR* measurements of the inclusive electron energy spectrum in $B \rightarrow X_u e \nu$ decays [9], and of the inclusive photon-energy spectrum in $B \rightarrow X_s \gamma$ decays [10]. These measurements are based on a data sample corresponding to a total integrated luminosity of about 80 fb^{-1} . The measured spectra will be integrated above an energy E_0 , measured in the B -meson rest frame.

A. Inclusive Lepton Spectrum in $B \rightarrow X_u e \nu$ Decays

The inclusive electron-energy spectrum above 2.0 GeV , measured in the $\Upsilon(4S)$ rest frame, is shown in Fig.1. The data are fully corrected for detection efficiencies as well as final state radiation and bremsstrahlung. They are normalized to the total number of charged and neutral B -meson decays and are presented as differential branching fraction.

For the extraction of $|V_{ub}|$ we need to transform the measured partial branching fraction from the $\Upsilon(4S)$ rest frame to the branching fraction in the B -meson rest frame, integrated over the spectrum above an energy cut-off at E_0 . This is done using correction factors de-

rived from the predicted electron spectrum [11], calculated with the shape-function parameters determined by a global fit [12] to moments of inclusive distributions in semileptonic and radiative B -meson decays. The systematic uncertainties for this transformation are estimated by varying the shape-function parameters within their experimental uncertainties. The resulting partial branching fractions for different values of the electron-energy cut-off, E_0 , measured in the $\Upsilon(4S)$ and the B -meson rest frames, and correction factors relating the two, are listed in Table I.

TABLE I: Partial branching fractions for $B \rightarrow X_u e \nu$ decays in units 10^{-3} , integrated over the energy range from E_0 to $2.6 \text{ GeV}/c$, both in the $\Upsilon(4S)$ rest frame and the B -meson rest frame, as well as the correction factor relating the two. The uncertainty of the correction factor reflects the uncertainty in the assumed shape of the spectrum.

E_0 [GeV]	$\Delta\mathcal{B}(E_0) \cdot 10^3$ $\Upsilon(4S)$ rest frame	Correction factor	$\Delta\mathcal{B}(E_0) \cdot 10^3$ B rest frame
2.0	$0.572 \pm 0.041 \pm 0.065$	1.002 ± 0.005	0.573 ± 0.077
2.1	$0.392 \pm 0.023 \pm 0.038$	0.994 ± 0.008	0.390 ± 0.044
2.2	$0.243 \pm 0.011 \pm 0.020$	0.973 ± 0.016	0.236 ± 0.023
2.3	$0.148 \pm 0.006 \pm 0.010$	$0.915^{+0.041}_{-0.034}$	0.135 ± 0.012
2.4	$0.075 \pm 0.004 \pm 0.006$	$0.772^{+0.075}_{-0.084}$	0.058 ± 0.008

B. Inclusive Photon Spectrum in $B \rightarrow X_s \gamma$ Decays

The inclusive photon-energy spectrum [10] in $B \rightarrow X_s \gamma$ decays, above 1.9 GeV , measured in the B -meson rest frame, is shown in Fig. 2. This spectrum is measured as the sum of the photon spectra in 38 exclusive decay modes. The data are fully corrected for detection efficiencies and energy resolution. They are normalized to the total number of charged and neutral B -meson decays and presented as differential branching fractions.

III. THEORETICAL FRAMEWORK FOR THE EXTRACTION OF $|V_{UB}|$

A. Method I by Leibovich, Low, and Rothstein

A. K. Leibovich, I. Low, and I. Z. Rothstein [3, 4] proposed a method for the extraction of the ratio $|V_{ub}|^2/|V_{tb}V_{ts}^*|^2$ without invoking knowledge of the shape function. Their calculation relates $|V_{ub}|^2/|V_{tb}V_{ts}^*|^2$ to the experimentally measured differential branching fractions for $B \rightarrow X_u \ell \nu$ and $B \rightarrow X_s \gamma$ decays,

