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Gravi-Leptogenesis: Leptogenesis from Gravity Waves in Pseudo-scalar Driven Inflation Models

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In this talk we present a mechanism for leptogenesis which is based on gravity waves produced during inflation. We show that when inflation is driven by a pseudo-scalar field the metric perturbations generated during inflation can become birefringent, therefore giving a non-vanishing contribution to the gravitational triangle anomaly and sourcing lepton anti-lepton asymmetry. As this asymmetry is sourced by the fields which are active during inflation, it is not washed out or diluted by inflation. The amount of matter asymmetry generated in our model can be of realistic size for the parameters within the range of some inflationary scenarios and grand unified theories. This talk is based on [1] which has appeared on the arXiv as hep-th/0403069.

I. INTRODUCTION

One of the puzzles of astroparticle physics and cosmology, which has been around for more than half a century and since the existence of antimatter and anti-particles established in collider experiments, has been to explain, if our universe is mostly made out of matter, why and how this has happened during the course of the evolution of the universe starting from a symmetric soup of matter and antimatter soon after the Big Bang.

The fact that in the parts of the universe visible to us there is an excess of matter over antimatter has now been backed by the recent determinations of the cosmological parameters from the cosmic microwave background observations and the WMAP experiment. Quantitatively this asymmetry is usually given through the ratio of excess of baryon density to the photon density [2]

$$\frac{n_B}{n_\gamma} = (6.5 \pm 0.4) \times 10^{-10} \ , \qquad (1)$$

where $n_B = n_b - n_{\bar{b}}$ and n_γ is the number density of photons. This ratio has the nice property that is time independent, as the evolution of the n_b and n_γ with the cosmic Hubble expansion are the same. This is a small number, but at the same time it is large enough to be a puzzle for models of particle physics.

About forty years ago Sakharov stated the three necessary conditions to generate a matter-antimatter asymmetry dynamically from a symmetric initial conditions [3]:
i) our particle physics model should have baryon number violating vertices.
ii) CP should be violated.
iii) CP and baryon number violating interactions should be active at a time when the universe is out of thermal equilibrium.

A baryon excess this large cannot be produced in the early universe within the Standard Model (SM) of particle physics [4]. This is due to the fact that baryon number violating interactions in the Standard Model are loop suppressed and the only source of CP violation in the hadronic sector is in the Dirac phase of the CKM mixing matrix, which is not enough for explaining the baryon asymmetry observed today. Moreover, assuming that the scale of inflation is larger than TeV, *i.e.* the SM is at work after inflation is ended, the out-of-equilibrium condition can be created at phase transitions or through late decay of massive particles. The most attractive choice for a phase transition is that associated with electroweak symmetry breaking. However, that phase transition is probably not sufficiently strongly first-order.

Although what is observed is a baryon asymmetry, since the 1980's it has been realized that the standard weak interactions contain processes, mediated by *sphalerons* ($SU(2)$ instantons), which interconvert baryons and leptons and are thermally activated at temperatures greater than 1 TeV. Thus, we can also create the baryon asymmetry by creating net lepton number at high temperature through out-of-equilibrium and CP-asymmetric processes [5, 6]. Scenarios of this type are known as *leptogenesis*.

To use the possibility of lepton asymmetry and sphalerons we, however, need to fulfill Sakharov conditions for leptons. Within the usual SM this does not solve the issue and we are hence forced to associate the observed baryon asymmetry of the universe to physics beyond the SM.

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Most of the allowed parameter space of the minimal supersymmetric Standard Model has already been excluded and large CP violating phases are strongly constrained in supersymmetric models [7] though they still could appear in the neutrino Yukawa couplings that are used in the Fukugita-Yanagida scenario for leptogenesis [6]. Models that can explain the baryon excess typically involve exotic nonstandard physics, CP violating couplings in the Higgs or supersymmetry sectors or in the couplings of the heavy neutral leptons associated with neutrino mass [8]. In any event, there is good reason to seek more effective sources of CP-violating out-of-equilibrium physics.

Here we present a new mechanism for the creation of the matter-antimatter asymmetry, one associated with gravitational fluctuations created during cosmological inflation [1].

II. OUTLINE OF THE MECHANISM

Let us first spell out how the three Sakharov conditions are realized in our model of matter-antimatter asymmetry, the *gravi-leptogenesis*.

