

# TeV-Scale Black Hole Lifetimes in Extra-Dimensional Lovelock Gravity <sup>\*</sup> <sup>†</sup>

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## Abstract

We examine the mass loss rates and lifetimes of TeV-scale extra dimensional black holes (BH) in ADD-like models with Lovelock higher-curvature terms present in the action. In particular we focus on the predicted differences between the canonical and microcanonical ensemble statistical mechanics descriptions of the Hawking radiation that results in the decay of these BH. In even numbers of extra dimensions the employment of the microcanonical approach is shown to generally lead to a significant increase in the BH lifetime as in case of the Einstein-Hilbert action. For odd numbers of extra dimensions, stable BH remnants occur when employing either description provided the highest order allowed Lovelock invariant is present. However, in this case, the time dependence of the mass loss rates obtained employing the two approaches will be different. These effects are in principle measurable at future colliders.

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# 1 Introduction and Background

The large extra dimensions picture of Arkani-Hamed, Dimopoulos and Dvali(ADD)[1] suggests that the fundamental scale of gravity,  $M_*$ , may not be far above the weak scale  $\sim \text{TeV}$ . In this scenario, gravity propagates in the  $D = 4 + n$  dimensional bulk while the Standard Model(SM) is confined to a three-dimensional 'brane' which is assumed to be flat. In such a scenario one finds that  $M_*$  is related to the usual 4-d (reduced) Planck scale,  $\overline{M}_{Pl}$ , via the expression

$$\overline{M}_{Pl}^2 = V_n M_*^{n+2}, \quad (1)$$

where  $V_n$  is the volume of the compactified extra dimensions. Assuming for simplicity that they form an  $n$ -dimensional torus, if all compactification radii,  $R_c$ , are the same, then  $V_n = (2\pi R_c)^n$ . This basic ADD picture leads to three essential predictions [2]: *(i)* the emission of graviton Kaluza-Klein(KK) states during the collision of SM particles leading to signatures with apparent missing energy[3, 4, 5]; *(ii)* the exchange of graviton KK excitations between SM fields leading to dimension-8 contact interaction-like operators with distinctive spin-2 properties[3, 4, 6]; *(iii)* the production of black holes(BH) at colliders and in cosmic rays with geometric cross sections,  $\simeq \pi R_s^2$ , with  $R_s$  being the BH Schwarzschild radius, once collision energies greater than  $\sim M_*$  are exceeded[7, 8, 9, 10].<sup>‡</sup> It has been noted that while *(i)* and *(ii)* are the result of an expansion of the  $D$ -dimensional Einstein-Hilbert(EH) action to leading order in the gravitational field and are in some sense perturbative, *(iii)* on the other-hand relies upon the full non-perturbative content of the EH action. Thus TeV scale BH production is actually testing  $D$ -dimensional General Relativity and not *just* the ADD picture. This is important as many other alternative theories of gravity in extra dimensions can lead to the same leading order graviton interactions. Within the ADD scenario, future collider measurements of the *(i)* and *(ii)* type processes should be able to tell us the values of both the quantities  $n$  and  $M_*$ [2] rather precisely.

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<sup>‡</sup>Note that in the simplest picture the BH production threshold is just a simple step-function.

Of course ADD is at best an effective theory that operates at energies below the scale  $M_*$ . It is reasonable to expect that at least some aspects of the full UV theory may leak down into these collider tests and may lead to potentially significant quantitative and/or qualitative modifications of simple ADD expectations that can be probed experimentally. We have recently begun an examination of the effect of the presence of higher curvature invariants in the  $D$ -dimensional action of ADD-like models [11] as well as in models with a warped metric[13]. We note that since the ADD bulk is flat and the SM fields are confined to a brane, the predictions for (i) and (ii) above are not influenced by the addition of such extra terms in the action[12] as the analogous predictions would be in the case of warped extra dimensions. Motivated by string theory[14, 15, 16], we examined a very special class of such invariants with interesting properties first described by Lovelock[17], called Lovelock invariants.

Lovelock invariants come in fixed order,  $m$ , which we denote as  $\mathcal{L}_m$ , that describes the number of powers of the curvature tensor out of which they are constructed. We can express the  $\mathcal{L}_m$  as

$$\mathcal{L}_m \sim \delta_{C_1 D_1 \dots C_m D_m}^{A_1 B_1 \dots A_m B_m} R_{A_1 B_1}{}^{C_1 D_1} \dots R_{A_m B_m}{}^{C_m D_m}, \quad (2)$$

where  $\delta_{C_1 D_1 \dots C_m D_m}^{A_1 B_1 \dots A_m B_m}$  is the totally antisymmetric product of Kronecker deltas and  $R_{AB}{}^{CD}$  is the  $D$ -dimensional curvature tensor. For a space with an even number of dimensions, the  $D = 2m$  Lovelock invariant is topological and leads to a total derivative, *i.e.*, a surface term, in the action. All of the higher order invariants,  $D \leq 2m - 1$ , can then be shown to vanish identically by using curvature tensor index symmetry properties. On the otherhand, for the cases with  $D \geq 2m + 1$ , the  $\mathcal{L}_m$  are truly dynamical objects that once added the action can significantly alter the field equations usually associated with the EH term. However, it can be shown that the addition of any or all of the  $\mathcal{L}_m$  to the EH action still results in a theory with only second order equations of motion as in ordinary Einstein gravity. In particular, variation of the action leads to modifications of Einstein's equations by the addition of new terms which are second-rank symmetric tensors with vanishing covariant derivatives, depending only on the metric and its first and second derivatives, *i.e.*, they

have the same general properties as the Einstein tensor itself but are higher order in the curvature. These are quite special properties not possessed by arbitrary invariant structures which usually lead to equations of motion of higher order, *i.e.*, more co-ordinate derivatives of the metric tensor and graviton field, *e.g.*, terms with quartic derivatives. Such theories would, in general, have serious problems with the presence of tachyons and ghosts as well as with perturbative unitarity[14]. The Lovelock invariants are constructed in such a way as to produce an action which is free of these problems. In addition, as might be expected, the introduction of Lovelock terms into the action does not modify the number of degrees of freedom encountered by studying the EH action. <sup>§</sup>

In our earlier work[11] we showed that the presence in the action of Lovelock invariants can lead to TeV-scale BH in ADD-like models with thermodynamical properties that can significantly differ from the usual EH expectations. This includes the possibility that BH may be stable in  $n$ -odd dimensions and that have production cross sections with calculable mass thresholds. In a more general context, BH in theories with Lovelock invariants have been discussed by a large number of authors[15]. The usual thermodynamical description of the Hawking radiation produced by TeV-scale BH decays is via the canonical ensemble(CE)[10] which has been employed in most analyses in the literature (in particular, our previous analysis of ADD-like BH). However, as pointed out by several groups[19], though certainly applicable to very massive BH, this approach does not strictly apply when  $M_{BH}/M_*$  is not much greater than  $O(1)$  or when the emitted particles carry an energy comparable to the BH mass itself due to the back-reaction of the emitted particles on the properties of the BH. This certainly happens when the resulting overall BH Hawking radiation multiplicity is low. In the decay of TeV-scale BH that can be made at a collider, the energy of the emitted particles is generally comparable to both  $M_*$  as well as the mass of the BH itself. In the CE approach the BH is treated as a large heat bath whose temperature is not significantly influenced by the emission of an individual particle. While this is a very good approximation for reasonably heavy BH it becomes worse as the BH mass approaches the  $M_*$  scale. Furthermore, the BH in an

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<sup>§</sup>As is well known, the addition of *arbitrary* curvature invariants to the EH action can lead to new propagating degrees of freedom in the resulting equations of motion.

asymptotically flat space (which we can assume here since the BH Schwarzschild radii,  $R_s$ , are far smaller than  $R_c$ ) cannot be in equilibrium with its Hawking radiation.