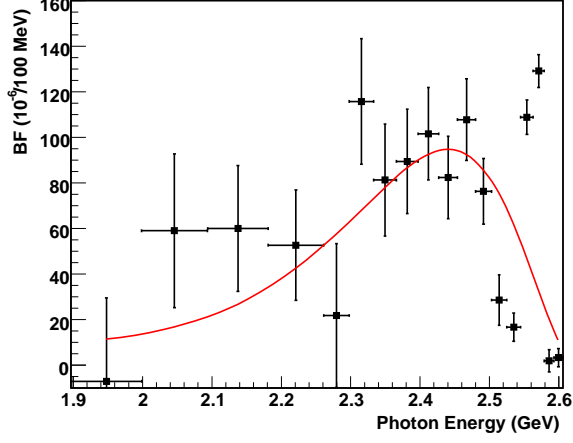


FIG. 2: The differential branching fraction of inclusive $B \rightarrow X_s \gamma$ decays as a function of the photon energy in the B rest frame. The error bars indicate the statistical errors. The data are compared to the theoretical prediction [11] using shape function parameters $m_b = 4.67$ GeV, $\mu_\pi^2 = 0.16$ GeV² that have been obtained from the fit to E_γ spectrum [10].

$$\frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} = \frac{3\alpha|C_7^{(0)}(m_b)|^2}{\pi}(1 + H_{\text{mix}}^\gamma) \times \left(\int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B} \right) / \left(\int_{x_B^c/\rho}^1 du_B \frac{d\Gamma^\gamma}{du_B} w(u_B, x_B^c, \rho) \right). \quad (1)$$

The integration variables are

$$x_B = 2E_l/M_B, \quad u_B = 2E_\gamma/M_B,$$

with limits x_B^c and x_B^c/ρ ; M_B refers to the B -meson mass. The weight function is approximately linear as a function of u_B ,

$$w(u_B, x_B^c, \rho) = u_B^2 \int_{x_B^c}^{\rho u_B} dx_B K\left(x_B; \frac{4}{3\pi\beta_0} \ln(1 - \alpha_s\beta_0 l_{x_B/u_B})\right), \quad (2)$$

and

$$H_{\text{mix}}^\gamma = \frac{\alpha_s}{2\pi C_7^{(0)}} \left[C_7^{(1)} + C_2^{(0)} \Re(r_2) + C_8^{(0)} \left(\frac{44}{9} - \frac{8\pi^2}{27} \right) \right],$$

$$\Re(r_2) \approx -4.092 + 12.78(m_c/m_b - 0.29),$$

$$K(x; y) = 6 \left\{ \left[1 + \frac{4\alpha_s}{3\pi}(1 - \psi'(4+y)) \right] \frac{1}{(y+2)(y+3)} - \frac{\alpha_s}{3\pi} \left[\frac{1}{(y+2)^2} - \frac{7}{(y+3)^2} \right] - \frac{4\alpha_s}{3\pi} \left[\frac{1}{(y+2)^3} - \frac{1}{(y+3)^3} \right] \right\} - 3(1-x)^2,$$

$$\psi(z) = \frac{1}{\Gamma(z)} \frac{d}{dz} \Gamma(z), \quad l_{x/u} = -\ln[-\ln(x/u)].$$

