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A Twin Higgs Model from Left-Right Symmetry

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We present twin Higgs models based on the extension of the Standard Model to left-right symmetry that protect the weak scale against radiative corrections up to scales of order 5 TeV. In the ultraviolet the Higgs sector of these theories respects an approximate global symmetry, in addition to the discrete parity symmetry characteristic of left-right symmetric models. The Standard Model Higgs field emerges as the pseudo-Goldstone boson associated with the breaking of the global symmetry. The parity symmetry tightly constrains the form of radiative corrections to the Higgs potential, allowing natural electroweak breaking. The minimal model predicts a rich spectrum of exotic particles that will be accessible to upcoming experiments, and which are necessary for the cancellation of one-loop quadratic divergences. These include right-handed gauge bosons with masses not to exceed a few TeV and a pair of vector-like quarks with masses of order several hundred GeV.

I. INTRODUCTION

In the Standard Model (SM) the Higgs mass parameter receives quadratically divergent quantum corrections that tend to destabilize the weak scale. This suggests the existence of new physics near a TeV that resolves this problem. However, precision electroweak measurements performed at LEP over the past decade have lead to an apparent paradox [1]. The problem is that these experiments indicate

- the existence of a light Higgs with mass less than about 250 GeV, and also that
- the cutoff Λ for non-renormalizable operators that contribute to the precision electroweak observables must be greater than about 5 TeV.

However Standard Model loop corrections from scales of order 5 TeV are sufficiently large so as to generate a Higgs mass much larger than 250 GeV. This is called the 'LEP paradox'. While we cannot rule out accidental cancellations between different contributions to the LEP measurements, the LEP paradox seems to suggest that whatever the new physics is that addresses the hierarchy problem, it does not contribute significantly to the precision electroweak observables. More concretely, there seem to be the three distinct possibilities below.

- There is no new physics below 5 TeV. In this case the Standard Model is simply fine-tuned at the 2-3% level or worse.
- The new physics which stabilizes the weak scale does contribute significantly to precision electroweak observables, but satisfies the current bounds. In this case the fact that the Standard Model with a light Higgs is a good fit to the data is merely a coincidence.
- The new physics which stabilizes the weak scale does not contribute significantly to precision electroweak observables.

Any complete solution to the LEP paradox must fall into the last category. One such solution is weak scale supersymmetry, where R-parity suppresses contributions to precision electroweak observables.

One interesting approach to the hierarchy problem, first proposed in [2, 3], is that the Higgs mass parameter is protected against radiative corrections because the Higgs is the pseudo-Goldstone boson of an approximate global symmetry. In the last few years several interesting realizations of this idea, based on the little Higgs mechanism [4, 5] (for a clear review and more references see [6]), have been constructed. These theories stabilize the weak scale up to 5 - 10 TeV. The underlying concept behind little Higgs theories is the idea of 'collective symmetry breaking' - the global symmetry is broken only when two or more couplings in the Lagrangian are non-vanishing. This is a significant restriction on the form of the quantum corrections to the pseudo-Goldstone potential, which can be used to realize natural electroweak symmetry breaking. Models based on this idea where the corrections to precision electroweak observables are small have been constructed [7], (see also [8]), and these naturally resolve the LEP paradox.

Recently twin Higgs models, an alternative class of realizations of the Higgs as a pseudo-Goldstone boson, have been proposed [9],[10],[11]. These theories also protect the weak scale from radiative corrections up to scales of order 5 - 10 TeV, but in a manner completely distinct from little Higgs theories. In the ultra-violet these theories respect a discrete Z_2 interchange symmetry in addition to an approximate global symmetry of the Higgs sector. The Standard Model Higgs field emerges as the pseudo-Goldstone boson associated with the breaking of the global symmetry. The discrete symmetry is enough to ensure that any quadratically divergent contribution to the Higgs potential accidentally respects the global symmetry. The pseudo-Goldstone mass is then at most logarithmically divergent, allowing natural electroweak breaking to be realized. Corrections to precision electroweak observables can be naturally

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small, providing a resolution of the LEP paradox. In the original incarnation of this idea the discrete symmetry corresponded to the interchange of every SM particle with the corresponding particle transforming under a mirror SM. These models have the intriguing feature that all of the new physics beyond the SM (or beyond a minimal extension of the SM [11]) is a singlet under the SM gauge groups. Such new physics will then appear in upcoming experiments purely as missing energy, posing an interesting challenge for the LHC.

In this paper we present a more minimal realization of the twin Higgs mechanism that does not involve adding a whole new mirror copy of the SM. Instead, we identify the discrete symmetry with the parity symmetry associated with the extension of the SM to a left-right symmetric model [12]. This directly leads to a class of interesting models with exciting implications for upcoming experiments.

This paper is organized as follows. In Section II we illustrate these ideas by presenting a simple model where the symmetries are realized linearly. In Section III we present a more general non-linear realization. In Section IV we discuss some of the phenomenology and demonstrate that natural electroweak breaking can be obtained.

