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Collider Signals of Gravitational Fixed Points * †

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Abstract

Recent studies have shown that the poor perturbative behavior of General Relativity in the ultraviolet regime may be ameliorated by the existence of a non-Gaussian fixed point which renders the theory asymptotically safe and possibly non-perturbatively renormalizable. This results in a running of the (effective) gravitational coupling such that gravity becomes weaker at high energies. We parameterize this effective coupling with a form factor and study its consequences at the LHC and ILC in models with large extra dimensions or warped extra dimensions. We find significant effects in the processes of Kaluza-Klein (KK) graviton exchange or resonant KK graviton production in both the Drell-Yan reaction as well as in $e^+e^- \to f\bar{f}$. On the otherhand, processes leading to KK graviton emission show qualitatively less sensitivity to the presence of a form factor. In addition, we examine tree-level perturbative unitarity in $2 \to 2$ gravity-mediated scattering and find that this form factor produces a far better behaved amplitude at large center of mass energies.

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1 Introduction

Over the past 20 years we have come to learn that all strong and electroweak phenomena below the scale of a few hundred GeV can be well described [1] with high precision by renormalizable Yang-Mills gauge theories as encoded in the Standard Model (SM) Lagrangian. Of course within this realm some issues remain to be addressed, such as the origin of electroweak symmetry breaking (which we hope to resolve from data provided by the LHC) and the origin of the fermion family structure. Similarly, the description of gravity via Einstein's General Relativity (GR), as encoded in the Einstein-Hilbert (EH) action, has been proven remarkably successful over a wide range of scales from the sub-millimeter [2], to interplanetary [3], and even cosmological distances [4]. Here, too, some issues remain to be addressed such as the nature of the (apparent) dark energy/cosmological constant. The next broad question to answer is how to unify our description of gravity with those of the other three forces, *i.e.*, how do we construct a quantum theory of gravity. This problem is long-standing and has been the subject of much labor over the last half-century [5], and our perspective on possible ways to resolve this problem have evolved significantly over time.

The essential issue with constructing quantum gravity in the standard approach is that the EH action leads to a quantum field theory which is not perturbatively renormalizable, unlike the case of Yang-Mills theories. This can be most easily seen by examining the interaction of gravitons with matter (or their self-interactions) in any fixed metric background. Distinct from the case of Yang-Mills theories, where the interactions of gauge fields with matter or each other correspond to dimension-4 operators (in 4-dimensional spacetime), the gravitational interactions correspond to operators of dimension-5 (or higher) leading to non-renormalizability by simple power counting. This implies that the theory is not well-behaved in the ultraviolet (UV). In particular, this approach implies that (4-d) gravity becomes strong near the reduced Planck scale, $\overline{M}_{Pl}$, so that we usually treat GR as an effective theory at energies far below that scale. As we have been recently reminded [6], Weinberg [7] long ago pointed out that there are only four known approaches to dealing with this issue, and they have not evolved qualitatively since that time:

- The EH action in 4 dimensions is incomplete; new physics must be added that somehow tames the poor perturbative UV behavior of GR. This is the path followed by String Theory (where extra dimensions, supersymmetry, new fields and a new scale enter) [8] as well as in Loop Quantum Gravity (which also introduces a new scale) [9]. This approach has received the most attention in recent years and has met with a number of successes.

- Gravitons are not fundamental objects but are composite; this possibility has been addressed by a number of authors [10].

- The poor UV behavior of GR in 4-d can be controlled by a re-ordering of the conventional perturbation expansion employing a modified version of the Yennie, Frautschi and Suura [11] resummation techniques. This approach has recently been advocated by Ward [6].

- The poor UV behavior of GR is an artifact of perturbation theory. General Relativity is non-perturbatively renormalizable having the property of being asymptotically safe due to the existence of a non-Gaussian fixed point; such an approach has also met with a number of recent successes [12]. The existence of such a fixed point has been demonstrated in both field theoretical and in lattice studies [13] in 4-d as well as in higher dimensions [14]. In such an approach, the running gravitational coupling becomes weaker as the fundamental gravity scale is reached.

A common feature of the last two approaches is that the strength of the gravitational interaction, usually expressed through Newton's constant, G_N, runs in such a way that the effective coupling actually becomes *weaker* in the energy regime near $\overline{M}_{Pl}$. This would imply good high energy behavior and, perhaps, the restoration of unitarity in graviton scattering amplitudes. Could such ideas be tested in, *e.g.*, collider experiments? Clearly, to do so collision energies must approach the fundamental scale of gravity, which is unattainable 4-dimensional gravity. However, the property of asymptotic safety has been demonstrated to persist in higher dimensions [14] where we know that we can construct scenarios where the (true) fundamental scale of gravity is of order $\sim$ TeV, such as in the models of Arkani-Hamed, Dimopoulos and Dvali(ADD) [15] with large extra dimensions and of Randall and Sundrum(RS) [16] with warped geometries. In this paper, we will demonstrate that if either the ADD or RS models are realized they, will provide a framework for testing the hypothesis of asymptotic safety at the LHC and ILC through the appearance of gravitational form-factors which will damp the strength of gravity at high energies. As we will see below, such form-factors will lead to significant modifications in the traditional predictions for both of these scenarios. These deviations from the standard predictions can then be used to test the nature of the form factor and to determine if a consistent theory along such lines can be successfully constructed.

The outline of this paper is as follows: In Section II, we will provide the essential background formalism for applying the approach of asymptotic safety to both the ADD and RS extra-dimensional models. In Sections III and IV we will examine how the traditional signals of the ADD and RS models, respectively, at future colliders are modified and the parameter space range over which they may be observed. A discussion and a summary of our results can be found in Section V.

2 Background Formalism

Here we present the formalism that is relevant to our analysis. A general review of non-perturbative renormalizability and asymptotic safety is clearly beyond the scope of the present work, but can be found in the introductory survey by Niedermaier [17]. We work in Euclidean space within the context of an effective average action that consists of a truncated list of operators which are part of a more generalized gravitational action. The simplest choice for this fixed set of operators is the Einstein-Hilbert truncation. This action is simply given by the familiar expression in D-dimensions,

$$S_{EH} = \int d^D x \sqrt{-g}\left[\frac{M^{D-2}}{2}\, R - \Lambda\right], \tag{1}$$

where M is the D-dimensional Planck scale, R is the Ricci scalar, Λ corresponds to the cosmological constant, and $D = \delta + 3 + 1$ where δ is the number of additional spatial dimensions.[‡] Newton's constant in D-dimensions can then be defined in analogy with 4-dimensional expression as $G_D = 1/(8\pi M^{D-2})$. For convenience, we define a corresponding dimensionless quantity,

$$g(\mu) = \mu^{D-2} G_D\,, \tag{2}$$

where μ is an arbitrary mass scale. Our goal is to obtain the renormalization group equations (RGEs) for this dimensionless gravitational coupling and we are particularly interested in the RGE behavior of g in the ultraviolet (UV). It has been found that the qualitative nature of this behavior is not very sensitive to the truncation in the gravitational theory employed above where only the EH term appears in the action [12].

Following Bonanno and Reuter [18], the relevant RGE corresponding to the action

[‡]For consistency with the structure of the ADD model, we drop the cosmological constant term from the above action in our ensuing discussion.

above is found to be

$$\frac{dg}{dt} = [D-2+\eta]g\,, \tag{3}$$

where $t = \log(\mu)$ and η is the non-perturbative anomalous dimension of the EH operator given by

$$\eta = \frac{gB_1}{1-gB_2}\,. \tag{4}$$

Here, the constants $B_{1,2}$ can be expressed in terms of a set of integrals which effectively result from loop summations:

$$\begin{aligned} B_1 &= \frac{1}{3}\,(4\pi)^{1-D/2}\Big[D(D-3)\phi_1 - [6D(D-1)+24]\phi_2\Big] \\ B_2 &= -\frac{1}{6}\,(4\pi)^{1-D/2}\Big[D(D+1)\tilde{\phi}_1 - 6D(D-1)\tilde{\phi}_2\Big]\,, \end{aligned} \tag{5}$$

where the integrals are explicitly given by

$$\begin{aligned} \phi_1 &= \frac{1}{\Gamma(D/2-1)}\int_0^\infty dz\ z^{D/2-2}\ \frac{Q-zQ'}{z+Q} \\ \phi_2 &= \frac{1}{\Gamma(D/2)}\int_0^\infty dz\ z^{D/2-1}\ \frac{Q-zQ'}{(z+Q)^2} \\ \tilde{\phi}_1 &= \frac{1}{\Gamma(D/2-1)}\int_0^\infty dz\ z^{D/2-2}\ \frac{Q}{z+Q} \\ \phi_1 &= \frac{1}{\Gamma(D/2)}\int_0^\infty dz\ z^{D/2-1}\ \frac{Q}{(z+Q)^2}\,. \end{aligned} \tag{6}$$

Here, Q is an essentially arbitrary smooth cutoff function with the properties $Q(0) = 1$ and $Q(z) \to 0$ as $z \to \infty$, and the derivative is defined by $Q' = dQ/dz$. We take $Q = z/(e^z - 1)$ in explicit computations and define the parameters $\omega = -B_1/2$ and $\omega' = \omega + B_2$ as suggested in [18]. The RGE is now seen to exhibit two fixed points where $\beta = D-2+\eta = 0$: (i) an attractive infrared (IR) Gaussian (or perturbative) fixed point at $g^{IR} = 0$ and (ii) an UV attractive non-Gaussian fixed point where $g^{UV} = 1/\omega'$.