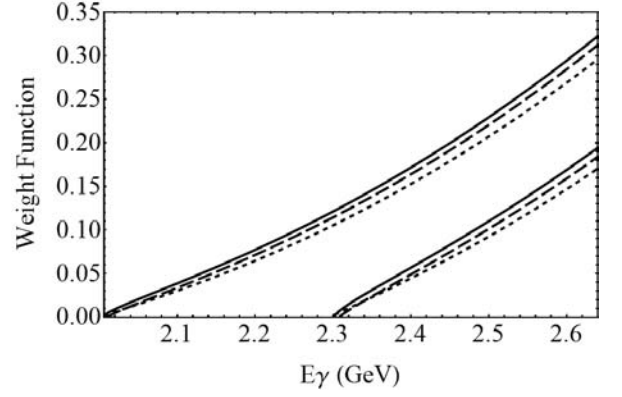


FIG. 3: The weight functions for Method I (LLR) for two values of $E_0 = 2.0$ GeV and $E_0 = 2.3$ GeV, corresponding to different values of $\rho = 0.9983$ (solid lines), $\rho = 0.995$ (dashed lines), and non-resummed weight function (dotted lines).

The Wilson coefficients $C_7^{(0)}(m_b)$, $C_8^{(0)}(m_b)$, *etc.*, computed in NLO, can be found in [13, 14]. The argument of K , $y = \frac{4}{3\pi\beta_0} \ln(1 - \alpha_s\beta_0 l_{x_B/u_B})$, diverges for $\alpha_s\beta_0 l_{x_B/u_B} = 1$. To avoid this pole, the integration limits are set to $x_B \leq \rho u_B$ with $\rho < 0.999$. For smaller values of ρ , the weight function deviates and thus the extracted value of $|V_{ub}|$ changes, and the uncertainty of this scheme increases.

To check the impact of the resummation of the Sudakov logarithms we also tried a non-resummed version of the LLR weight function [5]:

$$w(u_B, x_B^c) = u_B^2 \int_{x_B^c}^{u_B} dx_B \left(1 - 3(1-x_B)^2 + \frac{\alpha_s}{\pi} \left(\frac{7}{2} - \frac{2\pi^2}{9} - \frac{10}{9} \ln(1 - \frac{x_B}{u_B}) \right) \right). \quad (3)$$

This calculation (Eq. 1 – 3) includes NLO perturbative corrections, but does not take into account power corrections. Estimates of the theoretical uncertainties are discussed in Section 4. The weight function is shown in Fig. 3, for $E_0 = 2.0$ GeV and $E_0 = 2.3$ GeV and different values of the parameter ρ .

M. Neubert [15] performed calculations that are quite similar and give results that are very close to the ones obtained for this method.

B. Method II by Lange, Neubert, and Paz

The second method for the extraction of $|V_{ub}|$ is an application of the more recent two-loop calculations by B. Lange, M. Neubert, and G. Paz [6], and by B. Lange [7], performed for the high end of the lepton and photon energy spectra in $B \rightarrow X_u \ell \nu$ and $B \rightarrow X_s \gamma$ decays.

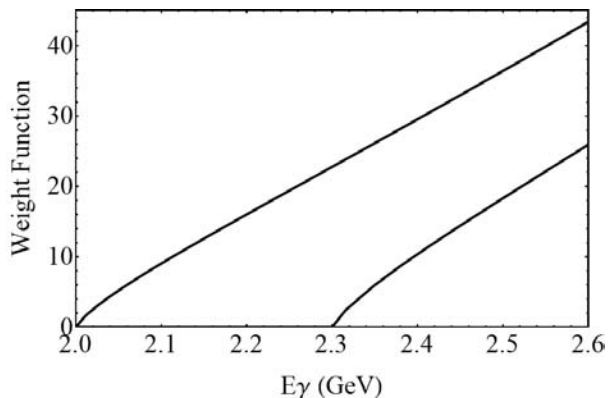


FIG. 4: The weight function (in units of ps^{-1}) for Method II (LNP), for $E_0 = 2.0 \text{ GeV}$ (upper curve) and $E_0 = 2.3 \text{ GeV}$ (lower curve).