A. Lepton number violation

The lepton number violation in our model comes from triangle anomaly. As it is well-known [9], the lepton number current, and hence the total fermion number current, has a gravitational anomaly in the Standard Model. Explicitly,

$$\partial_\mu J^\mu_\ell = \frac{N}{16\pi^2} R\tilde{R} \tag{2}$$

where

$$J^\mu_\ell = \sum_{i=L,R} \bar{\ell}_i \gamma^\mu \ell_i + \bar{\nu}_i \gamma^\mu \nu_i \ , \tag{3a}$$

$$R\tilde{R} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}{}^{\rho\sigma} \ , \tag{3b}$$

and $N = N_L - N_R$, which is three in the Standard Model. The anomaly is a consequence of an imbalance between left- and right-handed leptons. In general when heavy right-handed neutrinos are also added to the Standard Model, as is done in the seesaw mechanism for explaining the smallness of the neutrino mass, (2) will be correct in an effective theory valid below a scale μ, of order of the right-handed neutrino mass. More concretely N can in general be a function of energy. At low energies, below the right-handed neutrino mass scale $N = 3$. At higher energies, N could be anywhere between zero to three, depending on the details of the particle physics model invoked. In the usual seesaw scenarios with three right handed neutrinos, μ can be as large as 10^{14}, for energies below μ $N = 3$, and above that $N = 0$ [11, 12]. In any case, here we do not restrict ourselves to a specific particle physics model, and keep an open mind on larger values of μ.

B. CP violation

The need for CP violation manifests itself in our model through the fact that a non-zero lepton number generation can be achieved when $\langle R\tilde{R}\rangle$ is non-vanishing. As we will show explicitly, $R\tilde{R}$ receives a contribution with a definite sign from gravitational fluctuations produced during inflation, which is driven by a *pseudo-scalar field*. In other words, CP-violation in our model arises from the inflaton field with a CP-odd component. Note that during inflation the expectation value of the inflaton is non-vanishing and is rolling in time.

Such models of inflation can be naturally achieved if the inflaton is a complex modulus field such as one finds in supergravity or superstring models. In order to use these models, however, we need to make sure that they have flat enough potentials required for (slow-roll) inflation. The simplest model of this kind is when we have a single field inflation and a pseudo-scalar ϕ as the inflaton, known as natural inflation, however it can be incorporated to have multiple axions such as in N-inflation models [16]. In fact, these models of inflation fit quite nicely into extensions of the standard model and in string inspired inflation.

The imaginary part ϕ of this field (which we henceforth call an 'axion') can couple to gravity through an interaction

$$\Delta\mathcal{L} = F(\phi) R\tilde{R} \ , \tag{4}$$

where F is odd in ϕ. Under P and CP

$$\phi \to -\phi \ \& \ \ F(\phi) \to -F(\phi).$$

Terms like (4) would generically appear once we integrate out heavy fermions axially coupled to ϕ or as a result of the Green-Schwarz mechanism [13]. In the Appendix we will explicitly show how a linear F of the form

$$F(\phi) = \frac{\mathcal{N}}{(16\pi^2 M_{\rm Pl})}\phi, \tag{5}$$

with $\mathcal{N}$ depending on the details of string compactification, arises from heterotic string theory compactified to four dimensions [14].

C. Out-of-equilibrium

Out of equilibrium in our model is achieved noting that we apply the interaction in (4) to the dynamics of

metric fluctuations during *inflation* where due to (exponential) growth of the background space-time, the lepton number production is naturally out of equilibrium.

We are now ready to compute the amount of the lepton number generated during inflation. In our analysis, we assume that we have a given successful model of inflation which involves an axion field and do not elaborate on the details of the inflationary model. The axion driven models of inflation, the natural inflation, are preferred from a particle physics perspective. Natural inflation gives a small self coupling for the inflaton field without fine tuning [10]. Throughout this note we use notations and conventions of [15], in particular $M_{\rm Pl} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass.

