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New kinematical constraints on cosmic acceleration

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We present and employ a new kinematical approach to 'dark energy' studies. We construct models in terms of the dimensionless second and third derivatives of the scale factor $a(t)$ with respect to cosmic time t, namely the present-day value of the deceleration parameter q_0 and the cosmic jerk parameter, $j(t)$. An elegant feature of this parameterization is that all ΛCDM models have $j(t) = 1$ (constant), which facilitates simple tests for departures from the ΛCDM paradigm. Applying our model to redshift-independent distance measurements, from type Ia supernovae and X-ray cluster gas mass fraction measurements, we obtain clear statistical evidence for a late time transition from a decelerating to an accelerating phase. For a flat model with constant jerk, $j(t) = j$, we measure $q_0 = -0.81 \pm 0.14$ and $j = 2.16^{+0.81}_{-0.75}$, results that are consistent with ΛCDM at about the 1σ confidence level. In comparison to dynamical analyses, the kinematical approach uses a different model set and employs a minimum of prior information, being independent of any particular gravity theory. The results obtained with this new approach therefore provide important additional information and we argue that both kinematical and dynamical techniques should be employed in future dark energy studies, where possible.

1. Introduction

Late-time acceleration of the Universe is now an observed fact.[2,4,9,11] Most current analyses of cosmological data assume General Relativity and the Friedmann equations and employ the mean matter density of the Universe, Ω_m, and the dark energy equation of state, w, as model parameters. Other dynamical analyses employ modified Friedmann equations for a particular gravity model. However, a purely kinematical approach is also possible that does not assume any particular gravity theory. Kinematical models provide important, complementary information when seeking to understand the origin of the observed late-time accelerated expansion.[1,6–8,11–14]

In Rapetti et al. (2007)[10] we develop an improved method for studying the kinematical history of the Universe. Instead of using parameterizations constructed in terms of the deceleration parameter $q(z)$, we introduce a new kinematical framework using the cosmic jerk,[5] the dimensionless third derivative of the scale factor with respect to cosmic time. We apply our method to the 'gold' sample of type Ia supernovae (SNIa) measurements,[11] the SNIa data from the first year of the Supernova Legacy Survey (SNLS) project,[4] and the X-ray galaxy cluster distance measurements of Allen et al. (2007).[3]

2. A new kinematical framework

We rewrite the defining equation for the jerk parameter, $j(a) = (a^2H^2)''/2H^2$, in a more convenient form[5] $a^2V''(a) - 2j(a)V(a) = 0$ where prime denotes derivative with respect to a and $V(a)$ is defined as $V(a) = -a^2H^2/2H_0^2$. We specify the two constants of integration required by this differential equation in terms of the present

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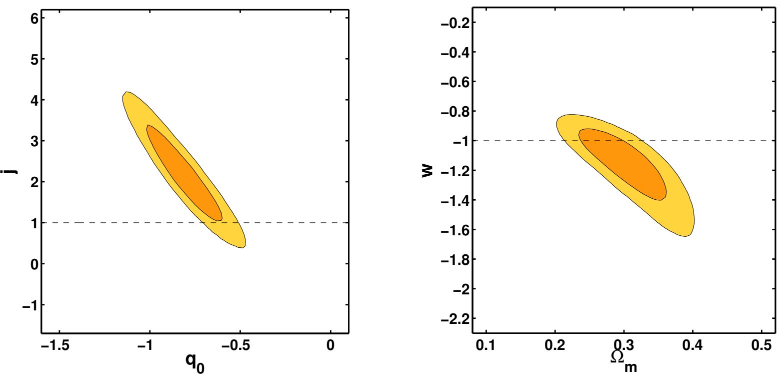

Fig. 1. The left panel shows the 68.3 and 95.4 per cent confidence limits in the (q_0,j) plane for the kinematical model with a constant jerk, j, obtained using all three data sets: both SNIa data sets[4,11] and the X-ray clusters data.[3] The right panel shows the results in the standard ($\Omega_{\rm m}$,w) plane obtained using the same three data sets and assuming HST, BBNS and b priors. (Note that the kinematical analysis does not use the HST, BBNS and b priors). The dashed lines show the expectation for a cosmological constant model in both formalisms ($j=1$, $w=-1$, respectively).

Hubble parameter H_0 and the present deceleration parameter q_0, $V(1)=-1/2$ and $V'(1)=q_0$ where $a(t_0)=1$ at the present time t_0. Here the first condition comes from $H(1)=H_0$ and the second from $V'(1)=-(H_0'/H_0)-1=q_0$. Allowing a constant deviation from ΛCDM ($j=1$), i.e. a constant j model (for more complicated $j(a)$ models see Rapetti et al. (2007)[10]), we solve the jerk differential equation analytically obtaining

$$V(a)=-\frac{\sqrt{a}}{2}\left[\left(\frac{p-u}{2p}\right)a^{p}+\left(\frac{p+u}{2p}\right)a^{-p}\right] \tag{1}$$

where $p\equiv(1/2)\sqrt{(1+8j)}$ and $u\equiv 2(q_0+1/4)$. The Hubble parameter, $H(a)$, obtained from (1) is used to calculate the angular diameter ($d_{\rm A}$) and luminosity ($d_{\rm L}$) distances for a flat Friedmann-Robertson-Walker-Lemaître (FRWL) metric $d_{\rm A}(a)=a^2\,d_{\rm L}(a)=c/H_0\,a\int_a^1 1/(a^2E(a))\,da$, where c is the speed of light and $E(a)=H(a)/H_0$. These theoretical distances $d_{\rm L}(a)$ and $d_{\rm A}(a)$ are then used to fit the data (for details about the data analysis see Rapetti et al. (2007)[10]).

3. Results

Combining all three data sets, we obtain tight constraints on $q_0=-0.81\pm0.14$ and $j=2.16^{+0.81}_{-0.75}$. Our result represents the first measurement of the jerk parameter from cosmological data. Our dynamical analysis of the same three data sets gives $w=-1.15^{+0.14}_{-0.18}$ and $\Omega_{\rm m}=0.306^{+0.042}_{-0.040}$. Figure 1 shows the constraints for both the kinematical (q_0,j; left panel) and dynamical ($\Omega_{\rm m},w$; right panel) models, using all three data sets combined. In both cases, the dashed lines indicate the expected range of results for ΛCDM models (i.e. a cosmological constant). It is important to recognise that the results from the kinematical and dynamical analyses constrain different sets of departures from ΛCDM. We are using two simple, but very different parameterizations based on different underlying assumptions.

4. Conclusions

We have developed a new kinematical approach[10] to study the expansion of the history of the Universe. Our technique uses the parameter space defined by the current value of the cosmic deceleration parameter q_0 and the jerk parameter j. The use of this (q_0, j) parameter space provides a natural framework for kinematical studies. In particular, it provides a simple prescription for searching for departures from ΛCDM, since the complete set of ΛCDM models are characterized by $j = 1$.

Using type Ia supernovae and X-ray galaxy clusters data and assuming geometric flatness, we measure $q_0 = -0.82 \pm 0.14$ and $j = 2.16^{+0.81}_{-0.75}$ (Figure 1). Note that this represents the first measurement of the cosmic jerk parameter, j. We suggest that future studies should endeavour to use both kinematical and dynamical approaches where possible, in order to extract the most information from the data. The combination of techniques may be especially helpful in to distinguish between an origin for cosmic acceleration that lies with dark energy (i.e. a new energy component to the Universe) from modifications to General Relativity.

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