Unlike the previously described methods, this one relates $|V_{ub}|$ to the measured partial $B \rightarrow X_u l \nu$ branching fraction and normalized photon spectrum in the $B \rightarrow X_s \gamma$ decay,

$$|V_{ub}|^2 = \frac{\frac{1}{\tau_B} \int_{E_0}^{M_B/2} dE_\ell (dB(B \rightarrow X_u l \nu)/dE_\ell)}{\int_{E_0}^{M_B/2} dE_\gamma w(E_\gamma, E_0) S(E_\gamma) + \Gamma_{\text{rhc}}(E_0)}, \quad (4)$$

$$S(E_\gamma) = \frac{1}{\Gamma(B \rightarrow X_s \gamma)_{E_\gamma > E_{\text{min}}}} \frac{d\Gamma(B \rightarrow X_s \gamma)}{dE_\gamma}, \quad (5)$$

where $\Gamma_{\text{rhc}}(E_0)$ represents residual hadronic power corrections, which in [6] were absorbed into the weight function. $E_{\text{min}} = 1.90 \text{ GeV}$ is chosen as the lower limit for the normalization of the photon energy spectrum; the corresponding branching fraction is 3.95×10^{-4} .

This calculation contains perturbative corrections at NNLO at the so-called “jet scale”, $\mu_i \sim \sqrt{m_b \Lambda_{\text{QCD}}}$, and at NLO at the so-called “hard scale”, $\mu_h \sim m_b$. Also included are the first-order power corrections, which are separated into two parts: kinematic corrections and residual hadronic corrections. The kinematic corrections do not introduce hadronic uncertainties and are applied directly to the weight function. The residual hadronic corrections include subleading shape functions for which the functional form is unknown. This introduces significant theoretical uncertainties. The weight functions, calculated for $E_0 = 2.0 \text{ GeV}$ and $E_0 = 2.3 \text{ GeV}$, are shown in Fig 4.

IV. DETERMINATION OF $|V_{UB}|$

A. Calculation of weighted integrals of $B \rightarrow X_s \gamma$ photon spectrum

For each of the two methods we extract $|V_{ub}|$ from a ratio of the $B \rightarrow X_u l \nu$ partial branching fraction and a weighted integral over the photon spectrum for $B \rightarrow X_s \gamma$. Each method introduces a specific weight function $w(E_\gamma, E_0)$. The value of the weighted integral of the photon spectrum above a minimum energy E_0 ,

$$I(E_0) = \int_{E_0}^{M_B/2} dE_\gamma w(E_\gamma, E_0) d\Gamma(E_\gamma)/dE_\gamma, \quad (6)$$

is taken as a sum over bins in the photon energy in the B -rest frame,

$$\tilde{I}(E_0) = \sum_i w(E_{\gamma i}, E_0) \left(\frac{d\Gamma(E_\gamma)}{dE_\gamma} \right)_i \Delta E_{\gamma i}. \quad (7)$$

The uncertainty for this sum is estimated using the standard error propagation for a linear combination of random variables:

$$\sigma^2 = \sum_{ij} w(E_{\gamma i}, E_0) w(E_{\gamma j}, E_0) \Delta E_{\gamma i} \Delta E_{\gamma j} V_{ij}, \quad (8)$$

where V_{ij} is a covariance matrix of the differential rate $d\Gamma(E_\gamma)/dE_\gamma$ for different photon energy bins. Assuming uncorrelated statistical errors and correlated systematic errors with known correlation matrix R_{ij} , the covariance matrix is of the form

$$V_{ij} = \sigma_{\text{stat}}^i \sigma_{\text{stat}}^j \delta_{ij} + \sigma_{\text{syst}}^i \sigma_{\text{syst}}^j R_{ij}. \quad (9)$$

The correlation matrix R_{ij} was provided by the BABAR Collaboration [10].

