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Higher Curvature Gravity in TeV-Scale Extra Dimensions * †

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Abstract

We begin a general exploration of the phenomenology of TeV-scale extra-dimensional models with gravitational actions that contain higher curvature terms. In particular, we examine how the classic collider signatures of the models of Arkani-Hamed, Dimopoulos and Dvali (missing energy and new dimension-8 contact interactions) and of Randall and Sundrum (TeV-scale graviton Kaluza-Klein resonances) are altered by these modifications to the usual Einstein-Hilbert action. We find that not only are the detailed signatures for these gravitationally induced processes altered but new contributions are found to arise due to the existence of additional scalar Kaluza-Klein states in the spectrum.

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1 Introduction and Background

The question as to why the Planck and electroweak scales differ by so many orders of magnitude remains mysterious. In recent years, attempts have been made to address this hierarchy issue within the context of theories with extra spatial dimensions that lower the effective scale of gravity to the TeV region. In both of the Arkani-Hamed, Dimopoulos and Dvali (ADD)[1] and Randall-Sundrum (RS)[2] models, new effects of gravitational origin are expected to occur near the TeV scale which should be observable at future colliders such as the LHC and ILC. Though these two models are very different in detail they do have some common features the most important of which are (*i*) in their original versions they both assume that Standard Model matter is confined to a 4-dimensional brane; (*ii*) they both assume that D-dimensional gravity is described by the Einstein-Hilbert (EH) action plus a possible cosmological constant and (*iii*) the background metric is either strictly flat, *i.e.*, Minkowskian as in the ADD model with toroidally flat compactification, or has constant curvature and is conformally flat, *i.e.*, AdS_5 as in the RS model.[‡] How would the predictions of these two models change if we surrendered the EH action and considered something more general? This is the discussion we would like to begin in this paper.

General Relativity (GR) as described by the EH action is considered to be an effective theory below the fundamental Planck scale, M. Once energies approaching or beyond the scale M begin to be probed one might expect to observe deviations from the expectations arising from the EH action. In the case of both the ADD and RS models, future colliders will probe near or above their (effective) fundamental scales so that non-EH aspects of the true gravitational theory, whatever its form, should become apparent. Since the ultraviolet form of the true gravity theory is as of yet unknown one may hope to capture some of its aspects by considering how the presence of new higher curvature (and higher derivative) invariants in the actions of the ADD and RS models may lead to variations in the well-known predictions of these theories. Many authors have considered

[‡]In the original RS model the space is not truly one of constant curvature, *i.e.*, the curvature invariant R is not strictly co-ordinate independent as it also has delta-function singularities due to the presence of the TeV and Planck branes at the two S^1/Z_2 orbifold fixed-points. we will return to this issue later.

the possibility of higher curvature invariants and how their existence would modify the predictions arising from the EH action within other contexts, *e.g.*, the properties of black holes[3, 4], deviations in solar system tests of GR[5] and in cosmology[6] to possibly avoid the need for dark energy. Some analyses along these lines for the potential modifications of the collider predictions of both the ADD and RS models have already been performed[4, 7]. In the present paper, we wish to both extend and generalize these results to get a feeling for the possible variation in the gravitational phenomena as predicted by these classic models all of which will be potentially observable at future colliders. In particular we are interested in how the well known signatures of both the ADD and RS models are morphed if we keep the basic setups intact but modify the actions on which the corresponding equations of motion are derived. A further generalization to such an analysis is possible if the original ADD/RS setups are simultaneously surrendered resulting in entirely new equations of motion; while this is an interesting possibility to consider it lies beyond the scope of the present paper.

Of course a completely general study of how these possible modifications to the effective gravity action may morph TeV collider signatures is an obviously immense task and here we aim only at a first round analysis in the discussion that follows. The major signatures arising in both ADD and RS models originate from graviton exchange and the production of black holes; the ADD model also leads to missing energy signatures from graviton emission. Fortunately, apart from issues associated with black holes, the relevant graviton properties (couplings, wavefunctions and propagators) necessary to extract experimental signatures for either model can be obtained from the expansion of the rather general action considered here to quadratic order in the curvature. (This would no longer be true if we wanted to consider, *e.g.*, the triple graviton coupling as then an expansion to third order would be required.) This simplifying observation forms the basis of the analysis that follows and allows us to determine the relevant graviton properties in both ADD- and RS-like models for a wide class of effective actions.

The general outline of our analysis in presented in Section 2 where our basic assumptions and notations are also given. In Section 3 we apply our analysis to the ADD model; we then apply it to the RS case in Section 4. Our summary and conclusions are given in Section 5.

2 Analysis

When going beyond the EH action there are many possibilities to consider especially when we are living in extra dimensions. In the literature various forms have been assumed for the potential structures of higher curvature invariants that may appear in the gravity action. The fairly general action that we will assume for the D-dimensional action in the present analysis takes the form (with $D = 4 + n$ below):

$$S_g = \frac{M^{D-2}}{2} \int d^D x \sqrt{g}\; F(R, P, Q)\,, \tag{1}$$

where F is a continuous, differentiable and generally mathematically well-behaved function; in particular we will assume that F is non-singular when all of its arguments are zero. Here R is the usual D-dimensional Ricci scalar while P and Q are quadratic invariants constructed from the curvature tensor R_{ABCD}: $P = R_{AB}R^{AB}$, with R_{AB} being the Ricci tensor, while $Q = R_{ABCD}R^{ABCD}$. M is the D-dimensional fundamental gravity scale which is $\sim$ TeV in ADD and $\sim \overline{M}_{Pl}$ in RS. In the low energy, small curvature limit we expect $F \to R$ (plus a possible cosmological constant) and so the overall dimensionful factor in the expression above allows us to make direct contact with the EH action in this limit. This specific form for F, though not completely general, covers a wide array of possibilities and has been considered (some only in $D = 4$) in may different contexts for a multitude of purposes in the literature[3, 4, 5, 6]. Many of the higher curvature models previous considered by other authors form subcases of this more general action.

As is by now well-known[8, 9] the gravity theories described by an action of the form S_g can potentially have several serious problems since they, amongst other things, lead to equations of motion which are generally fourth order in the derivatives of the metric. In particular, in addition to the usual massless D-dimensional tensor graviton which results in the familiar resulting 4-d graviton and graviscalar Kaluza-Klein (KK) tower excitations, there may also be present in the linearized D-dimensional theory additional massive scalar and tensor excitations. These fields will in 4-d have KK towers without zero modes and which can be ghostlike and/or tachyonic. (We can think of these new D-dimensional fields as having bulk masses which influence their corresponding 4-d KK tower masses.) Furthermore, the equations of motion naturally involving higher derivatives of the fields can lead to problems with unitarity. The new massive tensor excitations are potentially the most serious issue to deal with as they are ghost fields that must be eliminated from the spectrum (though they may help in dealing with the theories renormalizability and bad high-energy behavior). It has been noted[8, 9] that we one can remove these states from the spectrum (*i.e.*, by giving their bulk masses an infinite value) if a tuning occurs such that the quantities P and Q only appear in the special combination $P - 4Q$ in the action. There has been some discussion in the literature, however, that these ghost states may not be as dangerous as one would imagine from lowest order perturbation theory[10] so that we should perhaps keep an open mind about the possible forms for F. We will return to this point in what follows.

Given a general action of the kind above there are several issues that one normally wants to address in order to extract information that can be compared with experimental data. From studies of both the ADD and RS models there are certain things we want to know, *e.g.*, (*i*) the spectrum, wave functions, propagators and Standard Model (SM) matter couplings of the KK graviton (and other possible) excitations and (*ii*) the relationship between M, the volume of the compactified dimensions and the (reduced) 4-d Planck scale $\overline{M}_{Pl}$. [§] To obtain this information, as well as to make contact with several other analyses[4, 7], it is sufficient to expand the general action S_g above around the background metric to quadratic order in the curvature to obtain an effective action for the graviton (and like) excitations. At this level, one can extract the relevant 2-point functions as well as the differential equation for the KK wavefunctions which then yield the KK mass spectrum

[§]As in the ADD and in the original RS models, in what follows we will assume that the SM matter fields are localized at a fixed value of the extra-dimensional co-ordinates.

as well as the the desired graviton couplings to SM fields. If, however, one wanted to probe, *e.g.*, graviton 3- or 4-point functions then we would need to expand to at least cubic or quartic order in the curvature, respectively; these will not be of interest to us here but might be of interest in future experiment[11] which would tell us more about the underlying theory of gravity.