It has been suggested[19] that all these issues can be dealt with simultaneously if we instead employ the microcanonical ensemble(MCE) approach in the statistical mechanics treatment for BH decay. As  $M_{BH}/M_*$  grows larger,  $\gtrsim 10 - 20$ , the predictions of these two treatments will be found to agree, but they differ in the region which is of most interest to us since at colliders we are close to the BH production threshold where  $M_{BH}/M_*$  is not far above unity. Within the framework of the EH action it has been emphasized[19] that TeV-scale BH lifetimes will be increased by many orders of magnitude when the MCE approach is employed in comparison to the conventional CE expectations. This is not due to modifications in the thermodynamical quantities, such as the temperature, themselves but how they enter the expressions for the rate of mass loss in the decay of the BH. Here we will address the issue of how these two statistical descriptions may differ when the additional higher-curvature Lovelock terms are present in the action. In particular, we need to address what the combination of Lovelock terms plus the MCE description do to the BH mass loss rates and lifetimes. In the case of  $n$  even we will show that BH lifetimes are significantly increased as was found in the EH case. In the case of odd  $n$  the Lovelock BH will of course be found to produce stable remnants using either prescription for the same set of parameters as we will see below.

The outline of the paper is as follows: In Section 2 we will present the basic ingredients associated with the Lovelock invariant extended action and the altered expectations for the Schwarzschild radius, temperature and entropy of TeV-scale BH in ADD-like models where the bulk is essentially flat. We then will provide a brief overview of the general formalism for calculating BH lifetimes using the MCE, contrasting with the conventional CE approach. In Section 3 we will perform a numerical comparison of the predictions for the BH mass loss rate and lifetime in both the CE and MCE frameworks. For purposes of comparison we first analyze the results when only the EH term is present in the action. We also address the issue as to whether the dominance of BH

decays to brane fields is influenced by the choice of thermodynamic description. Section 4 contains a discussion and our conclusions.

## 2 Formalism

Based on the discussion above and the definition of the  $\mathcal{L}_m$  we see that the most general action with Lovelock invariants in 4-d is just EH plus a cosmological constant, *i.e.*, ordinary General Relativity. In 5-d, all of the  $\mathcal{L}_{m \geq 3}$  still vanish as in 4-d but  $\mathcal{L}_2$ , which is the familiar Gauss-Bonnet(GB) invariant, is no longer a total derivative and its presence will modify the results obtained from Einstein gravity. The generalization is clear: for  $D = 5, 6$  only  $\mathcal{L}_{0-2}$  can be present in the action. For  $D = 7, 8$  only  $\mathcal{L}_{0-3}$  can be present while for  $D = 9, 10$  only  $\mathcal{L}_{0-4}$ . Since ADD assumes that the compactified space is *flat* the coefficient of  $\mathcal{L}_0$  is taken to be zero in the present framework and, to reproduce the correct limit, the coefficient of  $\mathcal{L}_1$  is normalized so that it can be identified with the usual EH term. Thus in the Lovelock-ADD picture for  $D \leq 10 (n \leq 6)$  there are at most three new pieces to add to the EH action and so the general form relevant for the extended ADD model we consider is given by

$$S = \frac{M_*^{n+2}}{2} \int d^{4+n}x \sqrt{-g} \left[ R + \frac{\alpha}{M_*^2} \mathcal{L}_2 + \frac{\beta}{M_*^4} \mathcal{L}_3 + \frac{\gamma}{M_*^6} \mathcal{L}_4 \right], \quad (3)$$

where  $\alpha, \beta$  and  $\gamma$  are dimensionless coefficients which we take to be *positive* in the discussion below. (If we consider  $D > 10$  it is quite easy to extend this parameterization by including potential  $\mathcal{L}_5$  contributions.) If we expect this expansion to be the result of some sort of perturbation theory, some algebra suggests that  $\alpha D^2, \beta D^4$ , and  $\gamma D^6 \leq 1$  which yields the (only) suggestive values  $\alpha \sim 10^{-2}$ ,  $\beta \sim 10^{-3} - 10^{-4}$  and  $\gamma \sim 10^{-5}$ . ¶

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¶It is important to note that the fundamental mass parameter,  $M_*$  appearing in the above action is the *same* as the one appearing in the ADD  $M_*-\bar{M}_{Pl}$  relationship Eq.(1). It is also the parameter appearing in the 5-d coupling of the graviton to matter fields.  $M_*$  can be directly related to several other mass parameters used in the literature. The fundamental scale employed by Dimopoulos and Landsberg[8] is given by  $M_{DL} = (8\pi)^{1/(n+2)} M_*$  while that of Giddings and Thomas[9] is found to be  $M_{GT} = [2(2\pi)^n]^{1/(n+2)} M_*$ ; moreover, Giudice *et al.*[3] employ a different scale,  $M_{GRW} = (2\pi)^{n/(n+2)} M_*$ .  $M_*$  is thus correspondingly *smaller* than all of these other parameters with consequently

In order to calculate BH lifetimes we will need the relationships between the BH mass, the Schwarzschild radius,  $R_s$ , temperature,  $T$  and entropy,  $S$ . When Lovelock terms are present in the action these relations can be significantly modified from their conventional EH expectations. Here we simply summarize results from our earlier work. (For details, see [11]). The  $M_{BH} - R_s$  relationship is given by

$$m(x) = c \left[ x^{n+1} + \alpha n(n+1)x^{n-1} + \beta n(n+1)(n-1)(n-2)x^{n-3} \right. \\ \left. + \gamma n(n+1)(n-1)(n-2)(n-3)(n-4)x^{n-5} \right], \quad (4)$$

where  $m = M_{BH}/M_*$ ,  $x = M_*R_s$ , and the numerical constant  $c$  is given by

$$c = \frac{(n+2)\pi^{(n+3)/2}}{\Gamma(\frac{n+3}{2})}. \quad (5)$$

Since what we really want to know is  $x(m)$  and not  $m(x)$  as given above we must find the roots of this polynomial equation; this must be done numerically in general except for some special cases. Fortunately, for the range of parameters of interest to us and with  $\alpha, \beta, \gamma \geq 0$ , we find that this polynomial has only one distinct real positive root. Similarly, the BH temperature is found to be given by

$$T = \frac{(n+1)U(x)}{4\pi V(x)}, \quad (6)$$

where  $T = T_{BH}/M_*$  and

$$U(x) = x^6 + \alpha n(n-1)x^4 + \beta n(n-1)(n-2)(n-3)x^2 \\ + \gamma n(n-1)(n-2)(n-3)(n-4)(n-5) \\ V(x) = x \left[ x^6 + 2\alpha n(n+1)x^4 + 3\beta n(n+1)(n-1)(n-2)x^2 \right. \\ \left. + 4\gamma n(n+1)(n-1)(n-2)(n-3)(n-4) \right]. \quad (7)$$

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far weaker experimental bounds[2]. For example, if  $n = 2(6)$  and  $M_{GRW} > 1.5$  TeV then  $M_* > 0.60(0.38)$  TeV; existing direct bounds on  $M_*$  are thus well below 1 TeV at present.

The corresponding BH entropy can then be calculated using the familiar thermodynamical relation

$$S = \int_0^x dx T^{-1} \frac{\partial m}{\partial x}, \quad (8)$$

which yields

$$\begin{aligned} S &= \frac{4\pi c}{n+2} \left[ x^{n+2} + 2\alpha(n+1)(n+2)x^n + 3\beta n(n+1)(n+2)(n-1)x^{n-2} \right. \\ &\quad \left. + 4\gamma n(n+1)(n+2)(n-1)(n-2)(n-3)x^{n-4} \right]. \end{aligned} \quad (9)$$

Note that here we have required that the entropy vanish for a zero horizon size.

In order to calculate the BH mass loss rates we follow the formalism in [19]; to simplify our presentation and to focus on the differences between thermodynamical treatments we will ignore the effects due to grey-body factors[10] in the present analysis. <sup>||</sup> In this approximation, the rate of BH mass loss (time here being measured in units of  $M_*^{-1}$ ) into *bulk* fields employing the MCE approach is given by

$$\left[ \frac{dm}{dt} \right]_{bulk} = \frac{\Omega_{d+3}^2}{(2\pi)^{d+3}} \zeta(d+4) x^{d+2} \sum_i N_i \int_{m_{crit}}^m dy (m-y)^{d+3} \left[ e^{S(m)-S(y)} + s_i \right]^{-1}, \quad (10)$$

where,  $x = M_* R_s$  as above,  $S$  is the corresponding entropy of the BH,  $d$  labels the number of bulk dimensions into which the particles are emitted,  $d \leq n$ ,  $i$  labels various particle species with  $N_i$  degrees of freedom which live in the bulk and obey Fermi-Dirac(Boltzmann, Bose-Einstein) statistics, corresponding to the choices of  $s_i = 1(0, -1)$ ,  $\zeta$  is the familiar Riemann Zeta Function and as usual

$$\Omega_{d+3} = \frac{2\pi^{(d+3)/2}}{\Gamma((d+3)/2)}. \quad (11)$$

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<sup>||</sup>It is interesting to note that the presence of Lovelock invariants in the action can alter the usual values obtained for grey-body factors by terms of order unity. However, for the range of parameters of interest to us it has recently been shown that  $\alpha \neq 0$  does not lead to any significant change in the bulk or brane grey-body factors from those obtained from EH[20]. Our expectation is that similar results will hold when  $\beta, \gamma \neq 0$  contributions are included but this needs to be explored directly.