B. Results on $|V_{ub}|$ and Error Estimation

1. Method I (LLR)

The results of the extraction of $|V_{ub}|/|V_{tb}V_{ts}^*|$ based on Method I are presented in Table II. The theoretical uncertainties of the NLO calculations have been estimated [3, 4] to be $\mathcal{O}(\Lambda_{\text{QCD}}/M_B)$, resulting in a relative error of about 6% for $|V_{ub}|/|V_{tb}V_{ts}^*|$. The extraction of $|V_{ub}|/|V_{tb}V_{ts}^*|$ was done with resummed Sudakov logarithms. As a cross check, the extraction was also performed without resummation, resulting in decreases of the weight function by $\sim 6\%$ at $E_0 = 2.0 \text{ GeV}$ and by up to $\sim 12\%$ at 2.4 GeV .

To translate these results into $|V_{ub}|$ we exploit the constraints of the unitarity of the CKM matrix resulting in $|V_{tb}| \cong 1 + \mathcal{O}(\lambda^4)$, where $\lambda = 0.226$ is the sine of the

TABLE II: The results for $|V_{ub}|/|V_{tb}V_{ts}^*|$ and $|V_{ub}|$ for Method I (LLR). The extraction uses the weight functions based on resummed Sudakov logarithms with $\rho = 0.9983$. The first error represents the error from the measured $B \rightarrow X_u e \nu$ partial branching fraction, the second error is from the measured $B \rightarrow X_s \gamma$ spectrum, and the third is the estimated theoretical uncertainty. The fourth error on $|V_{ub}|$ is from the $|V_{ts}|$ uncertainty.

E_0 [GeV]	$ V_{ub} / V_{tb}V_{ts}^* $	$ V_{ub} \cdot 10^3$
2.0	$0.106 \pm 0.007 \pm 0.007 \pm 0.006$	$4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28$
2.1	$0.100 \pm 0.006 \pm 0.006 \pm 0.006$	$4.06 \pm 0.23 \pm 0.25 \pm 0.24 \pm 0.27$
2.2	$0.093 \pm 0.005 \pm 0.005 \pm 0.006$	$3.78 \pm 0.18 \pm 0.21 \pm 0.23 \pm 0.25$
2.3	$0.091 \pm 0.004 \pm 0.005 \pm 0.006$	$3.69 \pm 0.16 \pm 0.19 \pm 0.22 \pm 0.25$
2.4	$0.090 \pm 0.006 \pm 0.004 \pm 0.006$	$3.64 \pm 0.25 \pm 0.17 \pm 0.22 \pm 0.24$

Cabibbo angle. Using $|V_{ts}| = (40.6 \pm 2.7) \cdot 10^{-3}$ [16], we calculate $|V_{ub}|$ (see Table II).

The results show high stability with respect to variations of the lepton energy cut in the range from 2.0 to 2.4 GeV. The partial charmless semileptonic branching fraction decreases in this range from 25% to 2.3% of the total $B \rightarrow X_u l \nu$ branching fraction [9]. The observed stability is quite surprising because near the endpoint of the lepton energy spectrum the theoretical calculations are expected to break down, and this should lead to increasing theoretical uncertainties.

2. Method II (LNP)

The results on the extraction of $|V_{ub}|$ based on Method II are presented in Table III. The estimated individual theoretical uncertainties [7] are added in quadrature to obtain the total theoretical error. The first order non-perturbative hadronic power corrections, not considered in Method I, are split into two contributions: kinematic corrections, which are included in the weight function and depend on the scale for the calculation of kinematic power corrections, $\bar{\mu}$, but are independent of intermediate scale, μ_i , the hard scale, μ_h , and residual hadronic corrections. These corrections depend on unknown subleading shape functions, which affect the $B \rightarrow X_u l \nu$ and $B \rightarrow X_s \gamma$ spectra in different ways and thus do not cancel in the weighted ratio in the integrals. This uncertainty, σ_{had} , is estimated as suggested by Lange [7], namely by varying the shape and the parameters of the subleading shape functions.