Once we make this expansion, there are various equivalent ways of expressing the resulting effective action, S_{eff}, depending upon the basis of invariants we choose to employ; the most obvious form is simply

$$S^{(1)}_{eff} = \frac{M^{D-2}}{2} \int d^D x \sqrt{g}\, \left[\Lambda + dR + aR^2 + bP + cQ\right], \tag{2}$$

where P, Q have been defined above. Λ is an effective cosmological constant and $a - d$ are (in some cases dimensionful) constants all of which are functions of F and its derivatives evaluated employing the relevant background metric. To relate this back to the EH action in the limit of small curvature, one can think of the (necessarily positive) parameter d as a 'renormalization' of the fundamental mass scale M: $M \to M' = Md^{1/(D-2)}$.

A second version of S_{eff} is given by

$$S^{(2)}_{eff} = \frac{M^{D-2}}{2} \int d^D x \sqrt{g}\, \left[\Lambda + dR + a'R^2 + b'P + c'G\right], \tag{3}$$

where G is the well-known Gauss-Bonnet(GB) invariant:

$$G = R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} = R^2 - 4P + Q\,, \tag{4}$$

The co-efficients a', b' and c' can be easily converted to $a - c$ above by some straightforward algebra: $a = a' + c', b = b' - 4c'$ and $c = c'$. In 4-d, the GB invariant is just a total derivative but this is no longer true for arbitrary values of $D > 4$. The GB invariant is just (the quadratic) one of a general class called Lovelock invariants, constructed of various powers of the curvature tensor, which lead to special properties for the equations of motion[12]. Generally the existence in the action of higher curvature terms, as discussed above, leads to higher order equations of motion that produce tachyonic and/or ghost excitations in the spectrum as well as potentially non-symmetric and/or non-conserved pieces of the corresponding Einstein equations. Having an action consisting solely of Lovelock invariants avoids all of these potential difficulties as well as those associated with the massive tensor ghosts. The D-dimensional scalar excitation discussed above is also absent in this case. It is interesting to note that the GB term is the leading correction to the EH action in perturbative string theory[13, 14]. Higher order Lovelock invariant may also be present in the action (when $D > 6$) but these cannot be described by the function F as employed here since they are constructed out of cubic or higher order combinations of the curvature tensor. The effect of the presence of general Lovelock invariants in the action of the ADD model has been discussed within the black hole context in Ref.[4].

A further possible form for the quadratic action commonly used in the literature is

$$S_{eff}^{(3)} = \frac{M^{D-2}}{2} \int d^D x \sqrt{g} \left[\Lambda + a_1 R + a_2 R^2 + a_3 C + a_4 G \right], \tag{5}$$

where C is the square of the Weyl tensor which can be expressed as[15]:

$$C = C_{ABCD} C^{ABCD} = Q - \frac{4P}{n+2} + \frac{2R^2}{(n+3)(n+2)} \,. \tag{6}$$

where $n = D - 4$ is the number of extra dimensions; the a_i are linearly related to the coefficients $a - d$ above, *e.g.*, $a_1 = d$. This translation is simplified via the use of the identity[15]

$$aR^2 + bP + cQ = -\Big[\frac{(n+2)b + 4c}{4(n+1)}\Big] G + \Big[\frac{4(n+3)a + (n+4)b + 4c}{4(n+3)}\Big] R^2 + \frac{n+2}{n+1}\Big(c + \frac{b}{4}\Big) C \,. \tag{7}$$

Noting that the a_{2-4} have dimensions of mass^{-2} it is sometimes common in the literature to write

$$\begin{aligned} a_1/a_2 &= 2(n+3) m_0^2 \\ a_1/a_3 &= -(n+2) m_2^2 \,, \end{aligned} \tag{8}$$

where $m_{0,2}$ are two mass parameters which are naturally $\sim M$ in the theory. One then finds that $m_{0,2}$ are directly related to the bulk masses of the D-dimensional massive scalar and tensor excitations discussed above. To avoid tachyons we apparently must demand that $m_{0,2}^2 > 0$ but even in such a case this the massive tensor field remains a ghost since the kinetic term for this field would have the wrong sign.

Clearly all these forms for S_{eff} are simply related. In what follows we will make use of all of the above forms of S_{eff} and treat them interchangeably.

Our first goal will be to explicitly calculate S_{eff} in one of these 'bases' from the more general S_g in terms of F and its derivatives. To begin we perform a Taylor series expansion of F to quadratic order in all three arguments evaluating the result in the background metric, *e.g.*,

$$F = F_0 + (R - R_0)F_R + (P - P_0)F_P + (Q - Q_0)F_Q + \text{quadratic terms}\,, \tag{9}$$

where F_0 is a constant corresponding to the evaluation of F itself in the fixed curvature background metric and $F_X = \partial F/\partial X$; X_0 means that X is to be evaluated in terms of the background metric which we here assume to be a space of constant curvature, *i.e.*, a maximally symmetric space as is the case in both the ADD and RS models. Thus the quantities R_0, P_0, Q_0, F_X and F_{XY} are just numbers which depend on the explicit form of the metric and possibly the number of extra dimensions.¶ In such a maximally symmetric space the Weyl tensor and corresponding invariant both vanish identically, *i.e.*, $C_0 = 0$ and one further finds that

$$\begin{aligned} P_0 &= \frac{R_0^2}{n+4} \\ Q_0 &= \frac{2R_0^2}{(n+4)(n+3)} \\ G_0 &= \frac{(n+2)(n+1)R_0^2}{(n+4)(n+3)}\,. \end{aligned} \tag{10}$$

Note that in ADD $R_0 = 0$ since the metric is Minkowskian whereas in the AdS_5 RS bulk $R_0 = -20k^2$ (away from the two branes) where the parameter k originates from the usual RS metric

¶Note that the quadratic terms in the Taylor expansion naturally involve factors of P^2 and Q^2 which are actually fourth order in the (dynamical) curvature; we drop these terms for consistency in the discussion which follows.

$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$. Without making further assumptions we obtain

$$\begin{aligned}
\Lambda &= F_0 - R_0F_R + R_0^2(F_{RR}/2 - \sigma F_P - \tau F_Q) + R_0^3(\sigma F_{PR} + \tau F_{QR}) \\
&+ R_0^4(2\sigma\tau F_{PQ} + \sigma^2 F_{PP} + \tau^2 F_{QQ})/2 \\
a_1 &= F_R - R_0F_{RR} - R_0^2(\sigma F_{RP} + \tau F_{RQ}) \\
a_2 &= \beta F_P + \epsilon F_Q + F_{RR}/2 - R_0(\beta F_{RP} + \epsilon F_{RQ}) - R_0^2[(\tau\beta + \epsilon\sigma)F_{PQ} + \sigma\beta F_{PP} + \tau\epsilon F_{QQ}] \\
a_3 &= \alpha F_P + \delta F_Q - R_0(\alpha F_{RP} + \delta F_{RQ}) - R_0^2[(\tau\alpha + \delta\sigma)F_{PQ} + \sigma\alpha F_{PP} + \tau\delta F_{QQ}] \\
a_4 &= -\alpha F_P + \gamma F_Q + R_0(\alpha F_{RP} - \gamma F_{RQ}) + R_0^2[(\tau\alpha - \gamma\sigma)F_{PQ} + \sigma\alpha F_{PP} - \tau\gamma F_{QQ}]\,, \qquad (11)
\end{aligned}$$

where $\sigma = (n+4)^{-1}$, $\tau = 2(n+4)^{-1}(n+3)^{-1}$, $\delta = 4\alpha = (n+2)/(n+1)$, $4\beta = (n+4)/(n+3)$, $\gamma = -(n+1)^{-1}$ and $\epsilon = (n+3)^{-1}$. For the case of $n = 0$ this reproduces the results give by, *e.g.*, Navarro and Van Acoleyen in [5]. Note that if we make the assumption that F is a function only of R and the combination $P - 4Q$ then $F_P = -4F_Q$ *etc* and, also noting that $\delta - 4\alpha$=0, we obtain $a_3 = 0$ so that the remaining expressions greatly simplify; we now obtain

$$\begin{aligned}
\Lambda &= F_0 - R_0F_R + F_{RR}R_0^2/2 + \lambda R_0^2[F_Q - R_0F_{QQ} + \lambda F_{QQ}R_0^2/2] \\
a_1 &= F_R - R_0F_{RR} + \lambda R_0^2 F_{RQ} \\
a_2 &= -F_Q + F_{RR}/2 + R_0F_{RQ} - \lambda R_0^2 F_{QQ} \\
a_4 &= -a_2 + F_{RR}/2\,, \qquad (12)
\end{aligned}$$

where the parameter λ is given by

$$\lambda = \frac{2(2n+5)}{(n+3)(n+4)}\,. \qquad (13)$$

Note that having $a_3 = 0$ implies that the Weyl term, C, in the effective action is absent in second order which is equivalent to taking $m_2 \to \infty$ thus eliminating the massive tensor ghost issue. This field is now removed from the spectrum though the D-dimensional scalar remains in general.