The RGE differential equation above can be solved analytically. Taking $\mu = \mu_0$ as a boundary condition and defining $g_0 \equiv g(\mu_0)$ we obtain

$$\frac{g}{(1-\omega' g)^{\omega/\omega'}} = \frac{g_0}{[1-\omega' g_0]^{\omega/\omega'}} \left(\frac{\mu}{\mu_0}\right)^{D-2}. \tag{7}$$

This solution, in itself, cannot be solved analytically for $g(\mu)$ in closed form. However, a numerical analysis shows that $\omega \simeq \omega'$ to order $\sim 10\%$; this allows for an approximate analytical solution to be obtained which is given by

$$g(\mu) \simeq \frac{(\mu/\mu_0)^{D-2} g_0}{[1+\omega(\mu/\mu_0)^{D-2} g_0 - 1]}. \tag{8}$$

Rewriting this result in terms of the D-dimensional Planck scale and taking the limit $\mu_0 \to 0$ yields the effect of this RGE evolution on the gravitational coupling in the ADD model. This leads to a mapping into an effective D-dimensional Planck scale of

$$\frac{1}{M^{D-2}} \to \frac{1}{M_{eff}^{D-2}} = \frac{1}{M^{D-2}} \Big[1 + \frac{\omega}{8\pi} (\frac{\mu^2}{M^2})^{D/2-1}\Big]^{-1}, \tag{9}$$

which is then used in the D-dimensional coupling of the graviton field H^{AB} to the matter stress-energy tensor, T_{AB}, *i.e.*,

$$\mathcal{L} = -T_{AB} H^{AB}/M_{eff}^{D-2}. \tag{10}$$

We can instead write this rescaling relation as

$$\frac{1}{M^{D-2}} \to \frac{1}{M^{D-2}} F(\mu^2), \tag{11}$$

where F can be treated as a form factor now appearing in the effective coupling which we can rewrite as

$$F = \left[1 + \left(\frac{\mu^2}{t^2 M^2}\right)^{1+\delta/2}\right]^{-1}, \tag{12}$$

where $\delta = D - 4$ is the number of additional dimensions. Numerically, we find that the parameter t (which is trivially related to ω) is quite close to unity assuming that $5 \leq D \leq 11$ as is true for all the cases of interest to us here. In our analysis below we will treat t as an $\mathcal{O}(1)$ free parameter to allow for uncertainties in the calculation above which arise from, *e.g.*, the truncation of the EH action, our specific choice for the function Q, and the small violation of the $\omega \simeq \omega'$ relation.

Note that this form factor ensures that the gravitational coupling is unaffected at low energies and retains the value of the fundamental Planck scale, but then runs with increasing energy. The derivation of this form factor has not relied on the background metric and hence it can be equally well applied to the case of warped geometries as well as flat dimensions, provided that, for simplicity, the possibility of a running cosmological constant is neglected.

In order to quantify the effect of this form factor in the collider signatures of either the ADD or RS models, we need to relate the quantity μ to physical parameters in the production process; this issue is similar to that of the apparent scale ambiguity in QCD in computations performed at finite order in perturbation theory. In reactions mediated by s-channel kinematics, which are typical of graviton exchange or resonant production processes in the ADD or RS models, it is natural to take $\mu^2 = s$ so that in such cases the form factor becomes

$$F = \left[1 + \left(\frac{\sqrt{s}}{tM}\right)^{\delta+2}\right]^{-1}. \tag{13}$$

Thus in the graviton exchange process in the ADD model, which is described by a cutoff parameter Λ_H [19, 20], the corresponding result for the cross section including the form factor can be obtained by making the replacement

$$\Lambda_H^4 \to \Lambda_H^4 \left[1 + \left(\frac{\sqrt{s}}{tM}\right)^{\delta+2}\right] . \tag{14}$$

Note that this renders the predictions for graviton exchange explicitly dependent on δ, which does not occur within this formalism in the traditional ADD scenario. Since $\Lambda_H \simeq M$ in ADD, we can also make the substitution $tM \to t'\Lambda_H$, where t' is another $O(1)$ parameter. (In our analyses below we will not make the distinction between t and t'.) Note that for fixed Λ_H, the effect of the form factor increases as t (or t') take on smaller values. In other processes, such as graviton emission in ADD, there is more ambiguity in the identification of the scale μ. Here, one may choose from several different kinematic quantities such as the emitted graviton's energy or p_T. This ambiguity will affect the numerical values of the cross sections in detail and we will thus present results for several different choices for this scale. However, as we will see below, our qualitative results are insensitive to this uncertainty.

We now examine the effects of this form factor in the collider signatures for the ADD and RS models and determine whether the resulting suppression in the strength of the gravitational coupling is observable in these scenarios.

3 Large Extra Dimensions

In this scenario [15], the Standard Model fields are confined to a 3-brane in the higher dimensional bulk and gravity alone progrates in the additional dimensions. The 4-dimensional Planck scale is related to the fundamental scale M via

$$\overline{M}_{Pl}^2 = V_\delta M^{2+\delta} , \tag{15}$$

where V_δ is the volume of the extra dimensional space. Taking $M \sim$ TeV eliminates the hierarchy between $\overline{M}_{Pl}$ and the electroweak scale. A vast number of studies have been performed investigating the consequences of this framework [19]. Here, we explore the modifications in the high energy collider signals that are introduced by the presence of a running gravitational coupling as represented by the form factor in Eq. (12).

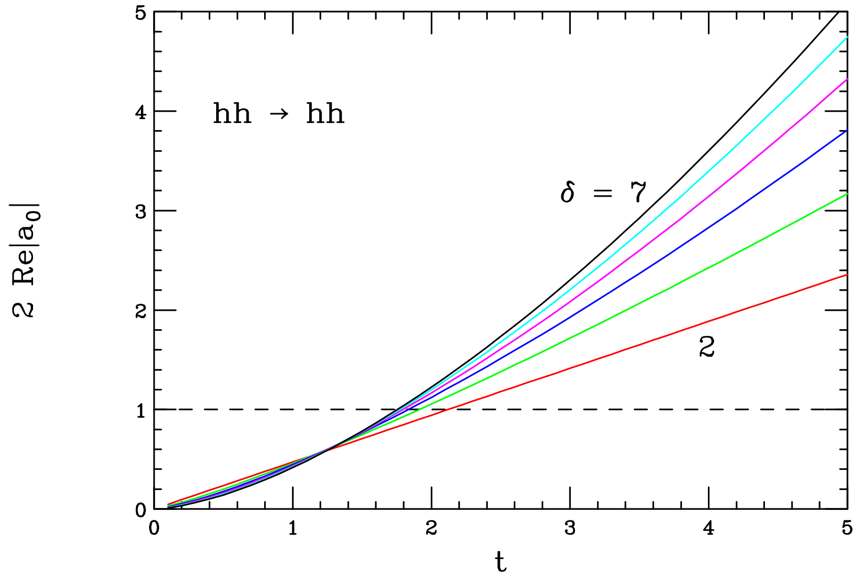
We first examine the effects of the form factor in unitarity considerations in high energy $2 \to 2$ scattering. In this model, the Kaluza-Klein (KK) tower of gravitons contribute to such processes via virtual multi-channel exchanges. The operator for this transition takes the generic form [20, 21]

$$\mathcal{L} \simeq i\frac{4}{\Lambda_H^4} T^{\mu\nu} T_{\mu\nu} + h.c. \,, \tag{16}$$

where the scale Λ_H represents a naive cutoff that regulates the integral over the sum of KK graviton propagators weighted by the density of KK states. Λ_H is of order the fundamental scale M, with the exact relationship between the two being governed by the full UV theory. At high energies, the $2 \to 2$ s-channel graviton exchange amplitude grows as, *e.g.*, s^2/Λ_H^4, and thus exhibits extremely poor behavior in the UV limit, violating perturbative unitarity when $\sqrt{s} \gtrsim \Lambda_H$. Here, we explore whether the μ/tM term in the form factor governing the running gravitational coupling can regulate this amplitude at high energies for some values of the parameter t. We first examine the case of $2 \to 2$ Higgs boson scattering, *i.e.*, $hh \to hh$, as that is claimed [22] to be the most sensitive process to the UV behavior of this theory. Including the form factor as discussed in the previous section, setting $\mu^2 = s$, and computing the tree-level $J = 0$ partial wave amplitude a_0 for this scattering process, we obtain the results displayed in Fig. 1. Here, we show the maximum value that $2Re|a_0|$ obtains as the ratio $\sqrt{s}/\Lambda_H$ is varied, while holding t and δ, the number of extra dimensions, fixed. A good test of unitarity in $2 \to 2$ scattering is that the zeroth partial wave amplitude be bounded by $|a_0| < 1/2$ for all values of s. We see that for all values of δ this condition is satisfied once the form factor is included, provided the parameter $t \lesssim 2$. As a second example, we also examine the process $e^+e^- \to f\bar{f}$, following the same procedure. The results for the maximum value of the zeroth order partial wave amplitude for this process are shown in the bottom panel of Fig. 1. We see that in this case, the bounds from perturbative unitarity are somewhat weaker, requiring only that $t \lesssim 8$ or so. Note that the values of t which allow for good UV behavior of this theory agree with the more theoretical expectations discussed in the previous section, which predicted t to be of order unity. Following these guidelines, we will take $t \leq 2$ in our ensuing calculations.

