

Gauge – Mediated Supersymmetry Breaking in String Compactifications

Duiliu-Emanuel Diaconescu^{∇1}, Bogdan Florea^{#2}, Shamit Kachru^{‡2} and Peter Svrček^{♭2}

¹*Department of Physics and Astronomy, Rutgers University,
Piscataway, NJ 08855-0849, USA*

²*Department of Physics and SLAC, Stanford University,
Stanford, CA 94305/94309, USA*

We provide string theory examples where a toy model of a SUSY GUT or the MSSM is embedded in a compactification along with a gauge sector which dynamically breaks supersymmetry. We argue that by changing microscopic details of the model (such as precise choices of flux), one can arrange for the dominant mediation mechanism transmitting SUSY breaking to the Standard Model to be either gravity mediation or gauge mediation. Systematic improvement of such examples may lead to top-down models incorporating a solution to the SUSY flavor problem.

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[∇] duiliu@physics.rutgers.edu

[#] bflorea@slac.stanford.edu

[‡] skachru@stanford.edu

[♭] svrcek@slac.stanford.edu

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1. Introduction

The Minimal Supersymmetric Standard Model (MSSM), and its natural extension into SUSY GUTs, provides perhaps the most compelling viable extension of the Standard Model [1,2]. Supersymmetric models can stabilize the hierarchy between the electroweak and Planck scales, and also successfully incorporate gauge coupling unification [3]. One can further hope that the correct theory of supersymmetry breaking explains the small scale of breaking via a dynamical mechanism [4], so that the weak scale is not only radiatively stable, but is also explained in a theory with no (very) small dimensionless parameters.

From the top down, supersymmetric GUTs have seemed very natural in the context of heterotic string theory [5]. It has been known for some time that explicit models with pseudo-realistic matter content can be constructed in this framework; the state of the art models are presented in e.g. [6]. In the heterotic M-theory framework [7], one can also accommodate gauge-gravity unification in a fairly natural way [8].

One missing ingredient in many of the stringy constructions has been an explicit model of supersymmetry breaking. In the old heterotic framework, one fruitful approach was to simply parameterize the SUSY breaking by assuming (without microphysical justification) that the dominant SUSY-breaking F-term arises in the dilaton or a given modulus multiplet. Then, using the structure of the low-energy supergravity, one can work out the patterns of soft terms in different scenarios [9]. More recently, type II flux vacua with intersecting D-branes have become a popular arena for phenomenological constructions as well [10]. In these models, the fluxes generate calculable SUSY breaking F-terms in various circumstances [11,12,13,14]. A full model of the soft terms must necessarily also solve the problem of moduli stabilization; by now there exist type IIB and type IIA constructions where this problem is solvable [15,16,17,18,19]. In all such constructions, of course, one must fine-tune the cosmological constant after SUSY breaking. This tune has always been performed in the phenomenological literature by a shift of the constant term in the superpotential W , and the flux discretuum [20] seems to microphysically allow the same procedure in string theory.¹

¹ Since in realistic models incorporating the MSSM or its extensions, the primordial scale of SUSY breaking is always $\leq 10^{11}$ GeV, the required constant in W is always parametrically small relative to the Planck or string scale. This can be viewed as a “bottom up” motivation for the small W_0 tune which is performed in many models of moduli stabilization [15], where the same tune allows one to stabilize moduli in a calculable regime.

One notable problem with the soft-terms induced by the fluxes is that their natural order of magnitude is typically only suppressed from the string scale by a few powers of the Calabi-Yau radius. While this may lead to suitable soft-terms in models with significant warping [21,22] or with a low string scale, both of these solutions destroy one of the main attractive features of supersymmetry – its natural connection to grand unified models.

In this paper, we describe some simple pseudo-realistic string constructions, which incorporate both a toy model of the MSSM or a SUSY GUT, and also incorporate a sector which accomplishes dynamical SUSY breaking (for early attempts in this direction, see e.g. [23]). It will be clear that our basic setup is sufficiently modular that one can view the MSSM/GUT sector as a “black box,” and could presumably improve the realism in that sector without disturbing the basic mechanism of SUSY breaking or mediation. We give two classes of constructions: one based on the so-called “non-calculable” SUSY breaking models of [24,25], and another based on the recent insight that there are plentiful examples of simple quiver gauge theories which exhibit dynamical supersymmetry breaking (DSB) [26], and which can be easily embedded in Calabi-Yau compactification. By tuning closed-string parameters (in particular, choices of background flux), we will argue that one can find in each case two different regimes: one regime where the dominant mediation mechanism transmitting SUSY breaking to the Standard Model is gravity mediation, and another where it is gauge mediation.