III. GRAVITY WAVE EVOLUTION

To begin, we must compute the production of gravitational waves during inflation under the influence of the coupling (4). The action which describes the gravity waves is hence

$$\mathcal{L} = \frac{1}{2} M_{pl}^2 \sqrt{-\det\ g}\ R + F(\phi) R\tilde{R}. \tag{6}$$

In general metric perturbations about an FRW universe can be parameterized as

$$\begin{aligned} ds^2 &= -(1+2\varphi)dt^2 + w_i dt dx^i \\ &\quad + a^2(t)\left[((1+2\psi)\delta_{ij} + h_{ij})\, dx^i dx^j\right] \end{aligned} \tag{7}$$

where φ, ψ, w_i and h_{ij} respectively parameterize the scalar, vector, and tensor fluctuations of the metric. It is straightforward to show that the scalar and vector perturbations do not contribute to $R\tilde{R}$, and so we ignore these fluctuations in the following discussion. We can also fix a gauge so that the tensor fluctuations are parameterized by the two physical transverse traceless elements of h_{ij}. For such physical gravity waves which are moving in the z direction, the metric takes the form

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)\big[(1-h_+)dx^2 \\ &\quad + (1+h_+)dy^2 + 2h_\times dx dy + dz^2\big] \end{aligned} \tag{8}$$

where $a(t) = e^{Ht}$ during inflation and h_+, $h_\times$ are functions of t, z.

To see the CP violation more explicitly, it is convenient to use a helicity basis

$$h_L = \frac{1}{\sqrt{2}}(h_+ - ih_\times)\ , \quad h_R = \frac{1}{\sqrt{2}}(h_+ + ih_\times)\ . \tag{9}$$

Here h_L and h_R are complex conjugate scalar fields. To be very explicit, the negative frequency part of h_L is the conjugate of the positive frequency part of h_R, and both are built from wavefunctions for left-handed gravitons.

A. The equations of motion

Plugging (8) into the action (6), up to second order in h_L and h_R, we obtain

$$\begin{aligned} \mathcal{L} &= -(h_L \Box h_R + h_R \Box h_L) \\ &\quad + 16iF(\phi)\Bigg[\left(\frac{\partial^2}{\partial z^2}h_R \frac{\partial^2}{\partial t \partial z}h_L - \frac{\partial^2}{\partial z^2}h_L \frac{\partial^2}{\partial t \partial z}h_R\right) \\ &\quad + a^2\left(\frac{\partial^2}{\partial t^2}h_R \frac{\partial^2}{\partial t \partial z}h_L - \frac{\partial^2}{\partial t^2}h_L \frac{\partial^2}{\partial t \partial z}h_R\right) \\ &\quad + Ha^2\left(\frac{\partial}{\partial t}h_R \frac{\partial^2}{\partial t \partial z}h_L - \frac{\partial}{\partial t}h_L \frac{\partial^2}{\partial t \partial z}h_R\right)\Bigg] + \mathcal{O}(h^4) \end{aligned} \tag{10}$$

where

$$\Box = \frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{1}{a^2}\frac{\partial^2}{\partial z^2}.$$

As it is explicitly seen from (10), if h_L and h_R have the same *dispersion relation*, $R\tilde{R}$ vanishes. Thus, nonzero $R\tilde{R}$ requires "cosmological birefringence" during inflation. Such an effect is induced by the addition of (4) to the gravitational equations. Lue, Wang, and Kamionkowski (LWK) [18] and Alexander and Martin [19] have studied the effects of such an interaction in generating observable parity-violation in the cosmic microwave background. In a future work, this coupling will be related to an observable to detect birefringence in binary systems for the LISA and Advanced LIGO gravitational wave detectors [17].

It is straightforward to obtain equations of motion for h_L and h_R:

$$\Box h_L = -2i\frac{\Theta}{a}\dot{h}'_L\ , \qquad \Box h_R = +2i\frac{\Theta}{a}\dot{h}'_R\ , \tag{11}$$

where

$$\begin{aligned} \Theta &= \frac{4}{a^2}\frac{d}{dt}(\dot{F}a^2)/M_{Pl}^2 \\ &\simeq 4(F''\dot{\phi}^2 + 2F'H\dot{\phi})/M_{Pl}^2 \end{aligned} \tag{12}$$

dots denote time derivatives, and primes denote differentiation of F with respect to ϕ. To obtain the above equations we have used the fact that the inflaton field is only a function of time t. In the second line of the expression for Θ, assuming the slow-roll inflation, we have dropped the terms proportional to $\ddot{\phi}$ (explicitly we have dropped $4F'\ddot{\phi}/M_{Pl}^2$).