II. A LINEAR REALIZATION

We illustrate how the symmetries are implemented in these models by considering first a simplified model where the global symmetry is realized linearly. Consider a complex scalar field, H, that transforms as a fundamental under a global $U(4)$ symmetry. The potential for this field is given by

$$V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2 \,. \tag{1}$$

Since the mass squared of H is negative it will develop a VEV, $\langle |H| \rangle = m/\sqrt{2\lambda} \equiv f$, that breaks $U(4) \to U(3)$ yielding 7 massless Nambu-Goldstone bosons. We now break the $U(4)$ explicitly by gauging an $SU(2)_L \times SU(2)_R$ subgroup. Here $SU(2)_L$ generates the weak interactions of the SM, while $SU(2)_R$ generates the corresponding right-handed interactions associated with the extension of the SM to the left-right symmetric model. (We defer a discussion of the $U(1)_{B-L}$ gauge symmetry.) The field H transforms as $((H_L, H_R)$ where H_L is a doublet under $SU(2)_L$ that is to be identified with the SM Higgs and H_R is a doublet under $SU(2)_R$. This Higgs structure is characteristic of Alternative Left-Right Symmetric Models [13], (see also [14]).

Since $U(4)$ is now broken explicitly, we expect that the would-be Goldstones pick up a mass that is proportional to the explicit breaking. Specifically, gauge loops contribute a quadratically divergent mass to the components of H as

$$\Delta V = \frac{9 g_L^2 \Lambda^2}{64\pi^2} H_L^\dagger H_L + \frac{9 g_R^2 \Lambda^2}{64\pi^2} H_R^\dagger H_R + \dots \,, \tag{2}$$

a loop factor below the cutoff Λ of the theory. If we now impose parity symmetry the two gauge couplings have to be equal, $g_L = g_R \equiv g$, so that

$$\Delta V = \frac{9 g^2 \Lambda^2}{64\pi^2} (H_L^\dagger H_L + H_R^\dagger H_R) = \frac{9 g^2 \Lambda^2}{64\pi^2} H^\dagger H \tag{3}$$

which is invariant under $U(4)$ and therefore will not contribute a mass to the Goldstones. In other words, left-right symmetry constrains the quadratically divergent mass terms to have an $U(4)$ invariant form. The Goldstones are therefore completely insensitive to quadratic divergences from gauge loops.

Gauge loops will however contribute a logarithmically divergent term to the potential that is not $U(4)$ symmetric and has the general form $\kappa \left(|H_L|^4 + |H_R|^4 \right)$ where κ is of order $g^4/16\pi^2 \log(\Lambda/gf)$. Provided Λ is not very much larger than f this leads to the would-be Goldstones acquiring a mass of order $g^2 f/4\pi$ which is of order the weak scale for f of order a TeV.

At this point we note that the Higgs potential of Eq. (1) actually possesses a larger global $O(8)$ symmetry of which $U(4)$ is merely a sub-group, and the 7 Goldstone bosons we have identified can also be thought of as emerging from the breaking of $O(8)$ to $O(7)$. In particular, this $O(8)$ symmetry includes the custodial $SU(2)$ of the Higgs potential in the Standard Model.

This approach to stabilizing the weak scale against quantum corrections from gauge loops can be generalized to include all the other interactions in the SM by making the entire theory left-right symmetric. The fermionic content of the theory is then three generations of

$$\begin{array}{ll} Q_L = (u,d)_L = [2,1,1/3] & L_L = (\nu,e)_L = [2,1,-1] \\ Q_R = (u,d)_R = [1,2,1/3] & L_R = (\nu,e)_R = [1,2,-1] \end{array} \tag{4}$$

where the square brackets indicate the quantum numbers of the corresponding field under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We see that in addition to the SM fermions the theory includes right-handed neutrinos as required by left-right symmetry. The Higgs fields have quantum numbers

$$H_L = [2,1,1] \qquad H_R = [1,2,1] \tag{5}$$

under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The down-type Yukawa couplings of the SM emerge from non-renormalizable couplings of the form

$$\left(\frac{\overline{Q}_R H_R H_L^\dagger Q_L \;+\; \overline{L}_R H_R H_L^\dagger L_L}{\Lambda} \right) \;+\; \text{h.c.} \tag{6}$$

The up-type Yukawa couplings of the SM emerge from non-renormalizable couplings of the form

$$\left(\frac{\overline{Q_R}\; H_R^\dagger H_L Q_L \;+\; \text{h.c.}}{\Lambda} \right) \tag{7}$$

When the field H_R acquires a VEV of order f breaking $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$ these non-renormalizable couplings reduce to the familiar Yukawa couplings of the SM. Unfortunately, although this works well for the smaller Yukawa couplings, it is not satisfactory for the top Yukawa coupling which is required to be order one. We address this difficulty by introducing a vector-like pair of quarks T_L and T_R which have the quantum numbers

$$T_L = [1, 1, 4/3] \qquad T_R = [1, 1, 4/3] \tag{8}$$

under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We can then write the interactions

$$\left(y\, \overline{Q_R}\, H_R^\dagger T_L \;+\; y\, \overline{Q_L}\, H_L^\dagger T_R \;+\; M \overline{T_L} T_R \right) \;+\; \text{h.c.} \tag{9}$$

The right-handed top quark of the SM then emerges as a linear combination of T_R and the third generation up-type quark in Q_R, while the orthogonal linear combination is heavy. Provided $M \lesssim f$ and y is of order one the physical top Yukawa will then also be of order one. The parameter M controls the mixing of the left-handed top with the $\mathrm{SU}(2)_L$ singlet T_L , and is therefore constrained by $Z \rightarrow b\, \bar{b}$. However, nothing prevents M from simply being set to zero and therefore this is not a particularly tight constraint.

The fact that the entire theory is now left-right symmetric ensures that any quadratically divergent contribution to the Higgs mass has a form $\propto \Lambda^2(|H_L|^2 + |H_R|^2)$ which is harmless due to its accidental $U(4)$ symmetry. Although quantum corrections to the quartic are in general not $U(4)$ invariant, once again these only lead to logarithmically divergent contributions to the mass of the pseudo-Goldstone Higgs field, allowing for a natural hierarchy between f and the weak scale.