We now turn to the conventional collider signatures of this scenario. The first class of processes that we consider is the contribution of virtual KK graviton exchange in Drell-Yan production, $pp \to \ell^+\ell^- + X$, at the LHC. This contribution proceeds through the operator given in Eq. 16, and involves $q\bar{q}$ and gg initial parton states. The same cutoff scale Λ_H is employed and when including the form factor parameterizing running gravitational couplings, the scale μ in the form factor is set to $\sqrt{\hat{s}}$ as discussed above. The unmodified graviton exchange amplitude behaves as $\sim \hat{s}^2/\Lambda_H^4$ and we expect the form factor to modify the production cross section at high energies. Figure 2 shows the number of events for 100 fb^{-1} of integrated luminosity as a function of the invariant mass of the final state lepton pair, with and without the form factor, taking $t = 1$ and $\delta = 3$. We see that the form factor has a dramatic effect on the production cross section and results in a sharp reduction of the event rate at high invariant masses.

We examine these effects in more detail in Fig. 3. Here, we take $\Lambda_H = 2.5$ and 4.0 TeV and separately study the variation in the Drell-Yan production cross section due to t and δ. In this computation, the size of the statistical errors in the event rate are shown by the size of the fluctuations in the binned rate. Holding the parameter t fixed at $t = 1$ and varying the number of extra dimensions, we see that the effect of the form factor is more pronounced and separated for the various values of δ at high invariant masses. In



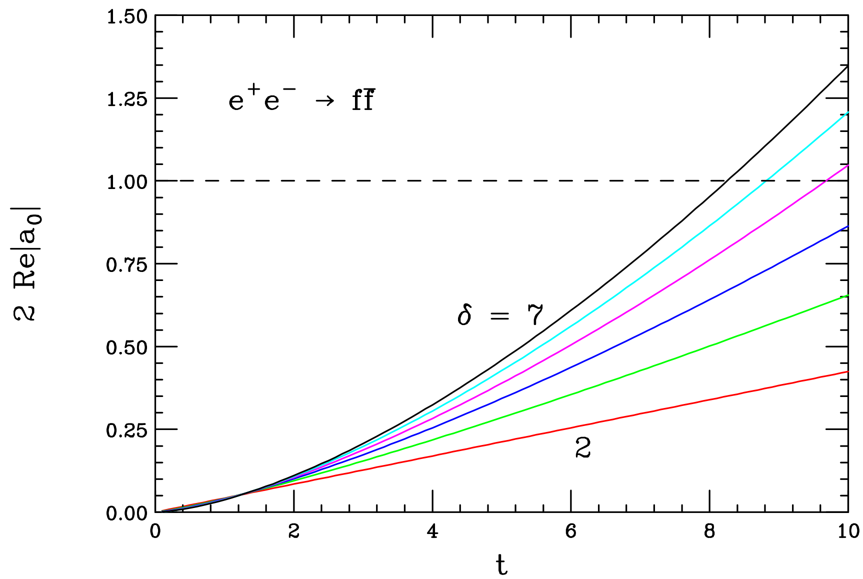


Figure 1: The maximum value of the $J = 0$ partial wave scattering amplitude as $\sqrt{s}/\Lambda_H$ is varied for (top panel) $hh \to hh$ and (bottom panel) $e^+e^- \to f\bar{f}$ as a function of t. The curves correspond to $\delta = 2, 3, 4, 5, 6, 7$ from bottom to top on the right-hand side.

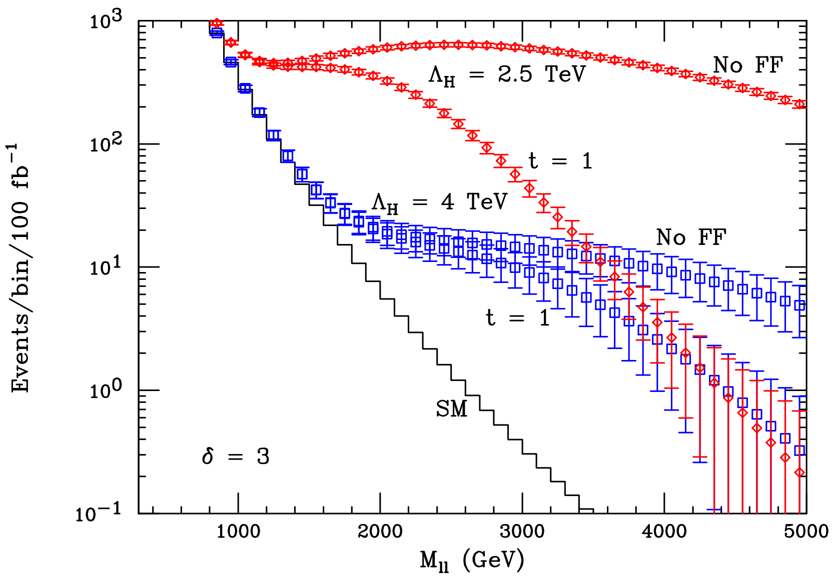


Figure 2: The event rate per bin with 100 fb^{-1} of integrated luminosity for Drell-Yan production at the LHC as a function of the lepton pair invariant mass with $\delta = 3$. The red (blue) curves correspond to scale $\Lambda_H = 2.5$ (4.0) TeV. In each case, the top curve is the result in the ADD model without the form factor [20], and the bottom curve includes the form factor with $t = 1$. The error bars represent the statistical errors. The black histogram corresponds to the Standard Model event rate.

the formalism employed here, the cross section for graviton exchange is insensitive to the number of extra dimensions. However, note that here, due to the form factor, the production rate is larger with increasing (decreasing) values of δ when the lepton pair invariant mass is less than (greater than) $t\Lambda_H$. Holding δ fixed, we see that decreasing the value of t sharply increases the effects of the running gravitational coupling, as we would expect. In fact, for $t = 1/2$, the contribution of virtual KK graviton exchange is damped to the point where the event rate lies not far above that of the Standard Model, particularly at large invariant masses. However, for $t = 2$, the change in the event rate is only minor in comparison to that of the standard ADD model. Hence, as t is varied over its theoretically expected range, the size of the form factor effect on virtual graviton exchange in models with large extra dimensions differs greatly.

Given the striking effect of a running gravitational coupling in the Drell-Yan invariant