Note that with a constant ϕ, (4) is the Gauss-Bonnet term and being a total divergence (in four dimensions), cannot affect the equations of motion; thus, all terms in Θ involve derivatives of ϕ. These equations should be compared to those for evolution in flat space given by LWK [18]. The new term proportional to $H\dot{\phi}$ leads to a substantial enhancement

in the size of Θ. With this simplification, and the approximate form (5),

$$\Theta = \frac{\sqrt{2\epsilon}}{2\pi^2}\left(\frac{H}{M_{Pl}}\right)^2 \aleph \ , \tag{13}$$

where $\epsilon = \frac{1}{2}(\dot{\phi})^2/(HM_{\rm Pl})^2$ is the slow-roll parameter of inflation [15].

B. Gravitational birefringence

To see gravitational birefringence we need to solve the equations of motion explicitly. Let us focus on the evolution of h_L and, more specifically, on its positive frequency component. It is convenient to introduce conformal time

$$\eta = \frac{1}{Ha} = \frac{1}{H}e^{-Ht} \ . \tag{14}$$

(Note that conformal time η runs in the opposite direction from t.) The evolution equation for h_L then becomes

$$\frac{d^2}{d\eta^2}h_L - 2\frac{1}{\eta}\frac{d}{d\eta}h_L - \frac{d^2}{dz^2}h_L = -2i\Theta\frac{d^2}{d\eta dz}h_L \ . \tag{15}$$

If we ignore Θ for the moment and let $h_L \sim e^{ikz}$, this becomes the equation of a spherical Bessel function:

$$\frac{d^2}{d\eta^2}h_L - 2\frac{1}{\eta}\frac{d}{d\eta}h_L + k^2 h_L = 0 \tag{16}$$

for which the positive frequency solution is

$$h_L^+(k,\eta) = e^{+ik(\eta+z)}(1-ik\eta) \ . \tag{17}$$

We now look for solutions to (15) with $h_L \sim e^{ikz}$. To do this, let

$$h_L = e^{ikz}\cdot(-ik\eta)e^{k\Theta\eta}g(\eta) \tag{18}$$

where $g(\eta)$ is a Coulomb wave function, we then have

$$\frac{d^2}{d\eta^2}g + \left[k^2(1-\Theta^2) - \frac{2}{\eta^2} - \frac{2k\Theta}{\eta}\right] g \ = \ 0 \ . \tag{19}$$

This is the equation of a Schrödinger particle with $\ell = 1$ in a weak Coulomb potential. When $\Theta = 0$, the Coulomb term vanishes and we find the spherical Bessel function (17). For h_L, the Coulomb term is repulsive; for h_R, with the opposite sign of the Θ term, the Coulomb potential is attractive. This leads to attenuation of h_L and amplification of h_R in the early universe. This is the anticipated cosmological birefringence which was also discussed by LWK [18]. (For a more detailed treatment see [19].)

As we will see generation of the matter asymmetry is dominated by modes at short distances (subhorizon modes) and at early times. This corresponds to the limit $k\eta \gg 1$. In this region, we can ignore the potential terms in (19) and take the solution to be approximately a plane wave. More explicitly,

$$g(\eta) = \exp[ik(1-\Theta^2)^{1/2}\eta(1+\alpha(\eta))] \ , \tag{20}$$

where $\alpha(\eta) \sim \log\eta/\eta$.

IV. THE GREEN'S FUNCTION

We would now like to use (18) to compute the expectation value of $R\tilde{R}$ in an inflationary space-time. It turns out that it will be dominated by the subhorizon, *quantum* part of the gravity-wave evolution. Hence, to compute $\langle R\tilde{R}\rangle$ we only need the two point (Green's) function $\langle h_L h_R\rangle$:

$$\begin{aligned} G(x,t;x',t') &= \langle h_L(x,t)h_R(x',t')\rangle \\ &= \int \frac{d^3k}{(2\pi)^3}e^{ik\cdot(x-x')}G_k(\eta,\eta') \ . \end{aligned} \tag{21}$$

For k parallel to z, the Fourier component G_k satisfies (15) with a delta-function source

$$\left[\frac{d^2}{d\eta^2} - 2(\frac{1}{\eta}+k\Theta)\frac{d}{d\eta} + k^2\right]G_k(\eta,\eta') = i\frac{(H\eta)^2}{M_{\rm Pl}^2}\delta(\eta-\eta'). \tag{22}$$