In different parts of the paper, we will use the language and techniques of heterotic string model building and of F-theory (or IIB string theory) constructions. A wide class of 4d $\mathcal{N} = 1$ models admit dual heterotic and F-theory descriptions, and we simply use whichever description is more convenient in a given circumstance. In many cases, one should be able to use the dictionary of [27] to translate back and forth. Because most of the models of supersymmetry breaking in the string literature have involved gravity mediation, in §2 we briefly review some elementary phenomenology, explaining the chief differences between gravity and gauge mediation. In §3, we recall basic features of the “non-calculable” models of DSB [24,25] and give examples where these can be embedded into string theory along with a toy-model of a SUSY GUT. In §4, we briefly review the construction of SUSY breaking quiver gauge theories in non-compact Calabi-Yau manifolds [26]. We turn to the embedding into a compact geometry containing both a toy-model SUSY GUT and a SUSY breaking sector in §5, while in §6 we construct compact models incorporating a SUSY breaking sector and the MSSM-like theory of [28] (presumably, similar constructions could be given incorporating the semi-realistic models of [29,30] or

other attempts at constructing the SM on intersecting branes or branes at singularities). In a concluding section, we describe several promising directions for further research.

We should state clearly at the outset that our “semi-realistic” explicit constructions are not close to being fully realistic. However, it seems clear that systematic further work along these lines could produce increasingly realistic models. Similarly, the constructions even at this level of realism are rather complicated, and many issues beyond those which we discuss (related to both gauge theory model building and to moduli stabilization) could be explored in each toy model. We will explicitly point out our assumptions, and our justification for making these assumptions, at various points in the text.

2. Gauge Mediation versus Gravity Mediation

In gauge mediated models of SUSY breaking, SUSY is broken in a hidden sector with gauge group G_H . The SUSY breaking in the hidden sector leads to splittings for a vector-like set of messenger chiral multiplets $\tilde{\phi}_i, \phi_i$, which carry Standard Model gauge charges. Standard Model gauge interactions then lead to a one-loop gaugino mass and a two-loop mass² for the other sparticles. In many models, the ϕ_i are neutral under G_H but are coupled to the SUSY breaking by additional gauge singlets; in other models of “direct mediation,” the ϕ_i can be charged under G_H . Some classic references include [31,32].

The primary virtue of models of gauge mediation is that they solve the SUSY flavor problem, the problem of why the soft-breaking terms do not introduce new sources of flavor violation which violate present experimental bounds. The fact that gauge mediation generates e.g. universal squark masses is clear, because the only coupling of the squarks to the messengers occurs through (universal) gauge interactions. While a similar slogan might naively be applied to gravity mediation, in fact it has been understood that generic gravity mediated models do *not* enjoy such universality. Suppose, in such a scenario, X is the modulus whose F-term breaks SUSY, and Q_i denote generic SM fields with i running over generations. Then, couplings of the form

$$K = \int d^4\theta \frac{c_i}{M_P^2} X^\dagger X Q_i^\dagger Q_i + \dots \quad (2.1)$$

in the Kähler potential exist in generic models. In string constructions, they arise by integrating out massive fields to write down the 4d effective Lagrangian. These operators

occur with different $\mathcal{O}(1)$ coefficients c_i , and generally yield non-universal squark masses.² From an effective field theory perspective, this failure of universality is easily understood [33]: gravity mediation is sensitive to Planck-scale physics, and the physics of flavor (which is presumably determined by the geometry of the compactification manifold) is visible to the massive fields which are integrated out to yield (2.1).

In gauge mediated models, on the other hand, one begins with an effective superpotential

$$W = \tilde{\phi}_i X \phi_i + W_{MSSM} \quad (2.2)$$

where X is a spurion superfield whose vev

$$X = M + \theta^2 F \quad (2.3)$$

both gives the messengers a mass, and breaks supersymmetry. One can think of (2.2) as an effective theory that parameterizes the piece of hidden sector physics relevant to the Standard Model; only the ϕ_i are charged under $SU(3) \times SU(2) \times U(1)$, so one can “integrate out” the hidden sector, parameterizing its effects via (2.2), (2.3). A standard analysis [32] then yields the sparticle masses and A -terms in terms of F/M and the “messenger index” N (basically, the number of messenger fields). Very roughly, one finds squark and slepton masses $m_Q^2 \sim \alpha^2 F^2/M^2$ and gaugino masses $m_\lambda \sim \alpha F/M$; we shall describe the results in more detail for our particular toy models in later sections.

The messenger fields are charged under the Standard Model, and contribute to running of the gauge couplings above their mass M . Hence, in gauge mediated models, one finds a shifted value of the unified gauge coupling:

$$\delta\alpha_{GUT}^{-1} = -\frac{N}{2\pi} \ln \frac{M_{GUT}}{M} \quad (2.4)$$

where

$$N = \sum_i n_i \quad (2.5)$$

and n_i is twice the Dynkin index of the observable sector gauge representation \mathbf{r}_i of the i th messenger. One can determine the maximal value of N (consistent with weakly coupled unification) as follows.

² Exceptions exist: for instance, dilaton domination in the weakly coupled heterotic string, can yield universal soft masses [9].

