SLAC-PUB-11607
December 2005

Measurement of the CKM angle α with the B-factories.

Adrian Bevan[1]
[1] *Physics Department, Queen Mary, University of London, Mile End Road, E1 4NS, UK.*[†]
Physics Department, University of Liverpool, Oxford Street, Liverpool, L69 7ZE, UK.
[†] *current address.*
bevan@slac.stanford.edu
(Dated: December 20, 2005)

B-meson decays involving $b \to u$ transitions are sensitive to the Unitarity Triangle angle α (or ϕ_2). The B-factories at SLAC and KEK have made significant progress toward the measurement of α in recent years. This paper summarizes the results of the B-factories' constraints on α.

PACS numbers: 13.25.Hw, 12.15.Hh, 11.30.Er
Keywords: *CP* Violation; B-meson decays; α.

I. INTRODUCTION

CP violation (CPV) was first seen in the decay of neutral kaons [1]. It was shown some time ago that CPV is a necessary but insufficient constraint in order to generate a net baryon anti-baryon asymmetry in the universe [2]. In the ensuing years there has been a tremendous amount of activity by the high energy physics community to better understand the role of CPV in our model of nature − the Standard Model of Particle Physics (SM).

CPV in the SM is described by a single complex phase in a 3×3 quark-mixing matrix, V_{CKM}, called the CKM [3] matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{1}$$

V_{CKM} describes the coupling of the u, c and t quarks to d, s and b quarks, which is mediated by the exchange of a W boson. In B-meson decays the interesting *CP* violating parameters of the SM are related to the angles (β, α, and γ)[1] and sides of the so-called Unitarity Triangle (as shown in Figure 1). The angles are defined as:

$$\alpha \equiv \arg\left[-V_{td}V_{tb}^{\star}/V_{ud}V_{ub}^{\star}\right], \tag{2}$$

$$\beta \equiv \arg\left[-V_{cd}V_{cb}^{\star}/V_{td}V_{tb}^{\star}\right], \tag{3}$$

$$\gamma \equiv \arg\left[-V_{ud}V_{ub}^{\star}/V_{cd}V_{cb}^{\star}\right]. \tag{4}$$

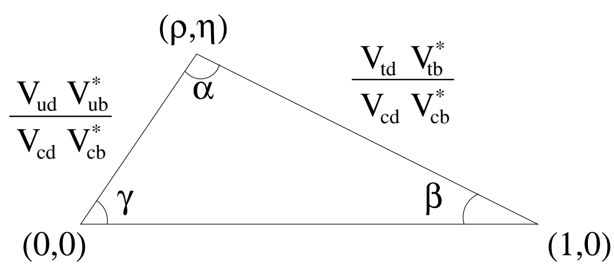

FIG. 1: The Unitarity Triangle.

This review is a summary of the experimental constraints on the Unitarity Triangle angle α obtained from B-meson decays involving $b \to u$ transitions. The CKM angle β is well known and consistent with SM predictions [4]. Any

[1] The Belle Collaboration use the notation ϕ_1, ϕ_2, and ϕ_3 for the angles of the Unitarity Triangle.

Submitted to Modern Physics Letters A

Work supported in part by Department of Energy contract DE-AC02-76SF00515

constraint on α constitutes a test of the SM description of quark mixing and CPV in B-meson decays. A significant deviation from SM expectation would be a clear indication of new physics. It is possible to obtain a SM prediction of α from indirect constraints by combining measurements of the CKM matrix elements $|V_{us}|$, $|V_{ud}|$, $|V_{ub}|$, and $|V_{cb}|$, CPV in mixing from neutral kaons, B-$\overline{B}$ mixing in B_d and B_s mesons and the measurement of $\sin 2\beta$ from $b \to c\overline{c}s$ decays. The SM predictions for α are $(98.2 \pm 7.7)^\circ$ from [5] and $(97^{+13}_{-19})^\circ$ [6].

Measurements of α have recently been performed at the B-factories, the *BABAR* experiment [7] at SLAC and the Belle experiment [8] at KEK. The B-factories study the decays of B-mesons produced in the process $e^+e^- \to \Upsilon(4S) \to B\overline{B}$, where $B\overline{B}$ is either B^+B^- or $B^0\overline{B}^0$. The neutral B-mesons are produced in a correlated P-wave state. Until one of the B-mesons in an event decays, there is exactly one B^0 and one $\overline{B}^0$ meson. The main goals of the B-factories include the study of CPV in B-meson decay, and to over-constrain the position of the apex of the Unitarity Triangle (ρ, η). The final states of interest for the study of α are $B \to hh'$, where $h = \rho, \pi$. Figure 2 shows the Feynman diagrams corresponding to the dominant amplitudes contributing to the $h^+h'^-$ decays.

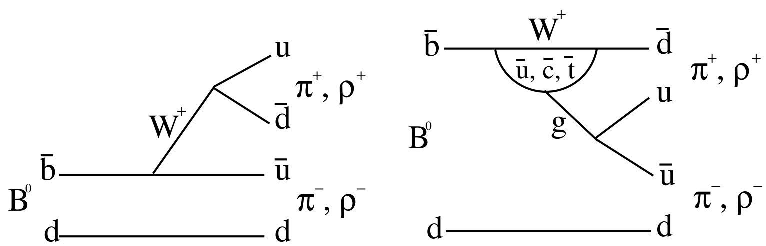

FIG. 2: The tree (left) and gluonic loop (right) contributions to $B \to h^+h'^-$ decays.