For $\Theta = 0$, the solution of this equation is

$$G_{k0}(\eta,\eta') = \begin{cases} \aleph h_L^+(k,\eta)h_R^-(-k,\eta') & \eta < \eta' \\ \aleph h_L^-(k,\eta)h_R^+(-k,\eta') & \eta' < \eta \ , \end{cases} \tag{23}$$

where

$$\aleph \equiv \left(\frac{H}{M_{Pl}}\right)^2\frac{1}{2k^3} \tag{24}$$

and h_L^- is the complex conjugate of (17), and h_R^+, h_R^- are the corresponding solutions of the h_R equation. For $\Theta = 0$, these solutions are the same as for h_L, but the structure of (23) will be preserved when we go to the case $\Theta \neq 0$. The leading effect of Θ is to introduce the exponential dependence from (18),

$$G_k = e^{-k\Theta\eta}e^{+k\Theta\eta'}G_{k0} \tag{25}$$

for both $\eta > \eta'$ and $\eta < \eta'$. The prefactor is modified in order Θ^2, and the wavefunctions acquire additional corrections that are subleading for $k\eta \gg 1$. Neither of these effects will be important for our result.

The Green's function (25) can now be used to contract h_L and h_R to evaluate the quantum expectation value of $R\tilde{R}$. The result is

$$\langle R\tilde{R}\rangle = \frac{16}{a}\int\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3M_{\rm Pl}^2}(k\eta)^2\cdot k^4\Theta + \mathcal{O}(\Theta^3) \tag{26}$$

where we have picked up only the leading behavior for $k\eta \gg 1$.

We would like to emphasize that our expression for $\langle R\tilde{R}\rangle$ is nonzero because of the effect of inflation in producing a CP asymmetry out of equilibrium. The original quantum state for the inflaton might have had nonzero amplitude for a range of values of ϕ and might even have been CP-invariant. However, inflation collapses the wavefunction onto a particular value of ϕ that is caught up in the local expansion of the universe. This value gives us a classical background that is CP-asymmetric.

The above result and computations seems to be crucially dependent on the form of the Green's function or the vacuum state we have used. To resolve the possible ambiguity in this regards, one may perform the above computation using a different method, the fermion level crossing, e.g. following [20]. This computation confirms the above results [21].

V. COMPUTING THE RATIO OF LEPTON TO PHOTON DENSITIES

To complete our leptogenesis model, there are two remaining steps. First, we need to compute the net lepton anti-lepton asymmetry generated. This, in our model, should be computed during inflation. We then need to have a reheating model through which we can compute the temperature and hence the entropy of the Universe after the inflation ended.

A. Lepton number density

We are now ready to evaluate the lepton density that arises through the gravitational anomaly (2). Inserting (26) into (2) and integrating over the time period of inflation, we obtain

$$n = \int_0^{H^{-1}} d\eta \int \frac{d^3k}{(2\pi)^3} \frac{1}{16\pi^2} \frac{8H^2k^3\eta^2\Theta}{M_{\rm Pl}^2} , \qquad (27)$$

where n is the lepton number density. The integral over k runs over all of momentum space, up to the scale μ at which our effective Lagrangian description breaks down. The dominant contribution comes not from the modes which have left the horizon by the end of inflation (the super-horizon modes), $k/H < 1$, but rather from very short distances compared to these scales. The integral over η is dominated at large values of η, toward the end of inflation (*cf.* (14)). The integral represents a compromise between two effects of inflation, first, to blow up distances and thus carry us to larger physical momenta and, second, to dilute the generated lepton number through expansion. It is now clear that the dominant contribution to the right-hand side comes from $k\eta \gg 1$, as we had anticipated.

We can now perform the integrals to find the lepton number density produced by the end of inflation

$$n = \frac{1}{72\pi^4}\left(\frac{H}{M_{\rm Pl}}\right)^2 \Theta H^3 \left(\frac{\mu}{H}\right)^6 . \qquad (28)$$

Let us now analyze each factor in n:

• The factor $(H/M_{\rm Pl})^2$ is the magnitude of the gravity wave power spectrum. We should stress that the usual gravity wave power spectrum is coming from the super-horizon modes, while the main contribution to n has come from the sub-horizon modes.

• The factor Θ is a measure of the effective CP violation caused by birefringent gravity waves.