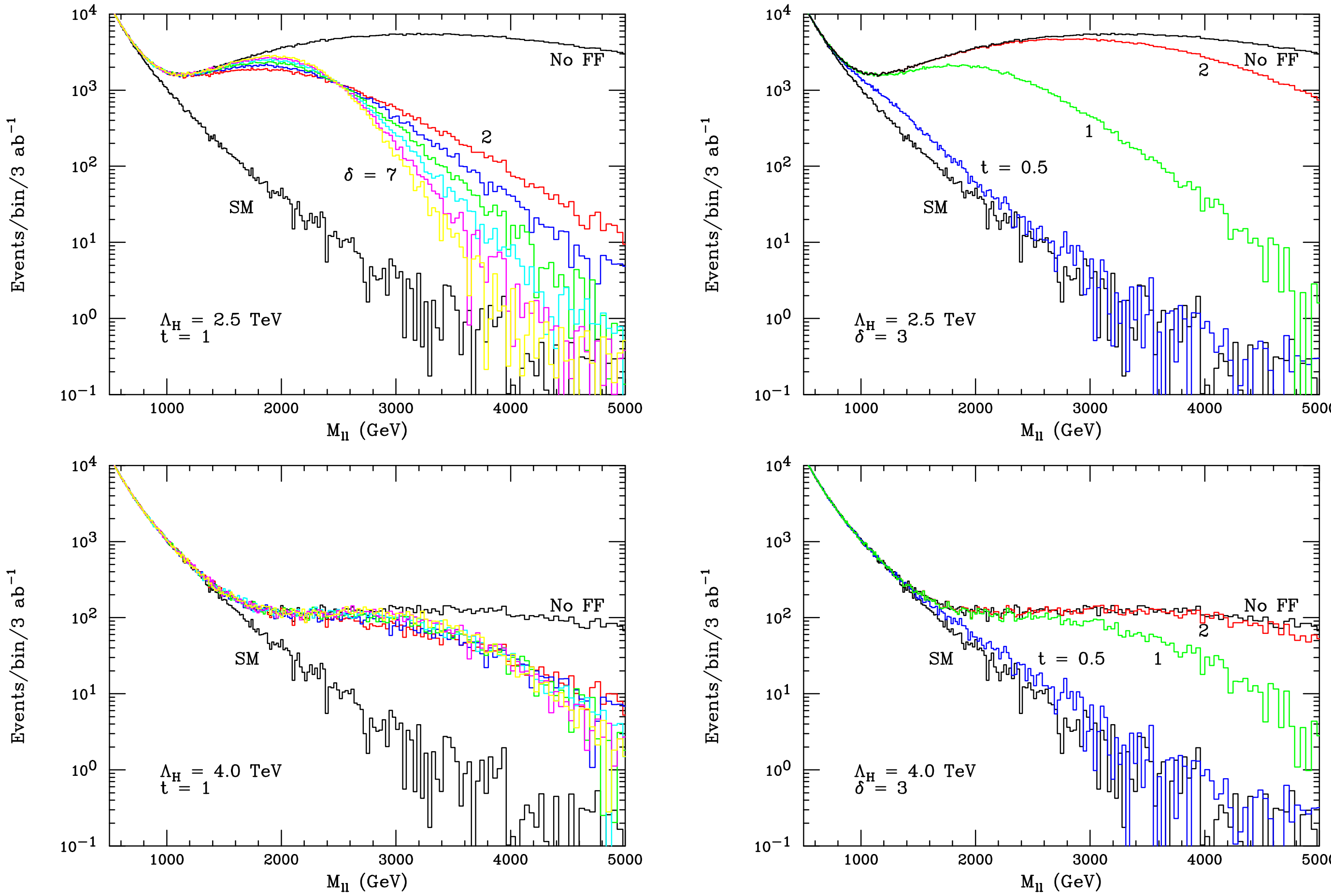


Figure 3: The event rate per bin with 3 ab^{-1} of integrated luminosity for Drell-Yan production at the LHC as a function of the lepton pair invariant mass taking the scale $\Lambda_H = 2.5$ TeV (top panels) and $\Lambda_H = 4.0$ TeV (bottom panels). In the left panels, $t = 1$ and $\delta = 2, 3, 4, 5, 6, 7$ from top to bottom on the right-hand side as labeled. In the right panels, $\delta = 3$ and the red, green, blue curves correspond to $t = 2, 1, 0.5$. In all panels, the bottom black histogram corresponds to the Standard Model result, and the top black histogram is the conventional ADD result [20] without the form factor.

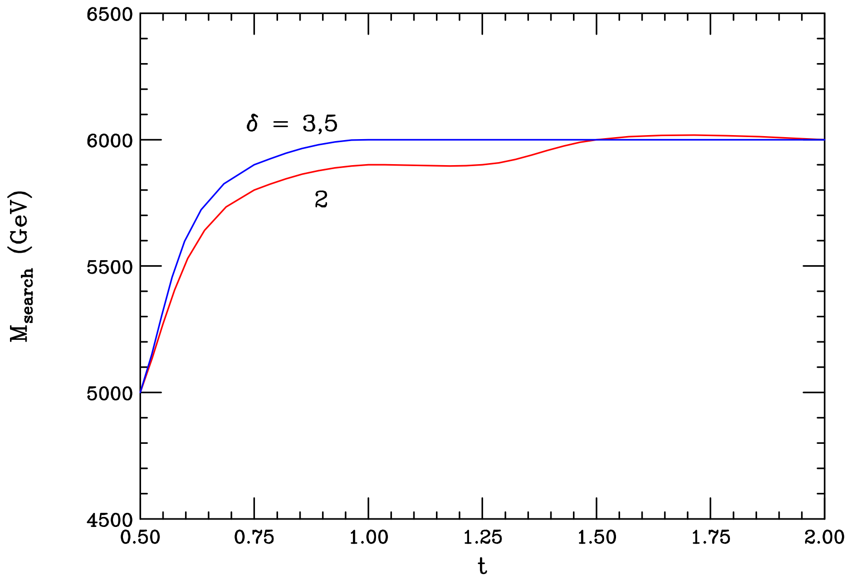


Figure 4: The 95% C.L. search reach for the fundamental scale M in Drell Yan production at the LHC as a function of the parameter t. The red curve corresponds to $\delta = 2$, while the $\delta = 3$ and 5 curves lie on top of each other and are represented by the blue curve.

mass distribution at the LHC, we now address the question of whether the search reach for the scale Λ_H is modified in the presence of the form factor. The 95% C.L. search reach for the cutoff scale Λ_H in this process is presented in Fig. 4 as a function of t for various values of δ. We see that for $t \gtrsim 1$, the search reach is unaffected by the presence of the form factor, since the cross section is independent of δ for large values of t. For $t \lesssim 1$, the reach degrades with decreasing t, but not substantially. For example, the reach in Λ_H decreases by only $\simeq 1$ TeV when t takes on the value of 0.5. This is due to the large statistics available at the LHC for lower values of the lepton pair invariant mass, which coincides with the region where the running coupling has the smallest effect.

We now consider the class of collider processes that involve the real emission of KK graviton states at the LHC, *i.e.*, the scattering process $pp \to jet + G_n$, where G_n represents a state in the graviton KK tower [21, 23]. The produced graviton behaves as if it were a massive, non-interacting, stable particle and once the KK states are summed, it yields a distribution of missing energy. The specific process kinematics regulate this reaction and there is no need to introduce a cutoff. This jet plus missing energy signature arises from the three sub-processes $gg, q\bar{q} \to gG_n$ and $gq \to qG_n$, and results in a $\delta-$dependent reach directly on the fundamental scale M. The search reach for this reaction, together with the SM backgrounds, have been well studied by the authors in [24]; they find, for example, that if $\delta = 2\ (3,\ 4,\ 5,\ 6)$ the maximum reach on M at the LHC is 9.1 (7.0, 6.0, 5.5, 5.2) TeV, respectively, for an integrated luminosity of 100 fb^{-1}. As discussed in the previous Section, a study of the effects of the running gravitation coupling is more complicated for this reaction as the various sub-processes do not take place at a fixed center of mass energy and several different scales are present. This introduces an ambiguity in the choice of the scale μ in the form factor. Here, as examples, we examine two possibilities: $\mu = E_{jet}$ or $p_{T,jet}$.

Figure 5 displays the missing energy distribution for the signal at the LHC for this process with $t = 0.5,\ 1,\ 2$. The top two panels compare the choices $\mu = E_{jet}$ and $p_{T,jet}$ for fixed values of M and δ. We see that the choice of $\mu = p_{T,jet}$ yields a much smaller deviation from the conventional result without the form factor than does the case of $\mu = E_{jet}$. In both cases, sizeable modifications of the distribution only occur when t takes on the value of 0.5, and become more pronounced at large values of missing E_T. This is to be expected since in general $p_{T,jet} < E_{jet}$ and the effect of the form factor grows as the scale μ increases in magnitude. The bottom two panels explore the form factor effects when M and δ are varied, taking $\mu = E_{jet}$. Here, we see that this effect is amplified for lower values of M and larger values of δ, yielding a significantly smaller missing E_T distribution than in the standard case.

Next we examine the total event rate for the signal above a cut on missing E_T, as a function of that cut. Note that strong cuts are required on missing E_T in order to suppress the Standard Model background. Figure 6 compares the choices $\mu = E_{jet}$ and $p_{T,jet}$ for $M = 5$ TeV for various values of the parameter t and the number of extra dimensions δ. Again, larger deviations from the conventional ADD result are obtained in the case

