SLAC-PUB-12745
August , 2007

ABCD formalism and attosecond few-cycle pulse via chirp manipulation of a seeded free electron laser

Juhao Wu[1], Paul R. Bolton[1], James B. Murphy[2], and Kelin Wang[3,4]

[1]*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309*
[2]*National Synchrotron Light Source, Brookhaven National Laboratory, Upton, NY 11973-5000*
[3]*Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China*
[4]*School of Sciences, Southwest University of Science and Technology, Mianyang 621010, China*
jhwu@slac.stanford.edu (J.W.)

Abstract: An *ABCD* formalism is identified to characterize a seeded Free Electron Laser (FEL) with three chirps: an initial frequency chirp in the seed Laser, an energy chirp in the electron bunch, and an intrinsic frequency chirp due to the FEL process. A scheme of generating attosecond few-cycle pulses is proposed by invoking an FEL seeded by high-order harmonic generation (HHG) from an infrared laser. The HHG seed has generic attosecond structure. It is possible to manipulate these three chirps to maintain the attosecond structure via post-undulator chirped pulse compression.

OCIS codes: (140.2600) Free electron lasers; (320.1590) Chirping; (030.1640) Coherence; (320.5520) Pulse compression; (260.2030) Dispersion; (140.3280) Laser amplifiers

---

References and links

1. T. Brabec and F. Krausz, “Intense few-cycle laser fields: Frontiers of nonlinear optics,” Rev. Mod. Phys. **72**, 545 (2000).
2. P. Emma, K. Bane, M. Cornacchia, Z. Huang, H. Schlarb, G. Stupakov, and D.Walz, “Femtosecond and subfemtosecond x-ray pulses from a Self-Amplified Spontaneous-Emissionbased free-electron Laser,” Phys. Rev. Lett. **92**, 074801 (2004).
3. A.A. Zholents and W.M. Fawley, “Proposal for intense attosecond radiation from an x-ray free-electron Laser,” Phys. Rev. Lett. **92**, 224801 (2004).
4. E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, “Self-amplified spontaneous emission FEL with energy-chirped electron beam and its application for generation of attosecond x-ray pulses,” Phys. Rev. ST Accel. Beams **9**, 050702 (2006).
5. J. Wu, P.R. Bolton, J.B. Murphy, and X. Zhong, “Free electron laser seeded by ir laser driven high-order harmonic generation,” Appl. Phys. Lett. **90**, 021109 (2007).
6. B.W.J. McNeil, J.A. Clarke, D.J. Dunning, G.J. Hirst, H.L. Owen, N.R. Thompson, B. Sheehy, and P.H. Williams, “An XUV-FEL amplifier seeded using high harmonic generation,” New J. Phys. **9**, 82 (2007).
7. J. Wu, J.B. Murphy, P.J. Emma, X.J. Wang, T. Watanabe, and X. Zhong, “Interplay of the chirps and chirped pulse compression in a high-gain seeded free-electron laser,” J. Opt. Soc. Am. B **24**, 484 (2007).
8. S.P. Dijaili, A. Dienes, and J.S. Smith, “*ABCD* Matrices for dispersive pulse propagation,” IEEE J. Quantum Electron. **26**, 1158 (1990).
9. R. Ortega-Martinez, C.J. Roman-Moreno, and A.L. Rivera, “The Wigner function in paraxial optics I. Matrix methods in Fourier optics,” Rev. Mex. de Fisica **48**, 565 (2002).
10. J.B. Murphy, J. Wu, X.J. Wang, and T. Watanabe, “Longitudinal coherence preservation and chirp evolution in a high gain Laser seeded free electron Laser amplifier,” Brookhaven National Laboratory Report BNL-75807-2006-JA, and SLAC-PUB-11852 (2006).

* The work of JW and PRB was supported by the US Department of Energy under contract DE-AC02-76SF00515. The work of JBM was supported by the US Department of Energy under contract DE-AC02-98CH10886 and the Office of Naval Research.
This work was performed in support of the LCLS project at SLAC.