3 Application I: ADD

In this section we will apply the above analysis to the general ADD framework where we now require (since the space is flat) $R_0 = F_0 = 0$ so that $\Lambda = 0$ automatically. This significantly reduces the possible deviations from the classic ADD picture. In this specific case the general second order expansion of F is rather simple and is given by

$$F \to F_R R + \big[\beta F_P + \epsilon F_Q + F_{RR}/2\big] R^2 + \alpha\big[F_P + 4F_Q\big] C + \big[-\alpha F_P + \gamma F_Q\big] G\,. \tag{14}$$

Note that if we also demand that F be a function only of R and the combination $P - 4Q$ in order to avoid the massive tensor ghost issue this expression simplifies even further to

$$F \to F_R R + \big[-F_Q + F_{RR}/2\big] R^2 + F_Q G\,. \tag{15}$$

There has been some discussion about other ways to circumvent this tensor ghost problem than by completely eliminating it from the perturbative spectrum. As we are working only to lowest non-trivial order perhaps we should keep an open mind about the forms for F. Note that since F_R will essentially rescale the overall mass factor in the action we must demand that $F_R > 0$ to insure that the usual D-dimensional massless tensor gravitons not be ghost-like.

How are the predictions of ADD modified by these additional curvature terms? [‖] The basic ADD picture leads to three essential predictions [16]: (i) the emission of graviton KK states during the collision of SM particles producing signatures with apparent missing energy[17, 18, 19]; (ii) the exchange of graviton KK excitations between SM brane fields leading to dimension-8 contact interaction-like operators with distinctive spin-2 properties[17, 18, 20]; (iii) the production of black holes(BH) at colliders and in cosmic rays with geometric cross sections, $\simeq \pi R_s^2$, with R_s being the BH Schwarzschild radius, once collision energies greater than $\sim M$ are exceeded[21, 22, 23, 24].

The production and properties of D-dimensional, TeV-scale BH in higher curvature theories has been partially explored within the context of Lovelock extended gravity[4] though not yet so in

[‖] This subject has already begun to be addressed by Demir and Tanyildizi in Ref.[7] in the case where F is only a function of R.

the fully general quadratic gravity case described by the function F considered here. Such a study, which would be very interesting, is far beyond the scope of the present analysis. However, it is interesting to make several observations: (i) Consider the vacuum solution; if we expand the general action F above to only quadratic order and *if* we also assume that all interesting solutions must satisfy $R_{AB} = 0$ in the vacuum, *then* the only deviations from the conventional Schwarzschild form arise from the GB term in the action. This can be seen immediately by examining the equations of motion resulting from the general quadratic action, *e.g.*, in Ref.[26]. This result does not remain valid when F is treated exactly. (ii) If $F = F(R)$ only and is treated exactly without expansion then the equations of motion allow for the conventional external BH result with $R_{AB} = 0$ and will appear as an ordinary D-dimensional Schwarzschild solution. This is *not* the most general solution as can be see by considering the simple case of $F = R + \beta R^2/M^2$ with β a dimensionless parameter. Here there is also exists a solution with $R \sim -M^2/\beta$, which is a constant corresponding to deSitter or anti-deSitter space depending on the sign of β^{**}. (iii) If $F = R + \alpha G/M^2$, with α being a constant, then the general BH solution has neither R nor G equal to zero as is well-known from the exact solution[3, 27, 28]. A more detailed study here would be worthwhile.

Let us now consider the situation of graviton exchange where it is well-known[17, 18, 20] that ADD leads to new dimension-8 contact interactions. To obtain the analogous quantities here we must expand the integrand of the action, *i.e.*, the Lagrangian $\sqrt{g}F$, to second order in the fluctuations, h_{AB}, around the flat background metric:

$$\mathcal{L}_2 = \frac{1}{2} h^{AB} O_{ABCD} h^{CD} \,. \tag{16}$$

Here we have expressed the original metric as

$$g_{AB} = g^0_{AB} + 2h_{AB}/M^{1+n/2} \,, \tag{17}$$

with g^0_{AB} being the background metric, here identified in the ADD model as the Minkowski metric η_{AB}. The propagator is then just the inverse of the operator O; once the propagator is known

**See, however, Ref.[25]

we sandwich it between two 4-d localized (at the origin of the extra-dimensional co-ordinates) and conserved SM stress-energy sources, $T_{\mu\nu}$, to find the relevant scattering amplitude remembering to KK decompose the various towers. Fortunately, much of this work has been done for us by Accioly, Azeredo and Mukai[8] from which, with some modifications, we obtain the expression for the D-dimensional graviton exchange amplitude (before performing the KK sums)

$$\mathcal{A} \sim \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+2)}{k^2 - m_n^2} - \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+3)}{k^2 - (m_2^2 + m_n^2)} + \frac{T^2}{(n+2)(n+3)[k^2 - (m_S^2 + m_n^2)]}, \quad (18)$$

where $m_n^2 = \sum_i n_i^2/R_c^2$ are the familiar (unaltered in terms of the radius R) flat space KK masses. The n_i label the various KK levels; $T = \eta^{\mu\nu}T_{\mu\nu}$ is the 4-d trace of the SM source stress-energy tensor and $m_{S,2}$ are just

$$\begin{aligned} m_S^2 &= \frac{(n+2)m_0^2}{2} = \frac{(n+2)F_R}{4(n+3)(\beta F_P + \epsilon F_Q + F_{RR}/2)} \\ m_2^2 &= \frac{-F_R}{(n+2)(F_P + 4F_Q)}, \end{aligned} \quad (19)$$

as described above. R_c is the compactification radius which sets the KK mass scale; here we have assumed a common value for this quantity for all n extra dimensions so that the volume of the compactified space is just $(2\pi R_c)^n$. In the expression above the first term in the amplitude is the usual one encountered in the ADD model which results from the D-dimensional EH action and combines the contributions of the 4-d spin-2 graviton and spin-0 graviscalar KK towers. The second and third terms correspond to the new D-dimensional massive tensor and scalar contributions, respectively. The difference in the factors of $n+2$ versus $n+3$ in the first two terms arises from the existence of a 5-d bulk mass for the tensor ghost field. It is interesting to note that this amplitude is very well behaved at large k^2 (in fact vanishing) due to the detailed cancellations between the various terms.

Note that here $m_{S,2}$ represent the bulk mass terms of the new fields which enter into the KK tower masses of the scalars (spin-0) and tensors (spin-2 and spin-0), respectively; here we see the effect of the tensor ghost KK tower exchange explicitly. From this point of view it appears that the only way to remove this ghost tower is to take the bulk mass $m_2^2 \to \infty$ implying that F is solely a function of R and the combination $P-4Q$, which we will assume from now on in our ADD discussion. Note that the existence of a GB term in the action will not yield a contribution to m_S. Since, as discussed above $F_R > 0$ is already required, tachyonic KK scalars are avoided when the denominator in the expression for m_S^2 above is positive; when F is assumed to be a function only of R and $P-4Q$, then this denominator simplifies to $F_{RR}/2-F_Q$. In the limit where $F = F(R)$ alone, and accounting for a sign factor in the definition of the above actions, our result for the squared scalar mass, m_S^2, agrees with that obtained by Demir and Tanyildizi[7]. As shown by these authors, the effect of the new scalar tower exchange is generally rather suppressed in comparison to the more familiar graviton exchange since the ratio $T^2/T_{\mu\nu}T^{\mu\nu}$ is small for most SM particle sources at TeV colliders. For example, for the process $WW \to WW$ this ratio is $\sim (M_W^2/s)^2$. This ratio of the of the KK summed scalar to graviton exchange amplitudes is somewhat further reduced by (i) the existence of the finite bulk scalar mass which implies that there are no light scalar KK exchanges with masses below m_S and (ii) the n-dependent numerical factor in the denominator of the scalar amplitude. Naturalness suggests that $m_S \sim M \sim$ TeV or larger unless the parameters of F are somehow fine-tuned. For example, if $F = R + \beta R^2/M^2$, then $m_S^2 = (n+2)M^2/(4\beta(n+3)) \sim M^2$ for all n *if* β is not too far from O(1). Interestingly we see here that as $\beta \to 0$ we recover the usual EH expectation as then $m_S \to \infty$. Thus we find that for many practical purposes the structure of the usual ADD results for graviton exchange are not modified when the action is generalized to the form considered here. However, with the existence of these additional scalars being a hallmark of the extended action, it behooves us to find a way to isolate their effects experimentally.