• The factor H^3 is the inverse horizon size at inflation; this gives the density n appropriate units.

• Finally, the factor $(\mu/H)^6$ gives the enhancement over one's first guess due to our use of strongly quantum, short distance fluctuations to generate $R\tilde{R}$, rather than the super-horizon modes which effectively behave classically.

B. Photon number density

To understand the significance of the lepton number density (28), we should compare it to the entropy density of the Universe just after reheating, or to the photon number density. (Recall that almost all of the entropy of the Universe is generated during the reheating time and is carried in the massless degrees of freedom, i.e. photons.) In order this we need a reheating model.

To illustrate the physical relevance of the result of our model, we take the simplest (and at the same time very naive) reheating model, the instant reheating model. That is, we assume that all of the energy of the inflationary phase has been converted to the heat of a gas of massless particles instantaneously. The reheating converts the energy density of the inflaton field ϕ

$$\rho = 3H^2M_{Pl}^2 \qquad (29)$$

to radiation with

$$\rho = \pi^2 g_* T^4/30 \qquad (30)$$

and to the entropy density s, $s = 2\pi^2 g_* T^3/45$, where T is the reheating temperature, g_* is the effective number of massless degrees of freedom and $s = 1.8 g_*\ n_\gamma$ [15]. (Here we have assumed that the evolution of the Universe after the reheating era has been adiabatic). This gives

$$n_\gamma = 1.28 g_*^{-3/4}(HM_{Pl})^{3/2}.$$

Recalling that the ratio of the present baryon number to the lepton number originally generated in leptogenesis is approximately $n_B/n_L = 4/11$ [5], in our model

we obtain

$$\frac{n_B}{n_\gamma} = 4.05 \times 10^{-5} g_*^{3/4} \left(\frac{H}{M_{Pl}}\right)^{7/2} \Theta \left(\frac{\mu}{H}\right)^6, \quad (31)$$

If we are less naive, we might follow the dilution of n and ρ with the expansion of the universe to the end of reheating. The final result is the same (see, however, [23] for a comment on this point). With the adiabatic expansion assumption, (31) can be compared directly to the present value of n_B/n_γ given in (1).

C. Numerical results

We should now estimate the numerical value of n_B/n_γ in our model to see if we have a viable leptogenesis model. In our final result (31) we have five dimensionless parameters, g_*, H/M_{Pl}, μ/M_{Pl} and the slow-roll parameter ϵ and $\mathcal{N}$ (the latter two appear through the CP violation parameter Θ). Inserting the expression for Θ we have

$$\frac{n_B}{n_\gamma} \simeq 2.9 \times 10^{-6} g_*^{3/4} \sqrt{\epsilon}\, \mathcal{N} \left(\frac{H}{M_{Pl}}\right)^{-1/2} \left(\frac{\mu}{M_{Pl}}\right)^6. \quad (32)$$

Within the usual supersymmetric particle physics models $g_* \sim 100$ is a reasonable choice. The WMAP data, through the density perturbation ratio $\delta\rho/\rho$ (for a single field inflation) leads to an upper bound on H/M_{Pl} ratio as [22]

$$\frac{H}{M_{Pl}} \lesssim 10^{-4}, \qquad \epsilon \lesssim 0.01, \quad (33)$$

or $H \lesssim 10^{14}\ GeV$.

The factor $\mathcal{N}$ in (5) is inferred from the string theory compactification and is proportional to the square of the four dimensional M_{Pl} to ten dimensional (fundamental) Planck mass [14] (see also the Appendix). Within string theory,

$$\mathcal{N} \simeq 10^2 - 10^{10}$$

could be a reasonable range. Therefore, assuming that the scale of inflation H saturates its current bound $H/M_{Pl} \simeq 10^{-4}$,

$$\Theta \simeq 10^{-8} - 10^0. \quad (34)$$

The physically viable range for the parameter μ depends on the details of the underlying particle physics model on which our gravi-leptogenesis is based. For example within the Standard Model + three heavy right handed neutrinos (and the seesaw mechanism) μ could be of order of the right handed neutrino mass, in which case $\mu \lesssim 10^{12} GeV$. If we do not restrict ourselves to the seesaw mechanism, μ can be larger. The upper bound on μ is then only coming from the range of validity of our effective field theory analysis. For example, we can accommodate μ's as large as the string scale where quantum gravity effects become important. Therefore, depending on the details of the model

$$10^{12} GeV \lesssim \mu \lesssim 10^{16} - 10^{17} GeV$$

or

$$10^{-6} \lesssim \frac{\mu}{M_{Pl}} \lesssim 10^{-2} - 10^{-1} \quad (35)$$

is a reasonable range for μ.