Accepted by Optics Express (2007)

11. R.L. Smith, “Velocities of light,” Am. J. Phys. **38**, 978-984 (1970).
12. R. Bonifacio, C. Pellegrini, and L.M. Narducci, “Collective instabilities and high-gain regime in a free electron Laser,” Opt. Commun. **50**, 373 (1984).
13. J.B. Murphy, C. Pellegrini, and R. Bonifacio, “Collective instability of a free electron laser including space charge and harmonics,” Opt. Commun. **53**, 197 (1985).
14. J.-M. Wang and L.-H. Yu, “A transient analysis of a bunched beam free electron Laser,” Nucl. Instrum. Meth. **A 250**, 484 (1986).
15. B.H. Kolner, “Space-Time duality and the theory of temporal imaging,” IEEE J. Quantum Electron. **30**, 1951 (1994).

Intense, coherent, attosecond (few-cycle) light pulses are very important for many scientific research fields [1]. Schemes have been proposed to generate attosecond Free Electron Laser (FEL) pulses [2, 3, 4]. In this paper, we propose an FEL scheme that is seeded by high-order harmonic generation (HHG) driven by an ir laser [5, 6], in which three chirps are manipulated in the HHG seeded FEL [7]. In a seeded FEL, the seed can have a frequency chirp, and the electron bunch can have an energy chirp. Besides these two externally imposed chirps, the FEL process introduces an intrinsic frequency chirp on the FEL pulse. In this paper, we identify an *ABCD* formalism [8, 9, 10] to characterize the interplay of the chirps and the evolution of the temporal duration, the spectral bandwidth, and the chirp of the FEL pulse. We consider a seeded FEL starting from a frequency chirped Gaussian laser pulse, *i.e.*, $E_s(t,z) = E_0 e^{i(k_s z - \omega_s t)} e^{-(\alpha_s + i\beta_s)\omega_s^2 (t - z/v_{g,0})^2}$ with E_0, k_s, ω_s, α_s, β_s, and $v_{g,0} = \omega_s/(k_s + 2k_w/3)$ characterizing the peak field, the pulse wavenumber, frequency, duration, chirp, and group velocity respectively. The chirp ($2\beta_s \omega_s^2 \equiv d\omega/dt$) is independent of the pulse duration [$\sigma_{t,s} = 1/(2\omega_s\sqrt{\alpha_s})$]. The FEL electric field in the (t,z) coordinates is [7],

$$E_{\rm FEL}(t,z) = E_{0,\rm FEL} e^{\rho(\sqrt{3}+i)k_w z} e^{i(k_s z - \omega_s t)} e^{-[\alpha_{s,f}(z) + i\beta_{s,f}(z)]\omega_s^2 (t - z/v_c)^2}, \quad (1)$$

where the initial conditions are $E_{0,\rm FEL}(z=0) = E_0$, $\alpha_{s,f}(z=0) = \alpha_s$, $\beta_{s,f}(z=0) = \beta_s$, and the centrovelocity [11] $v_c(z=0) \equiv (\langle t \rangle / z)^{-1}|_{z=0} = v_{g,0}$. In Eq. (1), ρ is the Pierce parameter [12, 13], $k_w = 2\pi/\lambda_w$ with λ_w being the undulator period, $\alpha_{s,f}(z) = [4\sigma_{t,s,f}^2(z)\omega_s^2]^{-1}$, $\beta_{s,f}^2(z) = \alpha_{s,f}(z)\sigma_{\omega,s,f}^2(z)/\omega_s^2 - \alpha_{s,f}^2(z)$ with $\sigma_{t,s,f}(z)$ being the FEL pulse rms temporal duration and $\sigma_{\omega,s,f}(z)$ being the FEL pulse rms frequency bandwidth. The *ABCD* matrix can be used to transform the complex Gaussian pulse parameter, $p(z)$ [8],

$$\frac{1}{p(z)} \equiv -2\beta_{s,f}(z)\omega_s + i2\alpha_{s,f}(z)\omega_s, \quad \text{as} \quad p(z) = \frac{Ap(0)+B}{Cp(0)+D}, \quad (2)$$

to obtain the new chirp and pulse duration after the FEL interaction. For a seeded FEL, the canonical transformation is a symplectic *ABCD* matrix,