The error σ_{pert} is the uncertainty of the NNLO approximation of the shape function and the LO approximation of the power corrections. It was estimated from the dependence of the sum of the weighted integral over the photon spectrum and residual hadronic correction on the hard scale, $\mu_h = m_b/\sqrt{2}$, on the intermediate scale, $\mu_i = 1.5$ GeV, and on the scale for the power corrections, $\bar{\mu} = 1.5$ GeV. All three scales are varied by factors of $\sqrt{2}$ and $1/\sqrt{2}$ relative to their default values, and the largest variation of the weighted integral is taken as the estimate of the perturbative uncertainty.

The errors σ_{m_b} and σ_{param} are due to uncertainties of

the parameters that are inputs to the calculation and were varied within their stated errors: $m_b = 4.61 \pm 0.06$ GeV and $m_c/m_b = 0.222 \pm 0.027$, $m_s = 90 \pm 25$ MeV. Here m_b and μ_π^2 are defined in the shape-function scheme at a scale $\mu_* = 1.5$ GeV, m_s and the ratio m_c/m_b are evaluated in the \overline{MS} scheme at 1.5 GeV, where the ratio is scale-invariant. The uncertainties on $\lambda_2 = 0.12$ GeV² and $\mu_\pi^2 = 0.25 \pm 0.10$ GeV² only enter into the subleading shape functions, for which the uncertainties are assessed separately.

The error σ_{norm} represents the uncertainty of the normalization of the photon spectrum, which is estimated to be about 6% [7].

We observe in Table III a significant increase in the extracted value of $|V_{ub}|$ for $E_0 \geq 2.2$ GeV, and the theoretical uncertainties also show a rapid growth for $E_0 > 2.1$ GeV, reaching 100% above $E_0 \simeq 2.3$ GeV. This is due to the power correction $\Gamma_{\text{rhc}}(E_0)$ in the denominator of the Eq.4. This correction is negative and almost independent of E_0 . As E_0 increases, the integral over the photon spectrum decreases and contribution from the $\Gamma_{\text{rhc}}(E_0)$ to the total uncertainty increases. This is not unexpected, because the effect of non-perturbative hadronic corrections increases in the region close to kinematic endpoints for both decays.

V. CONCLUSION

We have extracted the CKM matrix element $|V_{ub}|$ using published *BABAR* measurements of the inclusive lepton spectrum in $B \rightarrow X_u e \nu$ decays and inclusive photon spectrum in $B \rightarrow X_s \gamma$ decays. By using the ratios of the weighted spectra for these two decays, the results are expected to be less model dependent than previous measurements relying on the extraction of the shape functions from data and specific parameterizations of these functions.

For comparison, the results for $E_0 = 2.0$ GeV for both methods are presented in Table IV, together with the *BABAR* shape-function based measurement [9]. Figure 5 shows the dependence of the results on the lepton energy cut-off, E_0 .

The results agree well within their stated uncertain-

TABLE III: The results for $|V_{ub}|$ based on Method II (LNP). The first error represents the error from the measured $B \rightarrow X_u \ell \nu$ partial branching fraction, and the second from the measured $B \rightarrow X_s \gamma$ spectrum. The contributions to the theoretical error, as described in the text, are given together with the total theoretical error.

E_0 [GeV]	$ V_{ub} \cdot 10^3$	$\sigma_{\text{had}} \cdot 10^3$	$\sigma_{\text{pert}} \cdot 10^3$	$\sigma_{m_b} \cdot 10^3$	$\sigma_{\text{parm}} \cdot 10^3$	$\sigma_{\text{norm}} \cdot 10^3$	$\sigma_{\text{theory}} \cdot 10^3$
2.0	$4.40 \pm 0.30 \pm 0.41$	+0.08 -0.07	+0.03 -0.01	+0.13 -0.12	+0.01 -0.00	± 0.17	+0.23 -0.22
2.1	$4.55 \pm 0.26 \pm 0.45$	+0.15 -0.13	+0.11 -0.05	+0.16 -0.15	± 0.01	± 0.21	+0.32 -0.29
2.2	$5.01 \pm 0.24 \pm 0.60$	+0.42 -0.33	+0.40 -0.21	+0.25 -0.22	± 0.01	± 0.32	+0.71 -0.55
2.3	$6.99 \pm 0.31 \pm 1.60$	+3.90 -1.44	+3.02 -1.00	+0.75 -0.58	+0.04 -0.03	± 0.92	+5.07 -2.05