The lower limit of the integration  $m_{crit}=0$  for  $n$ -even but may be finite for  $n$ -odd corresponding to the mass threshold for BH production which can occur in the case of the Lovelock extended action[11]. The value of this parameter can in all cases be obtained by first fixing the value of  $n$  and then setting  $x = 0$  on the right-hand side of the expression for  $m(x)$  in Eq.(4) above. In the present analysis, we will assume that only gravitons live in the bulk, so for them  $d = n$ . For the case of decays into fields that live on the brane, the expression is the same as that given above but now with  $d = 0$  and we now must sum over all particles that live only on the brane, *i.e.*, those of the SM.

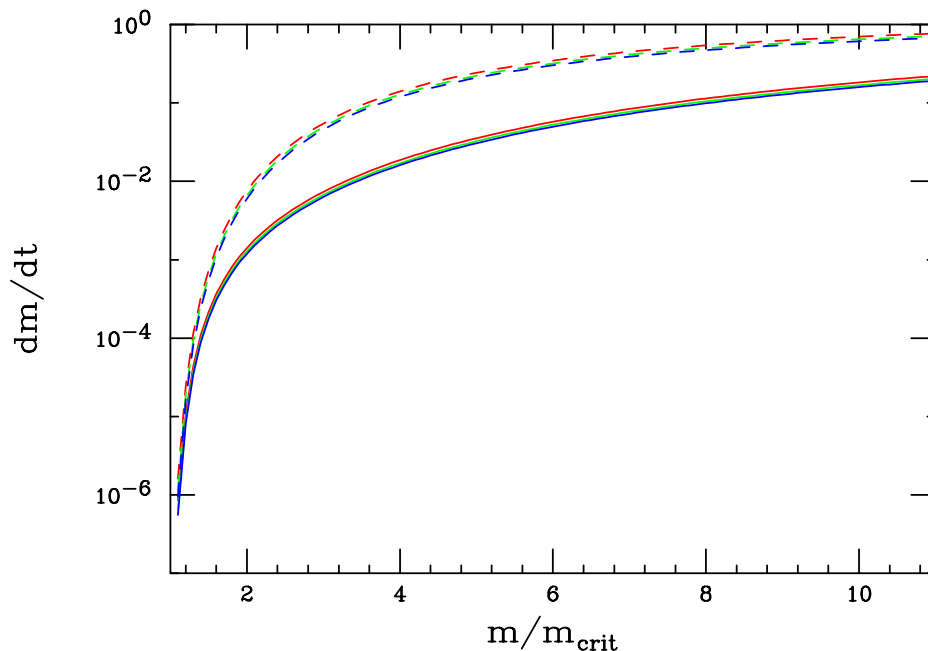


Figure 1: Comparison of the BH mass loss rates to brane fields following the MCE prescription assuming final states which, from top to bottom in each set of curves, are purely Fermi-Dirac, Boltzmann or Bose-Einstein. The solid(dashed) set corresponds to  $n = 5, \alpha = 0.005, \beta = 0.0003, \gamma = 1.14 \times 10^{-5} (n = 3, \alpha = 0.01, \beta = 0.005)$ .

In our recent analysis[11] of BH decay in the RS model where GB terms were present, we noted that following the MCE approach there is some potential sensitivity of the mass loss rate to the specific statistics mix of particles on the brane into which the BH decays. However, there it was shown that since we are concerned with decays to SM brane fields, where the numbers of fermionic degrees of freedom (48, assuming only light Dirac neutrinos) is somewhat larger than the number

for bosons (14), the value of  $s_i = 0$  was found to be a reasonable approximation given the other uncertainties in the calculation. The reason for this is that (i) the SM is mostly fermionic and the results for Boltzmann and FD statistics are found to lie numerically rather close to one another and (ii) the B distribution lies between the BE and FD ones. For the cases we study below it is important to check what happens in greater numbers of flat dimensions when additional Lovelock terms are present; Fig. 1 shows the results of two sample calculations. Here we see that for a flat bulk with larger values of  $n$  and when the new Lovelock terms are present with positive values of the coefficients  $\alpha, \beta,$  and  $\gamma$ , the differences between the predictions of the different statistics final states is significantly reduced. The corresponding integrated lifetimes are also found to agree within  $\sim 10\%$ . Thus in our numerical analysis that follows we will for simplicity take  $s_i = 0$  and assume that the total number of SM degrees of freedom is 60.

In the CE approach, the expression above, Eq.(10), simplifies significantly as in this case the integral can be performed analytically. The reason for this is that in the CE treatment, the factors  $S(m)$  and  $S(y = m - \omega)$ ,  $\omega$  being the scaled energy of the blackbody radiation, appearing in the exponential factor above are considered nearly the same since backreaction is neglected, *i.e.*, taking in the MCE expression above  $m \rightarrow \infty$  (the no recoil limit) and  $S(m) - S(m - \omega) \simeq \omega \partial_m S = \omega/T$ , then integrating over  $\omega$  yields the usual CE result. In particular, allowing for the different possible particle statistics in the CE case as well we obtain for bulk decay

$$\left[\frac{dm}{dt}\right]_{bulk} = \sum_i N_i Q_i \frac{\Omega_{d+3}^2}{(2\pi)^{d+3}} \zeta(d+4) x^{d+2} \Gamma(d+4) T^{d+4}, \quad (12)$$

where  $N_i$  is again just the appropriate number of degrees of freedom for each statistics type and  $Q_i$  takes the value  $\pi^4/90(1, 7\pi^4/720)$  for BE(B, FD) statistics. The corresponding expression for brane decays in the CE case is straightforwardly obtained by setting  $d = 0$ . Note that the values of  $Q_i$  differ from each other by less than  $\sim 10\%$  resulting in only small differences in lifetimes. It is interesting to note that since the Hawking radiation emitted from BH is generally softer in the MCE approach in comparison to the CE one, the average multiplicity for a fixed initial BH mass

and number of dimensions is found to be somewhat larger in the MCE case. This difference should be observable at future colliders.

Given the large number of SM brane fields, it is well known that for the EH action the brane modes tend to dominate over bulk modes by a factor of order  $\sim 100$  or more, a result which seems to continue to hold when GB terms (and the corresponding grey-body factors) are included[20]. The first question to address is whether these results remain valid when further Lovelock terms are present in the action.

### 3 Analysis

In order to address the above question of how the MCE vs CE choice might influence the ratio of bulk to brane BH mass loss rates, we construct the ratio

$$R = \left[ \frac{dm}{dt} \right]_{bulk} / \left[ \frac{dm}{dt} \right]_{brane} . \quad (13)$$

Using  $s_i = 0$ , we will assume  $N_{brane} = 60$  and  $N_{bulk} = 1$  in what follows and thus we might naively expect that  $R \sim 10^{-3} - 10^{-2}$  if EH provides a reasonable estimate over most of the parameter range. Does this estimate remain valid when Lovelock terms are present and does the result depend on whether one chooses to follow the MCE or CE approach? To be specific in addressing these issues, we consider the cases of  $n = 3, 5$  and examine the ratio  $R$  as a function of  $m$  for different values of the Lovelock parameters. The results of this analysis are shown in Figs. 2 and 3 where we see that  $R$  is a strong and increasing function of  $m$  and that  $R$  also increases with  $n$ . For large  $m$ , indeed  $R \sim 10^{-3}$  but the precise value depends in detail upon the value of  $n$  as well as the Lovelock parameters in a rather sensitive fashion. As the BH decays and  $m$  becomes smaller,  $R$  rapidly goes to zero in all cases implying a very strong suppression of the bulk modes. While the CE and MCE approach results do differ in detail they are qualitatively similar and in no cases do we see any significant BH decay into bulk states in our mass range of interest. (Note, however, that  $R$  is generally larger in the MCE case.) Thus we can concentrate on brane modes in what