Interference between the amplitude for direct decay, and decay after B^0-$\overline{B}^0$ mixing to a $h^+h'^-$ final state, results in a time-dependent decay-rate asymmetry between B^0 and $\overline{B}^0$ decays that is sensitive to the CKM angle α. To study the time-dependent asymmetry with neutral B-mesons, one needs to measure the proper time difference Δt between the decay of the two B-mesons in the event. On reconstructing the $h^+h'^-$ final state of interest ($B_{\rm rec}$), one needs to determine the flavor of the other B-meson $(B_{\rm tag})$[2] from its decay products. The time difference between the decays of the two neutral B-mesons in the event ($B_{\rm rec}$, $B_{\rm tag}$) is calculated from the measured separation Δz between the $B_{\rm rec}$ and $B_{\rm tag}$ decay vertices. The $B_{\rm rec}$ decay vertex is determined from the two charged-pion tracks in its final state. The $B_{\rm tag}$ decay vertex is obtained by fitting the other tracks in the event.

The signal decay-rate distribution of a *CP*-eigenstate decay, $f_+ (f_-)$ for $B_{\rm tag}= B^0(\overline{B}^0)$, is given by:

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau}[1 \pm S\sin(\Delta m_d \Delta t) \mp C\cos(\Delta m_d \Delta t)]\,, \tag{5}$$

where $\tau = 1.536 \pm 0.014\,{\rm ps}$ is the mean B^0 lifetime and $\Delta m_d = 0.502 \pm 0.007\,{\rm ps}^{-1}$ is the B^0-$\overline{B}^0$ mixing frequency [9]. This assumes that there is no difference between B^0 lifetimes, $\Delta\Gamma = 0$. The parameters S and C are defined as:

$$S = \frac{2Im\lambda}{1+|\lambda|^2}, \qquad\qquad C = \frac{1-|\lambda|^2}{1+|\lambda|^2}, \tag{6}$$

where $\lambda = \frac{q}{p}\frac{\overline{A}}{A}$ is related to the level of B^0-$\overline{B}^0$ mixing (q/p), and the ratio of amplitudes of the decay of a $\overline{B}^0$ or B^0 to the final state under study $(\overline{A}/A)$. CPV is probed by studying the time-dependent decay-rate asymmetry

$$\mathcal{A} = \frac{R(\Delta t) - \overline{R}(\Delta t)}{R(\Delta t) + \overline{R}(\Delta t)}, \tag{7}$$

where $R(\overline{R})$ is the decay-rate for $B^0(\overline{B}^0)$ tagged events. This asymmetry has the form

$$\mathcal{A} = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t). \tag{8}$$

Belle use a different convention to *BABAR* with $C = -\mathcal{A}_{CP}$.

[2] i.e. determine if $B_{\rm tag}$ is a B^0 or $\overline{B}^0$.

In this article, all results are quoted using the S and C convention. In the case of charged B-meson decays (and $\pi^0\pi^0$ as there is no vertex information) one can study a time integrated charge asymmetry

$$\mathcal{A}_{CP} = \frac{\overline{N} - N}{\overline{N} + N}, \tag{9}$$

where N ($\overline{N}$) is the number of B ($\overline{B}$) decays to the final state. A non-zero measurement of S, C or $\mathcal{A}_{CP}$ for any of the decays under study would be a clear indication of CPV.

In the absence of loop (penguin) contributions in B^0 decays to the CP eigenstate h^+h^-, $S = \sin 2\alpha$, and $C = 0$. The presence of penguin contributions with different weak phases to the tree level amplitude shift the experimentally measurable parameter $\alpha_{\rm eff}$ away from the value of α. In the presence of penguin contributions $\alpha_{\rm eff} = \alpha + \delta\alpha_{\rm (penguin)}$, $S = \sin 2\alpha_{\rm eff}$, and C can be non zero.

For B^0 decays to $\rho^+\rho^-$, S= S_L or S_T and C= C_L or C_T are the CP asymmetry parameters for the longitudinal and transversely polarized signal, respectively. The decay-rate distribution for B^0 decays to the $\rho^{\pm}\pi^{\mp}$ final state is more complicated than the above description as $\rho^{\pm}\pi^{\mp}$ is not a CP eigenstate (discussed below).

There are two classes of measurement that the B-factories are pursuing. The main goal is to use $SU(2)$ isospin relations to relate different hh' final states and limit $\delta\alpha_{\rm (penguin)}$ in each of the modes $B^0 \to \pi^+\pi^-$, $B^0 \to \rho^{\pm}\pi^{\mp}$ and $B^0 \to \rho^+\rho^-$. In some cases only weak constraints on α are obtained when using $SU(2)$ relations and one must use a model dependent approach to obtain a significant result.

The remainder of this article describes the isospin analysis used to constrain $\delta\alpha_{\rm (penguin)}$, and experimental techniques. There is a discussion of the results at the end of the article. Throughout this article, the experimental results are quoted with statistical errors preceding systematic errors unless otherwise stated.