$$M_{ABCD} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -\frac{2ik_w z}{9(i+\sqrt{3})\rho\omega_s} \\ C & D \end{pmatrix}, \quad (3)$$

with

$$C = \frac{(i\mathscr{V} - \mathscr{W})\,\omega_s}{2\mathscr{U}} - \frac{\left(i\,\mathscr{V}|_{\mu=0} - \mathscr{W}|_{\mu=0}\right)\omega_s}{2\,\mathscr{U}|_{\mu=0}}, \quad \text{and} \quad D = 1 + BC, \quad (4)$$

due to symplecticity. In Eq. (4),

$$\left\{ \begin{array}{lcl} \mathscr{U} & \equiv & 3 + P^2\left[Q + \left(6 + 4R^2\right)\alpha_s + \sqrt{3}\,(2\beta_s - \mu)\right] \\ \mathscr{V} & \equiv & 3\left[Q + 4\left(1 + R^2\right)\alpha_s\right] \\ \mathscr{W} & \equiv & 4\left(\sqrt{3}R^2\alpha_s + 3\beta_s\right) + P^2\mu\left[2Q + 4\left(3 + 2R^2\right)\alpha_s - \sqrt{3}\mu\right] \end{array} \right. \quad (5)$$

where

$$\left\{ \begin{array}{lcl} P & \equiv & \frac{\omega_s}{\sigma_{\omega,\mathrm{GF}}} \\ Q & \equiv & \frac{\mu(\mu-4\beta_s)\omega_s^2}{\sigma_{\omega,\mathrm{GF}}^2} \\ R & \equiv & \frac{\sigma_{\omega,s}}{\sigma_{\omega,\mathrm{GF}}} \end{array} \right. , \quad \text{with} \quad \mu \equiv \frac{2}{\gamma_0 \omega_s}\frac{d\gamma}{dt}, \quad \text{and} \quad \sigma_{\omega,\mathrm{GF}}(z) \equiv \sqrt{\frac{3\sqrt{3}\rho\omega_s^2}{k_w z}}, \tag{6}$$

being the rms bandwidth of the FEL Green function for a coasting electron beam without energy chirp [14], and μ characterize the energy chirp in the electron bunch. Notice that, $P > 0$ and $R > 0$, but Q can be negative or positive. For $\mu = 0$, we have $C = 0$ and $D = 1$ [10]. In this case, the form of the *ABCD* matrix ($A = D = 1, C = 0$) and B complex is characteristic of a system with group velocity dispersion (ReB) and gain (ImB). To illustrate the concept, imaging that a light pulse is represented by an ellipse in the t-ω space with t as the horizontal-axis and ω the vertical-axis. The group velocity dispersion is responsible for the "horizontal shearing" of the phase space ellipse. Since for $\mu = 0$, we have $C = 0$; there is no inherent time lensing or temporal chirping effect ("vertical shearing") in the FEL [15] without electron bunch energy chirp. An input pulse changes its overall chirp due to the group velocity dispersion and bandwidth reduction. It is possible to start with a negatively chirped seed and exit the FEL with no chirp. Contrarily, the energy chirp in the electron bunch provides a temporal chirping effect ("vertical shearing"). This leads to the complicated interplay of the three chirps and optimization of chirped pulse compression [7]. Indeed, the energy chirp in the electron bunch is the key to maintain a large bandwidth in the FEL pulse, so that an effective chirped pulse compression after the undulator is possible to generate an attosecond pulse train. The *ABCD* analysis is not limited to a simple chirped Gaussian seed pulse. Arbitrary seed pulses can be constructed from a complete set of chirped Hermite-Gaussian functions characterized by a single p-parameter and propagated through the system with the *ABCD* matrix formalism [8]. Indeed, this *ABCD* matrix in Eq. (3) is identified after the follow derivation by introducing the Wigner function; however, in order to highlight the physics first, we present it before the derivation.