In expressions for graviton exchange only the combination $M^{n+2}F_R$ will now appear. In the amplitude this will lead to a modification of the pure 'graviton' exchange cross section expectations by a factor of F_R^{-1}, which is likely to be of $O(1)$, *provided* M is considered to be held fixed. When the graviton tower interference term with the SM dominates, the effect in the gravitational part of the cross section will scale as F_R^{-1}. Given the previous results of Demir and Tanyildizi[7], this is not surprising.

We further note that since m_S is $\sim$ TeV or larger it has no effect on laboratory measurements of the strength of the gravitational interaction in the micron range when $n = 2$.

Before closing this part of the discussion we would like to remind the reader that it was pointed out long ago[9, 29] that we can take any action of the form $F = F(R)$ and map it over to the EH action coupled to an minimally coupled real scalar field with a rather complicated potential V, depending exponentially on the scalar field. This can be done via a special conformal transformation

$$\hat{g}_{AB} = |F_R|^{2/(n+2)} g_{AB} \,. \tag{20}$$

Going from the original (Jordan) to the new (Einstein) frame one explicitly sees the existence of the new scalar degree of freedom[††]. The mass of this scalar field is exactly that of the field S above and can be gotten directly from the canonically normalized potential V in the usual manner, *i.e.*, using $m_S^2 = \partial^2 V/\partial S^2$. This is a very powerful tool as it allows us to extend our previous flat space result for m_S to the much more general case where the metric is unspecified. For example, if $F = R + \beta R^2/M^2$, we find that the value of m_S^2 is the same as discussed above, *i.e.*, $m_S^2 = (n+2)M^2/(4\beta(n+3))$, in a space with an *arbitrary* metric. This will be an important result that we will employ when we discuss the case of the RS setup.

We now turn to the emission of gravitons in SM particle collisions. Since the compactifying space is flat in the ADD case the normalizations of the graviton (and scalar) wavefunctions which control their couplings are unaltered by the existence of the quadratic curvature terms but the relationship between M and $\overline{M}_{Pl}$ is modified. This was briefly mentioned above where we saw that in the small curvature limit the parameter d essentially renormalizes the fundamental scale. To see this in the present case it is sufficient to examine the tensor/spin-2 kinetic part of the 4-d

[††]It is even possible to make a generalized version of this transformation for more complicated forms of the action[30]

effective Lagrangian to second order in $h_{\mu\nu}$ (which has not yet been KK-expanded) in the familiar transverse traceless gauge, *i.e.*, $\partial_\mu h^{\mu\nu}=0$, $h^\mu_\mu=0$; one obtains[8]

$$\mathcal{L}_g = -\frac{d}{2}h^{\mu\nu}\Box h_{\mu\nu} + \frac{1}{2}\Big(\frac{b}{4}+c\Big)\Box h^{\mu\nu}\Box h_{\mu\nu}\,, \tag{21}$$

where here $\Box = \eta^{AB}\partial_A\partial_B$ and b, c, d are defined above[‡‡]. When we assume that F is only a function of R and the combination $P-4Q$ then the second term in $\mathcal{L}_g$ vanishes and we recover the familiar result of the standard EH scenario apart from the overall factor of d. Hence, to recover the conventional 4-d EH action when inserting the usual (extra dimensionally) flat zero mode graviton wavefunction into $\mathcal{L}_g$ the ADD relationship must be modified, as hinted above, to

$$\overline{M}^2_{Pl} = V_n M^{n+2} F_R\,, \tag{22}$$

where $\overline{M}_{Pl}$ is the 4-d reduced Planck scale and V_n is the volume of the compactified space. Of course, $d=F_R$ is just unity in the standard ADD model which employs the EH action. Since the lightest of the KK scalars has a mass which is naturally on the order of a TeV and have rather weak couplings to SM fields these particles will not play much of an important role in missing energy processes. *If* the cross section for graviton production, *i.e.*, missing energy, is expressed in terms of the original M with other parameters held fixed, then the presence of F_R leads to a modification of the production cross section by a factor of $1/F_R$. However, as M is not likely to remain a direct observable (only the product $M^{n+2}d$ is) there may be no way to experimentally disentangle this effect. Furthermore, for any given F_R, since $\overline{M}_{Pl}$ is numerically fixed and M is an input parameter the resulting derived value of R which sets the scale for the masses of the KK states is altered.

We thus conclude that if we assume that F is a function of only of R and the combination $P-4Q$ then the classic predictions (i) and (ii) of the ADD model will be essentially unaffected by going to the more general action considered here *except* for possible overall scalings by inverse powers of F_R when the parameter M is held fixed: graviton emission rates scale like $1/F_R$ while

[‡‡]Note that as usual Greek indices only run over 4-d.

graviton exchange cross sections scale as $1/F_R$ or $1/F_R^2$ depending on the presence of important SM contributions to the relevant process.

4 Application II: RS

The predictions of the classic RS model are the existence of TeV scale graviton resonances with fixed weak scale masses and couplings to the SM fields[32], the existence of a weak scale radion excitation[33], as well as the production of AdS_5 BH. In what follows we will be specifically interested in the nature of the KK gravitons so it is again sufficient to examine the quadratically expanded action. The classic RS model is not generally consistent with the assumed form of either the original action S_g or its quadratically expanded form S_{eff}. As is well-known, and as mentioned above, the equations of motion that follow from S_g and S_{eff} will generally be fourth order in the derivatives of the metric. In the usual 5-d RS model, one solves the Einstein equations of the form

$$G_{AB} = \frac{T_{AB}}{M^3}\,, \tag{23}$$

where G_{AB} is the Einstein tensor arising from the EH action involving no more than two derivatives of the metric. The problem is that RS completely specifies T_{AB}: a cosmological constant in the 5-d bulk plus two δ-function sources at the orbifold locations of the TeV and Planck branes. SM matter confined to the TeV brane is supposed to not be a large contributor to the stress-energy. To obtain this result the standard RS metric takes the form discussed above: $ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ with the linear exponential warp factor leading to the bulk AdS_5 and the two field derivatives acting on the absolute value leading to the brane δ-functions. (This is related to the comment above that R is not truly constant in RS and has brane δ-function singularities. Recall that these δ-functions are the results of assuming infinitely thin branes.) If an identical metric is assumed in our more general case we still can obtain AdS_5 but the fourth order equations would lead to the more singular derivatives of δ-functions at the brane locations. This amongst other reasons is what led Kim, Kyae and Lee[31] to consider only GB extensions of the EH action in RS since it alone only produces Einstein equations of second order in the derivatives. Thus if we keep the classic picture an analysis of RS given our assumed effective action expanded around a background of constant curvature is not relevant. (A possible way of dealing with these derivatives of δ-functions arising from orbifold singularities in higher dimensional effective field theories has been discussed in Ref.[34]. Implementing our scheme employing such techniques is, however, beyond the scope of the present paper.)

To avoid these issues for now we simplify our discussion of this problem (and to convince ourselves that an RS-like solution is possible in this framework) we consider a singularity-free, 'softened' version of RS where the orbifolded bulk space with branes is replaced by an interval, as has been suggested for other reasons[35], with SM matter placed at one singular end point possessing an ignorable amount of stress-energy. With a cosmological constant on the interval we can recover the background AdS_5 bulk; in addition by removing the absolute value sign of the co-ordinate y in the metric above we expunge the δ-functions as well as the possibility of any of their higher derivatives appearing in the equations of motion. The boundary conditions at the end points for the graviton KK states can then be freely chosen to be the same as that of the original RS model. This space is truly one of constant curvature and the general analysis we have presented above will now be applicable to this softened RS.