If one wants $\alpha F/M \sim \text{TeV}$, it follows one should take $F/M \sim 10 - 100 \text{ TeV}$. Then, for the highest F-term consistent with dominance of gauge mediation over gravity mediation $F \sim (10^{10} \text{ GeV})^2$, one would have $M \sim 10^{15} \text{ GeV}$. Perturbativity of gauge interactions requires

$$N \leq 150 / \ln \frac{M_{GUT}}{M} . \quad (2.6)$$

Hence, as the messenger scale and SUSY-breaking F-term increase, one is allowed a rather large number of messengers. However, phenomenological considerations (such as the desire to avoid a gravitino problem, in addition to the need to keep the relative significance of the gravity mediated contribution sufficiently small) generally favor F terms below 10^{10} GeV , and a correspondingly smaller number of messengers. For $M = 10^{10} \text{ GeV}$ one obtains a bound on N of about 10 (which would correspond to five $\mathbf{5} + \bar{\mathbf{5}}$ pairs if the SM is embedded in an $SU(5)$ GUT; for purposes of discussion we will always assume this) [32].

2.1. Strategy for embedding into string theory

Perhaps the most obvious place to try and construct a GUT model with supersymmetry breaking would be the strongly coupled heterotic string. Indeed, some of our toy models will have an explicit heterotic realization. However, we will also provide a dual F-theory description; in the F-theory formalism, the physics which controls moduli stabilization is better understood, since simple ingredients like NS and RR fluxes which generate potentials in type II strings, dualize to rather intractable ingredients in the heterotic theory.

It is well known that heterotic string compactifications on elliptically fibered Calabi-Yau n -folds, are dual to F-theory (type IIB) models on K3-fibered Calabi-Yau $(n+1)$ -folds. For $n = 1$, this was described in [34], while a detailed map for $n = 2$ was provided in [35]. For $n = 3$ the story is considerably more involved, but some important aspects involving nontrivial gauge bundles (relevant for constructing GUT models) were worked out in a series of papers by Friedman, Morgan and Witten [27]. It is important to note that many of the pseudo-realistic heterotic models constructed by the Penn group (see [36] and refs therein) involve the spectral cover construction on elliptic threefolds, and hence fall squarely into the class of constructions which admit dual F-theory descriptions. Because it will be easier to use the F-theory description to also make a model of gauge mediation in the later sections of this paper, we will mostly stick to the D-brane/F-theory language.

So, we will try to engineer a hidden SUSY breaking sector on a stack of D-branes in F-theory. The grand unified extension of the Standard Model or brane MSSM, will live

in different constructions on either a stack of D7-branes (dual to one of the heterotic E_8 walls, in some cases), or D3-branes at a singular point in the Calabi-Yau space. SUSY will be broken non-perturbatively at the dynamical scale of the hidden sector gauge theory, Λ_H . This can quite naturally be a scale which is parametrically low compared to M_P . The detailed geometries which accomplish this are discussed in later sections.

Transmission to the Standard Model

The dominant interactions which transmit SUSY breaking to the observable sector, will depend on the distance between the hidden and observable brane stacks. In the F-theory construction, there are complex structure moduli of the Calabi-Yau fourfold which control the brane positions. These can be interpreted as e.g. adjoints in the D7-brane gauge theory.³ These adjoints are generically stabilized (i.e. given a mass) by background flux in the F-theory description [37]. Depending on where in their moduli space the D7 adjoints are stabilized, the distance d between the SUSY breaking and observable sector brane stacks will vary. For flux choices which stabilize these stacks (i.e. the fourfold complex structure) in a regime where

$$\frac{d}{\alpha'} = M \ll M_s \tag{2.7}$$

the dominant interaction between the brane stacks is via zero modes of open strings stretching between them. In four dimensions, the open strings are described by chiral fields $\phi_i, \tilde{\phi}_i$ transforming in conjugate representations of the GUT gauge group. For example, if we assume an $SU(5)$ GUT coming from a stack of five D7-branes, $\phi_i, \tilde{\phi}_i$ transform in $\mathbf{5} \oplus \bar{\mathbf{5}}$ representation of $SU(5)$. The number of copies of the representations depends on the details of the hidden gauge group supported on the other D7-brane stack. In the effective four dimensional language, these fields are messengers of gauge mediated supersymmetry breaking. Hence, one obtains gauge mediation with messenger scale M equal to the mass of the stretched strings.

³ Note that on dualizing from the heterotic string to F-theory, the *chiral* spectrum of a gauge theory is guaranteed to stay the same, but extra vector-like representations can appear or disappear as one moves between the regimes of validity of the different descriptions. This explains why although GUT Higgses never appear in heterotic models with the standard level one realization of the E_8 current algebra, they can and often do appear in the dual F-theory picture. This means that more conventional GUT-breaking mechanisms may be invoked, as an alternative to the heterotic Wilson-line breaking mechanism.

To confirm the reasonableness of assuming flux stabilization in the parameter regime (2.7), one could do a simple statistical analysis along the lines of [38]. Existing results about similar mild tunes make it fairly clear that the regime (2.7) should be attainable for the phenomenologically relevant range of values of M , but a more detailed statistical analysis might be interesting.

