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Analysis of a Compact Circular $TE_{0,1}$ - Rectangular $TE_{0,2}$ Waveguide Mode Converter *

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Abstract

An analysis method for a three section mode transformer that converts a $TE_{0,1}$ circular waveguide mode to a $TE_{0,2}$ rectangular waveguide mode will be presented. Experimental results for this taper were earlier published in [1]. The middle section is a cylinder with a wall radius defined by $r_{wall} = a(1 + \epsilon \cos(2\theta))$, where a is the radius of the circular guide and ϵ is a design parameter. This cylinder is connected on either side to a circular waveguide and a rectangular waveguide section respectively, through tapered waveguide sections. In this analysis we used a perturbation technique where the rectangular waveguide section's wall radius is treated as a Fourier series expansion with a, the fundamental radius and ϵ the perturbation parameter. By applying the proper boundary conditions we optimize the taper dimensions to minimize conversion into spurious modes.

INTRODUCTION

In ultra high power RF systems such as those suggested for linear colliders, hundreds of megawatts of pulsed RF power is manipulated. Over-moded waveguides are widely used to increase the power handling capacity. Losses in the system are minimized by transporting power in circular waveguides in azimuthally symmetric modes such as the $TE_{0,1}$-mode. In many instances the RF power is easier to manipulate in rectangular waveguides than in circular waveguides [1]. Therefore, the power is often manipulated in rectangular waveguides and transported in circular waveguides.

In order to transport power between a circular waveguide and a rectangular waveguide, the two waveguides should be connected through a mode converter. It is possible to convert a $TE_{0,1}$-mode in a circular waveguide transitions into a $TE_{0,2}$-mode in a rectangular waveguide with a sufficiently smooth taper without scattering into other modes all along the taper. The length of the taper for such an adiabatic transition to a single mode at 11.424 GHz is about 18 inches which is excessive. We present a design method where the wave entering one end of the mode converter scatters into two modes and recombines into a single mode while coming out at the other end, leading to a much shorter mode converter.

The idea behind this mode converter is as follows. Let a circular waveguide be transitioned to another waveguide of a certain cross section (let us call it an "oval" waveguide) through a linear taper (taper1) such that a $TE_{0,1}$-mode traveling through the circular waveguide would scatter into two modes, say M_1 and M_2 in the oval waveguide. Let a rectangular waveguide be transitioned to the same oval waveguide through another linear taper (taper2) such that a $TE_{2,0}$-mode traveling through the rectangular waveguide is scattered into the same modes M_1 and M_2 in the oval waveguide. Then, we may be able to achieve perfect mode conversion from a circular $TE_{0,1}$-mode to a rectangular $TE_{2,0}$-mode by transitioning through the three sections *viz.*, taper1, oval waveguide and taper2, in that order and by optimizing the length of the three sections.

It is possible to design the mode converter using traditional numerical techniques like Finite Element Method (FEM). However, it would require a large amount of computational time to find an optimized solution for the design of the mode converter using FEM or other numerical techniques.

In this work we present a semi-analytical method to analyze a nonlinear waveguide whose cross section varies in two dimensions. We have used this method along with perturbation techniques to design the mode converter which needs much less computational time than FEM.

MODAL ANALYSIS

The wall radius of a waveguide that has a cross section with twofold symmetry ($r_w(\phi) = r_w(-\phi) = r_w(\pi+\phi) = r_w(\pi-\phi)$), may be expressed as,

$$r_w(\phi) = a_0 \left(1 + \epsilon \sum_{p=1}^{P} \delta_p \cos[2p\phi]\right), \qquad (1)$$

δ_p are Fourier expansion coefficients normalized to ϵ such that $\delta_1 = 1$. (If the waveguide wall cross section does not have twofold symmetry, then $\cos[2p\phi]$ should be replaced by $\cos[p\phi]$ in (1)).

For the oval waveguide, the cross section wall radius has a Fourier expansion only up to $P = 1$ which may be written as,

$$r_{w,oval}(\phi) = a(1 + \epsilon_{oval} \cos[2\phi]), \qquad (2)$$

where a is the radius of the circular waveguide and ϵ_{oval} is a small parameter which determines the deviation of the second section from a circular waveguide. The linear taper between the circular waveguide and the oval waveguide, taper1, also has a wall radius of the form of equation (2).

For a rectangular waveguide with side lengths c and d, the wall radius is given by

$$\begin{aligned} r_{w,rect}(\phi) &= \frac{d}{2\cos\phi}, \quad 0 < \phi < \tan^{-1}\left(\frac{c}{d}\right) \\ &= \frac{c}{2\sin\phi}, \quad \tan^{-1}\left(\frac{c}{d}\right) < \phi < \frac{\pi}{2}. \quad (3) \end{aligned}$$

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The Fourier expansion coefficients in (1) for a rectangular waveguide can be expressed as Fourier integrals of the wall radius given by (3). For the case of taper2, the expansion coefficients in (1) can be linearly interpolated in z in terms of the expansion coefficients of a rectangular waveguide. Thus, the mode converter's Fourier expansion coefficients in (1) may be determined anywhere along the axis in terms of the design parameters, *viz.,* a, ϵ_{oval}, and the dimensions of the rectangular waveguide.