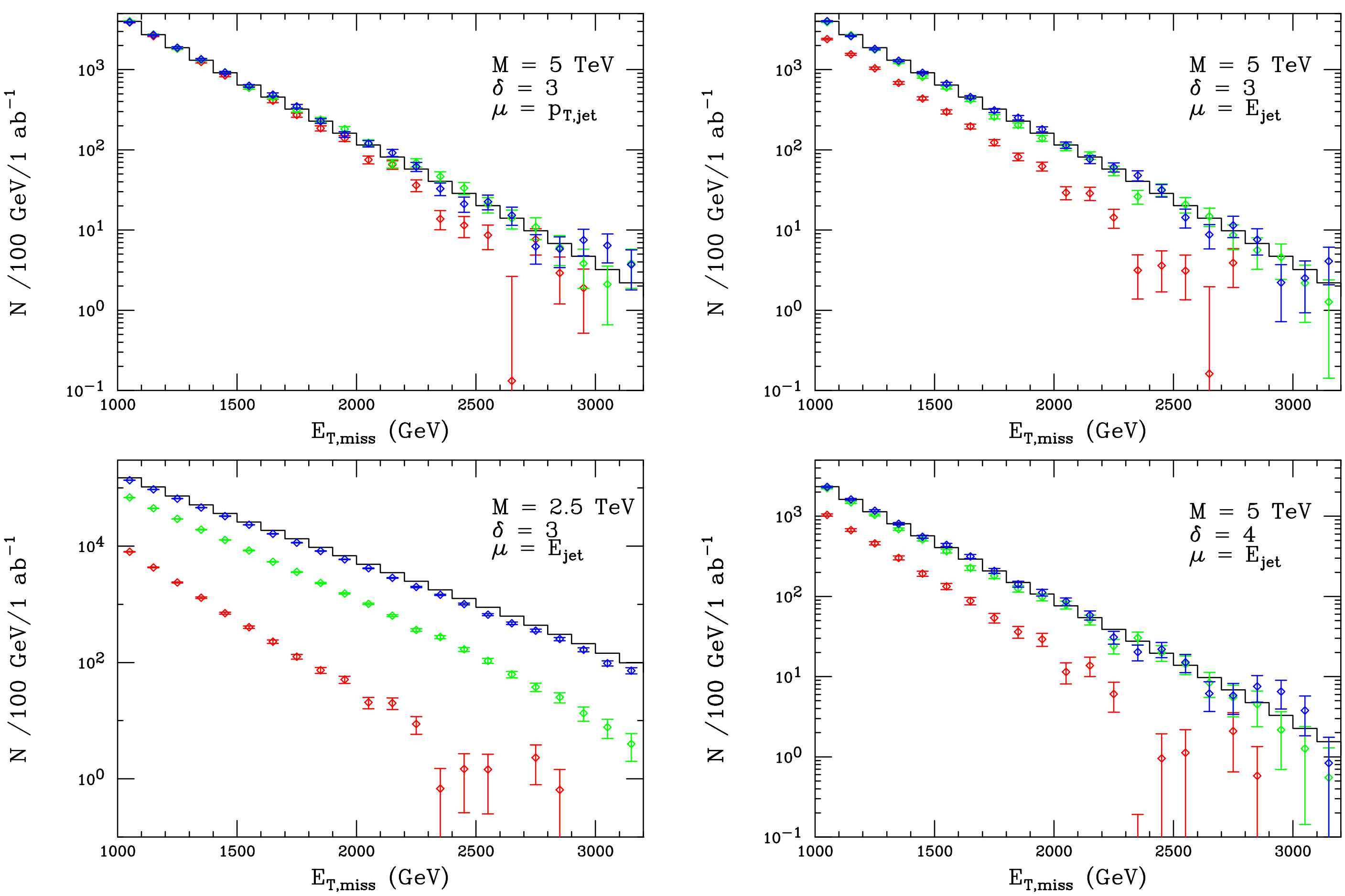


Figure 5: Missing transverse energy distribution for the signal process $pp \rightarrow jet + \not{E}_T$ assuming 1 ab^{-1} of integrated luminosity at the LHC. The standard ADD result is given by the black histogram, while the blue, green, and red data points correspond to the inclusion of the form factor with $t = 2,\ 1,\ 0.5$, respectively. The other parameters are as labeled. The errors bars represent the statistical errors.

$\mu = E_{jet}$ and the effect of the form factor increases as t becomes smaller. The case of $t = 2$ is essentially indistinguishable from the standard ADD result. Figure 7 shows the consequences for different values of the fundamental scale M. Here, we see that the effects of a running gravitational coupling are quite significant for smaller values of M, as would be expected.

We now study the modification to the search reach for large extra dimensions in this channel. Using the same search criteria as in [24], we examine the case which yields the largest deviation from the conventional ADD result, *i.e.*, we assume $t = 0.5$ and take $\mu = E_{jet}$, and find that the search reaches are reduced to 9.0 (6.6, 5.3, 4.2, 3.0) TeV, for $\delta = 2$ (3, 4, 5, 6), respectively. Note that the search reach degradation in comparison to the conventional ADD case increases for larger numbers of extra dimensions. Choosing $\mu = p_{T,jet}$ instead, and assuming the same value of t, we find that there are essentially *no* modifications in the search reach from the conventional results. For either choice of μ, taking $t \geq 1$, yields no reduction in the ADD search reach in this channel. Thus the ability to see the jet plus missing energy signature of large extra dimensions remains rather robust when form factor effects are included, as long as the parameter t is not too small.

Next, we examine the signatures of a running gravitational coupling at the ILC. The basic processes that are relevant for the ADD scenario are virtual KK graviton exchange and the direct production of KK gravitons via graviton emission as discussed above for the LHC.

We first consider the case of graviton exchange in the reaction $e^+e^- \rightarrow f\bar{f}$. At the $\sqrt{s} = 500$ GeV ILC, the search reach for the cutoff Λ_H in the conventional ADD model is approximately 5 TeV [20, 25], independent of the value of δ, assuming an integrated luminosity of 500 fb^{-1} and 80% electron beam polarization. Since $\sqrt{s}$ is fixed in this channel, the implementation of the form factor is straightforward and proceeds as discussed above. The modifications in the cross section as a function of $\sqrt{s}$ in the presence of the form factor

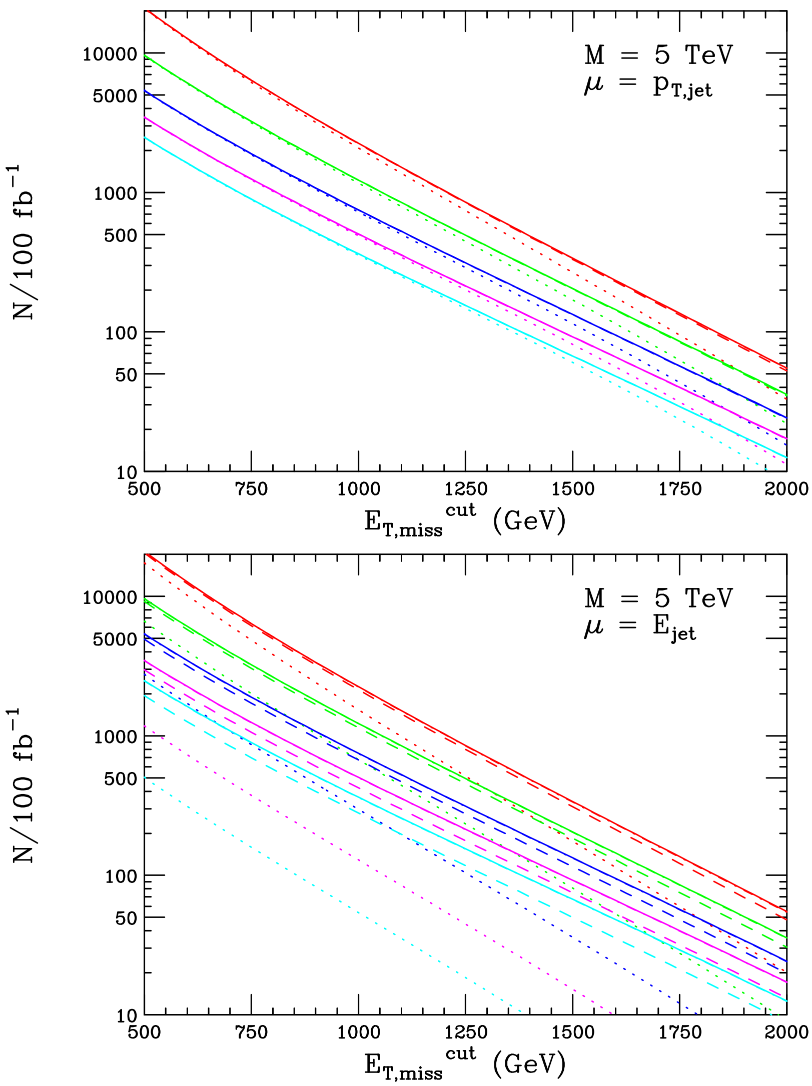
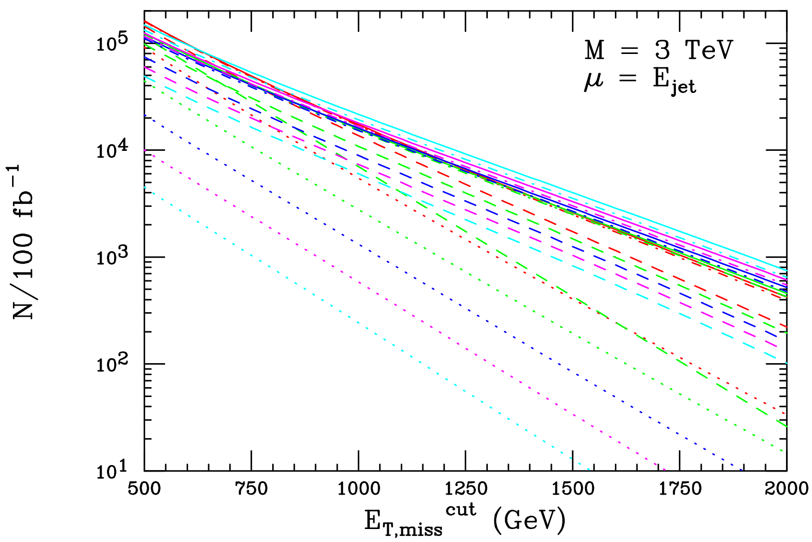


Figure 6: The excess signal event rate for $pp \to jet{+}G_n$ with 100 fb^{-1} of integrated luminosity at the LHC as a function of a cut on missing E_T. $M = 5$ TeV, $\delta = 2, 3, 4, 5, 6$ from top to bottom, and $\mu = p_{T,jet}$, (E_{jet}) in the top (bottom) panel. The solid curves correspond to the conventional ADD result and the (invisible) dash-dotted, dashed, and dotted curves are for $t = 2,\ 1,\ 0.5$, respectively.