Unfortunately the theory as described above is still not entirely satisfactory. The reason is that precision electroweak constraints on $SU(2)_R$ gauge bosons force the scale f to lie close to 2 TeV or above [15], which tends to reintroduce fine-tuning. While there may be several possible solutions to this problem, for the remainder of this paper we shall concentrate on only one. We introduce into the theory an additional Higgs field $\hat{H} = (\hat{H}_L, \hat{H}_R)$ where $\hat{H}_L$ and $\hat{H}_R$ have exactly the same gauge quantum numbers as H_L and H_R, but do not have the corresponding couplings to the SM fermions. We assume that $\hat{H}$ and H do not couple directly to each other at the scale Λ, and further that the potential for $\hat{H}$ at this scale has the $U(4)$ invariant form

$$V(\hat{H}) = -m^2 \hat{H}^\dagger \hat{H} + \lambda(\hat{H}^\dagger \hat{H})^2 \,. \tag{10}$$

Then the Higgs sector of the theory has an approximate $U(4) \times U(4)$ symmetry, or more precisely an approximate $O(8) \times O(8)$ symmetry, of which the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ sub-group is gauged. If $\hat{H}_R$ acquires a VEV $\hat{f} >$ 2 TeV breaking $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ then the precision electroweak constraints on this theory from the extra gauge bosons are satisfied. At the same time, the approximate $U(4) \times U(4)$ symmetry implies that the twin symmetric form of the potential for H is not significantly affected, so that electroweak symmetry breaking can still occur naturally.

III. A NON-LINEAR REALIZATION

We now construct a realistic twin symmetric model that implements these symmetries non-linearly. The pseudo-Goldstone fields of the non-linear model are those which survive after integrating out the radial modes of the fields H and $\hat{H}$ in the linear model. We parameterize these degrees of freedom as

$$\begin{aligned} H \;&=\; \exp(\frac{i}{f} h^a t^a) \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} + i \begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^0 \end{pmatrix} + \ldots \\ \hat{H} \;&=\; \exp(\frac{i}{\hat{f}} \hat{h}^a t^a) \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{f} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{f} \end{pmatrix} + i \begin{pmatrix} \hat{h}^1 \\ \hat{h}^2 \\ \hat{h}^3 \\ \hat{h}^0 \end{pmatrix} + \ldots \end{aligned} \tag{11}$$

where $h^{1,\ldots,3}$, $\hat{h}^{1,\ldots,3}$ are complex and h^0, $\hat{h}^0$ are real. The t^a are a suitably chosen set of broken generators. In general the effective theory for these fields will contain all of the operators allowed by the non-linearly realized $U(4) \times U(4)$ symmetry, suppressed by the cutoff scale Λ. However, in order to suppress custodial SU(2) violation we assume that the symmetry which is non-linearly realized is in fact $O(8) \times O(8)$. This provides additional restrictions on the form of the interactions in the effective theory below Λ, allowing precision electroweak constraints from higher dimensional operators to be naturally satisfied. If the theory is strongly coupled at the cutoff we can estimate $\Lambda \sim 4\pi f$. However, we do not exclude the possibility that Λ is less than this. For example, if the UV completion of the non-linear model is the linear model then Λ is simply the mass of the radial mode.

In general, any potential for the pseudo-Goldstone fields can only emerge from those interactions which violate the global symmetries, specifically the gauge and Yukawa couplings. In particular the electroweak gauge interactions and the top Yukawa contribute the most to the pseudo-Goldstone Higgs potential and must therefore be studied in detail. We will therefore calculate the contributions to the one loop Coleman-Weinberg (CW) potential [16] from these couplings. At one loop the gauge and top sectors contribute separately, simplifying the calculation.

As before, we gauge the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ sub-groups of the global symmetry. The VEVS f and $\hat{f}$ break $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$, giving W_R and Z_R masses of order $g\hat{f}$. One linear combination of the fields H_R and $\hat{H}_R$ is eaten. The $SU(2)_L$ doublet $h^T \equiv (h^1, h^2)$ is left uneaten and is identified as the SM Higgs. The couplings of the pseudo-Goldstone fields to the gauge fields is given by expanding out H and $\hat{H}$ in terms of the pseudo-Goldstones as given by eq. (11) in the interaction

$$\left[\left|\left(\partial_\mu + igW_{\mu,L} + \frac{i}{2}g'B_\mu\right)H_L\right|^2 + (L \to R)\right] +$$
$$\left[\left|\left(\partial_\mu + igW_{\mu,L} + \frac{i}{2}g'B_\mu\right)\hat{H}_L\right|^2 + (L \to R)\right] \quad (12)$$

where B_μ is the gauge boson of $U(1)_{B-L}$. A simple way of calculating the effective potential is to determine the vacuum energy as a function of the field dependent masses of all of the fields in the theory. In the absence of quadratic divergences this leads to the formula

$$V_{CW} = \pm\frac{1}{64\pi^2}\sum_i M_i^4\left(\log\frac{\Lambda^2}{M_i^2} + \frac{3}{2}\right) \quad (13)$$

where the sum is over all degrees of freedom, the sign being negative for bosons and positive for fermions. Writing the Higgs potential in the form