II. ISOSPIN ANALYSIS OF $B \to hh'$

One can use $SU(2)$ isospin to relate the amplitudes of B decays to $\pi\pi$ final states [10]. This results in two relations:

$$\frac{1}{\sqrt{2}} A^{+-} = A^{+0} - A^{00}, \tag{10}$$

$$\frac{1}{\sqrt{2}} \overline{A}^{+-} = \overline{A}^{-0} - \overline{A}^{00}, \tag{11}$$

where A^{ij} ($\overline{A}^{ij}$) are the amplitudes of B ($\overline{B}$) decays to the final state with charge ij. These two relations correspond to triangles in a complex plane as shown in Figure 3.

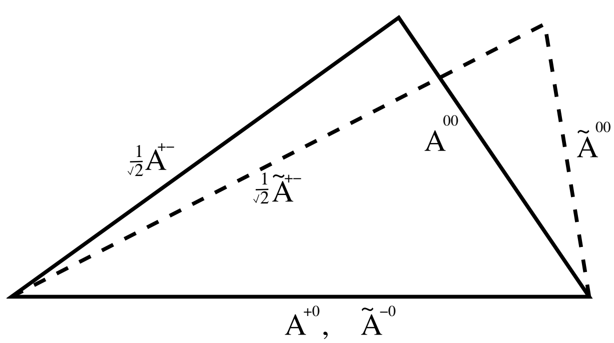

FIG. 3: The isospin triangle for $B \to \pi\pi$ decays.

This approach only considers tree and gluonic penguin contributions. Possible contributions from electroweak penguins (EWP) are ignored as they do not obey $SU(2)$ isospin symmetry. EWPs have the same topology as the gluonic penguin diagram in Figure 2, with the gluon replaced by γ or Z^0 bosons. In the absence of EWP contributions $|A^{+0}| = |\overline{A}^{+0}|$, i.e. $\mathcal{A}_{CP} = 0$ for $B^+ \to \pi^+\pi^0$. After aligning A^{+0} and $\overline{A}^{+0}$, the phase difference between A^{+-} and $\overline{A}^{+-}$ is $2\delta\alpha_{\rm (penguin)}$. In order to measure α one must measure the branching fractions ($\mathcal{B}$) and charge asymmetries ($\mathcal{A}_{CP}$, or C) of B decays to $\pi^+\pi^-$, $\pi^{\pm}\pi^0$, $\pi^0\pi^0$.

The decays $B \to \rho\rho$ are more complicated; one has an isospin triangle relation for each of the three amplitudes in the final state, one corresponding to the *CP* even longitudinal polarization and two corresponding to the *CP* even, and *CP* odd parts of the transverse polarization. The $\rho^{\pm}\pi^{\mp}$ final state is not a *CP* eigenstate. This results in the need for a pentagon isospin analysis [11] or a Dalitz Plot (DP) analysis of the $\pi^+\pi^-\pi^0$ final state [12] to measure α.

There are several assumptions implicitly used in the isospin-based direct measurements of α. The assumptions common to all decay modes are (i) to neglect EWP contributions and (ii) to neglect other $SU(2)$ symmetry breaking effects. In addition to this, possible I=1 amplitudes [13] in $B^0 \to \rho^+\rho^-$ decays are ignored. Several groups have estimated the correction due to the $SU(2)$ breaking effect of EWP contributions to be $\mathcal{O}(1.5-2)^\circ$ [14, 15]. These estimates consider contributions from the two EWP operators assumed to be dominant in the effective Hamiltonian. Some estimates of other expected $SU(2)$ symmetry breaking effects exist, where any correction is estimated to be much less than the current experimental precision [16].

III. EXPERIMENTAL TECHNIQUES

Continuum $e^+e^- \to q\overline{q}$ ($q = u, d, s, c$) events are the dominant background to $B \to hh'$ decays. Signal $B_{\rm rec}$ candidates are identified using two kinematic variables: the difference between the energy of the B candidate and the beam energy $\sqrt{s}/2$ in the center of mass (CM) frame ΔE; and the beam-energy substituted mass $m_{\rm ES} = \sqrt{(s/2 + \mathbf{p}_i \cdot \mathbf{p}_B)^2/E_i^2 - \mathbf{p}_B^2}$. As the B-factories collect data at the $\Upsilon(4S)$ resonance $\sqrt{s} = 10.58$ GeV. The B momentum $\mathbf{p}_B$ and four-momentum of the initial state $(E_i, \mathbf{p}_i)$ are defined in the laboratory frame. Event shape variables are used to further discriminate between signal and continuum background [17].

The flavor of $B_{\rm tag}$ is determined from its final state particles. Both *BABAR* and Belle characterize the final state particles of $B_{\rm tag}$ into events where there are leptons, kaons and π-mesons. The charge of these reconstructed tracks in the final state indicate if $B_{\rm tag}$ is a B^0 or a $\overline{B}^0$. The signal purity varies depending on the momentum and type of tracks in the final state. Events with leptons in the final state are the cleanest and those with charged kaons in the final state have less background than those with π-mesons. In addition to this, the probability of assigning the wrong flavor to $B_{\rm tag}$ increases as the background increases. *BABAR* separates events into mutually exclusive categories of events, whereas the Belle combines this information into a single variable r, and tunes event selection in bins of r. The two approaches are equivalent and are described in more detail in the following references [18, 19].