It is easy to verify that the form of the equations of motion[26] in this case (recalling that we are only searching for solutions with constant R backgrounds) are given by:

$$F_R R_{AB} - \frac{1}{2} g_{AB} F + 2F_P R_A^N R_{NB} + 2F_Q R_{MNSA} R^{MNS}{}_B = \frac{T_{AB}}{M^3} \,, \tag{24}$$

and that if we take stress-energy tensor in the 5d bulk to be of the usual RS form

$$T_{AB} = -\Lambda g_{AB} \,, \tag{25}$$

with $\Lambda > 0$, then indeed a space of constant curvature, $i.e. AdS_5$, *can* be an allowed solution. Taking the trace of the equations of motion above, evaluating it in the constant curvature bulk and relating the values of P_0, Q_0 to R_0^2 as before (and recalling that here $R_0 = -20k^2$ using the softened metric) results in the constraint equation

$$\frac{2}{5}R_0^2\big[F_P + \frac{1}{2}F_Q\big] + F_R R_0 - \frac{5}{2}F_0 = \frac{T}{M^3} = -\frac{5\Lambda}{M^3}\,. \tag{26}$$

It is interesting to note that if we *assume* that R =constant then this constraint equation automatically implies that T =constant also but *not necessarily* with the further requirement that T_{AB} are constants following from the equations of motion assuming that they are at most functions of y. When F is only a function of R and the combination $P - 4Q$, this constraint equation simplifies to

$$-\frac{7}{5}R_0^2 F_Q + F_R R_0 - \frac{5}{2}F_0 = -\frac{5\Lambda}{M^3}\,; \tag{27}$$

while in the specific RS background case this becomes, explicitly

$$224k^4 F_Q + 8k^2 F_R + F_0 = \frac{2\Lambda}{M^3}\,. \tag{28}$$

It is important to recall that F itself can be a complicated function of k so that this equation can be quite nontrivial. For the EH action limit this yields the usual relation that $\Lambda = -6k^2M^3$; here it in general provides an additional constraint on the allowed forms of the function F since we are requiring R_0 to be both real and negative. Given a specific function F for which a solution exists, this equation directly relates Λ and k^2 though the solution may not be unique. For example, if we assume the simple case of

$$F = R + \frac{\beta}{M^2}R^2\,, \tag{29}$$

as employed above, then there are two branches of solutions for $k^2(\Lambda)$:

$$k^2 = \frac{3M^2}{40\beta}\left[1 \pm \left(1 + \frac{40\Lambda}{9M^5}\beta\right)^{1/2}\right]\,, \tag{30}$$

one of which (the negative root) goes over to the usual EH result as the parameter $\beta \to 0$.

Allowing for the possibility of a RS-like solution with a softened metric it is interesting to think briefly about the previously analyzed effects of the GB term in the RS scenario. This analysis was originally performed for the classic RS[4] setup which employed the standard form of the RS metric; that result would now be modified by the changes in the model assumptions, *i.e.*, moving to an interval and removing the δ-function sources at the end points. The previous analysis of BH in RS with the added GB term would not be significantly affected if this transition were made. However, the properties and spectrum of the graviton KK states certainly would be influenced since the δ-function terms are now absent. The equation governing the masses and wavefunction of the graviton KK states for the present interval case can be obtained by expanding the equations of motion as before. Since we are here only interested in the tensor modes associated with the usual gravitons, we can employ the expansion

$$g_{\mu\nu} = e^{-2ky}\left(\eta_{\mu\nu} + \kappa h_{\mu\nu}\right), \tag{31}$$

where $\kappa = 2M^{-3/2}$. Applying the usual RS boundary conditions on the interval the most significant changes from the classic RS can be read off from Eqs.(15)-(28) in Ref.[4] by setting the parameter $\Omega = 0$ in appropriate places. At the end of the day we find that the only apparent difference from the classic EH based RS model would be a shift in the relationship between the fundamental scale and $\overline{M}_{Pl}$–remarkably similar to what we saw for the ADD model above. In the language employed in Ref.[4] we would now obtain

$$\overline{M}_{Pl}^2 = \frac{M^3}{k}\left[1 - 4\alpha\frac{k^2}{M^2}\right], \tag{32}$$

where α/M^2 is the coefficient of the GB term in the action. Otherwise the masses as well as the couplings of all of the KK gravitons to localized SM matter would be *identical* to those of the original RS model expressed in terms of the derived parameter k. The explicit coupling and spectrum changes found in Ref[4] for the graviton KK states in the presence of the GB term in the action were all found to due to the brane δ-function singularities.

How would these graviton KK results obtained in the GB extended action generalize to the case of S_{eff} above? Here we choose to begin our analysis with $S^{(3)}_{eff}$, setting $a_3 = 0$ from the beginning to avoid potential ghost fields, then taking $D = 5$ and making use the same curvature expansion as above. In order to make a connection with the previous discussion, the existing RS literature and to directly compare with the GB case, however, we massage our notation slightly and rewrite S^3_{eff} in the following form:

$$S^{(3)}_{eff} = \int d^5x\sqrt{g}\ \big[-\Lambda_b + a_1\frac{M^3}{2}R + \frac{\alpha M}{2}G + \frac{\beta M}{2}R^2\big]\,, \tag{33}$$

where the parameters a_1, α and β are dimensionless; the action employed in Ref.[4] is now directly recovered by taking the $a_1 \to 1$ and $\beta \to 0$ limits. It is important at this point to recall that to obtain the linearized graviton equations of motion it is sufficient to employ S_{eff} while the complete S_g needs to be examined in order to demonstrate the existence of the required AdS_5 solution. The equations of motion resulting from $S^{(3)}_{eff}$ are given by[26]

$$\begin{aligned}
-\frac{\Lambda_b}{M^3}g_{AB} &= a_1(R_{AB} - \frac{1}{2}g_{AB}R) + \frac{2\beta}{M^2}R(R_{AB} - \frac{1}{4}g_{AB}R) + \frac{2\beta}{M^2}(g_{AB}\Box - \nabla_A\nabla_B)R \\
&+ \frac{2\alpha}{M^2}\Big[RR_{AB} - 2R_{ASBP}R^{SP} + R_{ASPT}R^{SPT}{}_B - 2R_{AS}R^S_B - \frac{1}{4}g_{AB}G\Big]\,.
\end{aligned} \tag{34}$$

Here ∇_A is the covariant derivative operator and here $\Box = g^{AB}\nabla_A\nabla_B$. First we look at the $A, B = 5$ component of this equation, remembering that for the moment we will only be interested in the tensor excitations corresponding to the KK gravitons. In the usual gauge, R in this case is a constant to linear order so we arrive at a consistency condition

$$a_1 - \frac{2k^2}{M^2}\alpha - \frac{20k^2}{3M^2}\beta = -\frac{\Lambda_b}{6k^2M^3}\,. \tag{35}$$

Note that this reduces to the previously obtained purely quadratic GB extended RS result[4] when $a_1 = 1, \beta = 0$. In the more general case, this expression is not overly useful given the exact result in Eq.(27). Turning now to the $A, B = \mu, \nu$ terms which contain the 4-d graviton tensor excitation, we linearize employing the previously mentioned transverse, traceless gauge with constant R. This gives the standard equation of motion for the RS graviton found long ago[32] though scaled by an overall factor. Employing the standard KK decomposition

$$h_{\mu\nu}(x,y) = \sum_n h_{\mu\nu}^{(n)}(x)\chi_n(y)\,, \tag{36}$$

and recalling that $\eta^{\alpha\beta}\partial_\alpha\partial_\beta h_{\mu\nu}^{(n)} = -m_n^2 h_{\mu\nu}^{(n)}$, the χ_n are seen to satisfy

$$H\left[\partial_y^2 - 4k\partial_y + m_n^2 e^{2ky}\right]\chi_n = 0\,. \tag{37}$$

The overall factor H is given by

$$H = a_1 - \frac{4k^2}{M^2}\alpha - \frac{40k^2}{M^2}\beta\,, \tag{38}$$

or, more explicitly in the RS case,

$$H(k) = F_R + 36k^2 F_Q + 1000k^4 F_{RQ} + 10080k^6 F_{QQ}\,. \tag{39}$$

(Again recall that F itself can be a function of k.) This leads to a rescaling of the usual RS relationship

$$H(k)\frac{M^3}{k} = \overline{M}_{Pl}^2\,, \tag{40}$$

via the renormalization of the zero mode (*i.e.*, massless graviton) wavefunction, thus generalizing Eq.(31). Of course, $H > 0$ is required to avoid ghost states. This result reduces to that previously obtained in the RS case with just the added GB term[31, 4] once boundary effects are neglected. From this analysis we see immediately that the masses of the KK gravitons are identical to those obtained in the original RS model, *provided* we use the *same* value of the parameter k, as we might have expected. Here we are faced with the question of what are the independent parameters. k is clearly a *derived* parameter that is obtained by solving Eq.(27) for any given model. In that sense, the KK graviton spectrum would just be rescaled in comparison to the usual expectations given the same input values of $\Lambda, M, ...etc.$ As we have just seen, and as in the ADD case, the effect of a factor like H on the KK graviton couplings to 4-d SM matter again depends upon which model parameters are held fixed. At the very least, up to an overall constant, these couplings are identical to those of the standard RS model.