If the D-branes have separation d such that

$$\frac{d}{\alpha'} \gg M_s, \tag{2.8}$$

supersymmetry breaking is mainly communicated via zero modes of the closed strings. This corresponds to gravity or moduli mediation. For $d/\alpha' \sim M_s$, the D-branes stacks have string scale separation. The four dimensional effective description breaks down since the D-brane stacks interact via the whole tower of excited string oscillators. The supersymmetry breaking from the hidden sector to our braneworld is “string mediated.” In this way, string theory unifies different mechanisms of supersymmetry breaking in a single string compactification. The mediation is primarily via gauge or gravitational interactions, depending on the distance between the brane stacks.

3. Noncalculable Models of Supersymmetry Breaking

The standard lore about SUSY breaking in the hidden sector of the heterotic string, involves the assumption that hidden E_8 gaugino condensation can be responsible for SUSY breaking. (More complicated models with “racetrack” potentials are also commonly discussed). However, as argued convincingly in [39], and as is clear from analysis of the relevant effective potentials after the tree-level no-scale structure is broken (see e.g. [40,41]), generically hidden E_8 gaugino condensation does not guarantee supersymmetry breaking. This is not terribly surprising, since in the flat space limit gaugino condensation in $\mathcal{N} = 1$ field theory does not break supersymmetry. Here, and in subsequent sections, we describe some examples where the hidden sector breaks supersymmetry even in the flat space limit, and the embedding into string theory will not (in any model where closed-string moduli are stabilized) relax the SUSY breaking F-term.⁴

⁴ In the paper [15], an anti-D3 brane in a warped background [42] is used to induce an exponentially small scale of supersymmetry breaking. While for many purposes such models may be quite similar to models incorporating dynamical supersymmetry breaking [43], the field theoretic description of the the present class of models is certainly more transparent. Of course, many other ways of accomplishing supersymmetry breaking and yielding a positive contribution to the potential in the presence of moduli stabilization have also been studied in detail by now.

3.1. Simple alternatives to the hidden E_8

We would like to choose a simple mechanism of SUSY breaking that is easily embedded into string theory. One of the lessons of [15,16] is that it is possible to fix the geometrical moduli of F-theory compactifications *supersymmetrically*. (As mentioned previously, the small W_0 assumed there can perhaps be thought of as the small constant W that will be needed after SUSY breaking, to cancel the cosmological constant). We will therefore assume that all Calabi-Yau moduli are fixed as in those papers with masses close to the string scale (before any further ingredients which can yield dS vacua are considered), and look for our SUSY breaking F-term elsewhere.⁵

Studies of dynamical SUSY breaking models in the mid 1980s yielded a particularly simple class of models, often called “non-calculable models.” The simplest examples are the $SU(5)$ gauge theory with one generation of $\bar{\mathbf{5}} \oplus \mathbf{10}$, and the $SO(10)$ gauge theory with one generation of $\mathbf{16}$. These models were found by the following logic. Consider an $\mathcal{N} = 1$ supersymmetric gauge theory without flat directions at tree level, and with sufficiently little matter content that it is expected to undergo confinement. ’t Hooft anomaly matching for all global symmetries of the theory can then constrain the possible low-energy pion Lagrangians which describe the theory in the IR. In some cases, the possible spectra that saturate the anomalies (under the assumption that the symmetries are unbroken) are so contrived-looking that it is implausible that the theory generates such composites; in such a case, one must postulate that the global symmetries are broken in the IR. This means that there must be Goldstone bosons. But unbroken supersymmetry would require that they be complexified into full chiral multiplets whose scalar vevs are unconstrained. The existence of suitable partners to complexify the Goldstones is very implausible in a theory without tree-level flat directions. Hence, the theory must spontaneously break supersymmetry.

Following this logic, both the $SO(10)$ gauge theory with a single $\mathbf{16}$ [25], and the $SU(5)$ gauge theory with a single $\bar{\mathbf{5}} \oplus \mathbf{10}$ [24], should be expected to exhibit dynamical supersymmetry breaking. Further evidence that these theories do indeed dynamically break supersymmetry was provided in [44] by adding vector-like matter multiplets and computing the Witten index and vacuum structure. By now the case that SUSY breaking indeed occurs is quite compelling. Many other such models exist, but these are the simplest cases and we will be satisfied to use them for our toy constructions.

⁵ More precisely, the complex and dilaton moduli have masses which scale as $\frac{\alpha'}{R^3}$; the Kähler moduli may have masses which are significantly smaller and W_0 dependent.