The Δt distributions of Equations 5 and 13 are convoluted with a detector resolution description, which differs for signal and continuum background [19, 20], and also take into account dilution from incorrectly assigning the flavor of $B_{\rm tag}$. The signal and background parameters are extracted using an extended unbinned maximum-likelihood fit to the data for the analyses described here.

IV. RESULTS

A. $B \to \pi\pi$

The simplest decays to study in the pursuit of α are $B \to \pi\pi$. Both experiments have measured $B \to \pi^+\pi^-$, $\pi^+\pi^0$ and $\pi^0\pi^0$ decays [21, 22]. The results are summarized in the Table below. The *BABAR* measurements use a data sample of $210 fb^{-1}$, the Belle $\pi^+\pi^0$ result use $78 fb^{-1}$, and the other Belle results use $253 fb^{-1}$ of integrated luminosity. All of these modes are now well established experimentally and so it is possible to perform an isospin analysis. The charge asymmetry measurement of $B^0 \to \pi^0\pi^0$ accounts for the effect of B^0-$\overline{B}^0$ mixing.

mode	Expt.	$\mathcal{B}$ ($\times 10^{-6}$)	S	C
$\pi^+\pi^-$	*BABAR*	$5.5 \pm 0.4 \pm 0.3$	$-0.30 \pm 0.17 \pm 0.03$	$-0.09 \pm 0.15 \pm 0.04$
	Belle	$4.4 \pm 0.6 \pm 0.3$	$-0.67 \pm 0.16 \pm 0.06$	$-0.56 \pm 0.12 \pm 0.06$

mode	Expt.	$\mathcal{B}$ ($\times 10^{-6}$)	$\mathcal{A}_{CP}$
$\pi^\pm\pi^0$	*BABAR*	$5.8 \pm 0.6 \pm 0.4$	$-0.01 \pm 0.10 \pm 0.02$
	Belle	$5.0 \pm 1.2 \pm 0.5$	$-0.02 \pm 0.10 \pm 0.01$
$\pi^0\pi^0$	*BABAR*	$1.17 \pm 0.32 \pm 0.10$	$-0.12 \pm 0.56 \pm 0.06$
	Belle	$2.3^{+0.4}_{-0.5}\,^{+0.2}_{-0.3}$	$-0.44^{+0.53}_{-0.52} \pm 0.17$

One should note that the Belle measurement of $B^0 \to \pi^+\pi^-$ constitutes an observation of CPV in this decay at a level of 5.4σ, and evidence for direct CPV at a level of 4.0σ. This is the second observation of CPV in B-meson decays, the first being the measurement of a non-zero value for $\sin 2\beta$ from $b \to c\bar{c}s$ decays [4]. The *BABAR* data are consistent with the hypothesis of no CPV in this decay. In recent years the two sets of results have started to converge to a common value, but more statistics are required to resolve the controversy.

The $\pi\pi$ isospin analysis is limited by the value of $\mathcal{B}(B^0 \to \pi^0\pi^0)$. Unfortunately this branching fraction is neither large enough to provide sufficient statistics with the current data set to perform a precision measurement of $\delta\alpha_{(\mathrm{penguin})}$, nor small enough to enable a strong bound on this quantity. *BABAR* have performed an isospin analysis resulting in $|\delta\alpha_{(\mathrm{penguin})}| < 35^\circ$ (90% C.L.). Belle's data exclude values of α between 19 and 72° (95.4% C.L.), and constrain $|\delta\alpha_{(\mathrm{penguin})}| < 38^\circ$ (95.4% C.L.) [23]. One requires a significant increase in statistics to perform a precision measurement of α using $B \to \pi\pi$ decays. However, it is possible to extract a model dependent constraint on α from the current results assuming $SU(3)$ symmetry (i.e. exchanging u and s quarks), Gronau and Rosner obtain a value of $\alpha = (107 \pm 13)^\circ$ [24].

B. $B \to \rho\rho$

The decay $B \to \rho\rho$ is that of a spin zero particle decaying into two spin one particles (as shown in Figure 4). As a result, the *CP* analysis of B decays to $\rho^+\rho^-$ is complicated by the presence of three helicity states ($H = 0, \pm 1$). The $H = 0$ state corresponds to longitudinal polarization and is *CP*-even, while neither the $H = +1$ nor the $H = -1$ state is an eigenstate of *CP*. The longitudinal polarization fraction f_L is defined as the fraction of the helicity zero state in the decay. The angular distribution is

$$\frac{d^2\Gamma}{\Gamma d\cos\theta_1 d\cos\theta_2} = \frac{9}{4}\left(f_L \cos^2\theta_1 \cos^2\theta_2 + \frac{1}{4}(1-f_L)\sin^2\theta_1 \sin^2\theta_2\right) \tag{12}$$

where θ_i ($i = 1, 2$) is defined for each ρ-meson as the angle between the π^0 momentum in the ρ rest frame and the flight direction of the B^0 in this frame. The angle ϕ between the ρ-decay planes is integrated over to simplify the analysis. A full angular analysis of the decays is needed in order to separate the definite *CP* contributions of the transverse polarization; if however a single *CP* channel dominates the decay (which has been experimentally verified), this is not necessary [25].