To explore the importance of the energy chirp in the electron bunch, recall that the the *ABCD* matrix is applied to the following coordinate transformation

$$\begin{pmatrix} \tau \\ \frac{d\tau}{d\zeta} \end{pmatrix}_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{1\to 2} \begin{pmatrix} \tau \\ \frac{d\tau}{d\zeta} \end{pmatrix}_1, \text{where} \left\{ \begin{array}{lcl} \tau & = & t - z\left.\frac{dk}{d\omega}\right|_{\omega=\omega_s} = t - \frac{z}{v_{g,0}} \\ \zeta & = & \omega_s z \left.\frac{d^2k}{d\omega^2}\right|_{\omega=\omega_s} \end{array} \right. , \tag{7}$$

with the group velocity of the FEL light being $v_{g,0}^{-1} \equiv dk/d\omega|_{\omega=\omega_s} = (k_s + 2k_w/3)/\omega_s$ with or without energy chirp in the electron bunch. The vector $(\tau, d\tau/d\zeta)^T$ describes a time ray with τ standing for the delayed time and ζ the normalized propagation distance. Its position τ represents time deviation from a reference time, whereas its slope ($d\tau/d\zeta$) represents frequency sweep. A nonzero B in the system stands for a horizontal shearing along the t-axis, while a nonzero C stands for a vertical shearing along the ω-axis. It is now clear how important a nonzero C is. It is only with increment in the frequency bandwidth via a nonzero C, can a net temporal pulse compression be possible. For a seeded FEL with three chirps, the *ABCD* matrix is given in Eq. (3), we find that the only possibility to have a nonzero C element is to have a nonzero μ, *i.e.*, there has to be an energy chirp in the electron bunch. Indeed, a seeded FEL is represented by an integral Eq. [7]

$$A(\theta, Z) \cong e^{\rho(\sqrt{3}+i)Z} \int_0^\infty d\xi A(\theta - \xi, 0) e^{i\mu\theta(Z-\xi)} e^{-\rho(\sqrt{3}+i)\left[9(\xi - Z/3)^2/(4Z)\right]} e^{-i(\mu/2)(Z-\xi)\xi}, \tag{8}$$

where the slow varying envelope function $A(\theta, Z)$ is introduced by $E(t,z) \equiv A(\theta, Z)e^{i(\theta - Z)}$ with dimensionless variables as $Z = k_w z$ and $\theta = (k_s + k_w)z - \omega_s t$. This integral Eq. (8) which propagates an input seed $A(\theta,0)$ through the FEL via the Green function is of the general form of an integral representation of an *ABCD* canonical transformation, as such the phase space area and longitudinal coherence is preserved [9, 10]. Readers may refer to Ref. [7] for details.

We have emphasized the importance of an energy chirp in the electron bunch, and highlighted the physics of a seeded FEL in a compact *ABCD* formalism. We now study a HHG seeded FEL. We demonstrate that with properly introducing an energy chirp in the electron bunch, a few-cycle attosecond FEL pulse train can be retained by post-undulator chirped pulse compression. We will also provide the derivations which leads to this *ABCD* matrix in Eq. (3).

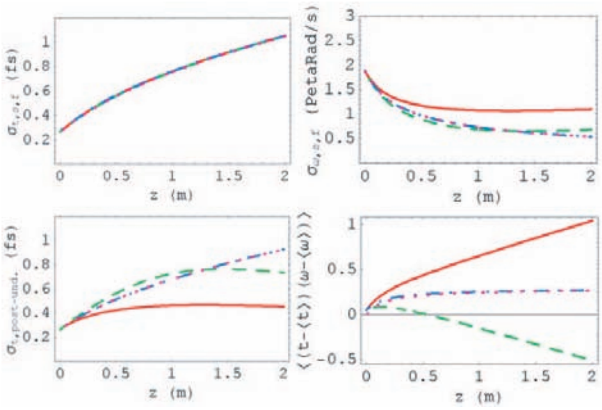

Fig. 1. The FEL pulse rms duration (upper left), the rms bandwidth (upper right), the rms duration after post-undulator compression (lower left), and the time-frequency correlation (lower right) as a function of the location into the undulator. The solid (red) curve is for $\mu = 2\beta_s$, the dashed (green) for $\mu = -2\beta_s$, and the dash-dotted (blue) for $\mu = 0$. For all these three cases, $\beta_s \approx 8.7\times10^{-5}$. The dotted (purple) curve is for $\mu = \beta_s = 0$.

