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INTRODUCTION

The nominal design gradient in the main linacs of the International Linear Collider (ILC) is 31.5 MV/m for a beam current of 9.0 mA. However, the superconducting cavities built to date have demonstrated a range in sustainable gradient extending well below this goal, being limited by Q drop-off and quenching. Thus, an economically feasible cavity acceptance rate will include a certain percentage of sub-performing cavities. An important question that needs to be addressed is, For a string of cavities rated to various levels of gradient and powered from a common source how can we optimize the overall gradient? Along with adjustable cavity coupling—or loaded Q factor—we assume adjustable RF power so that gradient can be leveled in non-nominal cavities, to avoid quench-inducing overshoots.

In the ILC an RF unit comprises three cryomodules containing a total of 26 nine-cell cavities, which are fed by one klystron that nominally feeds equal power to all cavities. One simple way of running such a unit is to set RF power, beam arrival time, and all loaded Q's so that the power is matched and the gradient in all cavities equals the gradient limit in the poorest performing cavity. This conservative strategy, however, sacrifices gradient and can be improved upon. One improvement strategy is to adjust the cavity couplings individually (possible, since circulators are assumed in the baseline ILC design) or in pairs (when circulators are not needed) using the movable antennae of the fundamental mode couplers. Another strategy is to use variable power tap-offs (VTO's)[1] by which the RF power to succeeding pairs of cavities can be made to differ. These solutions will not be matched, resulting in power inefficiency and a gradient variation along the beam that needs to be limited.

In this report we study the effect on overall gradient for various combinations of these ideas. Deciding which strategy is best can then be done by balancing gradient with the cost and complexity of hardware such as circulators and variable power tap-offs.

OPTIMIZATION

The gradient in a superconducting, standing wave cavity, assuming a square RF pulse, can be written in normalized parameters as (for the non-normalized equation see *e.g.* [2],[3])

$$g(t) = 2\sqrt{p\,q}(1-e^{-t/q}) - q(1-e^{-(t-\tau_b\ln 2)/q})H(t-\tau_b\ln 2)\ , \quad (1)$$

with $H(x)=0$ (1) for $x<0$ (>0). The first term is the RF power term, the second one the beam loading term. Here all parameters are normalized to the matched case (one with zero reflected power during the beam) for a particular gradient; the parameters gradient g (assuming on-crest operation), input power p, and loaded quality factor q are normalized to their matched values. We normalize time quantities to the nominal attenuation (or damping) time; these include the time t (with $t=0$ the RF turn-on time), the beam (head) arrival time (or cavity fill time) $\tau_b\ln 2$, and the beam tail arrival time (or RF pulse length) t_{max}. The cavity gradient limits g_{lim} are normalized like g. In this report we are interested in the behavior of $g(t)$ while the beam is in the cavity. Since Eq. 1 is monotonic during the beam passage, either the train head or tail experiences the maximum gradient (or the gradient is flat).

For our optimizations we randomly pick the gradient limits of the cavities from a uniform distribution over the interval 22–34 MV/m, which is representative of current cavity results with production-like processing. The mean gradient in this case is 28 MV/m. For our calculations we take the case $g=p=q=\tau_b=1$ to correspond to the matched case with gradient 34 MV/m. The relative gradient reduction compared to the case where every cavity is run at its g_{lim} is $\delta_{loss}=1-\langle g\rangle/\langle g_{lim}\rangle$, where $\langle\rangle$ means to average over the 26 cavities. We will assume that the premium is on gradient and not on RF power, so power efficiency will not be optimized. We set as boundary condition that the relative head-to-tail rms spread in energy gain (over an RF unit) $\sigma_\delta\leq 0.1\%$. In our calculations we take as (normalized) time of the beam tail $t_{max}=2.5$ (this is slightly conservative: for the ILC at 28 MV/m the real number is ~ 2.0).

The most general condition for being matched is $g=p=q=\tau_b$. Since the beam arrival time is one overall parameter, the only way to be matched over a 26-cavity unit is to match to the lowest g_{lim} in the distribution, in which case the relative loss in gradient $\delta_{loss}\sim 1-22/28\approx 20\%$.