Comments on the possibility of additional matter

We should point out here that in general, geometric engineering of the non-calculable models could also yield additional nonchiral flavors in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ of $SU(5)$, or the $\mathbf{10}$ of $SO(10)$. Since these representations are nonchiral, their appearance in the spectrum of fields with mass $< M_{string}$ will depend on the full details of moduli stabilization. We note that even in the presence of such fields, the supersymmetry breaking minimum persists [44]. Because they are vector-like, one generically expects worldsheet instantons (in the heterotic description) and/or fluxes to lift their masses to a relatively high scale. However, their presence below the GUT scale can change the RG running of the hidden sector gauge coupling, and hence the scale at which supersymmetry breaks.

3.2. Energy scales

The supersymmetry in the chiral $SU(5)$ or $SO(10)$ gauge theories is broken dynamically with F-term of the order of $F \sim (\Lambda_H/4\pi)^2$, where Λ_H is the strong coupling scale of the hidden gauge theory [45].

If the distance between the D7-brane stacks is much larger than the string length ℓ_s , supersymmetry is gravity mediated. The masses of the MSSM sparticles are of order

$$m_s \sim \frac{F}{M_P} \sim \frac{\Lambda_H^2}{(4\pi)^2 M_P}. \quad (3.1)$$

For $d < \ell_s$, supersymmetry breaking is predominantly mediated by open strings connecting the D-brane stacks. This is a stringy description of gauge mediation, hence the sparticle masses are $m_s \sim \alpha(F/M)$, where M is the mass of the open strings acting as messengers of supersymmetry breaking.

Hence, from the knowledge of gauge coupling α_H at the string scale and the matter content of the hidden gauge group, we can estimate the scale λ_H at which the hidden gauge group gets strongly coupled and breaks supersymmetry, and the masses of the sparticles. At the string scale, the gauge couplings of the GUT and of the hidden gauge group are approximately equal. One sees this most easily in the heterotic description of the models, where this comes from the equality of the gauge couplings of the two E_8 's of the $E_8 \times E_8$ heterotic string theory. In the following discussion, we assume $\alpha_H = \alpha_{GUT} \approx 1/25$ at the string scale $M_s = M_{GUT} = 2 \times 10^{16}$ GeV. At the end we discuss to what extent this approximation is valid in the F-theory compactification constructed in the following section.

The one-loop RG evolution of $\alpha = g^2/4\pi$ is governed by

$$\mu \frac{d\alpha^{-1}}{d\mu} = \frac{b}{2\pi}, \quad (3.2)$$

where b in a supersymmetric gauge theory is $b = 3C_2(G) - C(R)$. Evaluating this for $SU(5)$ with one generation of $\bar{\mathbf{5}} \oplus \mathbf{10}$ gives⁶ $b = 13$. Hence the gauge theory becomes strongly coupled at the scale

$$\Lambda_H = M_{GUT} \exp(-2\pi/13\alpha_{GUT}) \approx 10^{11} \text{ GeV}. \quad (3.3)$$

We estimate the F-term using naive dimensional analysis [45] to be $F \approx (\frac{\Lambda_H}{4\pi})^2 \approx (10^{10} \text{ GeV})^2$.

For gauge mediation $F \sim (10^{10} \text{ GeV})^2$ is at the upper end of the range values for the F-term for which gauge mediation dominates gravity mediation. This leads to sparticle masses of $\sim \text{TeV}$ if the mass of the messenger particles is $M \approx 10^{15} \text{ GeV}$. This is the largest M for which the flavor problem can plausibly be solved by gauge mediation (see eqn. (2.44) of [32]). The messengers are open strings stretched between the two stacks of D7-branes. If the branes are separated by distance l , the messenger mass is $M = 2\pi l/g_s \ell_s^2$. Assuming $\ell_s = M_s^{-1} \approx 2 \times 10^{16} \text{ GeV}$, the messenger mass of $M = 10^{15} \text{ GeV}$ corresponds to interbrane separation of $l = 10^{-2} g_s \ell_s$.⁷

In gravity mediation, this would lead to sparticle masses of the order of $m_s \sim \frac{F}{M_P} \sim 10^{-1} \text{ TeV}$ or less, which is a bit low (as it should be, for the gauge mediated contribution to dominate the mass squared matrix for the squarks). For gravity mediation, we would prefer $F \sim (10^{11} \text{ GeV})^2$. This actually might be the case because the hidden gauge group might be more strongly coupled than the GUT gauge group at the string scale, as we discuss later.

As we said, $M \sim 10^{15} \text{ GeV}$ is at the high end of the allowed range for gauge mediation. A scenario which would lead to lower M with essentially the same physics, is to imagine that there are some additional $\mathbf{5} \oplus \bar{\mathbf{5}}$ pairs which are present in the sigma model tree-level spectrum and receive a mass only from worldsheet instantons. Such extra vector-like states are quite common in heterotic constructions; their presence or absence depends on the full

⁶ Here we use that $C_2(SU(5)) = 5$, $C_2(\bar{\mathbf{5}}) = 1/2$ and $C_2(\mathbf{10}) = 3/2$.