We study an HHG seed with central frequency (carrier) being a single harmonic order s [5]

$$E_s(t,z) = E_{s,0}e^{i(k_s z-\omega_s t)}e^{-i\beta_s\omega_s^2 t^2}\sum_{n=-N}^{N} e^{-\frac{t_n^2}{4\sigma_{t,0}^2}}e^{-\alpha_s\omega_s^2[(t-t_n)-z/c]^2}. \tag{9}$$

where $E_{s,0}$ is the peak amplitude of the electric field. Even though, the HHG electric field consists of multiple orders, s with wavenumber, k_s and angular frequency, ω_s, and multiple pulselets, n as a double summation, only the resonant harmonic is relevant, since FEL is a narrow bandwidth filter [5]. The temporal structure is a sequence of $2N+1$ ultrashort pulselets with attosecond structure which is referred as the attosecond pulse train (APT). Pulselet peaks occur at times, $t_n = n\tau/2$ where τ is the ir laser period. The duration of a single attosecond pulselet (SAP) can be less than one femtosecond. The temporal envelope of the entire HHG pulse has an rms duration $\sigma_{t,0}$. We define the much shorter rms duration, $\sigma_{t,s}$ for the single attosecond pulselet with $\alpha_s\omega_s^2 \equiv 1/(4\sigma_{t,s}^2)$; and β_s characterizes the single harmonic seed chirp. The APT in Eq. (9) is simply the amplitude modulation of a single carrier frequency, ω_s. We use $\sigma_{t,0} = 10$ fsec, $\sigma_{t,s} = \tau/10 \approx 267$ attoseconds in the Fourier transform limit, and the ir laser wavelength $\lambda_{ir} = 800$ nm in this paper. According to the study in Ref. [5], the attosecond structure will be temporally smeared out quickly. However, if the energy chirp in the electron bunch is large enough, the pulselets will have a large chirp and a post-undulator chirped pulse compression will restore the attosecond few-cycle pulselets. For simplicity, we provide derivation for a single pulselet: the middle pulselet, *i.e.*, for $n=0$, so that $t_n = 0$. For this single pulselet, the electric field in the (t,z) coordinates is given in Eq. (1). To compute the second moments, especially,

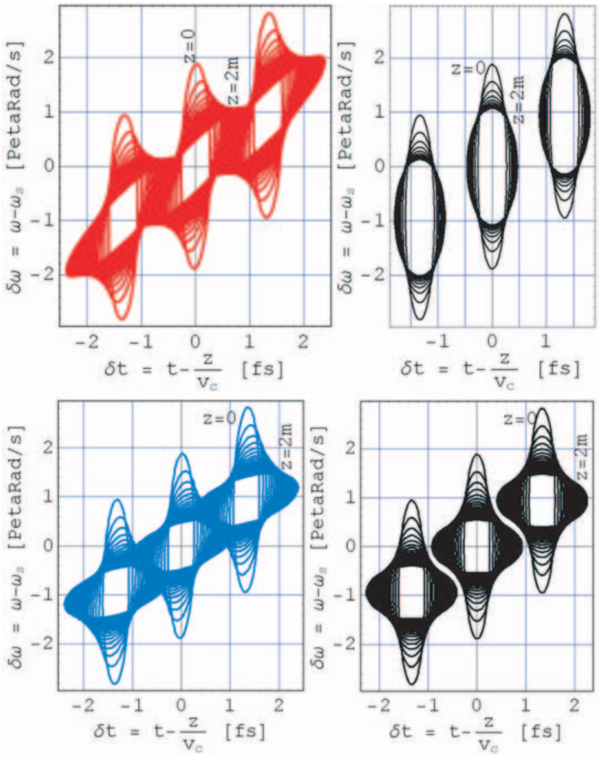

Fig. 2. The evolution of the Wigner function ellipse as a function of the location into the undulator is shown in the left subplots. For clarity, only three pulselets are shown. In the right subplots, each ellipse stands for experiencing a postundulator compression. In the upper row, the energy chirp in the electron bunch is $\mu = 2\beta_s$. In the lower row, $\mu = 0$. In all the plots, $\beta_s \approx 8.7\times10^{-5}$.