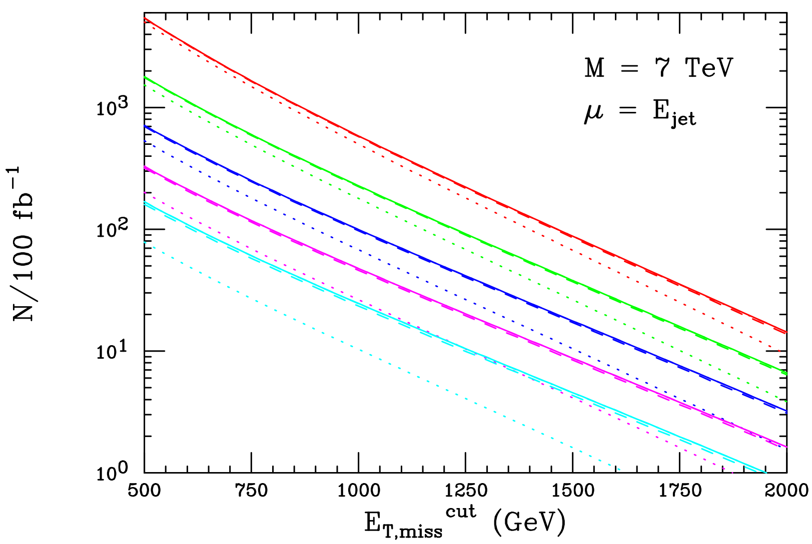


Figure 7: The excess signal event rate for $pp \to jet + G_n$ with 100 fb^{-1} of integrated luminosity at the LHC as a function of a cut on missing E_T. $\delta = 2, 3, 4, 5, 6$ from top to bottom, $\mu = E_{jet}$, and $M = 3$, (7) TeV in the top (bottom) panel. The solid curves correspond to the conventional ADD result and the (invisible) dash-dotted, dashed, and dotted curves are for $t = 2,\ 1,\ 0.5$, respectively

are illustrated in Fig. 8, for various values of δ and the parameter t, taking $\Lambda_H = 2$ TeV. We see that the behavior of the cross section is similar to that of the invariant mass distribution for Drell-Yan production at the LHC; the effects are more pronounced at small values of t and track the Standard Model result for $t = 0.5$. Note that again the dependence on δ reverses as the threshold $\sqrt{s} = t\Lambda_H$ is passed. At large enough values of $\sqrt{s}$, the cross section for $t = 2$ starts to turn over, displaying the onset of unitarity as discussed above. For this value of Λ_H, a higher center-of-mass energy is clearly beneficial in order to detect the presence of the form factor.

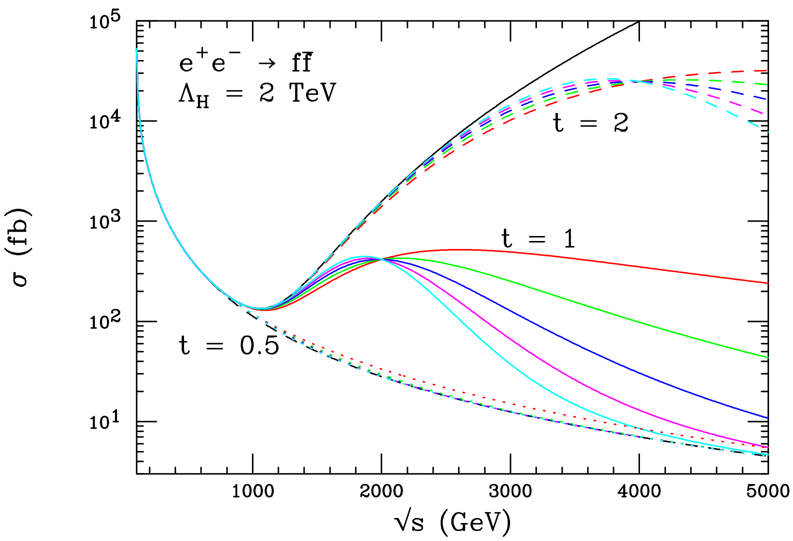


Figure 8: The cross section for $e^+e^- \longrightarrow f\bar{f}$ as a function of $\sqrt{s}$ with $\Lambda_H = 2$ TeV. The solid, dashed, and dotted sets of curves correspond to the parameter $t = 2,\ 1,\ 0,5$, respectively, and $\delta = 2,\ 3,\ ,4\ ,5,\ 6$ correspond to the curves in each set from top to bottom on the right-hand side. The solid black curves at the top (bottom) represent the conventional ADD (Standard Model) result for this channel.

Since the implementation of the form factor is straightforward, the modification to the search reach can be derived analytically. We find (denoting the search reach with running coupling as Λ_H^{FF})

$$\Lambda_H^4 = (\Lambda_H^{FF})^4 + (\Lambda_H^{FF})^{-\delta+2}\left(\frac{\sqrt{s}}{t}\right)^{\delta+2}, \tag{17}$$

which can be easily solved numerically. If the parameter $t \geq 1$, we find that Λ_H^{FF} does not differ from Λ_H by more than 0.5% for any value of δ. If, however, $t = 0.5$ and $\delta \geq 5$ there is a reasonable search reach degradation; in particular, we find, for $\delta = 5$ (6, 7) that $\Lambda_H^{FF} = 4.77$ (4.20, 3.44) TeV.

The last channel for us to consider is graviton emission at the ILC, which proceeds via the reaction $e^+e^- \to \gamma + G_n$. The effect of running gravitational couplings on the missing energy cross section are shown in Fig. 9 as a function of $\sqrt{s}$. In this figure, we take $\mu = E_\gamma$ as the analogous choice was found to lead to the largest deviations at the LHC. We see that deviations from the conventional result are only observable for $t = 0.5$, which somewhat lowers the cross section at larger values of $\sqrt{s}$. Since this process is typically employed as a means to determine the number of extra dimensions, the existence of a form factor would in principle interfere with this determination.

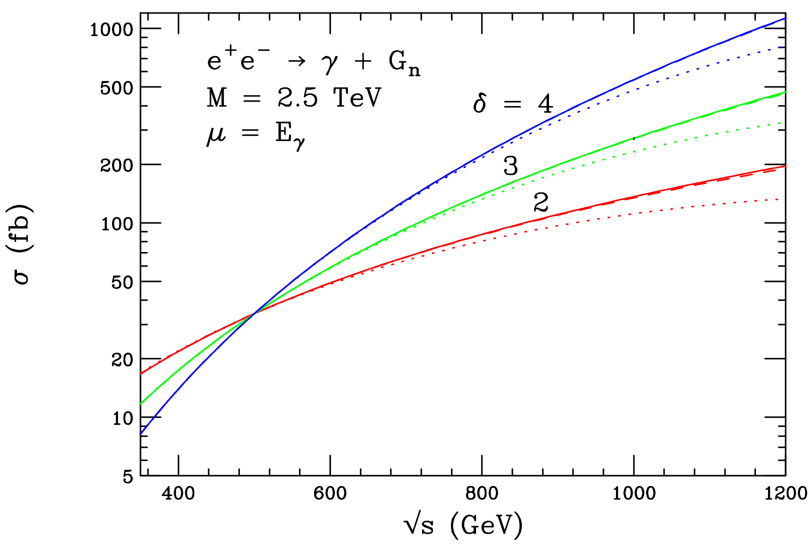


Figure 9: The cross section for $e^+e^- \to \gamma + \not{E}_T$ from KK graviton emission as a function of $\sqrt{s}$, taking $\mu = E_\gamma$. All curves are normalized to the value of the cross section at $\sqrt{s} = 500$ GeV with $M = 2.5$ TeV and $\delta = 2$. The (invisible) dash-dotted, dashed, and dotted curves correspond to $t = 2,\ 1,\ 0.5$, while the blue, green, and red sets of curves represent $\delta = 4,\ 3,\ 2$. The solid curves represent the conventional ADD cross section.