⁷ We have neglected the contribution of the messengers to the running of the hidden sector gauge coupling in computing Λ_H , since they decouple almost immediately given the high value of M . Including their effects would lower Λ_H slightly.

details of the point in moduli space chosen to study a given model. The presence of n extra pairs would reduce b to $13 - n$ for some portion of the RG running starting from the GUT scale, and hence decrease Λ_H .

The $SO(10)$ gauge theory with one chiral matter multiplet in the **16** has⁸ $b = 22$, so the gauge theory gets strongly coupled at a higher scale $\Lambda_{SO(10)} = M_{GUT} \exp(-2\pi/22\alpha_{GUT}) \approx 10^{13}$ GeV. The masses of the sparticles due to gravity mediation of supersymmetry alone are $\sim 10^3$ TeV which is rather high. The additional contribution to sparticle masses from gauge mediation would make the masses even larger. To make even a realistic model of gravity mediation with this hidden sector, one would need to invoke the assumption of some extra **10s** of $SO(10)$ with intermediate scale masses, to lower Λ_H .

In the above discussion, we neglected that the hidden gauge group may actually be more strongly coupled than the GUT gauge group at the string scale. In the heterotic description, which at finite string coupling can be viewed as a compactification of M-theory on a Calabi-Yau manifold times an interval, this is a consequence of embedding the GUT symmetry into the larger of the two end of the world-branes. The other world-brane has smaller volume because of the warping along the interval. But the gauge couplings are inversely proportional to the volumes of the world-branes $\alpha = \ell_{11}^6/V$, where $\ell_{11} = (4\pi\kappa_{11}^2)^{1/9}$ is the eleven-dimensional Planck length. Hence, the hidden gauge group is more strongly coupled at the string scale by a factor of V_{GUT}/V_H .

In the F-theory picture, the gauge symmetry comes from two stacks of D7-branes. The inverse gauge couplings of the observable and hidden sectors, are controlled by the volumes of the two divisors that the stacks wrap, which are two different sections of the \mathbb{P}^1 fibration of the base of the Calabi-Yau fourfold. The homology classes of the divisors cut out by the two sections differ by the class of a \mathbb{P}^1 fibration over some curve η in the base. Hence, the volumes of the two sections differ by an amount that grows with the size of the \mathbb{P}^1 fibre. In our example, the GUT symmetry comes from the divisor with larger volume.

One of the implications of this is that we can increase the strength of supersymmetry breaking effects by stabilizing the \mathbb{P}^1 fibres at larger volume. With larger \mathbb{P}^1 fibres, the hidden gauge group comes from a divisor with a smaller volume compared to the volume of the GUT divisor, hence it is more strongly coupled and dynamically breaks supersymmetry at a higher scale. In the one generation $SU(5)$ model, this effect could

⁸ This follows from the $SO(10)$ values of $C_2(SO(10)) = 8$, $C_2(\mathbf{16}) = 2$.

increase the supersymmetry breaking F-terms to the intermediate scale 10^{11} GeV preferred in the gravity mediated SUSY breaking solution to the hierarchy problem. The $SO(10)$ model of a hidden sector leads to a somewhat high scale of SUSY breaking if $\alpha_H = \alpha_{GUT}$ at the string scale. Increasing the volume of the \mathbb{P}^1 would only exacerbate this problem.

For gauge mediation, both the gravitino problem and the dominance over gravity mediation prefer a lower scale of supersymmetry breaking with $F \sim (10^{10}\text{GeV})^2$ being at the upper end of the allowed values. So while these models provide simple toy models where one can compare the strength of gauge and gravity mediation, with gauge mediation marginally winning in one case (and winning clearly if there are extra vector-like representations of the hidden sector gauge group at intermediate masses), a clear next step would be to identify analogous hidden sectors which are easy to engineer and give rise to much smaller Λ_H .

3.3. Some stringy embeddings

Following the above discussion, we will construct heterotic models with a three generation $SU(5)$ GUT sector and a one generation $SU(5)$ hidden sector.

Consider heterotic $E_8 \times E_8$ compactifications on smooth Weierstrass models $\pi : Y \rightarrow S$. The base S is a del Pezzo surface. We would like to construct a background bundle $V_1 \times V_2$ on Y so that both V_1 and V_2 are stable $SU(5)$ bundles and the following conditions are satisfied

I) V_1, V_2 yield one and respectively three generations i.e.

$$\text{ch}_3(V_1) = \pm 1, \quad \text{ch}_3(V_2) = \pm 3. \quad (3.4)$$

II) Anomaly cancellation:

$$c_2(V_1) + c_2(V_2) + \Lambda = c_2(Y) \quad (3.5)$$

where Λ is an effective curve on Y which supports background heterotic fivebranes [46,47,48]. Motivated by Kähler moduli stabilization, we would like to impose an additional condition on the fivebrane class Λ . Suppose Λ has a decomposition

$$\Lambda = \Xi + N_5 E$$

where Ξ is a horizontal curve on Y contained in the image of the canonical section $\sigma : S \rightarrow Y$, E is the elliptic fiber and N_5 is a non-negative integer. Note that Ξ can be naturally identified to an effective curve on S . Then we impose

III) The connected components of Ξ are smooth irreducible (-1) curves on S .