the chirp, we introduce a Wigner function [7],

$$W(t,\omega,z) = \int E_{\mathrm{FEL}}(t-\tau/2,z)E^*_{\mathrm{FEL}}(t+\tau/2,z)e^{-i\omega\tau}d\tau = 2\sqrt{2\pi}\sigma_{t,s,f}\,e^{-\frac{\frac{\delta t^2}{\sigma^2_{t,s,f}}-2r\frac{\delta t\delta\omega}{\sigma_{t,s,f}\sigma_{\omega,s,f}}+\frac{\delta\omega^2}{\sigma^2_{\omega,s,f}}}{2(1-r^2)}}, \tag{10}$$

where $\delta t \equiv t - z/v_c$, $\delta\omega \equiv \omega - \omega_s$, and

$$r \equiv 2\beta_{s,f}\frac{\sigma_{t,s,f}\omega_s^2}{\sigma_{\omega,s,f}} = \frac{\beta_{s,f}}{2\alpha_{s,f}}\frac{1}{\sigma_{t,s,f}\sigma_{\omega,s,f}} = \frac{\langle(t-\langle t\rangle)(\omega-\langle\omega\rangle)\rangle}{\sigma_{t,s,f}\sigma_{\omega,s,f}}. \tag{11}$$

With the Wigner function in Eq. (10), we can compute the second moments to have

$$\begin{cases} \sigma^2_{t,s,f}(z) & = \frac{\mathscr{U}}{\omega_s^2\mathscr{V}} \\ \sigma^2_{\omega,s,f}(z) & = \frac{\omega_s^2(\mathscr{V}^2+\mathscr{W}^2)}{4\mathscr{U}\mathscr{V}} \\ \langle(t-\langle t\rangle)(\omega-\langle\omega\rangle)\rangle \equiv \frac{\beta_{s,f}(z)}{2\alpha_{s,f}(z)} & = \frac{\mathscr{W}}{2\mathscr{V}} \end{cases}. \tag{12}$$

With the second moments given in Eq. (12), the *ABCD* matrix in Eq. (3) can be verified. The centrovelocity v_c [11] is $v_c^{-1}(z) \equiv \langle t\rangle/z = v_{g,0}^{-1} + \sqrt{3}\mu\omega_s k_w(2\alpha_s - 2\sqrt{3}\beta_s + \sqrt{3}\mu)/(2\mathscr{V}\sigma^2_{\omega,\mathrm{GF}})$. Equation (29) of Ref. [7] has a formatting typo in generating the final production version.

We now consider how the chirp can affect the system. We take parameters of those in Ref. [5]: $\omega_s = 2\pi c/\lambda_s$ with λ_s as the 27^{th} harmonic of the ir laser, $\rho = 6.3\times 10^{-3}$, undulator period $\lambda_w =$ 3 cm, and $z \in [0,2]$ m (the saturation length $L_{sat} \approx 4$ m). Following the resonance condition, initially the seed and the electron bunch are matched, *i.e.*,

$$\frac{\gamma(t)^2}{\gamma_0^2} = \frac{\omega(t)}{\omega_s} \Longrightarrow \frac{d\gamma}{dt} \approx \frac{\gamma_0}{2\omega_s}\frac{d\omega}{dt} = \gamma_0\beta_s\omega_s. \tag{13}$$