$$V(h) = m_h^2 h^\dagger h + \lambda_h (h^\dagger h)^2 + \dots \quad (14)$$

we find that the contribution to the Higgs mass term from the gauge sector is

$$\begin{aligned} m_h^2|_{gauge} &= \frac{3g^2 M_{W_R}^2}{32\pi^2}\left(\log\frac{\Lambda^2}{M_{W_R}^2} + 1\right) \quad (15) \\ &+ \frac{3g^2(2M_{Z_R}^2 - M_{W_R}^2)}{64\pi^2}\left(\log\frac{\Lambda^2}{M_{Z_R}^2} + 1\right) \\ \lambda_h|_{gauge} &= -\frac{m_h^2|_{gauge}}{3f^2} \end{aligned}$$

where $M_{W_R}^2 = g^2(f^2 + \hat{f}^2)/2$ and $M_{Z_R}^2 = (g^2 + g'^2)(f^2 + \hat{f}^2)/2$. Except the term proportional to the Higgs mass squared, all other contributions to the Higgs quartic from this sector are small and can be neglected.

We now turn to the top sector. The couplings of the pseudo-Goldstone fields to the top quark are obtained by expanding out H as in eq. (11) in the interactions of eq. (9) which generate the top Yukawa coupling. The h dependent masses of the fields in the top sector are determined from this and can be expressed as

$$m_Q^2 = \frac{y^4 f^2}{M^2 + y^2 f^2}h^\dagger h \qquad m_T^2 = M^2 + y^2 f^2 \quad (16)$$

to leading order in $|h|^2/f^2$, where we have assumed for simplicity that y is real. This leads to the following contributions to the Higgs potential of eq. (19).

$$\begin{aligned} m_h^2|_{top} &= -\frac{3}{8\pi^2}y_t^2 m_T^2\left(\log\frac{\Lambda^2}{m_T^2} + 1\right), \\ \lambda_h|_{top} &= -\frac{m_h^2|_{top}}{3f^2} + \frac{3}{16\pi^2}\left(y_t^4\log\frac{m_T^2}{m_Q^2} + 2y^4\log\frac{\Lambda^2}{m_T^2}\right) \\ &\quad -\frac{3}{32\pi^2}\left[y_t^4 - 4y^4\right] \end{aligned} \quad (17)$$

where y_t is defined by

$$y_t = \frac{y^2 f}{\sqrt{M^2 + y^2 f^2}} \quad (18)$$

This completes the determination of the one-loop potential for the SM Higgs. One may worry that corrections to the SM Higgs mass squared of order $g^2 f^2$ may arise at higher loop order [10]. However, we show in the appendix that this is not the case.

It is also necessary to show that the other pseudo-Goldstone fields in H and $\hat{H}$ also have positive mass squareds. It is straightforward to ensure that $\hat{H}_L$ has positive mass squared by adding to the potential a term $\hat{\mu}^2\hat{H}_L^\dagger\hat{H}_L$ where $\hat{\mu}$ is of order f. Such a term breaks both parity and the approximate $U(4)$ symmetry of the potential for $\hat{H}$, but only softly. It is therefore technically natural for $\hat{\mu}$ to be smaller than Λ.

What about the fields in H_R and $\hat{H}_R$? Of these six fields, three are eaten and become the longitudinal components of the right-handed gauge bosons while the remaining three remain light as pseudo-Goldstone bosons associated with the breaking of the approximate $U(2)_R \times U(2)_R$ symmetry of the Higgs potential. Of the light fields, two carry electric charges of +1 and -1 while the last is neutral. The electrically charged fields acquire positive mass squareds from the gauge interactions which violate $U(2)_R \times U(2)_R$, but the neutral state remains massless. In order to give it a mass we add to the potential a term $B\ H_R^\dagger\hat{H}_R$ where $\sqrt{B}$ is of order 100 GeV or so. Since this is the only term in the Lagrangian which breaks the discrete symmetry $\hat{H}_R \to -\hat{H}_R$ it is technically natural for it to be small.

In this non-linear model, the absence of quadratically divergent contributions to the Higgs mass can be understood as a consequence of cancellations between the familiar SM loop corrections and new loop corrections that arise from the (mostly non-renormalizable) couplings of the Higgs to the twin sector.

We are now in a position to estimate the fine-tuning in this class of models. Unfortunately, for a fixed value of the cutoff Λ, the precision of this estimate is necessarily limited by the fact that the answer is sensitive to the exact relation between f and Λ, which in a strongly coupled theory depends both on the detailed dynamics of the theory and also on the physical observable under consideration. A naive estimate gives $\Lambda \sim 4\pi f$. However, it was shown in [10] that in the linear model, in the absence of the radial mode, unitarity is saturated at $\Lambda \sim 2\pi f$. Therefore, in order to get some sense of the fine-tuning we will allow for both possibilities, considering points in parameter space satisfying the relation $\Lambda = 4\pi f$ as well as points in parameter space satisfying the relation $\Lambda = 2\pi f$.

For $f = 800$ GeV, $\Lambda \sim 4\pi f \approx 10$ TeV, $M = 150$ GeV, $\sqrt{B} = 50$ GeV we find that in order to obtain the SM values of M_W and M_Z we need $\hat{f} \approx 4.29$ TeV. The Higgs mass is then about 174 GeV. Estimating the fine-tuning as $\partial\ \log M_Z^2/\partial\ \log f^2$ we find that it is of order

12% (1 in 8). Similarly for $f = 800$ GeV, $\Lambda \sim 2\pi f \approx 5$ TeV, $M = 150$ GeV, $\sqrt{B} = 50$ GeV we find that in order to obtain the SM values of M_W and M_Z we need $\hat{f} \approx 4.68$ TeV. The Higgs mass is then about 155 GeV. Estimating the fine-tuning as $\partial \log M_Z^2/\partial \log f^2$ we find that it is of order 12% (1 in 8). These and other results are summarized in Table I. This shows that these models stabilize the weak scale up to about 5 TeV.