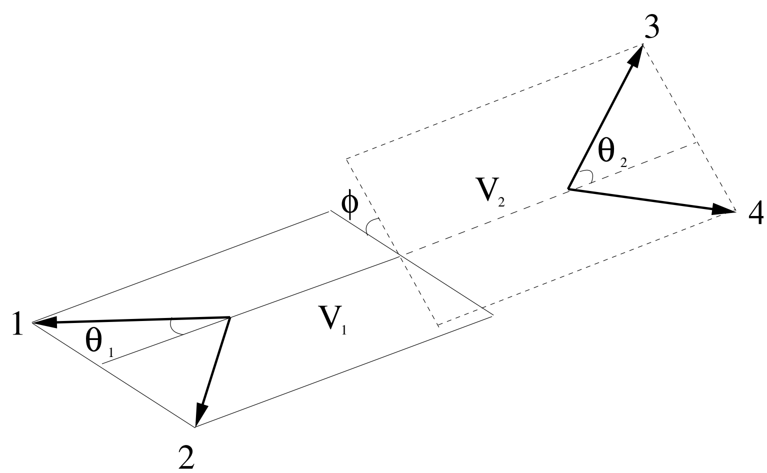

FIG. 4: A schematic diagram of the decay of a B-meson via two vector particles, V_1 and V_2, into a four-particle final state.

The longitudinal polarization dominates this decay [26, 27]. Not all of the $\rho\rho$ final states have been observed, however as $\mathcal{B}(B^0 \to \rho^0\rho^0)$ is small, one can conservatively assume that it is longitudinally polarized when performing an isospin analysis. The measured branching fractions, f_L, and C for $B \to \rho\rho$ are summarized in the Table below, and $S_L = -0.33 \pm 0.24^{+0.08}_{-0.14}$ ($0.09 \pm 0.42 \pm 0.08$) for *BABAR*. (Belle). The *BABAR* (Belle) $\rho^+\rho^-$ results use 210 (253)fb^{-1} of integrated luminosity, respectively[3]. The *BABAR* (Belle) $\rho^+\rho^0$ results use 82 (78)fb^{-1}, and the upper limit on $\rho^0\rho^0$

[3] The branching fraction reported by *BABAR* is from $82 fb^{-1}$.

mode	Expt.	$\mathcal{B}$ $(\times 10^{-6})$	f_L	C_L
$\rho^+\rho^-$	*BABAR*	$30 \pm 5 \pm 4$	$0.978 \pm 0.014^{+0.021}_{-0.029}$	$-0.03 \pm 0.18 \pm 0.09$
	Belle	$24.4 \pm 2.2^{+3.8}_{-4.1}$	$0.951^{+0.033}_{-0.039}{}^{+0.029}_{-0.031}$	$0.00 \pm 0.30^{+0.10}_{-0.09}$
mode	Expt.	$\mathcal{B}$ $(\times 10^{-6})$	f_L	$\mathcal{A}_{CP}$
$\rho^\pm\rho^0$	*BABAR*	$22.5^{+5.7}_{-5.4} \pm 5.8$	$0.97^{+0.03}_{-0.07} \pm 0.04$	$-0.19 \pm 0.23 \pm 0.03$
	Belle	$31.7 \pm 7.1^{+3.8}_{-6.7}$	$0.95 \pm 0.11 \pm 0.02$	$0.00 \pm 0.22 \pm 0.03$
$\rho^0\rho^0$	*BABAR*	< 1.1 (90% C.L.)	-	-

uses $210 fb^{-1}$ of integrated luminosity. Recent measurements of the $B^+ \to \rho^+\rho^0$ branching fraction and upper limit for $B^0 \to \rho^0\rho^0$ [28] indicate small penguin contributions in $B \to \rho\rho$, as predicted by some calculations [29]. The ultimate uncertainty on α from $B \to \rho\rho$ will depend on the branching fraction and *CP* content of $B^0 \to \rho^0\rho^0$. The decay $B^0 \to \rho^0\rho^0$ has an all charged track final state and it is possible to measure both S and C for the different *CP*-eigenstate components of the decay, unlike $B^0 \to \pi^0\pi^0$ decays where one can only measure C.

Given that penguin pollution is small, it is possible to perform an isospin analysis of the longitudinal polarization of the $B \to \rho\rho$ decays, and use the results of *BABAR*'s time-dependent *CP* analysis of $\rho^+\rho^-$ [27] to constrain α. If one does this, using the aforementioned assumptions, one obtains $\alpha = (100 \pm 13)^\circ$ using *BABAR* data [30]. The error on α is dominated by $|\delta\alpha_{(\mathrm{penguin})}| < 11^\circ$ (68% C.L.). Belle have recently produced a preliminary measurement of the polarization, branching fraction and *CP* parameters of $B^0 \to \rho^+\rho^-$ [31]. The Belle constraint is $\alpha = (87 \pm 17)^\circ$, which is slightly weaker than the *BABAR* result. Both $\rho\rho$ isospin analyses use the same experimental information for the decays $B^0 \to \rho^0\rho^0$, and $B^\pm \to \rho^\pm\rho^0$. Figure 5 shows the (1-C.L.) plot of α corresponding to the isospin analysis of the longitudinally polarized $\rho\rho$ data [6]. There is also a non-SM solution for α near 175°.