Individual p's, Individual q's

The case of individual p and q controls, while the most expensive, is of particular interest since a flat gradient profile can be achieved in each cavity (see *e.g.*[3]). By taking the derivative with respect to t of Eq. 1 (after the beam arrival time) and then setting the result to zero, we find that the condition for a flat gradient (in general, not matched) is $-2^{\tau_b/q}+2\sqrt{p/q}=0$. This equation has 0 (2) solutions for q/τ_b whenever $p/\tau_b<(>)\, e\ln(2)/2\approx 0.94$ (see Fig. 1).

The gradient of the flat solutions is given by $g=(-1+2^{\tau_b/q})q$ and the power by $p=4^{\tau_b/q-1}q$. We can plot q/τ_b, p/τ_b, and relative reflected power $\rho_r=1-g/p$ as functions of g/τ_b (see Fig. 2). We see that *e.g.* for $p/\tau_b=1$ the second (non-matched) solution has $q/\tau_b=2$,

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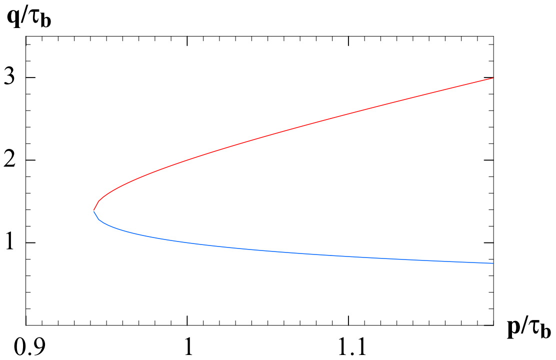

Figure 1: Parameters that yield a flat gradient along the bunch. The point $(1, 1)$ gives the matched case.

$g/\tau_b = 2(-1 + \sqrt{2}) \approx 0.83$, and $\rho_r = 3 - 2\sqrt{2} \approx 0.17$. Note that there is a minimum $g/\tau_b = \ln 2 \approx 0.69$, below which there is no solution.

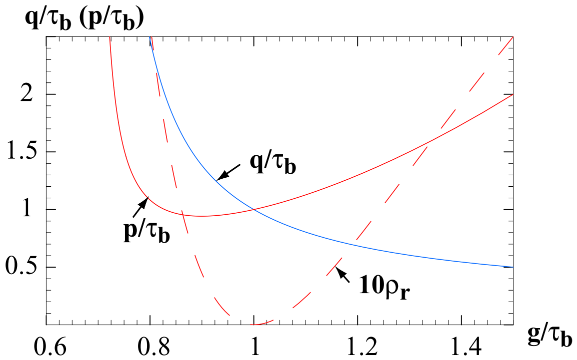

Figure 2: The values q/τ_b, input power p/τ_b, and reflected power ρ_r which yield a flat gradient, as functions of g/τ_b.

With individual p and q controls τ_b is an extra parameter, and we can choose it to minimize the average reflected power for the RF unit during the beam pulse. For our distribution of gradient limits this will occur if we choose τ_b so that the corresponding matched gradient is 26.5 MV/m. Note that, since $22/26.5 = 0.83 > \ln 2$, a flat solution exists for the entire range of g_{lim}'s. In this scenario, for each cavity, p and q are chosen from the curves in Fig. 2 with g taken as the cavity g_{lim} (only here normalized to 26.5 MV/m). The gradient loss for the RF unit is $\delta_{loss} = 0$, and relative reflected power $\rho_r \approx 5\%$.

1 p, Individual q's

In the scenario of one p and individual q's (and, of course, an overall τ_b) the implementation requires circulators. To optimize the gradient for an RF unit with gradient limits $(g_{lim})_i$, one, in principle, needs to solve a 27d optimization problem (where the 26 q_i and the beam time τ_b are adjustable) with one boundary condition ($\sigma_\delta \leq 0.1\%$). Once the adjustable parameters are set, p is chosen to be the highest value for which all $g_i(t) \leq (g_{lim})_i$ for t within the bunch train. Note that, unlike in the previous case, the RF unit voltage gain will not be completely flat along the bunch train (it will also, in general, not be monotonic).

Finding the global optimum of a 27d minimization problem using brute force can be quite challenging. Through analysis, however, we can convert this problem to one of only 3d, and we believe that the optimum for this problem is near the optimum for the original one. The three adjustable parameters in the new problem are p, τ_b, and q_{min} (to be described below). For given p and τ_b, the gradient g experienced by the head and tail of the bunch train in a cavity can be plotted as function of q; a typical case is shown in Fig. 3 (the red curve gives the head, the blue curve the tail). For a given q the maximum of these two curves—which we will denote by $\hat{g}(q)$—gives the maximum gradient in the cavity, since $g(t)$ between the head and tail is monotonic.