Λ(TeV)	f(GeV)	$\hat{f}$(TeV)	M(GeV)	$\sqrt{B}$(GeV)	m_h(GeV)	Tuning
10	800	4.29	150	50	174	0.117
6	500	2.27	150	50	172	0.270
5	800	4.68	150	50	155	0.124

TABLE I: A summary of the Higgs mass and fine tuning, $\partial \log M_Z^2/\partial \log f^2$, for sample points of parameter space. The largest fine tuning is associated with f.

To what extent does the absence of a tree-level quartic affect the fine-tuning in these theories? To understand this, consider a theory with a single light Higgs doublet at low energies and a scalar potential of the form

$$V(h) = m_h^2 h^\dagger h + \lambda_h (h^\dagger h)^2 \tag{19}$$

In terms of these parameters the electroweak VEV $v = \sqrt{|m_h|^2/2\lambda_h}$ and the physical Higgs mass, which we denote by $m_{h,\mathrm{phys}}$, is given by $\sqrt{2}|m_h|$. In our model, the dominant contribution to the Higgs mass parameter m_h^2 arises from the top Yukawa coupling. If we denote this contribution by $m_h^2|_{\mathrm{top}}$, a good estimate of the fine-tuning may be obtained by considering the ratio $m_h^2/m_h^2|_{\mathrm{top}}$. This is equal to $m_{h,\mathrm{phys}}^2/(2\ m_h^2|_{\mathrm{top}})$. Now, is clear from Table [I] that $m_{h,\mathrm{phys}}$ in our models is of order 150 GeV or larger. Since precision electroweak constraints require the Higgs to be lighter than about 250 GeV, the potential improvement in fine-tuning in our model from a tree-level quartic in the region of parameter space where the LEP paradox is addressed is at most of order $(250)^2/(150)^2 \approx 3$, and close to 2 for most of the points in the table.

The preceding analysis enables us to compare the fine-tuning in our model to that in little Higgs theories with a tree level quartic. For concreteness we focus on the little Higgs model of Kaplan and Schmaltz [17], for which the pattern of symmetry breaking, $\mathrm{SU}(4)^4 \to \mathrm{SU}(3)^4$, is most similar to ours, and for which f and Λ can therefore be defined in close analogy. In this theory the low energy spectrum contains two Higgs doublets, of which only one couples to the top quark. In the limit where this doublet is significantly lighter than the other we can obtain a simple estimate of the fine-tuning. To do this we calculate $m_h^2|_{\mathrm{top}}$ for the light doublet, and compute $m_{h,\mathrm{phys}}^2/2\ (m_h^2|_{\mathrm{top}})$, setting $m_{h,\mathrm{phys}}$ to its upper bound of 250 GeV. The gauge symmetry is $\mathrm{SU}(3)_C \times \mathrm{SU}(4) \times \mathrm{U}(1)_X$, where $\mathrm{SU}(4)\times\mathrm{U}(1)_X$ is broken down to the familiar $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ of the Standard Model. The top Yukawa coupling emerges from couplings of the form

$$\left[y_1 \overline{Q} H t_1 + y_2 \overline{Q} \hat{H} t_2 + \mathrm{h.c.}\right] \tag{20}$$

The gauge quantum numbers of these fields under $\mathrm{SU}(3)_C \times \mathrm{SU}(4)$ are $Q \equiv [3,4]$, $t_1, t_2 \equiv [3,1]$ and $H, \hat{H} \equiv [1,4]$. The third generation quark doublet of the Standard Model is contained in Q while the right-handed top quark emerges from a linear combination of t_1 and t_2. The light Higgs doublet emerges as the uneaten linear combination of the doublets in H and $\hat{H}$, which may be expanded out exactly as in Eq. (11). Then a simple calculation [18] shows that for this theory, the divergent part of $m_h^2|_{top}$ is bounded from below as

$$|m_h^2|_{top}| \geq 2\ \frac{3 y_t^2 f^2}{8\pi^2} \log\frac{\Lambda^2}{f^2} \tag{21}$$

where we have assumed $f < \hat{f}$ without loss of generality. This must be compared against the contributions to $m_h^2|_{\mathrm{top}}$ in the twin Higgs model as given by Eq. (17). We see that for small M, assuming fixed values of f and Λ, the value of $m_h^2|_{\mathrm{top}}$ in the little Higgs model is larger by a factor of 2 or more. From this it follows that in spite of the absence of a tree-level quartic in our model, for fixed values of f and Λ, the fine-tuning in the two models is in fact quite comparable. However precision electroweak constraints on the twin Higgs model are much weaker due to the absence of SU(4) gauge bosons, which means that much lower values of f are experimentally allowed than in this little Higgs model. This translates to a significant improvement in fine-tuning over the little Higgs case. Some models based on collective symmetry breaking where the bounds from precision electroweak measurements are weaker and which admit low values of f have been constructed, for example, [7], [19]. A study of the relative fine-tuning with respect to these models is left for future work.