We will discuss the relation between this condition and Kähler moduli stabilization in subsection §3.10.

Ideally one would like to construct both V_1, V_2 in terms of spectral data [27,49,50], but this approach may be too restrictive, given the constraints *(I) – (III)* above. In fact V_2 will be indeed constructed in terms of spectral data, but not V_1 , which will be constructed by extensions. Let us first review some aspects of the spectral cover construction.

3.4. Spectral covers

There is a one-to-one correspondence between bundles $V \rightarrow Y$, flat and semistable along the elliptic fibers, and spectral data $(\mathcal{C}, \mathcal{N})$. \mathcal{C} is an effective divisor on Y flat over S , and \mathcal{N} is a torsion free rank one sheaf on \mathcal{C} [27,49,50]. In order to construct the bundle V in terms of spectral data $(\mathcal{C}, \mathcal{N})$, first take the fiber-product

$$T = \mathcal{C} \times_S Y \subset Y \times_S Y.$$

Let $p_{\mathcal{C}}, p_Y$ denote the canonical projections onto the two factors and $\pi_T : T \rightarrow S$ denote the projection to S . Note that we have three natural divisor classes $\Delta, \sigma_1, \sigma_2$ obtained by restriction from $Y \times_S Y$. Δ is the restriction of the diagonal, and σ_1, σ_2 are restrictions of the canonical sections $\sigma_1 = \sigma \times_S Y, \sigma_2 = Y \times_S \sigma$. The relative Poincaré line bundle on $Y \times_S Y$ is defined by

$$\mathcal{P} = \mathcal{O}(\Delta - \sigma_1 - \sigma_2) \otimes \pi_S^* K_S \quad (3.6)$$

where $\pi_S : Y \times_S Y \rightarrow S$ denotes the natural projection. Then V is given by the push-forward

$$V = p_{Y*} (p_{\mathcal{C}}^* \mathcal{N} \otimes \mathcal{P}|_T) \quad (3.7)$$

The topological invariants of V are determined by the linear equivalence class of \mathcal{C} and the Chern class of \mathcal{N} . In particular, if the class of \mathcal{C} is of the form

$$\mathcal{C} = n\sigma + \pi^* \eta \quad (3.8)$$

with η a divisor class on S , V will have rank n .

Suppose \mathcal{C} is irreducible and meets the section σ along an effective curve $F \subset S$. Then we have $\mathcal{O}_T(\sigma_1) \simeq p_{\mathcal{C}}^* \mathcal{O}_{\mathcal{C}}(F)$. Moreover, the only generic line bundles on \mathcal{C} are $\mathcal{O}_{\mathcal{C}}(F)$ and line bundles pulled back from S . Therefore a generic bundle V will be of the form

$$V = p_{Y*} (\mathcal{O}_T(\Delta - \sigma_2) \otimes p_{\mathcal{C}}^* (\mathcal{O}_{\mathcal{C}}(-aF)) \otimes \pi_T^* \mathcal{M}) \quad (3.9)$$

for an integer a , and a line bundle \mathcal{M} on S . Following [49], we will denote by $V_{n,a}[\mathcal{M}]$ a rank n bundle of the form (3.9) and by $V_{n,a}$ a bundle of the form (3.9) with $\mathcal{M} \simeq \mathcal{O}_S$. Note that $V_{n,a}[\mathcal{M}] \simeq V_{n,a} \otimes \pi^* \mathcal{M}$.

The Chern character of a bundle of the form $V_{n,a}[\mathcal{M}]$ is given by [49] (Thm. 5.10)

$$\text{ch}(V_{n,a}[\mathcal{M}]) = \left[e^{-\eta} \left(\frac{1 - e^{(a+n)c}}{1 - e^c} \right) - \frac{1 - e^{ac}}{1 - e^c} + e^{-\sigma}(1 - e^{-\eta}) \right] \cdot e^{c_1(\mathcal{M})} \quad (3.10)$$

where $c = \pi^* c_1(S)$. In particular we have

$$\begin{aligned} \text{ch}_1(V_{n,a}[\mathcal{M}]) &= -(n+a-1)\eta + \left[an + \frac{n^2 - n}{2} \right] c_1(S) + nc_1(\mathcal{M}) \\ \text{ch}_3(V_{n,a}[\mathcal{M}]) &= \frac{1}{2}(\sigma^2\eta + \sigma\eta^2) - \sigma\eta c_1(\mathcal{M}). \end{aligned} \quad (3.11)$$

For future reference, note that for a bundle of the form $V_{n,a}[\mathcal{M}]$, the spectral line bundle \mathcal{N} in (3.7) is of the form

$$\begin{aligned} \mathcal{N} &\simeq \mathcal{O}_{\mathcal{C}}((1-a)F) \otimes \pi^*(K_S^{-1} \otimes \mathcal{M}) \\ &\simeq (\mathcal{O}_Y((1-a)\sigma) \otimes \pi^*(K_S^{-1} \otimes \mathcal{M}))|_{\mathcal{C}}. \end{aligned} \quad (3.12)$$