Our knowledge of $\delta\alpha_{(\mathrm{penguin})}$ is primarily determined from the experimental knowledge of the $\rho^0\rho^0$ and $\rho^\pm\rho^0$ branching ratios. As in the case of $B \to \pi\pi$, our knowledge on α is limited by the penguin contribution. However, for $B \to \rho\rho$ decays $\delta\alpha_{(\mathrm{penguin})}$ is sufficiently small to allow the B-factories to perform a meaningful measurement.

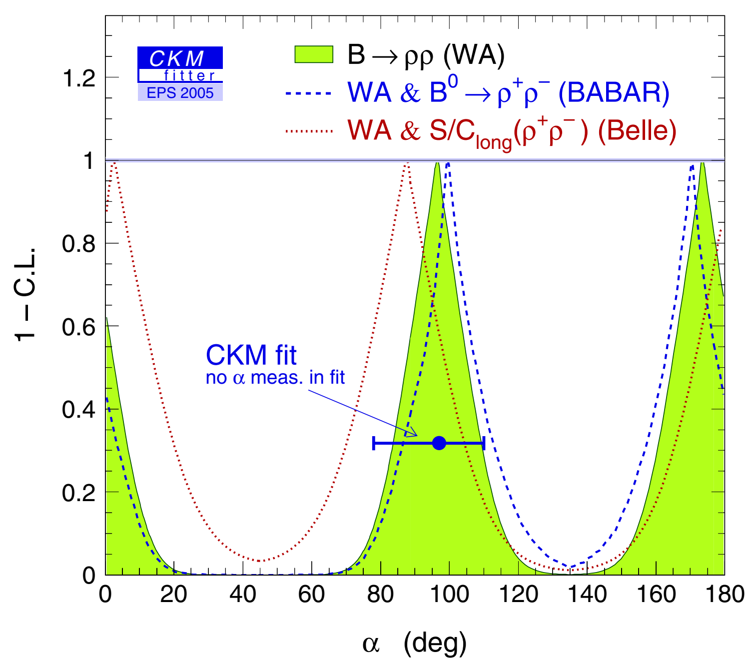

FIG. 5: A plot of (1 - C.L.) of α determined from the *BABAR* (dashed) and Belle (dotted) $\rho\rho$ isospin analyses [6]. The shaded region corresponds to the constraint obtained from the average of the two experiments. The point with error bars corresponds to the 1σ interval (68 % C.L.) obtained from the CKM fit when excluding direct measurements of α.

The $\rho\rho$ isospin triangle does not close when taking into account the measurements of the B-factories. Both the CKM Fitter and UTfit groups have noted that $SU(2)$ symmetry constraints prefer a smaller branching fraction for the $B^+ \to \rho^+\rho^0$ decay, at the level of $1.5 - 2\sigma$. The measurements did not test for possible non-resonant backgrounds, S-wave $\pi\pi$ contributions under the ρ^0, or account for exclusive B backgrounds to decays excluding charm particles. It is important to see that this decay mode is studied in more detail with higher statistics in the future.

C. $B \to \rho\pi$

The decays $B \to \rho\pi$ can be analysed in two different ways. The more straightforward approach is to cut away interference regions of the $\pi^+\pi^-\pi^0$ DP and analyze the regions in the vicinity of the ρ resonances. This is the so-called Quasi-2-body approach (Q2B) and it avoids the need to understand the interference regions. The drawbacks of the Q2B method are that one looses a lot of statistical power by cutting on the DP, and that interference is unaccounted for. A corollary of this is that one requires more statistics than the B-factories currently have in order to obtain a significant constraint from the pentagon isospin analysis of $B \to \rho\pi$. The alternative is to perform an analysis of the $B \to \pi^+\pi^-\pi^0$ DP, accounting for the interference between intersecting ρ resonance bands and other resonant structures. The Q2B approach has been studied by *BABAR* and Belle [32, 33], and the first time-dependent DP analysis has been performed by *BABAR* [34].

In the Q2B approach, one fits a time-dependence of

$$f_{\pm}(\Delta t) = (1 \pm \mathcal{A}_{CP}) \frac{e^{-|\Delta t|/\tau}}{4\tau} [(S \pm \Delta S \sin(\Delta m_d \Delta t) - (C \pm \Delta C)\cos(\Delta m_d \Delta t)] \,, \tag{13}$$

where there are three additional parameters in comparison with Equation 5. These parameters are a charged asymmetry, $\mathcal{A}_{CP}$, between decays to $\rho^+\pi^-$ and $\rho^-\pi^+$ final states, and two dilution parameters; ΔS and ΔC. The details of the DP analysis can be found in [34], where one varies a larger number of parameters in the nominal fit, and converts these to the same observables as the Q2B approach. The Table below summarizes the experimental constraints on $B \to \rho\pi$ (DP analysis from *BABAR* and Q2B analysis from Belle). The branching fractions measured for this decay are $(22.6 \pm 1.8 \pm 2.2) \times 10^{-6}$ and $(20.8^{+6.0}_{-6.3}{}^{+2.8}_{-3.1}) \times 10^{-6}$ by *BABAR* and Belle, respectively [35]. These results used 82 $(29.4) fb^{-1}$ of integrated luminosity and the time dependent results use 192 $(140) fb^{-1}$, respectively.