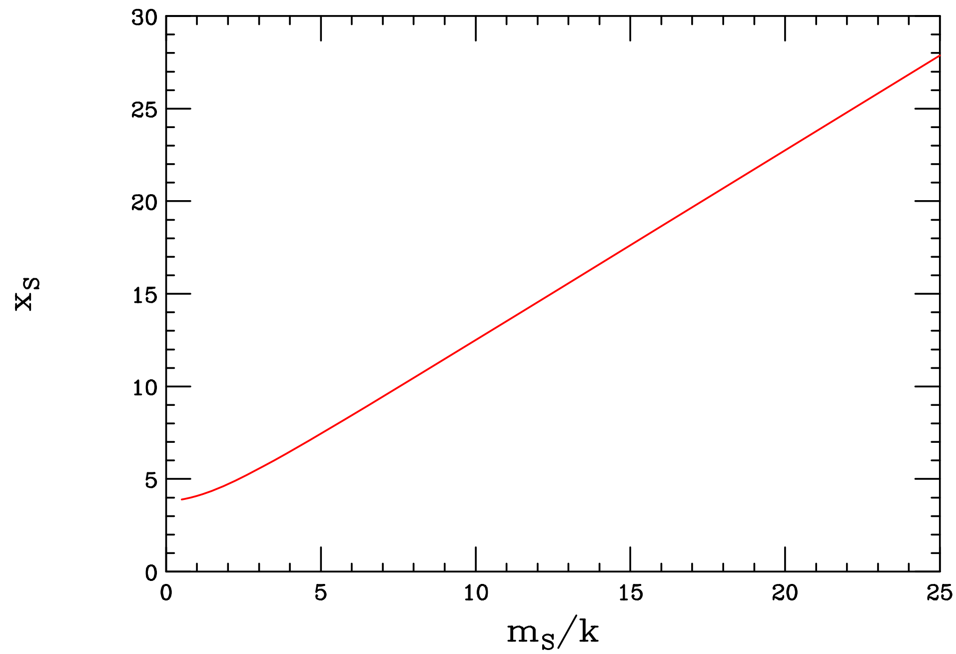

Figure 1: Root for the determination of mass of the lightest KK state corresponding to the new RS scalar as a function of the scaled bulk mass. As a point of comparison the first KK graviton has a root of $\simeq 3.83$.

So far we have only considered the 4-d graviton, spin-2 excitations. It is important to remember that our softened RS model now has an additional massive scalar in the 5d spectrum with a large bulk mass, m_S, so that no massless scalar zero mode will exist. Since the bulk scalar mass is naturally of order k the KK spectrum of the corresponding tower will begin with a KK scalar state whose mass is qualitatively comparable to that of the first graviton excitation. This bulk mass is explicitly calculable from expansion of the full action to quadratic order, $S^{(3)}_{eff}$, by going to the Einstein frame since we know that the GB term does not contribute. In that case, using the results from the previous section we find that

$$m_S^2 = \frac{3a_1}{16\beta} M^2 \,, \tag{41}$$

or, in terms of the original parameters of the action, evaluated in the RS background:

$$m_S^2 = \frac{3}{8} \, \frac{F_R + 20k^2 F_{RR} + 280k^4 F_{RQ}}{F_{RR} - 2F_Q - 40k^2 F_{RQ} - 560k^4 F_{QQ}} \,. \tag{42}$$

Note that $a_1, \beta > 0$ is required to avoid the scalar tachyons and graviton ghosts, consistent with our above analysis. Note further that this reproduces the results of Eq.(19) in the flat space, $k \to 0$, limit.

Given this bulk mass we can determine the mass(es) of the lightest KK scalar state(s), by following the standard RS manipulations[36]. These masses are essentially given by the first roots of the equation

$$(2 - \nu) J_\nu(x_S) + x_S J_{\nu-1}(x_S) = 0 \,, \tag{43}$$

where $\nu^2 = 4 + m_S^2/k^2$ and J is the usual Bessel function. The solution for the first KK state is provided by Fig.1; as stated above there are no massless modes. The lightest scalar mass is then $x_S k e^{-\pi k r_c}$. Here we observe that the mass of the first scalar KK scales almost linearly with the bulk mass when m_S gets large. Note that for $\beta/a_1 = 1$ and a typical value[32] of $k/M = 0.05$, we then find $m_S/k \simeq 8.7$ implying $x_S \simeq 11$ from Fig.1; this is about 3 times larger than that for the usual lightest massive KK graviton, $\simeq 3.83$. Here we see that unless β/a_1 takes on large values the first scalar KK state is always rather heavy. As is well-known, the x_S values for the more massive KK scalar states will be somewhat larger: approximately given by $x_S \to x_S + (p-1)\pi/2$ where p labels the KK level. Since these scalars will couple to the trace of the stress-energy tensor for the 4-d SM fields they will interact far more weakly than do the graviton KK states unless this is at least partially offset by ratios of 5-d wavefunction factors. A quick estimate of such factors, however, indicates that, if anything, these wave function ratios lead to a further suppression of the scalar couplings relative to those of the KK gravitons by $\simeq [12(1+(m_S^2/kx_S)^2)]^{-1/2} \simeq 1/4$ as shown in Fig.2. This overall picture of the scalar sector is qualitatively very similar to that of the existence of a very heavy tower of RS radions[33] or a tower of KK Higgs bosons as in the case of Universal Warped Extra Dimensions[37].

In the analysis as presented here we have ignored the possibility that the new scalar KK states may mix with the (usually eaten) RS graviscalars through cross-talk in the equations of motion, *i.e.*, we have assumed that the 5-d tensor and scalar KK decompositions can be performed independently, and this is something which needs further exploration. A fully detailed analysis of the such possibilities is, however, beyond the scope of the present paper.

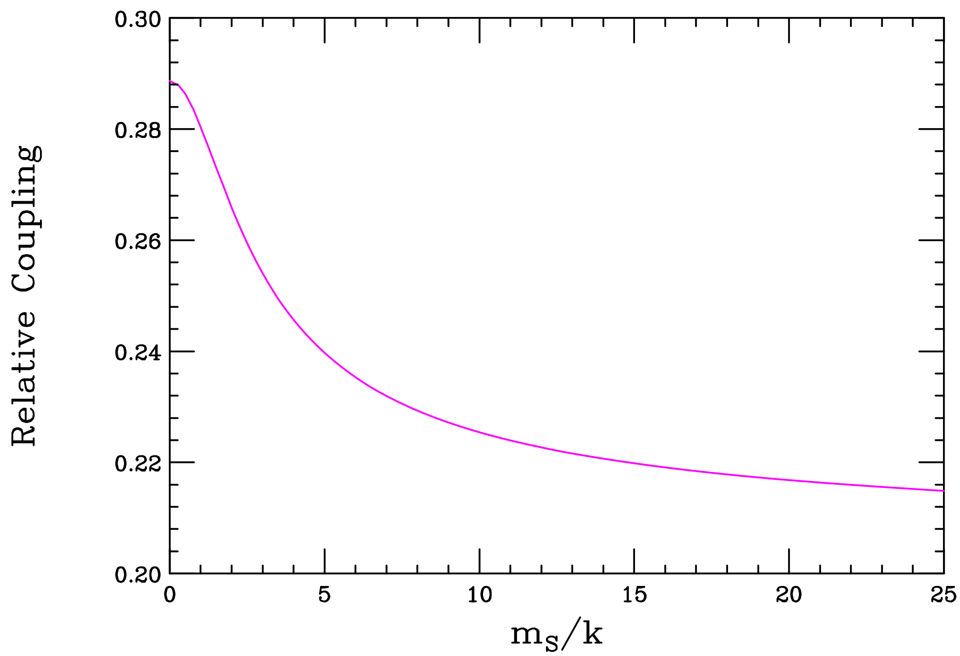

Figure 2: Estimate of the lightest scalar KK coupling strength relative to that of the lightest KK graviton. Recall that graviton KK states couple to $T_{\mu\nu}$ whereas the new scalar KKs couple to its trace, T.