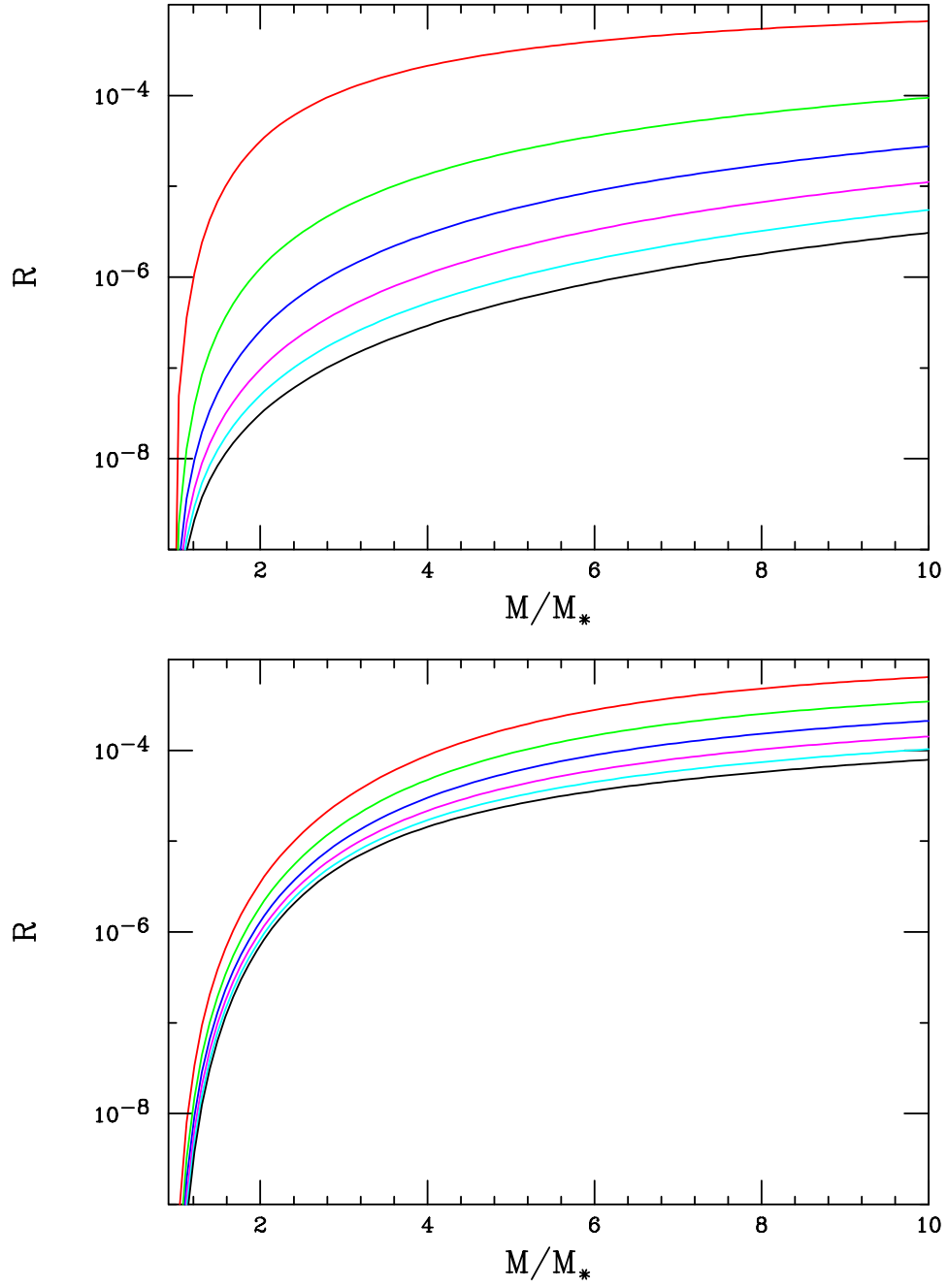


Figure 2: The ratio  $R$  as a function of  $m = M_{BH}/M_*$  assuming  $n = 3$  for  $\alpha = 0$  to  $\alpha = 0.025$  from top to bottom in steps of 0.005 assuming  $\beta = 0.0005$ . The top(bottom) panel is the CE(MCE) result.

follows ignoring the decays to bulk gravitons. Even if grey-body factors modify the values of  $R$  obtained above by an order of magnitude or more when Lovelock terms are present we expect the basic result of dominant brane field BH mass loss to remain valid.

Before addressing the more complex case of the extended action with Lovelock terms let us briefly review the numerical differences between the CE and MCE treatments of BH decay in the case of the EH action by setting  $\alpha = \beta = \gamma = 0$ ; the results are shown in Fig. 4. The first thing to examine is the mass loss rate of BH; to this end we consider the dimensionless quantity  $M_*^{-2}dM/dt = dm/dt$ , with time measured in Planck units, which is shown in the upper panel in the Figure for  $n = 3, 5$  for both the CE and MCE cases. For larger values of  $m$ , both the MCE and CE analyses yield identical results as expected but differ significantly once  $m \lesssim 4$ . In the CE case, the rates grow rapidly as the mass of the BH decreases below this range whereas in the MCE case the rate drops quite dramatically. The influence on the lifetimes of these significantly different behaviors is shown in the lower panel for a BH which begins life with  $m = 5$ , a reasonable value for production at the LHC. Here what is specifically shown is the time taken (in units of  $M_*^{-1}$ ) for an initial  $m = 5$  BH to decay to a BH with  $m < 5$  via Hawking radiation for various values of  $n$ . Note that while the state of complete BH evaporation,  $m = 0$ , is reached in the CE case for  $0.03 \lesssim M_*t \lesssim 1$ , for the MCE analysis one obtains values which are larger than this by many (6 to 8 or more) orders of magnitude. This shows, assuming the EH action, that the slowing of the BH evaporation rate in the MCE approach leads to a dramatic increase in the lifetime of BH that can be produced in TeV collisions as has been emphasized by several sets of authors[19].

We now turn to the case where Lovelock invariants are present in the action. In order to analyze a scenario where the number of extra dimensions is even, we consider a specific example with  $n = 2$  so that only the quadratic G-B term can be present, *i.e.*, only  $\alpha$  can be non-zero. Note that as  $n$  is even, the G-B invariant cannot lead to a BH mass threshold. Fig. 5 shows the results of the calculations which are analogous to those displayed in the previous Figure. Here in the upper panel we see that for both the MCE and CE analyses the presence of the G-B term leads to a