Expt.	S	C	$\mathcal{A}_{CP}$
BABAR	$-0.10 \pm 0.14 \pm 0.04$	$0.34 \pm 0.11 \pm 0.05$	$-0.088 \pm 0.049 \pm 0.013$
Belle	$-0.28 \pm 0.23^{+0.10}_{-0.08}$	$0.25 \pm 0.17^{+0.02}_{-0.06}$	$-0.16 \pm 0.10 \pm 0.02$

Using *BABAR*'s DP result, one obtains the following constraint; $\alpha = (113^{+27}_{-17} \pm 6)^\circ$. Figure 6 shows the corresponding (1 - C.L.) plot for α. This result is self-consistent as the strong phase differences and amplitudes are determined solely from the structure of the DP. The unique aspect of this result is that there is only a single solution between 0 and 180°, and thus a two fold ambiguity on α. As a result this measurement is an important constraint on α, ancillary to that from $B \to \rho\rho$.

It is possible to derive a model dependent constraint on α from these data as shown by Gronau and Zupan [36]. One can make several assumptions, including (i) the relative strong phase differences between tree level contributions for $\rho^+\pi^-$ and $\rho^-\pi^+$ being less than 90° (ii) $SU(3)$ being exact for penguin amplitudes (iii) neglecting EWP (iv) neglecting annihilation and exchange diagrams to derive a constraint on α. $SU(3)$ breaking effects are estimated in this model using CLEO, *BABAR* and Belle data. When doing this Belle obtains $\alpha = (102 \pm 11_{expt} \pm 15_{model})^\circ$. Gronau and Zupan compute the corresponding result for the *BABAR* analysis as $(93 \pm 4_{expt} \pm 15_{model})^\circ$. There are other models proposed in the literature, for example [37].

V. DISCUSSION

The indirect measurements of α from the SM are $(98.2 \pm 7.7)^\circ$ [5] and $(97^{+13}_{-19})^\circ$ [6] from the UTfit and CKM Fitter groups, respectively. Direct measurements from the B-factories are in good agreement with these predictions, where the uncertainty on α from $B \to hh'$ decays is known to $\mathcal{O}(9^\circ)$. The precision of this result is dominated by an isospin analysis of $B \to \rho\rho$ decays, with an important contribution from $B \to \rho\pi$ as the latter result suppresses a second solution for α near 175°. This solution is disfavored at the 1.3σ level by studies of $B^0 \to \pi^+\pi^-\pi^0$ decays. The B-factory constraints on α will be improved as additional data are analyzed over the next few years, and these measurements will start to test the closure of Unitarity Triangle with precision. A significant isospin analysis result

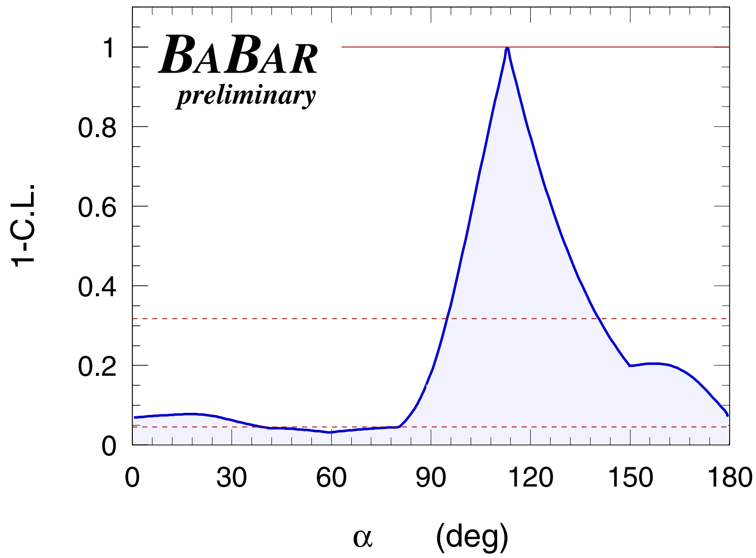

FIG. 6: A plot of (1 - C.L.) of α determined from *BABAR*'s $\pi^+\pi^-\pi^0$ DP analysis [34].

from $B \to \pi\pi$ requires a considerable increase in statistics. The measurement of α from $B \to \pi\pi$ remains a vital part of the program, as theoretical uncertainties in the extraction of α (other than the determination of $\delta\alpha_{(\text{penguin})}$) are the best understood of all the channels that have been discussed here.

Similarly, the model dependent calculations on α are in agreement with the SM. The constraints from $B \to \pi\pi$ and $\rho\pi$ decays have a precision of 13° and 15.5°, respectively. These constraints are dominated by theoretical uncertainties. The precision on the model dependent constraints obtained are comparable with the isospin analysis result from $B \to \rho\rho$.

The measurement of α using $B \to a_1\pi$ decays was proposed some time ago [38]. The recent observation of $B^0 \to a_1^+\pi^-$ with a branching fraction $\mathcal{O}(45 \times 10^{-6})$ [39] raises the question of how well one can ultimately measure α using this decay mode. The use of $SU(3)$ to measure α with $B \to a_1\pi$ decays has recently been proposed [40].