IV. PHENOMENOLOGY

These models predict a rich spectrum of light exotics which can be detected in the next generation of collider experiments. These include

- the right-handed gauge bosons, which have masses not to exceed a few TeV, and which couple with the same strength as the gauge bosons of $SU(2)_L$,
- the vector-like quarks T_L and T_R which are expected to have masses of several hundred GeV
- the charged pseudo-Goldstones from H_R and $\hat{H}_R$, which have masses not to exceed a few hundred GeV
- the neutral pseudo-Goldstone from H_R and $\hat{H}_R$ which also has mass of order a hundred GeV

A detailed study of the collider signatures of this model is left for future work. Since $\hat{H}_L \to -\hat{H}_L$ is an exact symmetry of the model the neutral component of this field is a natural dark matter candidate.

We now turn to the question of how neutrino masses are generated in this model. Dirac neutrino masses arise from the operator $[(\overline{L}_R H_R^\dagger H_L L_L/\Lambda) + \text{h.c.}]$ while the operator $[(L_R \hat{H}_R \hat{H}_R L_R)/\Lambda + \text{h.c.}]$ generates a Majorana mass for the right-handed neutrinos. This allows the SM neutrinos to get a Majorana mass of the right size through the see-saw mechanism [20], provided the coefficient of the operator which generates the Dirac neutrino mass is small $\sim 10^{-5}$.

In summary we have constructed a new class of twin Higgs models based on parity-symmetric left-right models which stabilize the weak scale against radiative corrections up to scales of order 5 TeV. These theories make definite predictions for exotic particles that can be detected in the next generation of collider experiments, and admit a natural dark matter candidate.

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APPENDIX A: 2 LOOP DIAGRAMS

In [10] it was pointed out that in the strong coupling limit one expects two loop contributions to the $U(4)$ violating quartic, $|H_L|^4 + |H_R|^4$, of order g^2. One can see that such contributions might be present by inspecting 2-loop diagrams such as those of Figure 1 that are of order $g^2\lambda^2/(16\pi^2)^2$ and taking λ to its NDA value of $(4\pi)^2$. The presence of such a large quartic would imply that in the strong coupling limit the Higgs mass in our model (and in the model of [9]) is close to the upper bound allowed by precision electroweak data. Moreover the exact Higgs mass would be uncalculable. In this appendix we show that summing all of the relevant 2-loop graphs in fact yields a $U(4)$ symmetric quartic. In the next appendix we show that this is not an accident but can be understood as a consequence of a symmetry argument which can be extended to all loop orders.

The 2-loop diagrams that will potentially contribute at order g^2 may be divided into four subcategories as follows: (a) the gauge boson connects between two external legs, (b) the gauge boson connects an internal and an external scalar leg, (c) the gauge boson connects to a common internal leg, thus correcting the scalar propagator, and (d) the gauge boson connects between two different internal legs. A representative of each group is shown in Figure 1. Because only the $SU(2)_L \times SU(2)_R$ subgroup of $U(4)$ is gauged, we only consider diagrams where the gauge bosons are exchanged between a pair of fields labeled L or R and not when the exchange is

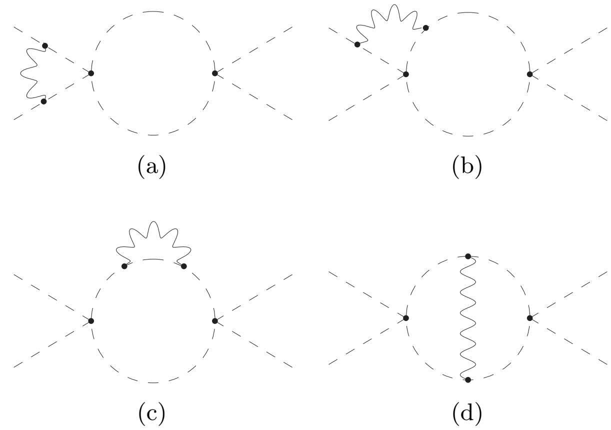

FIG. 1: Representative graphs that contribute at order $g^2\lambda^2/(16\pi^2)^2$. Summing all such diagrams yields an $SU(4)$ symmetric quartic.

between L and R. Note that we can ignore contribution form $B-L$ gauge boson exchange because the $B-L$ charges of the Higgs fields respect $U(4)$.

To show that the 2-loop quartic is $U(4)$ symmetric we can factor a common phase space integral from each category of diagrams and focus our attention to symmetry factors, signs, and group theory coefficients. In particular, a relative sign difference between diagrams arises when a gauge boson is exchanged between two H's versus the case where it is exchanged between an H and an $H^\dagger$. In order to show that the quartic is $U(4)$ symmetric we need to show that the $|H_L|^2|H_R|^2$ quartic is twice as large as the $|H_L|^4$.

The quartics arising from graphs of type (a) are 1-loop corrections to to an already $U(4)$ symmetric (1-loop) quartic due to the derivative interaction of the gauge boson. It is then sufficient to show that the $|H_L|^4$ and $|H_L|^2|H_R|^2$ are renormalized in the same way. There are 6 diagrams with identical kinematics that contribute to $|H_L|^4$, however two of them have a relative minus sign giving a total symmetry factor of $4-2=2$. The $|H_L|^2|H_R|^2$ receives contribution from just two graphs that have an identical phase space structure. The relative factor of 2 between the two types of factors is thus preserved.

The diagrams of type (b) can be shown to sum to zero. This can be seen by noting that for any given diagram where the gauge boson connects to an H_L (H_R) on the external leg, one can draw a different diagram in which the gauge boson connects to $H_L^\dagger$ ($H_R^\dagger$), yielding a relative minus sign. The sum of all such diagrams cancels in pairs.

The contribution from diagrams of type (c) are also trivially $U(4)$ symmetric since the gauge boson loop is merely a correction to the propagator which respects $U(4)$ due to the conserved Z_2 symmetry. $U(4)$ is thus not violated by such diagrams.