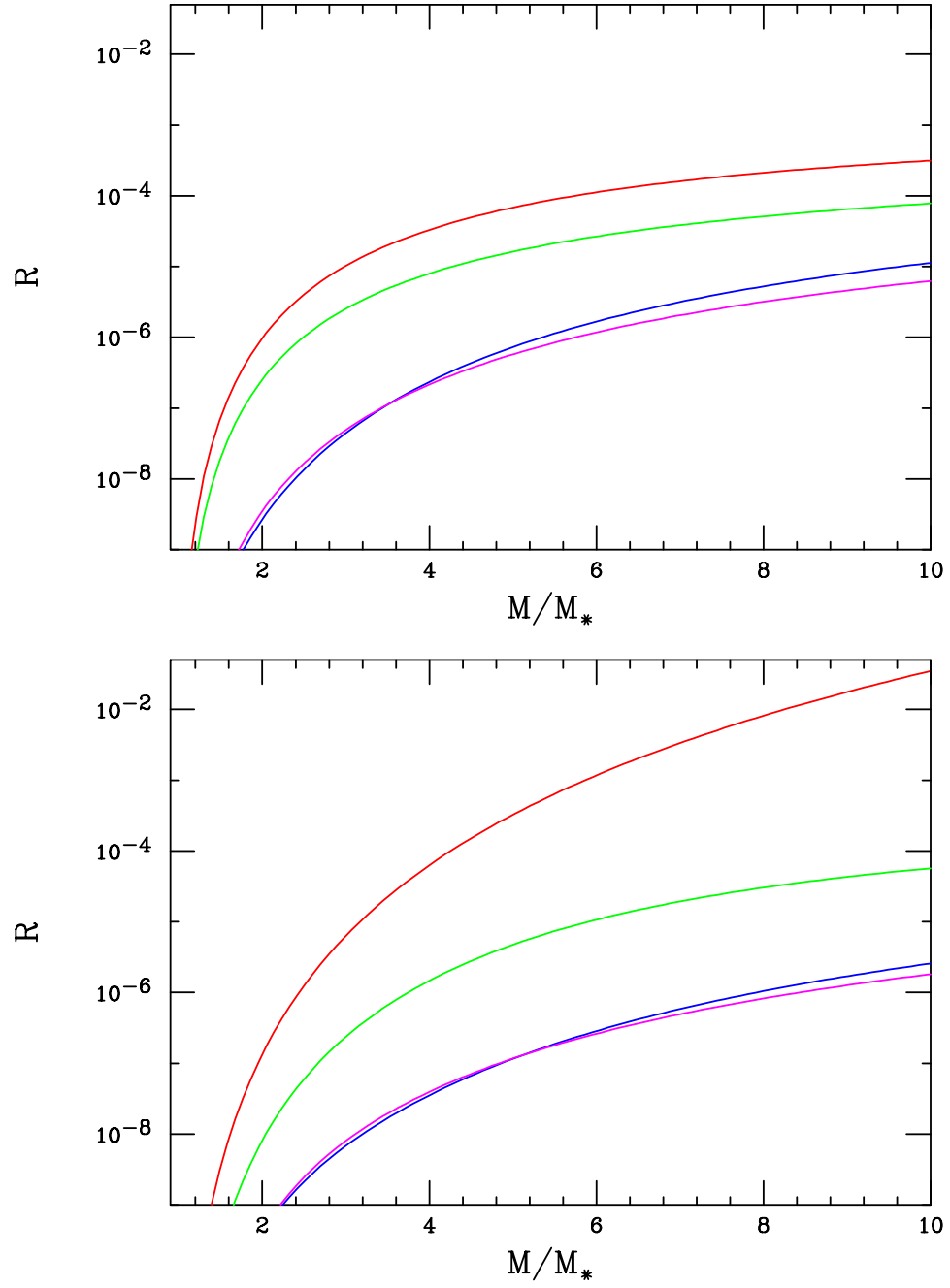


Figure 3: Same as in the previous figure but now for  $n = 5$  assuming  $\gamma = 1.14 \times 10^{-5}$ . From top to bottom on the right-hand side the figure the curves correspond to  $\alpha = \beta = 0$ ,  $\alpha = 0.005$  with  $\beta =$ ,  $\beta = 0.0003$  with  $\alpha = 0$  and  $\alpha = 0.005, \beta = 0.0003$ , respectively.

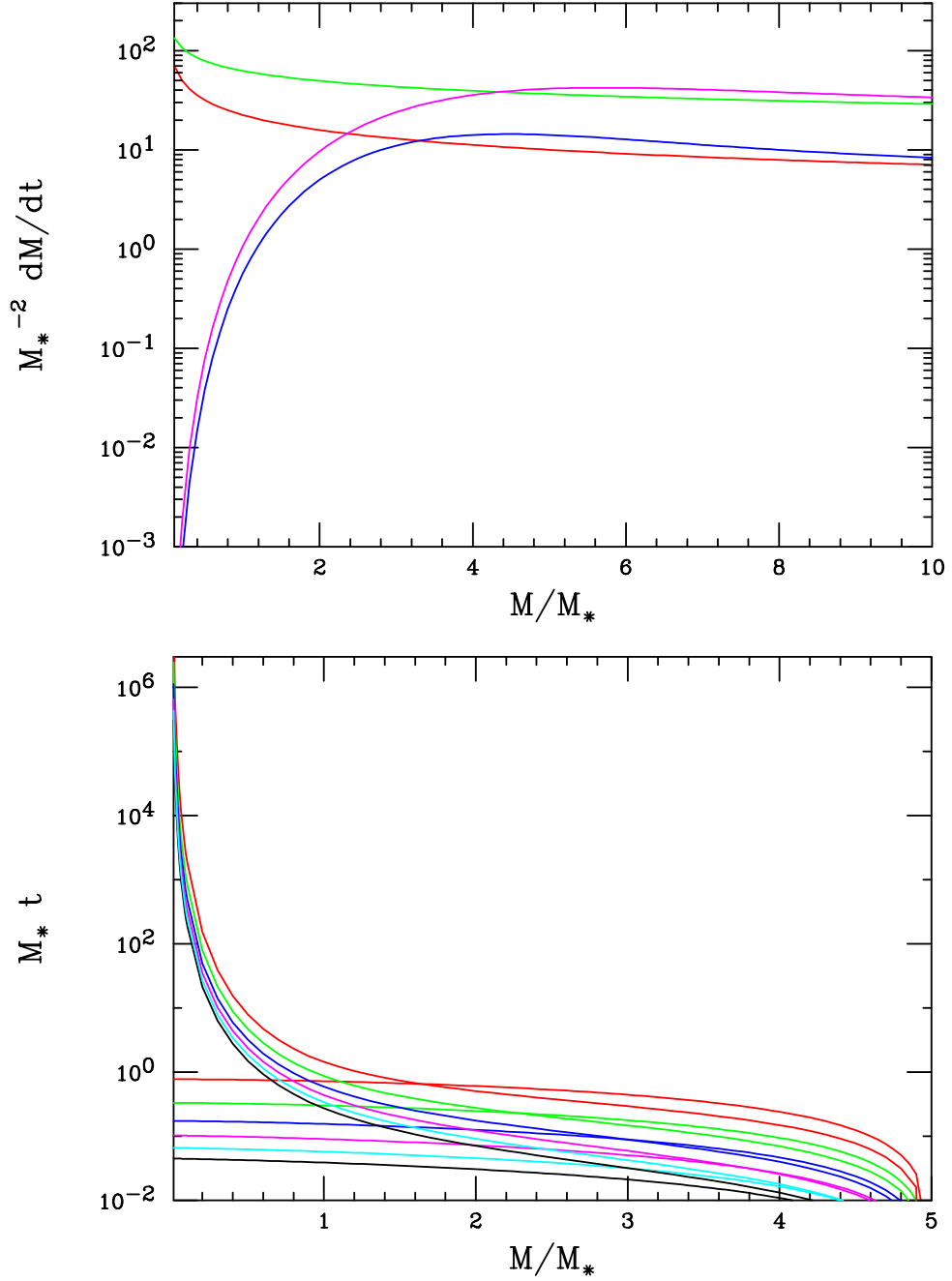


Figure 4: (Top) Rate of change of the BH mass through Hawking radiation on the brane assuming the EH action as discussed in the text; the CE(top) and MCE(bottom) results are represented by the two sets of curves with the case  $n = 5(3)$  being the upper(lower) curve. (Bottom) Decay times for a BH with an initial  $m = M_{BH}/M_* = 5$ ; the rising(flat) set of curves at low  $m$  corresponds to the MCE(CE) case. In each set of curves  $n$  ranges from 2 to 7 going from top to bottom.

suppression of the BH decay rate,  $M_*^{-2}dM/dt$ , which is active for all values of  $m$  but is somewhat magnified as  $m$  gets smaller. For larger  $m$  the CE and MCE analyses are seen to agree just as in the case of the EH action above. For the overall BH lifetime,  $M_*t$ , we see in the bottom panel that a non-zero  $\alpha$  for the case of  $n = 2$  can increase the value of this quantity up to 2 orders of magnitude in the CE analysis. Using the MCE combined with the non-zero values of  $\alpha$  leads to a further increase in the BH lifetime by two more orders of magnitude, *i.e.*, the presence of the G-B term is seen to further augment the BH lifetime obtained by employing the MCE analysis in comparison to the standard EH picture employing the CE. However, this relative enhancement is far smaller than that found in the EH case since the common enhancement arising from the Lovelock terms present in both cases is already large. We thus conclude that for  $n$  even the presence of Lovelock invariants together with the use of the MCE will significantly increase the BH lifetimes but only by a few orders of magnitude.