5 Discussion and Conclusions

In this paper we have begun an examination of how generic higher curvature terms in the gravitational action can alter the predictions of both the ADD model and the RS model defined on a interval to avoid possible brane singularities. We have assumed that the traditional assumptions of the two models, *e.g.*, SM localized matter in a conformally flat bulk, remain valid; we have not considered more complex setups that may now be allowed by the modified equations of motion. To be more concrete, we have further assumed that the EH action is generalized generalized to an action which is of the form $F(R, P, Q)$ where F is a well-behaved function, $P = R_{AB}R^{AB}$ and $Q = R_{ABCD}R^{ABCD}$. In D-dimensions this action results in a propagating massless tensor field (identified with the usual graviton), a massive ghost tensor field, as well as a massive (possibly tachyonic) scalar. The potentially dangerous ghost is removable from the spectrum, *i.e.*, it becomes infinitely massive, if we demand that $F = F(R, P - 4Q)$ only. The remaining new scalar field has a bulk mass whose value is naturally expected to be of order the fundamental scale, M, in either scenario. The resulting ADD and RS models are altered in similar ways from their traditional standard forms:

(i) New scalar KK excitations appear in the spectrum of both models in a rather benign fashion coupling to the trace of the stress-energy tensor of the localized SM fields. Since this trace is proportional to SM masses, the couplings of these scalars are relatively strongly suppressed in comparison to those for the KK gravitons at typical collider energies in both models. In the ADD model, the KK scalar excitations begin at a mass $\sim M \sim$ TeV. Consequently their contributions to missing-energy signatures as well as to the usual dimension-8 contact interactions are further kinematically suppressed. Thus at leading order these new scalars do not much influence ADD collider signatures. In RS, the bulk scalar mass tends to be large so that the lightest scalar KK is several times more massive than is the lightest KK graviton. Given their rather weak couplings such states will be difficult to observe at colliders.

(ii) The basic model relationships involving the fundamental and 4-d Planck masses in both models get rescaled by functions of F and its derivatives evaluated in the corresponding background metric of the two models: in ADD we obtain $\overline{M}_{Pl}^2 = V_n M^{n+2} F_R$ while in RS we obtain $\overline{M}_{Pl}^2 = H(k)\ M^3/k$ where $H(k)$ is explicitly given in Eq.(39). Assuming that M is a fixed fundamental parameter these modifications lead to changes in the graviton KK sectors of both models. In the ADD case, since $\overline{M}_{Pl}$ is known and M is an input parameter for any given F_R the volume of the compactified space and, hence, the value of the compactification radius which sets the graviton KK mass scale is altered. Due to the presence of the F_R factor the emission rate for gravitons in the collisions of SM particles and for the graviton exchange amplitude are both modified by potentially O(1) effects. Similarly in RS, k is a derived parameter which sets the scale for all the KK states. The constraint Eq.(28) allows us to calculate k in terms of the input parameters M, Λ and the function F thus providing for us with $H(K)$. In a manner similar to ADD, the presence of H rescales the coupling strengths of the of the KK graviton states to the SM fields thus modifying the widths and production cross sections at colliders by potentially O(1) factors.

As we have seen, the extension of the EH action to a more complicated structure can lead to significant modifications to both the ADD and RS model predictions in the simplest possible case. The observation of such effects at future colliders could tell us valuable information about the underlying theory of gravity.

Note Added: After this paper was essentially completed, Ref.[38] appeared which discusses generalised actions for the ADD model and thus has some common areas with the present work. Where the two papers overlap there is general qualitative agreement though the points of view are somewhat different.

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References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999) [arXiv:hep-ph/9807344] and Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[3] There is a vast literature on this subject. For a far from exhaustive list see, for example, R. G. Cai, Phys. Rev. D **65**, 084014 (2002) [arXiv:hep-th/0109133], Phys. Lett. B **582**, 237 (2004) [arXiv:hep-th/0311240] and Phys. Rev. D **63**, 124018 (2001) [arXiv:hep-th/0102113]; R. G. Cai and Q. Guo, Phys. Rev. D **69**, 104025 (2004); [arXiv:hep-th/0311020]. R. C. Myers and J. Z. Simon, Phys. Rev. D **38**, 2434 (1988); S. Nojiri, S. D. Odintsov and S. Ogushi, Phys. Rev. D **65**, 023521 (2002) [arXiv:hep-th/0108172]; Y. M. Cho and I. P. Neupane, Int. J. Mod. Phys. A **18**, 2703 (2003) [arXiv:hep-th/0112227] and Phys. Rev. D **66**, 024044 (2002) [arXiv:hep-th/0202140]; T. Clunan, S. F. Ross and D. J. Smith, Class. Quant. Grav. **21**, 3447 (2004) [arXiv:gr-qc/0402044]; R. G. Cai and Q. Guo, Phys. Rev. D **69**, 104025 (2004) [arXiv:hep-th/0311020]; N. Okuyama and J. i. Koga, arXiv:hep-th/0501044; J. Crisostomo, R. Troncoso and J. Zanelli, Phys. Rev. D **62**, 084013 (2000) [arXiv:hep-th/0003271]; M. Cvetic, S. Nojiri and S. D. Odintsov, Nucl. Phys. B **628**, 295 (2002) [arXiv:hep-th/0112045]; R. Aros, R. Troncoso and J. Zanelli, Phys. Rev. D **63**, 084015 (2001) [arXiv:hep-th/0011097]; E. Abdalla and L. A. Correa-Borbonet, Phys. Rev. D **65**, 124011 (2002) [arXiv:hep-th/0109129]; T. Jacobson, G. Kang and R. C. Myers, Phys. Rev. D **52**, 3518 (1995) [arXiv:gr-qc/9503020] and Phys. Rev. D **49**, 6587 (1994) [arXiv:gr-qc/9312023]; T. Jacobson and R. C. Myers, Phys. Rev. Lett. **70**, 3684 (1993) [arXiv:hep-th/9305016]; S. Deser and B. Tekin, Phys. Rev. D **67**, 084009 (2003); [arXiv:hep-th/0212292]. A. Barrau, J. Grain and S. O. Alexeyev, Phys. Lett. B **584**, 114 (2004) [arXiv:hep-ph/0311238]; R. Konoplya, Phys. Rev. D **71**, 024038 (2005)

[arXiv:hep-th/0410057]; J. Grain, A. Barrau and P. Kanti, arXiv:hep-th/0509128; S. Mignemi and D. L. Wiltshire, Phys. Rev. D **46**, 1475 (1992) [arXiv:hep-th/9202031].

[4] T. G. Rizzo, JHEP **0506**, 079 (2005) [arXiv:hep-ph/0503163], JHEP **0501**, 028 (2005) [arXiv:hep-ph/0412087], arXiv:hep-ph/0510420 and arXiv:hep-ph/0601029.

[5] See, for example, I. Navarro and K. Van Acoleyen, arXiv:gr-qc/0512109 and arXiv:gr-qc/0511045, Phys. Lett. B **622**, 1 (2005) [arXiv:gr-qc/0506096]; G. J. Olmo, Phys. Rev. Lett. **95**, 261102 (2005) [arXiv:gr-qc/0505101] and Phys. Rev. D **72**, 083505 (2005); M. T. Jaekel and S. Reynaud, Mod. Phys. Lett. A **20**, 1047 (2005) [arXiv:gr-qc/0410148]; A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003) [arXiv:astro-ph/0307285]; T. Chiba, Phys. Lett. B **575**, 1 (2003) [arXiv:astro-ph/0307338]; M. E. Soussa and R. P. Woodard, Gen. Rel. Grav. **36**, 855 (2004) [arXiv:astro-ph/0308114]; S. Capozziello and A. Troisi, Phys. Rev. D **72**, 044022 (2005) [arXiv:astro-ph/0507545].; S. Capozziello, arXiv:gr-qc/0412088; I. Quandt and H. J. Schmidt, Astron. Nachr. **312**, 97 (1991) [arXiv:gr-qc/0109005]. See, however, K. Tangen, arXiv:gr-qc/0602089.