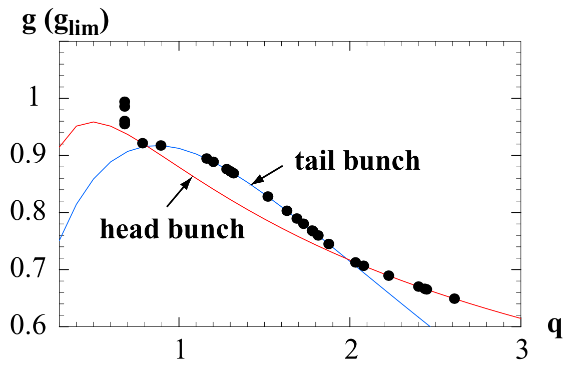

Figure 3: 1-p, individual q's: For one seed, where optimized $p = 0.92$ and $\tau_b = 0.885$: gradient g *vs.* q for the head (red) and tail (blue) bunch in the train. Also plotted are $(g_{lim})_i$ *vs.* optimized q_i for the 26 cavities (plotting symbols). For this seed $\delta_{loss} = 2.8\%$.

Now let us consider a 26-cavity distribution of gradient limits, $(g_{lim})_i$. Our prescription for choosing the q_i is: if $(g_{lim})_i < \hat{g}(q_{min})$ we choose it so that the point $(q_i,(g_{lim})_i)$ is on the curve $\hat{g}(q)$ (the monotonically decreasing part); otherwise we take $q_i = q_{min}$. The parameter q_{min} specifies a minimum for q; it was chosen as a degree of freedom because for low q the head-tail difference in gradient can become large. For given p, τ_b, and the q_i's, we can obtain δ_{loss} and σ_δ. Thus we can minimize δ_{loss}, with the boundary condition $\sigma_\delta < 0.1\%$, by adjusting p, τ_b, and q_{min} and following the above prescription.

Fig. 3 gives an optimized example. The gradient limits of the cavities were randomly generated from our uniform distribution. The plotting symbols give the $(q_i,(g_{lim})_i)$ following our prescription. Note that for the total head-to-tail variation in energy gain to be small, points must lie both between and outside the two flat-gradient points of the curves (where the two curves cross; the tail gradient being larger in some cavities must be canceled by it being smaller in others). The method works well, and only a few points have $q_i = q_{min}$ (for which cavities, neither the head nor the tail of the train sees the limiting gradient). For this seed $\delta_{loss} = 2.8\%$ and $\sigma_\delta = 0.04\%$. Note that this 1-p, individual q's method tends to work less well if many of the g_{lim} are at the two extremes of the distribution.

For 100 ensembles (seeds) of 26 cavities we have performed this 3d minimization of δ_{loss}, with boundary condition $\sigma_\delta < 0.1\%$. A histogram of δ_{loss} of the results is shown in Fig. 4. We find that $\delta_{loss} = 2.7{\pm}0.4\%$ (average $\pm$ rms deviation); the minimum is 1.6%, the maximum 3.9%. The variation σ_δ follows a rather uniform distribution up to 0.1%. The optimized $\tau_b = 0.885 \pm 0.003$, $p = 0.91 \pm 0.02$. The relative reflected power $\rho_r \approx 1 - \langle g \rangle / \langle p \rangle \approx 10\%$.

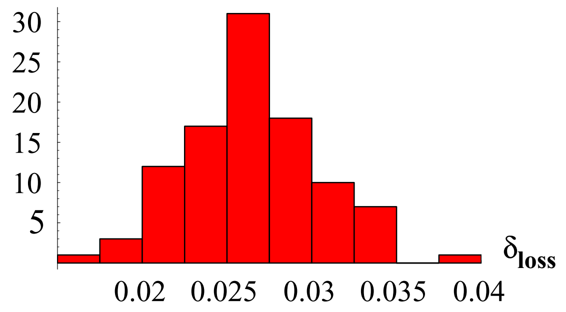

Figure 4: For optimized cases with 1 p and individual q's: histogram of δ_{loss} for 100 ensembles of 26 cavities.