According to Eq. (6), $\mu \approx 2\beta_s$. It is easy to check that with this set of parameters, we have $\sigma_{t,s,f} \to P/(\sqrt{3}\omega_s) = 1/(\sqrt{3}\sigma_{\omega,\text{GF}})$, $\sigma_{\omega,s,f} \to \omega_s P|\mu|/\sqrt{3} = \omega_s^2|\mu|/(\sqrt{3}\sigma_{\omega,\text{GF}})$, and $\langle(t-\langle t\rangle)(\omega-\langle\omega\rangle)\rangle \to P^2\mu/3 = \omega_s^2\mu/(3\sigma_{\omega,\text{GF}}^2)$ for $|\mu| \ll 1$ which is usually satisfied. With this set of parameters, it is difficult to manipulate the FEL pulse duration via chirps during the FEL process; however, the residual frequency chirp is apparently inherited from the initial energy chirp in the electron bunch. Hence, post-undulator compression will be possible to generate FEL pulse with attosecond duration. In the following, let us choose, $\omega(t = \sigma_{t,s}) = \omega_s + 0.1\sigma_{\omega,s} \Longrightarrow \beta_s \approx 8.7\times 10^{-5}$. This amount of chirp for the seed of rms duration of $\sigma_{t,0} = 10$ fs translates into about 3.3 nm rms bandwidth on the 27^{th} harmonic (*i.e.*, 30 nm) of the ir laser. For the energy chirp, we take $|\mu| = 2\beta_s \approx 1.7\times 10^{-4} \ll 1$, which translates to $|\Delta_\delta|_{t=\sigma_{t,s}} \equiv |\Delta\gamma/\gamma_0|_{t=\sigma_{t,s}} \approx 1.5\times 10^{-3}$. For this parameter set, we show the FEL pulse rms duration $\sigma_{t,s,f}$ as a function of location z into the undulator in the upper left subplot of Fig. 1. The solid (red) curve is for $\mu = 2\beta_s$, the dashed (green) curve is for $\mu = -2\beta_s$, and the dash-dotted (blue) curve is for $\mu = 0$. For all these three cases, $\beta_s \approx 8.7\times 10^{-5}$. The dotted (purple) curve is for $\mu = \beta_s = 0$. However, for this set of parameters, there is very little difference for $\sigma_{t,s,f}$, as we expect from the above mentioned limiting case of $|\mu| \ll 1$, *i.e.*, $\sigma_{t,s,f} \to 1/(\sqrt{3}\sigma_{\omega,\text{GF}})$. The frequency rms bandwidth $\sigma_{\omega,s,f}$ is shown as the upper right subplot in Fig. 1, and the cross moment $\langle(t-\langle t\rangle)(\omega-\langle\omega\rangle)\rangle$ is shown as the lower right subplot in Fig. 1. With the energy chirp in the electron bunch, the final FEL pulse inherits a frequency chirp, which is crucial for an effective post-undulator chirped pulse compression. Shown as the lower left subplot in Fig. 1 is the FEL pulse rms duration after post-undulator chirped pulse compression as a function of the undulator length. Such a post-undulator compression is a final "horizontal shearing", which results in a pulselet rms duration below the femtosecond. To illustrate this explicitly, we show as the upper row in Fig. 2 for the case of three pulselets in an HHG seed with the frequency chirp $\beta_s \approx 8.7\times 10^{-5}$ and the energy chirp to be $\mu = 2\beta_s$. Even though, the attosecond pulselets are temporally smeared out [5] in the undulator as shown in the left subplot; the inherited large frequency chirp enables the post-undulator compression to regenerate attosecond few-cycle pulselets as shown in the right subplot. In contrast to this, shown as the lower row in Fig. 2 are three puslelets in the HHG seed with the same chirp $\beta_s \approx 8.7\times 10^{-5}$, but there is no energy chirp in the electron bunch; the post-undulator chirped pulse comprssion is not adequate to restore an attosecond pulse train. Indeed, μ can either be negative or positive so that the FEL frequency chirp exiting the undulator can either be negative or positive.

In this paper, we study the possibility of generating an attosecond pulse train by manipulating the three chirps in a seeded FEL. An useful *ABCD* formalism is identified. This compact formalism illustrates clearly the importance of an energy chirp in the electron bunch. With a proper energy chirp in the electron bunch, it is possible to keep large frequency bandwidth, so that a post-undulator pulse manipulation can restore an attosecond pulse train inherited from the HHG seed. The work of JW and PRB was supported by the US Department of Energy under contract DE-AC02-76SF00515. The work of JBM was supported by the US Department of Energy under contract DE-AC02-98CH10886 and the Office of Naval Research.