In sum, assuming that scale of inflation H saturates its current bound $H \simeq 10^{14} GeV$, we have

$$\frac{n_B}{n_\gamma} \simeq 10^{+5}\ \Theta \left(\frac{\mu}{M_{Pl}}\right)^6. \quad (36)$$

Noting (34) and (35), it is readily seen that there is big parameter space for obtaining the observed baryon asymmetry of the Universe within our gravi-leptogenesis model.

VI. SUMMARY AND OUTLOOK

Here we have constructed a leptogenesis model in which the lepton asymmetry is a result of gravitational chiral anomaly. Indeed our model is a "module", which uses minimal ingredients and could be fitted into a successful inflation model in which the inflaton field(s) has a pseudo-scalar component. From the particle physics side, it is again like a module and many different particle physics models could be invoked. Here we did not discuss the details of neither the inflationary nor the underlying particle physics models. It is of course very important to explicitly fit our module in viable physical models.

As our construction is a leptogenesis model, we need a mechanism to convert the lepton asymmetry to baryon asymmetry. With the Standard Model this is usually performed via the thermally activated electroweak sphalerons, which in our model should be active after inflation. This happens for reheat temperature $T_r \gtrsim 1$ TeV. Within our naive "instant reheating" model that translates to $H \gtrsim 10^{-3}$ eV, which is not a serious constraint on the inflationary models used.

According to recent WMAP observations, the scalar metric perturbations generated during inflation have a size that gives density fluctuations with $\delta\rho/\rho \sim 10^{-5}$. On the other hand, noting (1), one has

$$\frac{n_B}{n_\gamma} \sim \left(\frac{\delta\rho}{\rho}\right)^2. \quad (37)$$

The above could be a simple numerical accident or there might be some underlying deeper physics. In any case, our model which invokes gravity waves as the source for the baryon asymmetry and operates during inflation, could be an interesting framework to uncover the possible deeper physics behind (37).

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Appendix A: Realization of the model within string theory [14]

To see how this term appears in string theory, consider e.g. the Heterotic SUGRA action:

$$S = M_{10}^8 \int d^{10}x \sqrt{\det g_{10}} \big(\mathcal{R} + \frac{1}{2}(\partial\Phi)^2 + \frac{1}{12} e^{-\Phi} H_{ABC}^2 + \frac{1}{4} e^{-\Phi/2} (F_{AB})^2\big)$$

where

$$H_3 = dB_2 - \frac{1}{4}\left(\Omega_3(A) + \alpha' \Omega_3(\omega)\right)$$

($\Omega_3(A)$ and $\Omega_3(\omega)$ are the gauge and gravitational Chern-Simons three-forms, respectively. Explicitly,

$$\Omega_3(\omega) = Tr(\omega \wedge d\omega + \frac{2}{3}\omega \wedge \omega \wedge \omega)$$

In particular note that

$$^*(d\Omega_3(\omega)) = R\tilde{R}.$$

Upon compactification to four dimensions, the H^2 term leads to

$$\begin{aligned} S &\sim M_{10}^8 \int d^6y \int d^4x \; e^{-\Phi} (dB + \alpha' \Omega_3(\omega))^2 \\ &= M_{10}^8 \frac{V_6}{g_s} \int d^4x [(dB)^2 + 2\alpha' (^*dB) \wedge \Omega_3(\omega) + \cdots] \\ &= \frac{M_{pl}^2}{g_s} \int d^4x ((\partial\phi)^2 - 2\alpha' \phi R\tilde{R}) \end{aligned}$$

where the pseudoscalar ϕ (our inflaton field) is dual to the two form B_2 and the 4d Planck length, M_{pl} is defined as

$$M_{pl}^2 = M_{10}^8 V_6.$$

And hence, finally:

$$\mathcal{N} = 8\pi^2 \frac{M_{pl}^2 \alpha'}{g_s} = 8\pi^2 \left(\frac{M_{pl}}{M_{10}}\right)^2 \frac{1}{\sqrt{g_s}}$$

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