TABLE IV: Comparison of the $|V_{ub}|$ extraction for $E_0 = 2.0$ GeV for the two methods. The first error reflects the uncertainty in the measurements of the $B \rightarrow X_u \ell \nu$ lepton spectrum, the second error is due to the measurement of the $B \rightarrow X_s \gamma$ photon spectrum. For the shape-function based analysis, the second error originates from the extraction of the shape-function parameters, in this case based on both the inclusive photon spectrum as well as hadron-mass and lepton-energy moments from $B \rightarrow X_c \ell \nu$ decays. The third is the theoretical uncertainty. The fourth error for Method I is due to the uncertainty of $|V_{ts}|$.

Method	$ V_{ub} \cdot 10^3$
LLR [3, 4]	$4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28$
LNP [6, 7]	$4.40 \pm 0.30 \pm 0.41 \pm 0.23$
SF-based analysis [9]	$4.44 \pm 0.25^{+0.42}_{-0.38} \pm 0.22$

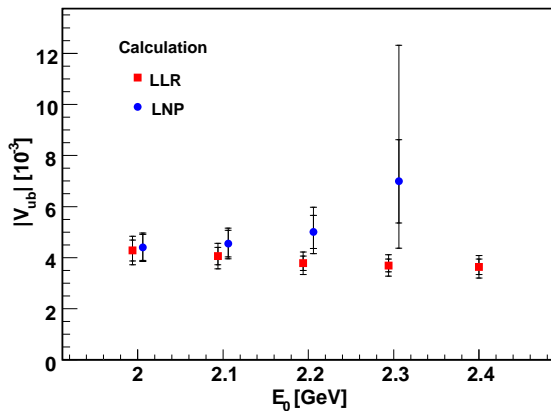


FIG. 5: Comparison of $|V_{ub}|$ values extracted from the two different calculations as a function of the lepton energy cut-off, E_0 . The errors bars represent the experimental and the total error.

ties. Method's I results in values of $|V_{ub}|$ and errors that appear to be unaffected by the restriction of the phase space near the kinematic limit. The theoretical errors are somewhat smaller than for Method II, and also smaller than for the previous extraction method, but there is an

additional error due to the uncertainty of $|V_{ts}|$.

The values of $|V_{ub}|$ extracted by using Method II for the lepton energy cut-off at $E_0 = 2.0$ GeV and 2.1 GeV agree well with those of Method I. The theoretical error contributions are estimated in detail. The smallest total theoretical error is obtained for $E_0 = 2.0$ GeV; it is about 5%. However, at larger E_0 , there are several increasing contributions to the theoretical uncertainties, in particular the first-order non-perturbative hadronic power corrections that are not dealt with in Method I. This is why the error contribution from the weighted integral over the photon spectrum increases very rapidly as the phase space gets more restricted.

At present, for $E_0 = 2.0$ GeV, the experimental errors on both the $B \rightarrow X_u \ell \nu$ branching fraction and the integral over $B \rightarrow X_s \gamma$ are about 12%; this means that there are opportunities to improve the accuracy of $|V_{ub}|$ significantly by reducing the experimental errors of these two spectra with more data available now and in the future. Also, extending E_0 to 1.9 GeV or lower, would allow us to establish more clearly the stability of the results in this region.

It would be of interest to perform a similar analysis for the hadron-mass distribution in $B \rightarrow X_u \ell \nu$ decays, with increasingly tighter restrictions on the mass of X_u , and assess whether this measurement is indeed more robust than the measurement of the lepton spectrum near the kinematic endpoint.

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