An analytical analysis can also be applied for this process since the center-of-mass energy is fixed. In the usual case, the rate for graviton emission scales as $M^{-(2+\delta)}$ and one can obtain the reach for any given value of δ in a straightforward manner [21, 23]. Equating the signal rate that yields the search limit in the conventional ADD case to the rate in the presence of the form factor, yields a relation between the search reaches with and without a running gravitational coupling. Denoting M^{FF} as the search reach with the form factor, we have

$$M^{\delta+2} = (M^{FF})^{\delta+2} - \left(\frac{\sqrt{s}}{t}\right)^{\delta+2}, \tag{18}$$

where here we have chosen $\mu^2 = s$ to maximize the effect of the form factor. Since values of M are in the multi-TeV range, we find that $M^{FF} = M$ to the high accuracy of $\sim 0.1\%$ or better for $t \geq 0.5$. For example, with $\delta = 2$ (6) the traditional reach is given by $M = 8.3$ (2.9) TeV [19, 21, 23]. Taking $t = 0.5$ for these cases we obtain $M^{FF} = 8.298$ (2.898) TeV when the form factor is present. Choosing a smaller and perhaps more realistic value of μ, such as the photon energy, we see that the corresponding change in the ILC discovery reach is even further reduced. Thus to a very good approximation, a running gravitational coupling is seen to have very little effect on ILC search reaches for ADD model signatures.

4 Warped Extra Dimensions

In the RS model [16], the S^1/Z_2 orbifolded, slice of 5-dimensional space bounded by two branes is described by a non-factorizable metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \tag{19}$$

with $\eta_{\mu\nu}$ being the flat Minkowski metric. The two branes are separated by a distance πr_c, with one being located at $y = 0$ (known as the UV brane), and the other at $y = \pi r_c$ (the IR brane). The warp factor, $\epsilon = e^{-\pi k r_c}$, generates the hierarchy between the Planck and electroweak scales when $kr_c \sim 11$. The curvature parameter k satisfies $k \sim M \sim \overline{M}_{Pl}$ with $0.01 \lesssim c = k/\overline{M}_{Pl} \lesssim 0.1$ [26].

In order to keep things simple, we limit ourselves to the case where the SM fields are localized to the IR brane; we can then concentrate on the gravitational sector of the theory and readily compare results with the classic RS model. In this case, the principle collider signal for the RS model is the resonant production of spin-2 KK gravitons [26]. The KK masses are given by $m_n = x_n k\epsilon$ where x_n are the roots of the J_1 Bessel function, and they couple to the Standard Model fields with electroweak strength. Since the KK gravitons are directly produced in the s-channel in this scenario, the form factor describing the running gravitational coupling can be written as

$$F^{-1} = 1 + \left(\frac{\sqrt{s}}{tM\epsilon}\right)^3 , \tag{20}$$

where we have set $\mu = \sqrt{s}$.

It is interesting to first consider the ratio of width to mass for the graviton KK states as n increases. In the standard RS picture,

$$\frac{\Gamma_n}{m_n} = Nc^2\left(\frac{m_n}{k\epsilon}\right)^2 = Nc^2 x_n^2 , \tag{21}$$

where N is a fixed numerical factor $\simeq 5/16\pi$. For a fixed value of c we observe that this ratio grows significantly as n increases, and at some point the very idea of a graviton resonance is lost. On the resonance peak for a KK graviton, the form factor given above can be written as

$$F^{-1} = 1 + \left(\frac{m_n}{tM\epsilon}\right)^3 = 1 + [x_n c^{2/3}/t]^3 , \tag{22}$$

where the last equality follows from the relation $M^3 = k\overline{M}_{Pl}^2$. We now see that width scales as

$$\Gamma_n = Nk\epsilon c^2 x_n^3 \Big[1 + c^2 x_n^3/t^3\Big]^{-1} . \tag{23}$$

As x_n gets large we now find

$$\Gamma_n \simeq Nk\epsilon t^3 , \tag{24}$$

which is *independent* of both c and n. Thus the form factor prevents the widths of the KK graviton states from growing too large and a well-defined resonance structure is maintained for every level in the KK tower. We also see that the parameter t plays an important role in determining the graviton width; however, a problem may still arise if the value of t is too large.

To determine the range of the parameter t that is allowed by perturbative unitarity in the RS model, we again study the $2 \to 2$ scattering process $hh \to hh$ at high energies. The KK tower of gravitons contribute to this process via s-, t- and u-channel exchanges. Including the form factor and summing over the first 10,000 states in the graviton KK tower, we obtain the results displayed in Fig. 10 for the $J = 0$ partial wave amplitude for this channel. In this figure, we show the value of $2Re|a_0|$ as a function of the ratio $\sqrt{s}/m_1$ (where m_1 is the mass of the first graviton KK state) for several values of t assuming $c = 0.05$. It is clear that this amplitude is well-behaved and $2Re|a_0|$ is always less than unity when $t \leq 2$. However, at very high energies, it appears that the amplitude starts to grow for $t \geq 3$ and will eventually violate perturbative unitarity. We find that these results do not change appreciably as c is varied. Note that the presence of the form factor greatly dampens this amplitude compared to the standard RS result.

The effects of the form factor in the production of graviton KK resonances in the Drell-Yan channel at the LHC are shown in Fig. 11. Here we see the familiar pattern that, in the standard RS picture, the resonances get wider and wider as the level increases in the

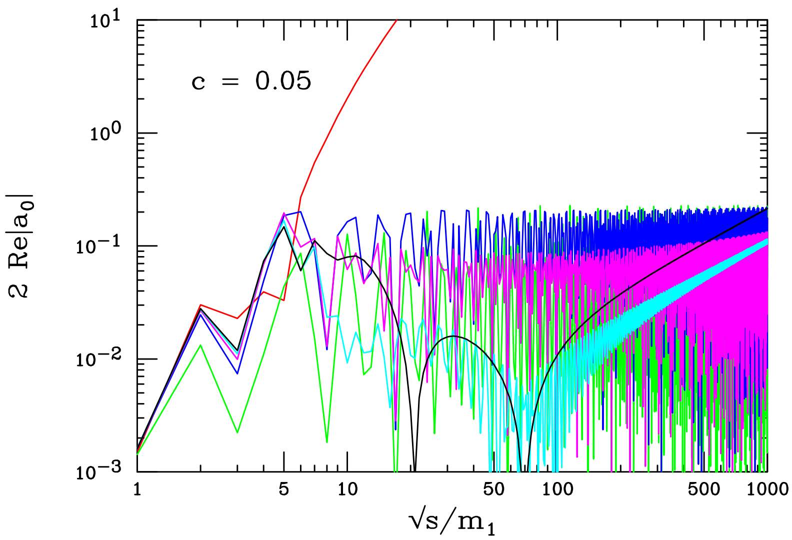


Figure 10: The $J = 0$ partial wave amplitude for $hh \to hh$ as a function of the ratio $\sqrt{s}/m_1$, summing over the first 10,000 KK graviton states in the RS model in the presence of the form factor. We set $c = 0.05$. The green, dark blue, magenta, cyan, and black curves correspond to the values $t = 1,\ 2,\ 3,\ 4$ and 5, respectively. The red curve represents the conventional RS rsult.

KK tower and the resonance structure is completely lost above $n = 3 - 4$ depending upon the value of c. Turning on the form factor and taking smaller values of t, we do not lose too much of the apparent signal peak, but the towers separate and become more narrow for large n. Certainly, for $t \lesssim 2$ the resonance structure is always quite clean for the range of KK tower masses shown in the Figure. Note that if RS graviton KK resonances are observed and such form factors are present, the value of t will be relatively easy to extract from the cross section data.