[6] S. M. Carroll, A. De Felice, V. Duvvuri, D. A. Easson, M. Trodden and M. S. Turner, Phys. Rev. D **71**, 063513 (2005) [arXiv:astro-ph/0410031]; S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004) [arXiv:astro-ph/0306438]; S. Nojiri and S. D. Odintsov, arXiv:hep-th/0601213; T. Clifton and J. D. Barrow, arXiv:gr-qc/0601118; N. Arkani-Hamed, S. Dimopoulos, G. Dvali and G. Gabadadze, arXiv:hep-th/0209227; T. Biswas, A. Mazumdar and W. Siegel, arXiv:hep-th/0508194; G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016]; G. Kofinas, R. Maartens and E. Papantonopoulos, JHEP **0310**, 066 (2003) [arXiv:hep-th/0307138]; R. A. Brown, R. Maartens, E. Papantonopoulos and V. Zamarias, JCAP **0511**, 008 (2005) [arXiv:gr-qc/0508116]; P. D. Mannheim and D. Kazanas, Astrophys. J. **342**, 635 (1989);

H. J. Schmidt, Int. J. Mod. Phys. A **5**, 4661 (1990); T. Biswas, A. Mazumdar and W. Siegel, arXiv:hep-th/0508194.

[7] D. A. Demir and S. H. Tanyildizi, arXiv:hep-ph/0512078.

[8] See, for example, A. Accioly, A. Azeredo and H. Mukai, J. Math. Phys. **43**, 473 (2002).

[9] For an historical overview and original references, see H. J. Schmidt, arXiv:gr-qc/0602017. For detailed results, see K. S. Stelle, Gen. Rel. Grav. **9**, 353 (1978) and Phys. Rev. D **16**, 953 (1977); T. Chiba, JCAP **0503**, 008 (2005) [arXiv:gr-qc/0502070]; A. Nunez and S. Solganik, Phys. Lett. B **608**, 189 (2005) [arXiv:hep-th/0411102].

[10] I. Antoniadis and E. T. Tomboulis, Phys. Rev. D **33**, 2756 (1986).

[11] H. Davoudiasl and T. G. Rizzo, Phys. Lett. B **512**, 100 (2001) [arXiv:hep-ph/0104199].

[12] D.Lovelock, J. Math. Phys.**12**, 498 (1971) . See also C. Lanczos, Z. Phys. 73, 147 (1932) and Ann. Math. **39**, 842 (1938).

[13] B. Zwiebach, Phys. Lett. B **156**, 315 (1985). See also D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985) and B. Zumino, Phys. Rept. **137**, 109 (1986).

[14] N. E. Mavromatos and J. Rizos, Phys. Rev. D **62**, 124004 (2000) [arXiv:hep-th/0008074].

[15] See, for example, A. L. Maroto and I. L. Shapiro, Phys. Lett. B **414**, 34 (1997) [arXiv:hep-th/9706179].

[16] J. Hewett and M. Spiropulu, Ann. Rev. Nucl. Part. Sci. **52**, 397 (2002) [arXiv:hep-ph/0205106].

[17] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **544**, 3 (1999) [arXiv:hep-ph/9811291].

[18] T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D **59**, 105006 (1999) [arXiv:hep-ph/9811350].

[19] E. A. Mirabelli, M. Perelstein and M. E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999) [arXiv:hep-ph/9811337].

[20] J. L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999) [arXiv:hep-ph/9811356].

[21] T. Banks and W. Fischler, hep-th/9906038.

[22] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001) [arXiv:hep-ph/0106295].

[23] S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002) [arXiv:hep-ph/0106219].

[24] For a recent review, see P. Kanti, arXiv:hep-ph/0402168. See also D. Stojkovic, arXiv:hep-ph/0409124; S. B. Giddings and E. Katz, J. Math. Phys. **42**, 3082 (2001) [arXiv:hep-th/0009176];

[25] A. Hindawi, B. A. Ovrut and D. Waldram, Phys. Rev. D **53**, 5597 (1996) [arXiv:hep-th/9509147] and Phys. Rev. D **53**, 5583 (1996) [arXiv:hep-th/9509142]; S. W. Hawking and T. Hertog, Phys. Rev. D **65**, 103515 (2002) [arXiv:hep-th/0107088].

[26] See, for example S. Carroll *et al.*in Ref [6] and S. Deser and B. Tekin in Ref.[3].

[27] J. T. Wheeler, Nucl. Phys. B **273**, 732 (1986) and Nucl. Phys. B **268**, 737 (1986). See also, B. Whitt, Phys. Rev. D **38**, 3000 (1988); D. L. Wiltshire, Phys. Lett. B **169**, 36 (1986) and Phys. Rev. D **38**, 2445 (1988).

[28] S. Mignemi and D. L. Wiltshire, Phys. Rev. D **46**, 1475 (1992) [arXiv:hep-th/9202031].

[29] K. i. Maeda, Phys. Rev. D **39**, 3159 (1989); B. Whitt, Phys. Lett. B **145**, 176 (1984); M. Mijic and J. A. Stein-Schabes, Phys. Lett. B **203**, 353 (1988); D. Wands, Class. Quant. Grav. **11**, 269 (1994) [arXiv:gr-qc/9307034].

[30] A. Jakubiec and J. Kijowski, Phys. Rev. D **37**, 1406 (1988); J. D. Barrow and S. Cotsakis, Phys. Lett. B **214**, 515 (1988); G. Magnano, M. Ferraris and M. Francaviglia, Class. Quant. Grav.

7, 557 (1990); G. Magnano and L. M. Sokolowski, Phys. Rev. D **50**, 5039 (1994) [arXiv:gr-qc/9312008].

[31] J. E. Kim, B. Kyae and H. M. Lee, Nucl. Phys. B **582**, 296 (2000) [Erratum-ibid. B **591**, 587 (2000)] [arXiv:hep-th/0004005] and Phys. Rev. D **62**, 045013 (2000) [arXiv:hep-ph/9912344].

[32] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. **84**, 2080 (2000) [arXiv:hep-ph/9909255].

[33] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **595**, 250 (2001) [arXiv:hep-ph/0002178]; C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001) [arXiv:hep-th/0008151]; D. Dominici, B. Grzadkowski, J. F. Gunion and M. Toharia, Acta Phys. Polon. B **33**, 2507 (2002) [arXiv:hep-ph/0206197] and Nucl. Phys. B **671**, 243 (2003) [arXiv:hep-ph/0206192]; J. L. Hewett and T. G. Rizzo, JHEP **0308**, 028 (2003) [arXiv:hep-ph/0202155].

[34] See, for example, F. del Aguila, M. Perez-Victoria and J. Santiago, arXiv:hep-ph/0601222.

[35] C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, Phys. Rev. D **69**, 055006 (2004) [arXiv:hep-ph/0305237]; C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. **92**, 101802 (2004) [arXiv:hep-ph/0308038]. G. Cacciapaglia, C. Csaki, C. Grojean and J. Terning, Phys. Rev. D **70**, 075014 (2004) [arXiv:hep-ph/0401160] and Phys. Rev. D **71**, 035015 (2005) [arXiv:hep-ph/0409126]; G. Cacciapaglia, C. Csaki, C. Grojean, M. Reece and J. Terning, arXiv:hep-ph/0505001. C. Csaki, C. Grojean, J. Hubisz, Y. Shirman and J. Terning, Phys. Rev. D **70**, 015012 (2004) [arXiv:hep-ph/0310355]; Y. Nomura, JHEP **0311**, 050 (2003) [arXiv:hep-ph/0309189]; R. Barbieri, A. Pomarol and R. Rattazzi, Phys. Lett. B **591**, 141 (2004) [arXiv:hep-ph/0310285]. H. Davoudiasl, J. L. Hewett, B. Lillie and T. G. Rizzo, Phys. Rev. D **70**, 015006 (2004) [arXiv:hep-ph/0312193] and JHEP **0405**, 015 (2004) [arXiv:hep-ph/0403300]; J. L. Hewett, B. Lillie and T. G. Rizzo, JHEP **0410**, 014 (2004) [arXiv:hep-

ph/0407059]; R. S. Chivukula, E. H. Simmons, H. J. He, M. Kurachi and M. Tanabashi, Phys. Rev. D **70**, 075008 (2004) [arXiv:hep-ph/0406077] and Phys. Lett. B **603**, 210 (2004) [arXiv:hep-ph/0408262].

[36] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [arXiv:hep-ph/9907447].

[37] H. Davoudiasl, B. Lillie and T. G. Rizzo, arXiv:hep-ph/0508279 and arXiv:hep-ph/0509160.

[38] O. Aslan and D. A. Demir, arXiv:hep-ph/0603051.