What happens when  $n$  is odd and a BH remnant can form? In this case we consider typical non-zero values for all of the allowed Lovelock parameters for the fixed number of extra dimensions considered. Figs. 6 and 7 show the values of  $M_*^{-2}dM/dt$  for  $n=3$  and 5 comparing the expectations of the CE and MCE approaches. For the case of  $n = 3$ , we hold  $\beta = 0.0005$  fixed and vary the values of  $\alpha$ ; for  $n = 5$ , we hold fixed  $\gamma = 1.14 \times 10^{-5}$  and vary the values of both  $\alpha$  and  $\beta$ . In both figures we see that the general patterns associated with a reduced rate of Hawking radiation employing the MCE analysis observed for  $n = 2$  repeated. However, unlike for even values of  $n$ , for  $n$  odd we see that  $M_*^{-2}dM/dt \rightarrow 0$  for *both* the MCE and CE approaches as  $m \rightarrow m_{crit}$ . In addition, for large values of  $m$ ,  $M_*^{-2}dM/dt$  is generally seen to be larger when the CE technique is employed than when one instead uses the MCE.

In order to access the influence on the BH decay time of the CE versus MCE choice in the presence of higher curvature terms, we show in Figs. 8 and 9 the integrated decay time for the above studied cases of  $n = 3$  and 5. As expected from the  $n = 2$  analysis, here we again see that the BH described by the MCE has a decay time for  $m > m_{crit}$  which is somewhat longer than the



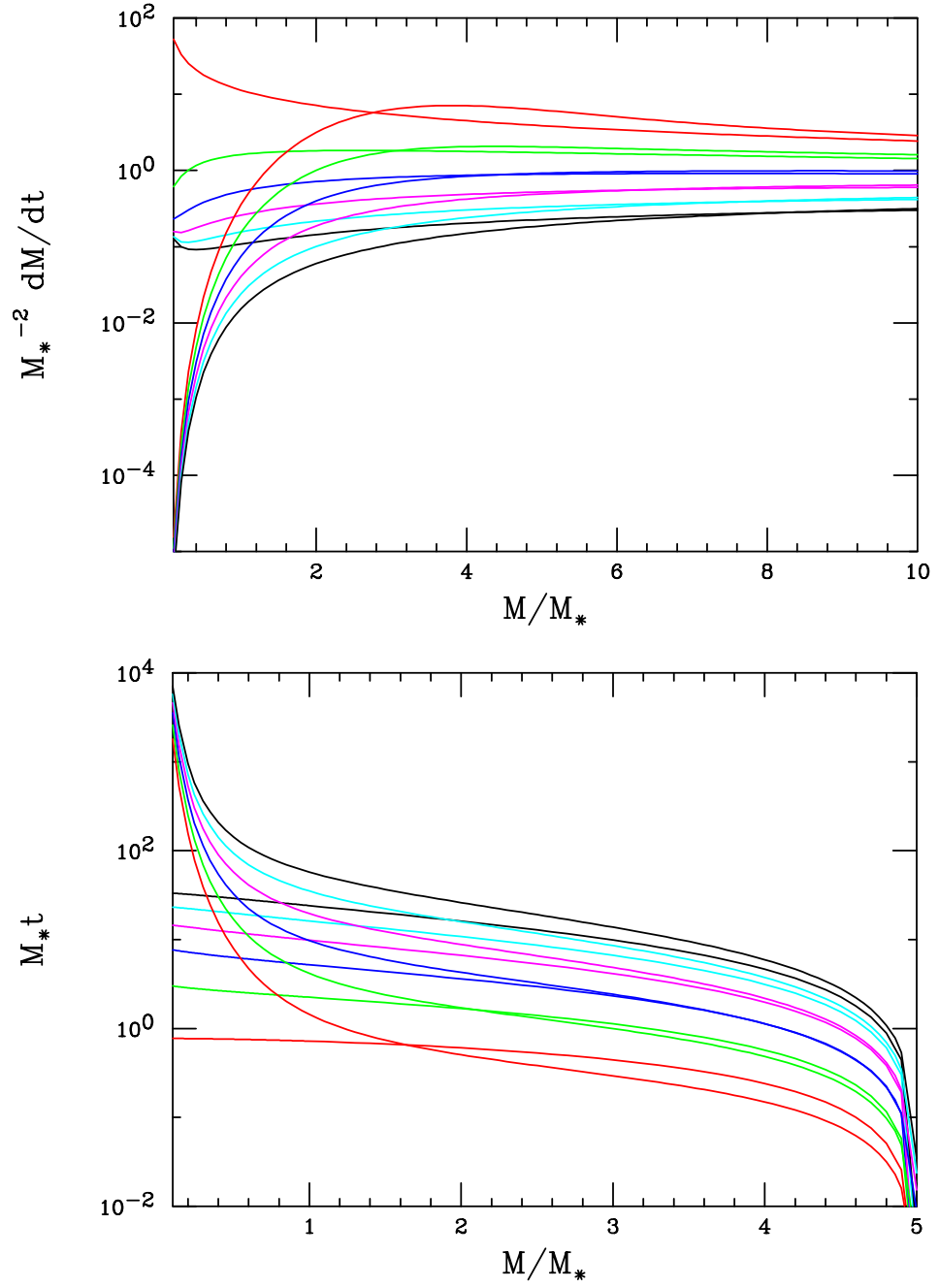


Figure 5: Same as the previous figure but now assuming  $n = 2$  for  $\alpha \neq 0$ . In the top panel, for both sets of curves,  $\alpha$  goes from 0 to 0.025 in steps of 0.005 going from top to bottom. In the bottom panel,  $\alpha$  goes over the same range but in opposite order.