The most non-trivial cancellation occurs in diagrams of type (d). For brevity we will simply quote the result here and present a more instructive proof in the next appendix. The $|H_L|^4$ receives a contribution proportional to $15\lambda^2 g^2$ (here the relative sign between some diagrams plays an important role). The contribution to $|H_R|^2|H_L|^2$ is proportional to $30\lambda^2 g^2$. The constants of proportionality in both cases are a common phase space integral. The overall quartic from this class of diagrams is thus $U(4)$ symmetric as well.

As mentioned above, the $U(1)_{B-L}$ contribution is $U(4)$ symmetric because these two groups commute. However in [9] the $U(1)$ gauge structure is different. There two sets of hypercharge, A and B were gauged. One can use the same arguments for diagrams of type (a)-(c) and explicitly calculate those of type (d) to find that diagrams with hypercharge gauge boson exchange do not contribute an $U(4)$ violating quartic either.

APPENDIX B: HIGHER ORDER CORRECTIONS

We now demonstrate that in the non-linear model there are no corrections of order $g^2 f^2$ to the mass of the pseudo-Goldstones at any order in perturbation theory. In particular the cancellation of the two-loop diagrams in the previous section is not accidental, but instead follows from a symmetry argument.

Consider first the linear model. We will show that no U(4) violating potential terms are generated for H at order g^2. We start first with $\mathrm{U}(1)_{B-L}$. At order g' we have the following interaction between H and B_μ.

$$\frac{i}{2} g' B_\mu \left[\partial^\mu H_L^\dagger\, H_L + \partial^\mu H_R^\dagger\, H_R - H_L^\dagger\, \partial^\mu H_L - H_R^\dagger\, \partial^\mu H_R\right] \tag{B1}$$

We see that the couplings of B_μ at order g' are invariant under the U(4) symmetry under which $H = (H_L, H_R)$ transforms as a fundamental. Then the only terms in the potential which can be generated from this interaction have the form of $H^\dagger H$ raised to some power, which is U(4) invariant.

At order g'^2 we have the interaction

$$\frac{g'^2}{4} B'_\mu B'^\mu \left[H_L^\dagger H_L + H_R^\dagger H_R\right] \tag{B2}$$

which is also manifestly U(4) invariant and will not generate U(4) violating terms in the potential.

We now turn to the interactions of H with the gauge bosons of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$. Decompose $W_{\mu,L} = 1/2\ W^a_{\mu,L}\tau^a$, $W_{\mu,R} = 1/2\ W^a_{\mu,R}\tau^a$ where a runs from one to three and the τ^a are the Pauli matrices. Since the gauge boson propagator is diagonal in the index a, to order g^2 we are free to consider $a = 1$, $a = 2$ and $a = 3$ separately. For now we therefore focus only on the interactions of H with $W^3_{\mu,L}$ and $W^3_{\mu,R}$. At order g these take the form

$$\begin{aligned} &\frac{i}{2}\, g\, W^3_{\mu,L}\ \left[\partial^\mu H_L^\dagger\, \tau^3\, H_L - H_L^\dagger\, \tau^3\, \partial^\mu H_L\right] \\ +&\frac{i}{2}\, g\, W^3_{\mu,R}\ \left[\partial^\mu H_R^\dagger\, \tau^3\, H_R - H_R^\dagger\, \tau^3\, \partial^\mu H_R\right] \end{aligned} \tag{B3}$$

We can rewrite this interaction in terms of a new set of variables. Expanding out $H_L = (H_{L1}, H_{L2})$, $H_R = (H_{R1}, H_{R2})$ we can define

$$\begin{aligned} H_{3,+} &= (H_{L1}, H_{R1}, H^*_{L2}, H^*_{R2}) \\ H_{3,-} &= (H_{L1}, H_{R2}, H^*_{L2}, H^*_{R1})\,. \end{aligned} \tag{B4}$$

Further, define

$$\begin{aligned} W_{\mu,+} &= \frac{1}{2}\left[W_{\mu,L} + W_{\mu,R}\right] \\ W_{\mu,-} &= \frac{1}{2}\left[W_{\mu,L} - W_{\mu,R}\right] \end{aligned} \tag{B5}$$

Note that the gauge boson propagators are diagonal in the $W_{\mu,+}, W_{\mu,-}$ basis. In terms of these new variables eq. (B3) becomes

$$\begin{aligned} &\frac{i}{2}\, g\, W^3_{\mu,+}\ \left[\partial^\mu H_{3,+}^\dagger\, H_{3,+} - H_{3,+}^\dagger\, \partial^\mu H_{3,+}\right] \\ +&\frac{i}{2}\, g\, W^3_{\mu,-}\ \left[\partial^\mu H_{3,-}^\dagger\, H_{3,-} - H_{3,-}^\dagger\, \partial^\mu H_{3,-}\right] \end{aligned} \tag{B6}$$

We see that the couplings of $W^3_{\mu,+}$ at order g are invariant under a U(4) symmetry under which $H_{3,+}$ transforms as a fundamental. Therefore the only potential terms which can be generated from this interaction at order g^2 have the form of $H_{3,+}^\dagger H_{3,+} = H^\dagger H$ raised to some power. Similarly the couplings of $W^3_{\mu,-}$ at order g are invariant under a different U(4) symmetry under which $H_{3,-}$ transforms as a fundamental. Again the only potential terms this allows at order g^2 have the form of $H_{3,-}^\dagger H_{3,-} = H^\dagger H$ raised to some power. Although the argument we have just given applies only to $W^3_{\mu,L}$ and $W^3_{\mu,R}$ it generalizes in a straightforward way to the other components of the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ gauge bosons, since the different contributions can be related to each other through SU(2) rotations.