Other Scenarios

In scenarios where the q's are adjusted in pairs, circulators are not needed; the cost, however, is loss in gradient. The gradient of these scenarios can be improved, however, by sorting the cavities, *i.e.* organizing cavities with similar g_{lim} into pairs. There is uncertainty, however, how stable these limits are: will they be affected by the installation process? will they remain stable over time?

In the case 1-p, q's in pairs the optimization is similar to the previous case. The difference, however, is that, for each pair, only the one with lower g_{lim} determines the q for the pair; *i.e.* in the g *vs.* q plot one point of each pair is on the curve $\hat{g}(q)$ while the other point is above it (see Fig. 5). For 100 ensembles of RF units the loss is $\delta_{loss} = 8.8{\pm}1.3\%$. If we sort, however, the loss decreases to $\delta_{loss} = 3.3{\pm}0.5\%$.

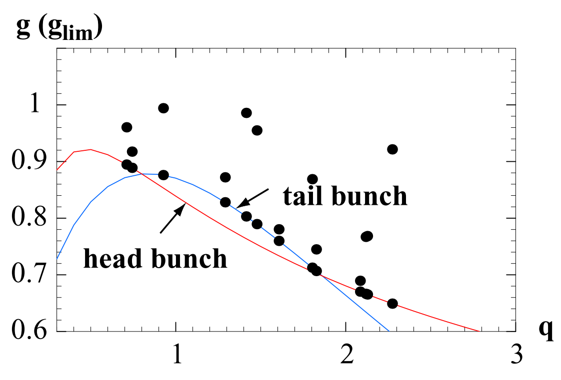

Figure 5: 1-p, q's in pairs: For the same $(g_{lim})_i$ as Fig. 3, where now optimized $p = 0.88$ and $\tau_b = 0.855$: gradient g *vs.* q for the head (red) and tail (blue) bunch in the train. Also plotted are $(g_{lim})_i$ *vs.* optimized q_i for the 26 cavities (plotting symbols). For this seed $\delta_{loss} = 8.8\%$.

With variable power tap-offs the p's of cavities are adjustable in pairs. In the case p's in pairs, q's in pairs, variable tap-offs are needed but circulators are not. This case was optimized with brute force beginning with the flat solution tuned to the lesser g_{lim} of each pair. The final solution turned out to be not very different from the initial solution (though it is difficult to know if it is, indeed, the global minimum). We find that, without sorting, $\delta_{loss} = 7.2 \pm 1.4\%$; with sorting, $\delta_{loss} = 0.8 \pm 0.2\%$.

To complete the picture we studied finally p's in pairs, individual q's, a scenario that requires variable tap-offs and circulators. This configuration was optimized with brute force, beginning with the 1-p, individual q's solution as a first guess. We found not much improvement over the initial state: $\delta_{loss} = 2.5 \pm 0.4\%$. With sorted cavities, however, we obtain $\delta_{loss} = 0.8 \pm 0.2\%$ (the same as the p's in pairs, q's in pairs case with sorted cavities). A summary of all our results is given in Table 1.

Table 1: Optimized gradient loss, δ_{loss}, in percent for various scenarios of p's and q's, where the overall beam time parameter τ_b is also adjusted. For 100 ensembles of 26 cavities, given are the average result and the rms deviation (the number after the $\pm$ sign).

Case	Not Sorted	Sorted
Individual p's and q's	0.0	0.0
p's in pairs, individual q's (VTO's and circulators)	2.5 ± 0.4	0.8 ± 0.2
1 p, individual q's (needs circulators)	2.7 ± 0.4	2.7 ± 0.4
p's in pairs, q's in pairs (needs VTO's)	7.2 ± 1.4	0.8 ± 0.2
1 p, q's in pairs	8.8 ± 1.3	3.3 ± 0.5
g_i set to lowest $(g_{lim})_i$	19.8 ± 2.0	19.8 ± 2.0

DISCUSSION

For the current distribution of gradient limits in the RF cavities of the ILC linacs we have optimized the overall gradient of a 26 cavity RF unit, assuming the availability of various combinations of circulators and variable power tap-offs. This has been a theoretical study. Besides the question of cost, the realizability of these solutions needs to be considered carefully. To name one example: Our solutions have q's varying by more than a factor of two. The question is, with the higher q's can the cavity frequencies still be kept sufficiently well regulated? After such considerations more iterations of a study such as this no doubt will be needed.

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