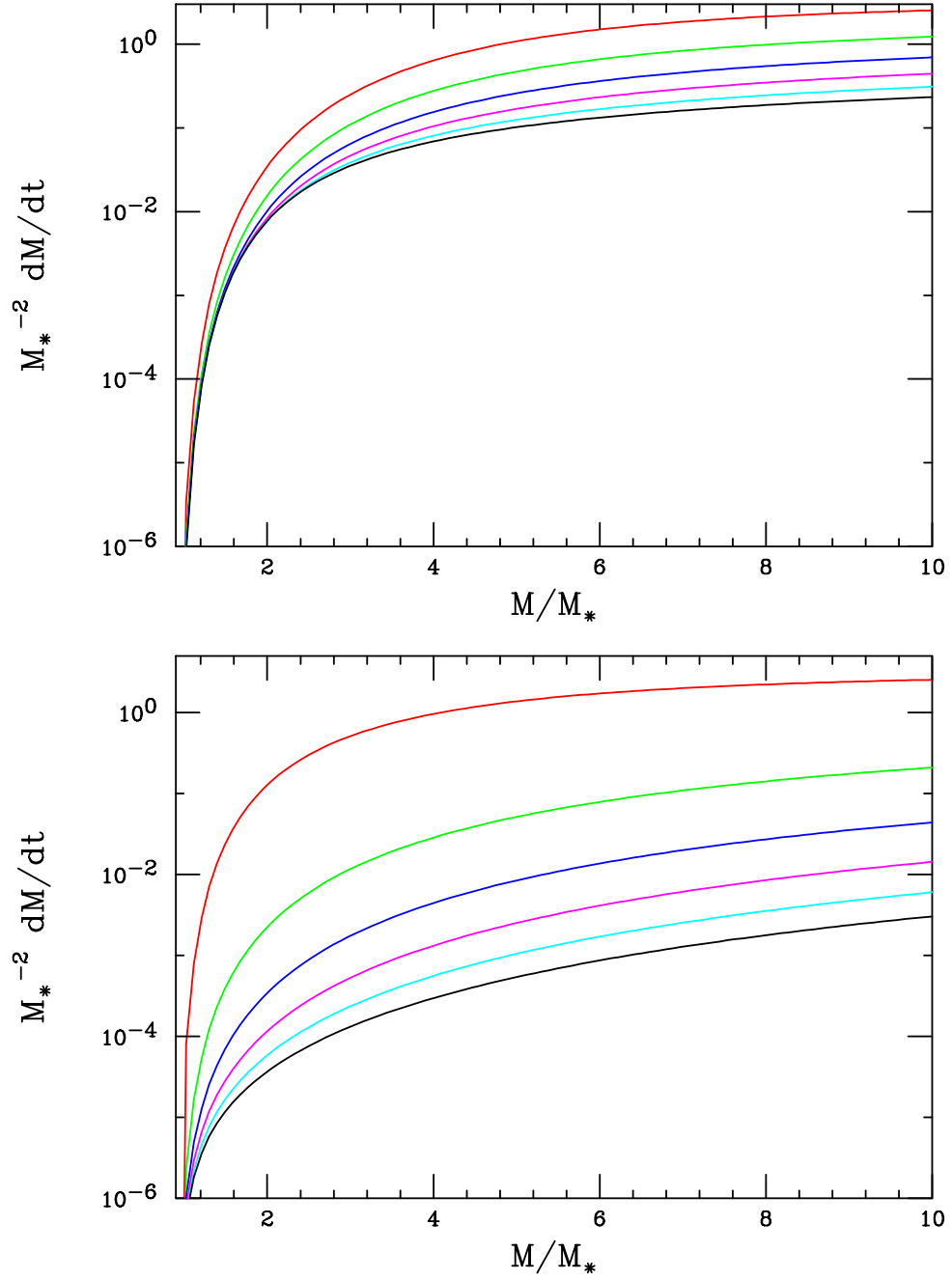


Figure 6: BH decay rate as a function of  $m = M/M_*$  for  $n = 3$  with  $\beta = 0.0005$ . From top to bottom the value of  $\alpha$  ranges from 0 to 0.025 in steps of 0.005. The upper(lower) panel employs the CE(MCE) analysis approach.

corresponding CE result. This enhancement in the decay time to a fixed mass final state, which can be up to a couple of orders of magnitude, is observed to be substantially less than that obtained in the pure EH scenario found above. We thus conclude, for  $n$  odd, that while the MCE approach always leads to an enhancement in the BH decay time relative to that obtained in the CE approach, the effect of the higher Lovelock terms is to reduce the degree of this enhancement in comparison to that obtained in the case of the pure EH action. Note that in *either* approach the resulting *total* BH lifetime is infinite as the BH decay still results in a stable remnant as was found in our earlier work.

It is clear from this analysis that the Lovelock extended action leads to significant modifications in the mass loss rates and lifetimes of BH and that the choice of MCE vs CE is critical. Theoretical arguments support the use of the MCE description but experiments will be able to distinguish these two approaches at colliders.

## 4 Discussion and Conclusions

Higher curvature invariants of the Lovelock type could be present in the extra-dimensional effective gravitational action and would make their presence known at energies of order  $M_*$  and above. If the higher dimensional bulk is flat (as in ADD-like models but not in RS-like models) there are few ways to directly probe the existence of these additional terms experimentally. The reason for this is that the conventional ‘perturbative’ graviton-related ADD signatures are found to be quite insensitive to the existence of Lovelock terms. On the otherhand, since they are non-perturbative structures, the properties of TeV-scale black holes in extra-dimensional models are potentially very sensitive to these new interactions which can be probed at future particle colliders.

In this paper we have considered how the existence of Lovelock invariant extensions to the Einstein-Hilbert action will modify the mass loss rates and lifetimes of TeV-scale BH. In particular we examined the sensitivity of both of these quantities to the choice made in the statistical mechanics treatment of BH. It had been shown, and we verified those results here, that in the case

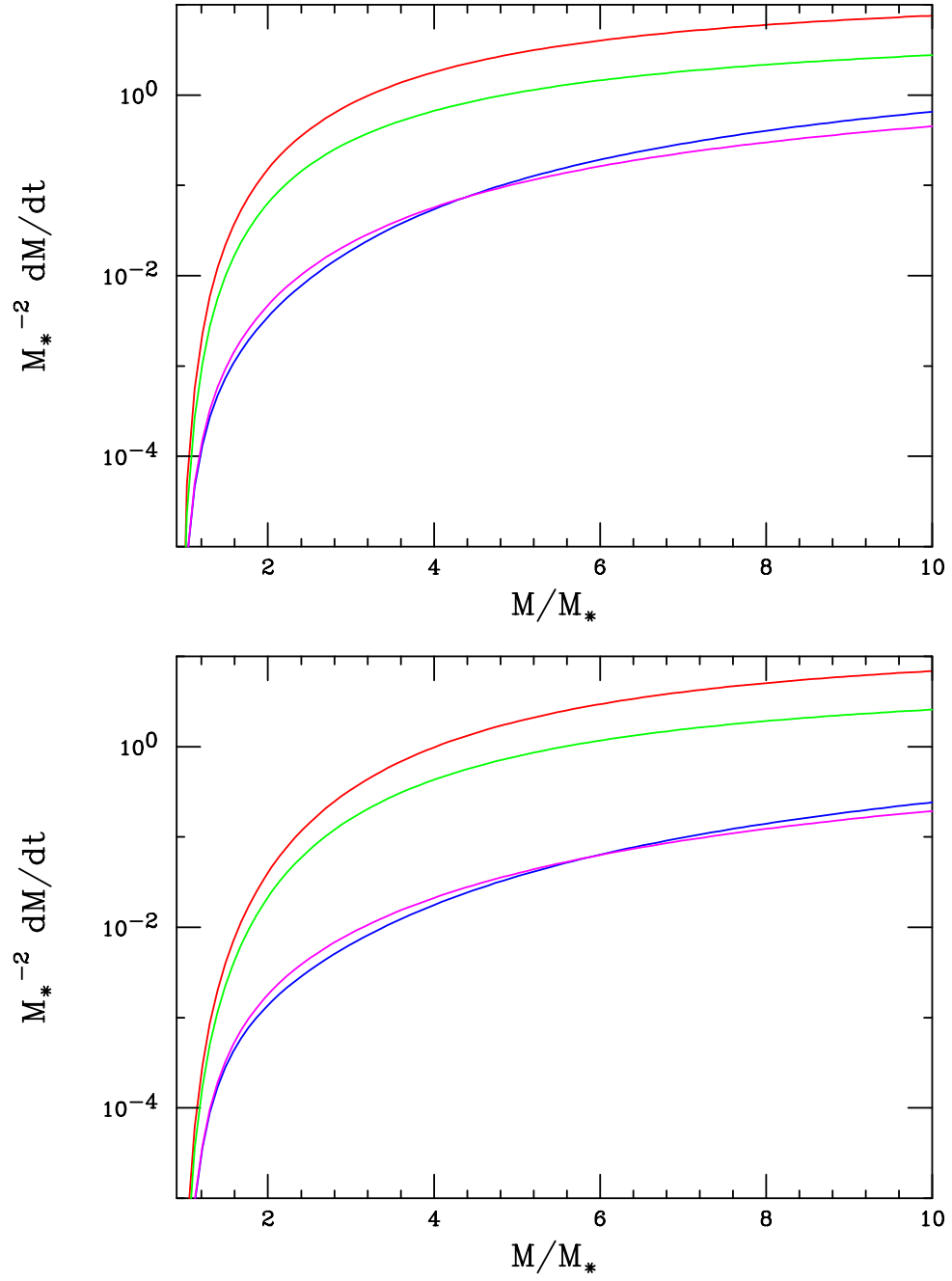


Figure 7: Same as the previous figure but now for  $n = 5$  with  $\gamma = 1.14 \times 10^{-5}$ . From top to bottom on the right-hand side of the plot the curves correspond to  $\alpha = \beta = 0$ ,  $\alpha = 0.005$ ,  $\beta = 0.0003$  and  $\alpha = 0.005$ ,  $\beta = 0.0003$ , respectively.