For TE modes, the mode vector function $\vec{e}_i$ which is proportional to the RF electric field inside the waveguide is given by [2],

$$\vec{e}_i = \hat{z} \times \nabla_\perp \Psi_i. \tag{4}$$

$\nabla_\perp$ is the gradient operator transverse to the waveguide axis and $\hat{z}$ is the unit vector in the direction of the waveguide axis and Ψ_i is the mode eigen function.

The eigen function for a mode in a nonlinear waveguide may be written as a Fourier series of the mode in a circular waveguide perturbed by a small expansion factor ϵ, which may be expressed as

$$\Psi_s = \sum_i^H \alpha_{2i} J_{2i} \left(k_{\perp s} r\right) \cos\left(2i\phi\right), \tag{5}$$

where $\alpha_{2i} = \sum_j A_{2i,j}\epsilon^j$ and $k_{\perp s} = \frac{\sum_j \chi_j \epsilon^j}{a}$. $A_{2i,j}$ and χ_j are the expansion coefficients of the mode amplitude and mode cutoff wave number at any given cross section, respectively. We use subscript s to represent any mode inside the mode converter.

By applying the boundary condition that the tangential component of the electric field at the waveguide wall is zero,

$$\vec{e}_s \cdot \frac{\partial \vec{r}_w}{\partial \phi} = 0, \tag{6}$$

where $\vec{r}_w = r_w \hat{r}$, we can determine all the expansion coefficients $A_{2i,j}$ and χ_j in (5) at any cross section along the axis of the nonlinear waveguide and hence determine the eigen function for both modes.

MODE COUPLING

This section describes a method developed by Solymar [3] to estimate the scattering of modes in a nonlinear waveguide.

The inter-mode coupling in a nonlinear waveguide may be accounted through Telegrapher's equations of the form,

$$\begin{aligned} \frac{dV_i}{dz} &= -jk_{z_i} Z_i I_i + \sum_m T_{im} V_m \\ \frac{dI_i}{dz} &= -j\frac{k_{z_i}}{Z_i} V_i - \sum_m T_{mi} I_m, \end{aligned} \tag{7}$$

where V_i and I_i are the mode voltage and current, $k_{z_i} = \sqrt{k^2 - k_{\perp_i}^2}$ is the uncoupled propagation constant for the ith mode, $k_{\perp_i}$ is the cutoff wave number of the ith mode, k is the propagation constant in free space, m denotes all other modes including the main mode in the waveguide, and Z_i is the mode wave impedance.

The mode coupling coefficients given in (7) are given by,

$$T_{mi} = \int_S \vec{e}_m \cdot \frac{\partial \vec{e}_i}{\partial z} dS, \tag{8}$$

where S is the cross sectional surface of the nonlinear waveguide.

Assuming that the modes considered are above cutoff, the mode voltage V_i and mode current I_i may be expressed in terms of forward and backward wave amplitudes, A_i^+ and A_i^-. We assume that only two modes, M_1 and M_2 are present inside the non-linear waveguide and there are no reflections. Under these conditions the amplitude of the ith mode due to the coupling with the mth mode is described by,

$$\frac{dA_i^+}{dz} + jk_{z_i} A_i^+ = S_{im}^+ A_m^+. \tag{9}$$

where

$$S_{im}^+ = \frac{1}{2}\left[\sqrt{\frac{k_{z_i}}{k_{z_m}}} T_{mi} - \sqrt{\frac{k_{z_m}}{k_{z_i}}} T_{im}\right], \tag{10}$$

is the transfer coefficient between the two modes.

RESULTS

Inside the mode converter the modes M_1 and M_2 may be considered as perturbations of circular waveguide $TE_{0,1}$-mode and $TE_{2,1}$-mode respectively. Then the known expansion coefficients in (5) for mode M_1 are $A_{0,0} = 1$, $A_{0,j} = 0$ for $j \neq 0$, $A_{2i,0} = 0$ for $i \neq 0$ and $\chi_0 = 3.832$ (eigen number corresponding to $TE_{0,1}$-mode in a circular waveguide) and for mode M_2 are $A_{0,0} = 0$, $A_{2,0} = 1$, $A_{2,j} = 0$ for $j \neq 0$, $A_{2i,0} = 0$ for $i \neq 1$ and $\chi_0 = 3.054$ (eigen number corresponding to $TE_{2,1}$-mode in a circular waveguide). Using these known expansion coefficients the unknown coefficients $A_{2i,j}$ and χ_j may be determined by expanding the left hand side of (6) in ϵ and equating the expansion coefficients to zero to find the eigen functions for the two modes M_1 and M_2 anywhere inside the mode converter. The accuracy of the solution increases as we consider more number of RF harmonics, H, as well as more number of waveguide wall radius expansion harmonics, P, to represent a rectangular waveguide in (1). As H and P are increased the number of expressions that need to be solved to determine the expansion coefficients increases rapidly into thousands. This necessitates the use of a symbolic solver like Mathematica [4] which we have used in our calculations.