Studies of the LHCb experiment's [41] sensitivity to α using $B \to \pi^+\pi^-\pi^0$ decays have been performed [42]. LHCb is expected to achieve a 10° precision on α within the first year of data taking. The physics reach of decays involving $b \to u$ transitions with all charged particles in the final states, such as $B^0 \to \rho^0\rho^0$ and $B^0 \to a_1^+\pi^-$ should also be investigated. The current B-factories will reach a precision of $7-10^\circ$ on α with 1 ab^{-1} of data using $B \to \rho\rho$ decays. The ultimate precision reached depends on the values of the branching fraction, f_L, S and C in $B^0 \to \rho^0\rho^0$. Sensitivity projections for the $\pi^+\pi^-\pi^0$ DP analysis are very dependent on the many input parameters involved. For this reason, the running experiments have not extrapolated the precision of α obtained from $\pi^+\pi^-\pi^0$ decays to higher luminosities.

Acknowledgments

This work is supported in part by the UK Particle Physics and Astronomy Research Council (PPARC), and the U.S. Department of Energy under contract number DE-AC02-76SF00515. The author wishes to thank K. K. Phua for the opportunity to write this review, and I. Bigi, R. Faccini and J. Zupan for useful discussions and comments during the preparation of this article.

[1] J. Christenson *et al.*, Phys. Rev. Lett. **13**, 138 (1964).
[2] A. Sakharov, JETP Lett. **5**, 24 (1967).
[3] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963);
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[4] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **94**, 161803 (2005);
Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **71**, 072003 (2005).
[5] UTfit Collaboration, M. Bona *et al.*, JHEP **0507** 028 (2005), updated results and plots available at: http://utfit.roma1.infn.it/.
[6] CKMfitter Group, J. Charles *et al.*, Eur. Phys. J. C **41**, 1-131 (2005), updated results and plots available at: http://ckmfitter.in2p3.fr.
[7] *BABAR* Collaboration, B. Aubert *et al.*, Nucl. Instr. Methods Phys. Res., Sect. A **479**, 1 (2002).
[8] Belle Collaboration, A. Abashian *et al.*, Nucl. Instr. Methods Phys. Res., Sect. A **479**, 117 (2002).
[9] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[10] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
[11] H. Lipkin *et al.*, Phys. Rev. D **44**, 1454 (1991).
[12] A. Snyder and H. Quinn, Phys. Rev. D **48**, 2139 (1993);
H. Quinn and J Silva, Phys. Rev. D **62**, 054002 (2000).
[13] A. Falk *et al.*, Phys. Rev. D **69**, 011502 (2004).
[14] e.g. see p73 of Ref. [6].
[15] M. Gronau and J. Zupan, Phys. Rev. D **71**, 074017 (2005).
[16] S. Gardner, hep-ph/9906269;
S. Gardner, Phys. Rev. D **72**, 034015 (2005).
[17] e.g. see the Fisher discriminant of Ref. [21] and likelihood ratio cut of of Ref. [22].
[18] H. Kakuno *et al.*, Nucl. Instr. Methods Phys. Res., Sect. A **533**, 516 (2004).
[19] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. D **66**, 032003 (2002).
[20] H. Tajima *et al.*, Nucl. Instr. Methods Phys. Res., Sect. A **533**, 370 (2004).
[21] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **95**, 151803 (2005);
BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **94**, 181802 (2005);
BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **89**, 281802 (2002).
[22] Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **95**, 101801 (2005);
Belle Collaboration, Y. Chao *et al.*, Phys. Rev. Lett. **94**, 181803 (2005);
Belle Collaboration, Y. Chao *et al.*, Phys. Rev. D **69**, 111102 (2004).
[23] Private communication with M. Hazumi.
[24] M. Gronau and J. Rosner, Phys. Rev. D **65**, 093012 (2002);
M. Gronau and J. Rosner, Phys. Lett. B **595**, 339 (2004).
[25] I. Dunietz *et al.*, Phys. Rev. D **43**, 2193 (1991).
[26] G. Kramer and W. Palmer, Phys. Rev. D **45**, 193 (1992);
D. Ebert *et al.*, Phys. Rev. D **56**, 312 (1997);
A. Ali *et al.*, Phys. Rev. D **58**, 094009 (1998);
Y.-H. Chen *et al.*, Phys. Rev. D **60**, 094014 (1999).
[27] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **93**, 231801 (2004).
[28] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 171802 (2003);
Belle Collaboration, J. Zhang *et al.*, Phys. Rev. Lett. **91**, 221801 (2003).
[29] R. Aleksan *et al.*, Phys. Lett. B **356**, 95 (1995).
[30] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **95**, 041805 (2005).
[31] Belle Collaboration, K. Abe *et al.*, hep-ex/0507039.
[32] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 201802 (2003).
[33] Belle Collaboration, C. C. Wang *et al.*, Phys. Rev. Lett. **94**, 121801 (2005).
[34] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408099.
[35] Belle Collaboration, A. Gordon *et al.*, Phys. Lett. B **542**, 183 (2002);
also see Ref. [32].
[36] M. Gronau and J. Zupan, Phys. Rev. D **70**, 074031 (2004).
[37] M. Gronau *et al.*, Phys. Lett. B **606**, 95 (2005).
[38] R. Aleksan *et al.*, Nucl. Phys. B **361**, 141 (1991).
[39] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0507029;
Belle Collaboration, K. Abe *et al.*, hep-ex/0507096.
[40] M. Gronau and J. Zupan, hep-ph/0512148.
[41] LHCb Collaboration, CERN-LHCC/98-4; CERN-LHCb/2003-030.
[42] O. Deschamps *et al.*, CERN-LHCb-2005-024.