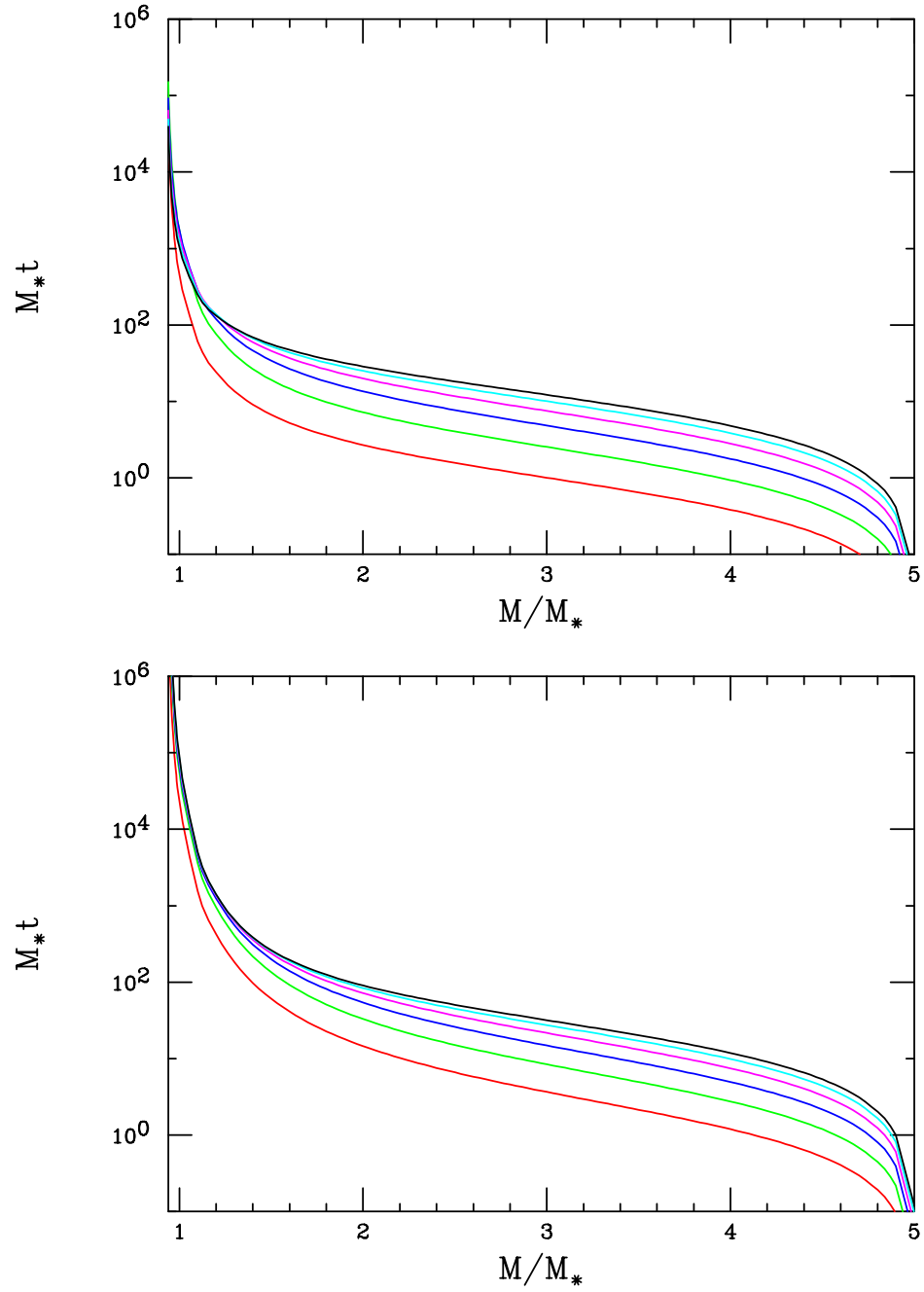


Figure 8: Integrated BH lifetimes corresponding to the decay rates shown in Fig.5 but with the curves labeled in the opposite order.

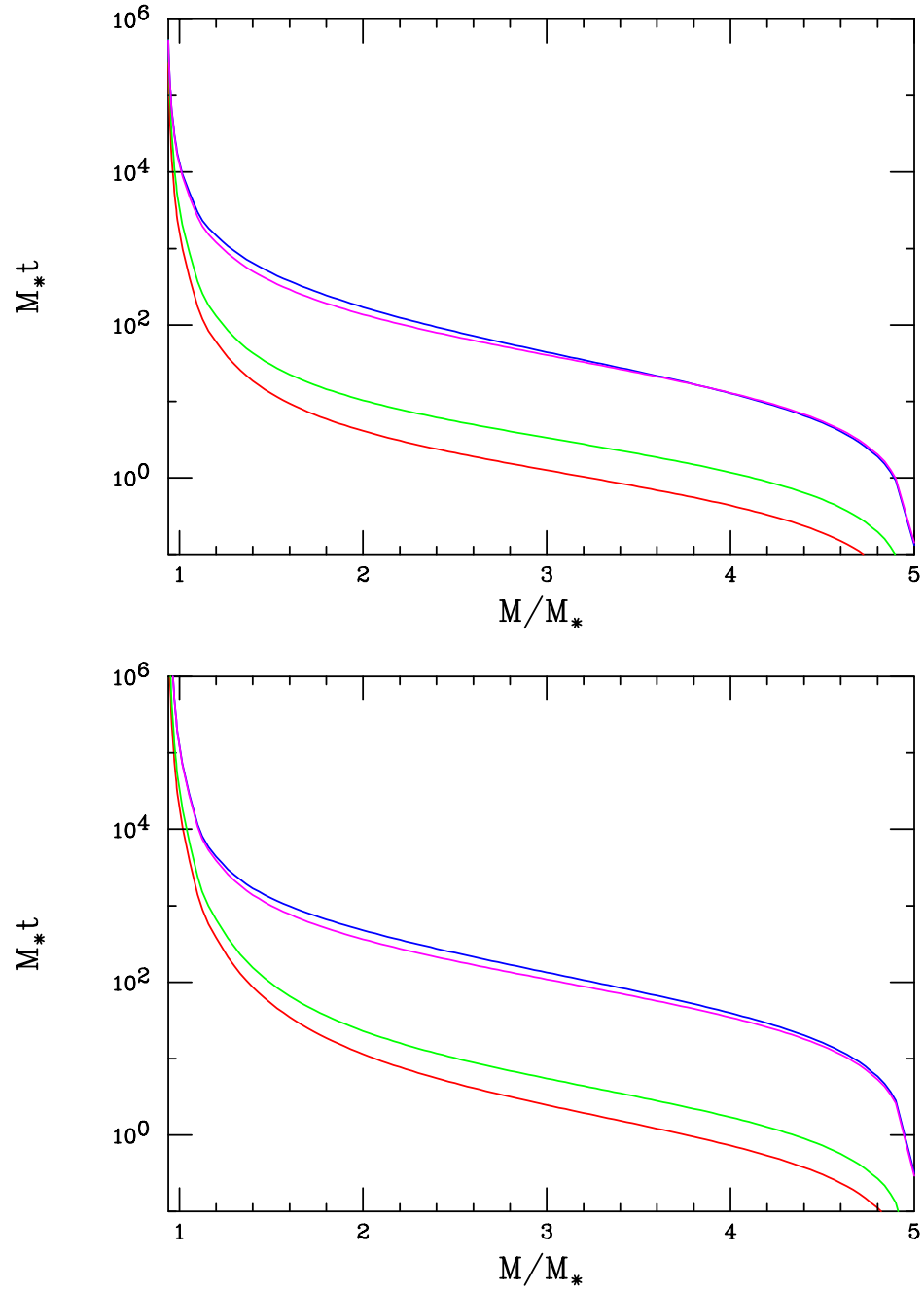


Figure 9: Integrated BH lifetimes corresponding to the decay rates shown in Fig.6 but with the curves labeled in the opposite order.

of the EH action, BH lifetimes are significantly enhanced by many orders of magnitude when the microcanonical ensemble description is employed in comparison to the more conventional canonical ensemble approach. There are several reasonable arguments in the literature as to why BH in the mass range of interest to us are in fact best described by the MCE.

Within this context, when Lovelock terms are present in the case of ADD-like flat extra dimensions we demonstrated in the present paper that: *(i)* BH decays to SM fields on the brane remain dominant over those to graviton bulk fields employing either the MCE or CE descriptions when Lovelock terms are present. However, in all cases the bulk/brane ratio was shown to grow as the number of extra dimensions increases. *(ii)* Unlike in the case of a single warped extra dimension, the BH decay rates and lifetimes for ADD-like extra dimensions are found to be insensitive to the statistics ‘mix’ of the particles on the brane. *(iii)* For *even* numbers of extra dimensions the lifetimes of BH described by the MCE are up to a few orders of magnitude larger than those obtained employing the CE. While this significant enhancement is large it is many orders of magnitude smaller than that obtained employing only the EH action. *(iv)* For odd numbers of extra dimensions, with the highest order allowed Lovelock term present, BH are found to decay to stable relics independent of the MCE/CE choice. However, the functional dependence of the mass loss rate in the two cases can be somewhat different but the details are sensitive to the particular values of the model parameters.

Black holes observed at future colliders may open an exciting window on gravity in extra dimensions.

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