We have considered the rectangular waveguide wall radius as a Fourier expansion given in (1) with $P = 5$. Also we have assumed that there are, $H = 6$, RF harmonics inside the mode converter. We have optimized the length of taper1, taper2 and oval waveguide as well as ϵ_{oval} to obtain the lowest level of the mode M_2 at the end of the mode converter at 11.424 GHz. The following are the dimensions of the mode converter that were obtained by this exercise. Radius of circular waveguide, $a = 1.905$ cm. Rectangular waveguide sides, $c = 1.52$ cm, $d = 1.82$ cm. Taper length between circular and oval waveguides (taper1), $L_1 = 2.43$ cm. Length of oval waveguide $= 1.975$ cm. Taper length between oval and rectangular waveguide (taper2)$= 3.0$ cm. $\epsilon_{oval} = 0.1132$.

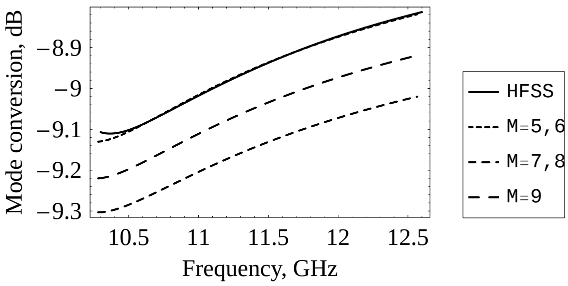

Figure 1: Level of mode M_2 at the end of taper1 calculated using HFSS and perturbation techniques.

We have studied the frequency response of the above geometry with different number of RF harmonics H. We first consider only taper1 for our study. Fig.1 shows the normalized amplitude of the mode M_2 (normalized to the amplitude of the main mode M_1) at the end of taper1 for $H = 5 - 9$, along with the results of HFSS field solver simulation (see reference [5]) for the same geometry as a function of frequency. We see that the frequency response has similar characteristics for all values of H except for a difference in the level of M_2. It is interesting to note that the level of M_2 changes very little for $H = 5, 6$ and for $H = 7, 8$. We also see from Fig.1 that the results of perturbation theory matches reasonably closely with that of HFSS field solver simulations at the end of taper1 (within 0.2 dB over the frequency range considered).

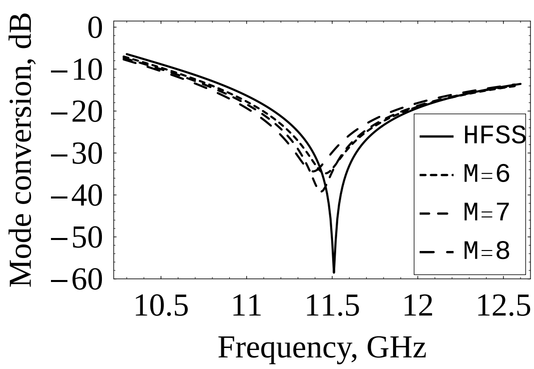

Figure 2: Level of mode M_2 at the end of mode converter. A fifth order Fourier approximation ($P = 5$) for the wall radius of the rectangular waveguide is used.

In Fig.2 we have shown the normalized amplitude of the mode M_2 (normalized to the amplitude of the main mode M_1) at the end of the mode converter (beginning of the rectangular waveguide) when the rectangular wall radius is expanded up to the 5th component of the Fourier series ($P = 5$). For a fair comparison between perturbation theory and HFSS simulations, we have used exactly the same geometry for our simulations in HFSS where the rectangular waveguide wall radius is approximated by a 5th order Fourier expansion.

In Fig.2 mode conversion into the mode M_2 was calculated using the perturbation theory with $H = 6$, 7 and 8 RF harmonics. A small change in the frequency response characteristics is observed with change in the number of RF harmonics considered in our calculations using perturbation techniques. The resonant frequency as predicted by our analytical method based on perturbation techniques is within 1% of the resonant frequency predicted by HFSS for all the three cases considered. However, the resonant frequency reduces with increase in the number of RF harmonics, H, constituting a "drift" from the resonant frequency predicted by HFSS, as can be seen from Fig.2. The reason for this descripency may be due to the approximations used in the perturbation theory where all reflections inside the mode converter are neglected.

CONCLUSIONS

We have developed an analytical perturbation technique that was used to design a compact circular to rectangular waveguide transition. This technique predicts the level of a spurious mode inside the mode converter which is a nonlinear waveguide within reasonable accuracy compared to HFSS field solver simulations. As the perturbation technique presented in this work leads to an analytical solution, the calculations are much faster than HFSS. Therefore this technique, which is quite general, can be an attractive tool in the design and analysis of a wide variety of nonlinear waveguides.

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