The effect of the form factor on the widths of the KK graviton resonances become even more obvious at multi-TeV scale e^+e^- colliders such as CLIC [27]. Figure 12 shows the cross section for the process $e^+e^- \to \mu^+\mu^-$ as a function of $\sqrt{s}$ with $m_1 = 600$ GeV. In the upper panel we assume $c = 0.05$ and see the loss of resonance structure and the potential unitarity violation in the conventional RS model and how this situation is tamed by the presence of a form factor. Certainly for $t \lesssim 2$ we see that the narrow resonance structure is

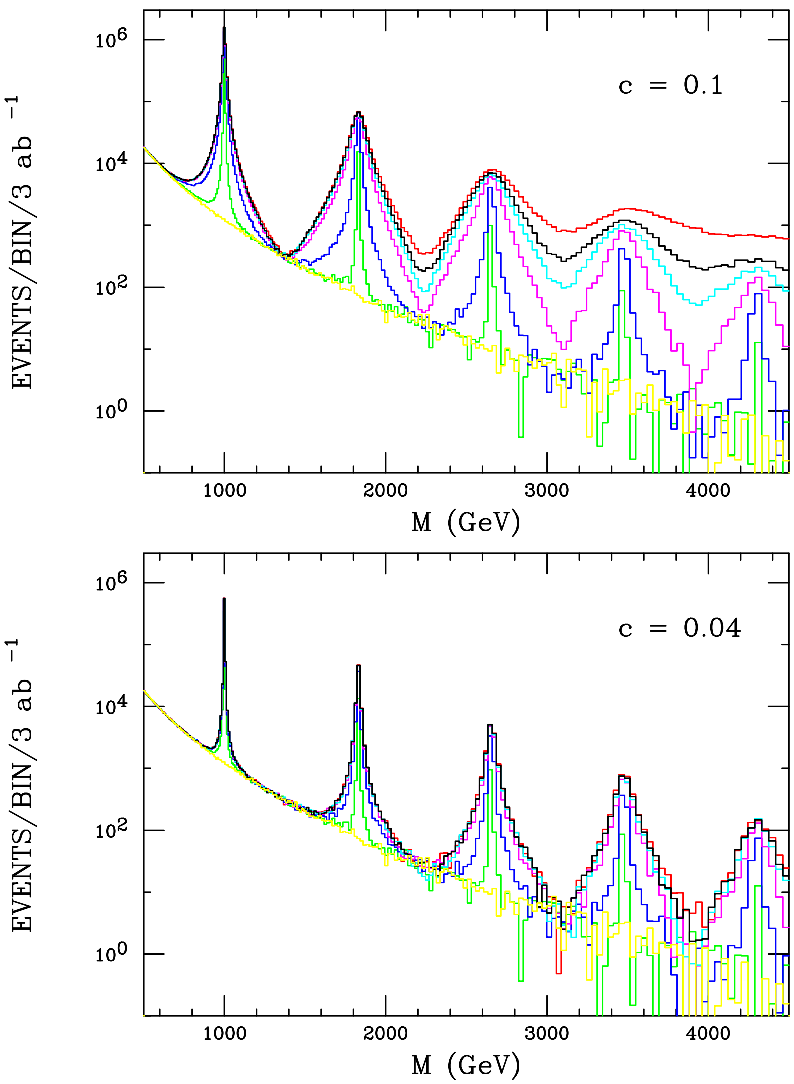


Figure 11: RS graviton resonance production in the Drell-Yan channel as a function of the dilepton pair invariant mass at a high luminosity LHC assuming $m_1 = 1$ TeV and $c = 0.1(0.04)$ in the upper(lower) panel. The lowest(yellow) histogram in the SM background while the outermost(red) histogram is the usual RS prediction. From outside going inwards the next five histograms correspond to $t = 4(3, 2, 1, 0.5)$, respectively. In all the form factor cases the scale is assumed to be the dilepton pair mass M.

maintained for all levels of the KK tower. In the lower panel, for fixed $t = 1$, it is clear that the graviton KK resonances are all quite narrow and are essentially c and n independent, even for high levels in the KK tower. Only for the lightest KK state do we see dependence on c and even in this case it is rather weak. From these figures we can see that the form factor modifies the cross section as advertised.

5 Summary and Discussion

The poor high energy behavior of General Relativity observed in perturbation theory may be cured if there exists a non-Gaussian fixed point rendering the theory asymptotically safe and potentially non-perturbatively renormalizable. If such a possibility is realized, the effective gravitational coupling at high energies becomes weaker and this running can be parameterized through the introduction of a form factor when calculating the interactions of gravitons with matter or each other. These form factors can also modify graviton exchange amplitudes rendering them unitary at tree-level. The evidence that such a situation may be realized in nature is reasonably strong and has improved theoretically in recent years. However, since the effect of the form factor is only significant once the relevant energies approach the fundamental scale of gravity, in 4-dimensions it will be difficult to test this possibility directly anytime in the near future.

Extra-dimensional scenarios of the ADD or RS type allow for the fundamental scale to be not far above ~ 1 TeV. If such scenarios are realized, the existence of gravitational form factors can then be probed at future colliders such as the LHC and/or the ILC. In this paper we have shown that this is indeed the case for both these scenarios.

The analysis of Reuter *et al.* and of Litim [12] suggests a very specific structure for this form factor which is totally determined in D-dimensions up to an order one coefficient,

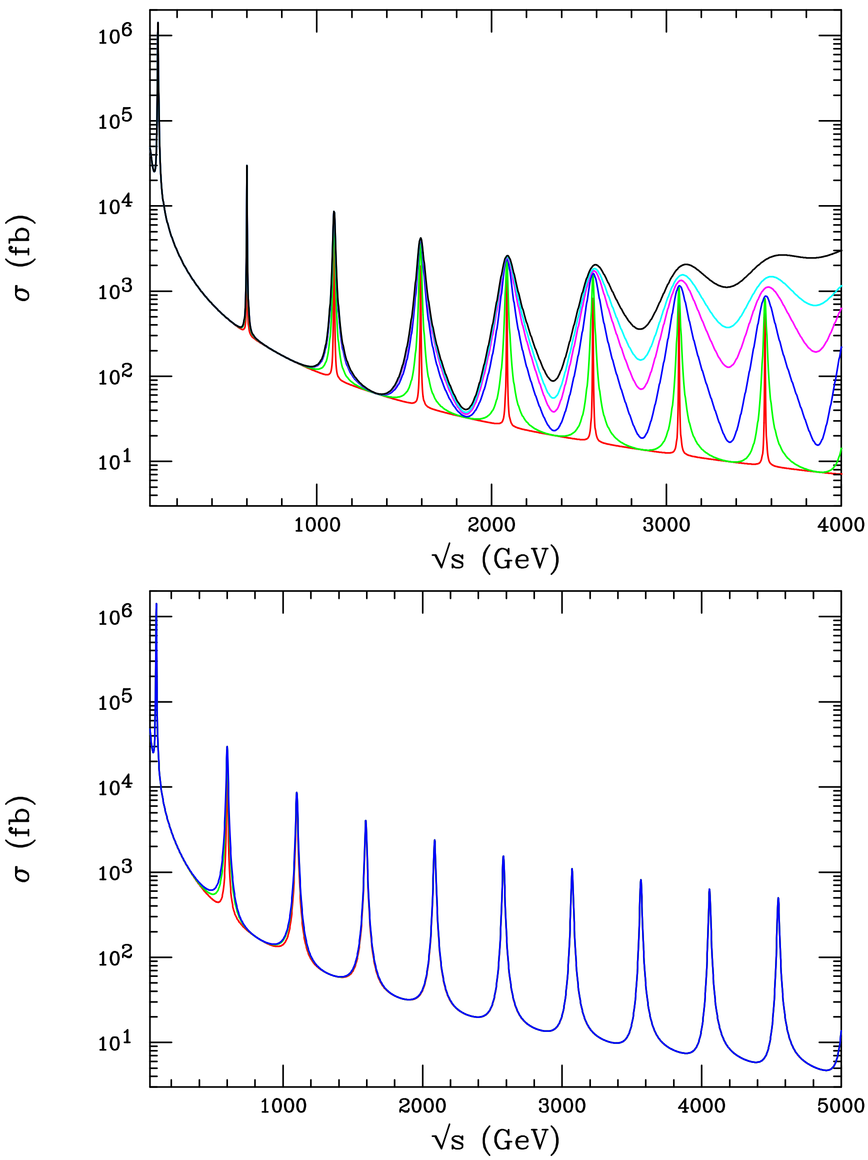


Figure 12: Cross section for $e^+e^- \to \mu^+\mu^-$ as a function of $\sqrt{s}$ in the RS model with $c = 0.05$ and m_1 =600 GeV showing the first 7(9) graviton KK excitations in the top(bottom) panel. In the top panel, the uppermost curve corresponds to the standard result. Moving inward the curves correspond to the case where a form factor is present assuming $t = 4(3, 2, 1, 0.5)$, respectively. In the lower panel, $t = 1$ has been assumed for very large values of $c = 0.1, 0.2$ and 0.3, with the narrowest resonance curve corresponding to the smallest value of c.

t. We find that imposing tree-level unitarity requirements on graviton exchange amplitudes in either the ADD or RS models implies that the range of t is restricted: $t \lesssim 2$. In both of these models, graviton exchange processes were shown to be particularly sensitive to the presence of these form factors. In particular, we demonstrated that measurements at both the LHC and ILC can be used to extract the value of t. In the RS model, the width of graviton resonances, which ordinarily increases at higher levels of the KK tower, was shown to asymptote to a constant value when form factors are employed. On the otherhand, the process of graviton emission which occurs in the ADD scenario, was shown to be rather insensitive to the presence of form factors. Interestingly, the collider search reaches for extra dimension was also shown to not be overly sensitive to form factor contributions in both the ADD and RS cases.

If these extra dimensional scenarios are realized in Nature, then hopefully the observation of form factor effects will be observed, thus providing us with an important handle on the underlying theory of quantum gravity.

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