We now consider the interactions of H with $W_{\mu,L}$ and $W_{\mu,R}$ at order g^2. These take the form

$$g^2 H_L^\dagger W_{\mu,L} W^{\mu,L} H_L + g^2 H_R^\dagger W_{\mu,R} W^{\mu,R} H_R \tag{B7}$$

We rewrite the relevant terms in this interaction in terms of $W_{\mu,+}$ and $W_{\mu,-}$ as

$$\begin{aligned} &g^2 H_L^\dagger \left(W_{\mu,+}W^{\mu,+} + W_{\mu,-}W^{\mu,-}\right) H_L \\ +&g^2 H_R^\dagger \left(W_{\mu,+}W^{\mu,+} + W_{\mu,-}W^{\mu,-}\right) H_R \end{aligned} \tag{B8}$$

where we have dropped 'mixed terms' such as $H_L^\dagger W_{\mu,+} W^{\mu,-} H_L$ which cannot contribute to the quartic at order g^2. From eq. (B8) we see that the remaining terms are invariant under a U(4) symmetry under which $H = (H_L, H_R)$ transforms as a fundamental and therefore only give rise to a U(4) invariant potential terms at order g^2. This completes the proof that in the linear model U(4) violating potential terms are not generated at order g^2 to any order in perturbation theory.

We now consider the effect of adding arbitrary non-renormalizable interactions to the linear model. The additional terms are assumed to be invariant under O(8), with the $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ subgroups gauged. In general these new terms can be constructed by making Lorentz invariant contractions or products from gauge invariants of the form

$$[D_\alpha D_\beta ... H]^\dagger [D_\lambda D_\sigma ... H] \; + \; \text{h.c.} \tag{B9}$$

where the number 'n' of gauge covariant derivatives $D_\alpha^\dagger$ acting on $H^\dagger$ and the number 'm' of gauge covariant derivatives D_λ acting on H are both arbitrary. Hermitian conjugation is necessary for O(8) invariance. Let us first consider the interactions between H and the SU(2) gauge bosons at order g. We take as a representative term

$$[\partial^n H_L^\dagger \partial^{m-1} (igW_L H_L) \; + \; \partial^n H_R^\dagger \partial^{m-1} (igW_R H_R)] \\ + \; [\text{h.c.}] \tag{B10}$$

where for simplicity we have suppressed all Lorentz indices. Once again, we go to the $W_{\mu,+}, W^{\mu,-}$ basis and restrict our consideration to the third component of the W's. Then the interaction above can be rewritten as

$$\frac{ig}{2}[\partial^n H_{3,+}^\dagger \partial^{m-1} \left(W_{\mu,+}^3 H_{3,+}\right) \; + \; \partial^n H_{3,-}^\dagger \partial^{m-1} \left(W_{\mu,-}^3 H_{3,-}\right)] \\ + \; [\text{h.c.}] \tag{B11}$$

We see that to order g all the interactions of $W_{\mu,+}^3$ with $H_{3,+}$ are invariant under a U(4) symmetry as in the simple linear model. Similarly all the interactions of $W_{\mu,-}^3$ with $H_{3,-}$ are also invariant under a U(4) symmetry to order g. Then to order g^2 the only potential terms that can be generated from these couplings have the form of the U(4) invariant $H^\dagger H$ raised to some power.

We now consider the interactions between H and the SU(2) gauge bosons at order g^2. We take as a representative term

$$[\partial^n H_L^\dagger \partial^{m-2} (gW_L)^2 H_L \; + \; \partial^n H_R^\dagger \partial^{m-2} (gW_R)^2 H_R] \\ + \; [\text{h.c.}] \tag{B12}$$

This can be rewritten

$$\left[\partial^n H_L^\dagger \partial^{m-2} \left(g^2 W_+ W_+ + g^2 W_- W_-\right) H_L\right] \; + \\ \left[\partial^n H_R^\dagger \partial^{m-2} \left(g^2 W_+ W_+ + g^2 W_- W_-\right) H_R\right] \; + \; [\text{h.c.}] \tag{B13}$$

where we have dropped the terms that involve products of W_+ and W_- because they do not contribute at order g^2. The remaining terms are invariant under a U(4) symmetry under which $H = (H_L, H_R)$ transforms as a fundamental and therefore only give rise only to U(4) invariant terms at order g^2, just as in the simple linear model.

We have therefore shown that the SU(2) gauge interactions do not generate a U(4) violating terms in the potential to order g^2 at any order in perturbation theory. It is straightforward to generalize this argument to include $U(1)_{B-L}$ using the same methods. It follows that adding arbitrary non-renormalizable interactions to the linear sigma model does not result in U(4) violating terms in the potential at order g^2. Since the general non-linear model with the symmetry properties we desire may be obtained by integrating out the radial mode from this model, it follows that in the non-linear model the pseudo-Goldstones do not acquire a mass at order g^2 to any order in perturbation theory. Although the pseudo-Goldstones do acquire a mass at order g^4 from states at the cutoff this is always further loop suppressed, and is expected to be smaller than the logarithmically enhanced contribution calculated in the body of the paper. The very same arguments can be applied to the mirror twin Higgs model of ref [9] to establish the absence of corrections to the pseudo-Goldstone potential at order g^2.

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