According to [49] (Thm.7.1) if \mathcal{C} is irreducible it follows that V is stable with respect to a polarization of the form

$$J = \epsilon J_0 + \pi^* H \quad (3.13)$$

where J_0 is a fixed ample class on Y , H is an ample class on S , and ϵ is a sufficiently small positive number. Sufficient criteria for the spectral cover to be irreducible have been formulated in [47,51,46,48,52]. They show that \mathcal{C} is irreducible if

- i*) $|\eta|$ is a base point free linear system on S , and
- ii*) $\eta - nc_1(S)$ can be represented by an effective curve on S .

By Bertini's theorem, the first criterion is satisfied if η is ample on S , which in turn amounts to the numerical condition

- i'*) $\eta \cdot \zeta \geq 0$ for all generators ζ of the Mori cone of S .

We will make heavy use of these criteria later in this section. In order to construct the hidden sector bundle we have to invoke a generalization of the above formalism dealing with reducible spectral covers.

3.5. Reducible spectral covers and extensions

Let assume now that \mathcal{C} is a reducible spectral cover with two smooth reduced irreducible components

$$\mathcal{C} = \mathcal{C}' + \mathcal{C}'' \quad (3.14)$$

intersecting along a smooth irreducible curve $C = \mathcal{C}' \cap \mathcal{C}''$. The two components are equipped with spectral line bundles $\mathcal{N}', \mathcal{N}''$ so that the restrictions $\mathcal{N}'|_C, \mathcal{N}''|_C$ are isomorphic. Note that if we choose an isomorphism $\phi : \mathcal{N}'|_C \rightarrow \mathcal{N}''|_C$, the data $(\mathcal{N}', \mathcal{N}'', \phi)$ determines a line bundle \mathcal{N} on the reducible spectral cover \mathcal{C} .

Following [53] (§5.1) to any such spectral data $(\mathcal{C}, \mathcal{N})$ we can associate a set (D, Q) of gluing data. D is a vertical divisor on Y constructed by projecting \mathcal{C} to the base S , and then taking the inverse image, $D = \pi^{-1}(\pi(\mathcal{C}))$. Let \mathcal{D} denote the intersection of the fiber products $\mathcal{C}' \times_S Y$ and $\mathcal{C}'' \times_S Y$ in $Y \times_S Y$. Note that we have natural projections $p_{\mathcal{D}} : \mathcal{D} \rightarrow D$ and $\pi_{\mathcal{D}} : \mathcal{D} \rightarrow C$. We can use $\pi_{\mathcal{D}}$ to pull back the restriction of \mathcal{N}' (or equivalently \mathcal{N}'') to \mathcal{D} , obtaining a line bundle on \mathcal{D} which will be denoted by $\pi_{\mathcal{D}}^* \mathcal{N}$. Then Q is defined by the following push-forward formula

$$Q = p_{\mathcal{D}*} (\pi_{\mathcal{D}}^* \mathcal{N} \otimes \mathcal{P}|_{\mathcal{D}}). \quad (3.15)$$

According to [53] (§5.1) the bundle $V \rightarrow Y$ corresponding to the spectral data $(\mathcal{C}, \mathcal{N})$ is given by the following elementary modification

$$0 \rightarrow V \rightarrow V' \oplus V'' \rightarrow Q \rightarrow 0 \quad (3.16)$$

where Q is regarded as a torsion sheaf on Y supported on D .

We will be interested in a special case of this construction when $\mathcal{C}' = \sigma$ with multiplicity 1, and \mathcal{C}'' is a smooth irreducible component which intersects σ along a smooth curve C . Note that in this case, C is identical to the curve F for \mathcal{C}'' , which was introduced below (3.8). Moreover, we take both \mathcal{N}' and \mathcal{N}'' to be restrictions of a line bundle \mathcal{N} on Y of the form (3.12). In particular, $\mathcal{N}' \simeq K_S^{(-a)} \otimes \mathcal{M}$. This case was treated in detail in [49] (§5.7). The bundle V' is isomorphic to $\pi^*(K_S^{-a} \otimes \mathcal{M})$ and the elementary modification (3.16) reduces to

$$0 \rightarrow V \rightarrow \pi^*(K_S^{-a} \otimes \mathcal{M}) \oplus V'' \rightarrow \pi^*(K_S^{-a} \otimes \mathcal{M})|_D \rightarrow 0. \quad (3.17)$$

As explained in [53] (§5.1), the elementary modification (3.17) has moduli parameterized by the linear space

$$\mathrm{Ext}^1(V'(-D), V'') \oplus \mathrm{Ext}^1(V'', V'(-D)) \quad (3.18)$$

up to \mathbb{C}^* identifications. The first direct summand in (3.18) parameterizes extensions of the form

$$0 \rightarrow V'(-D) \rightarrow V \rightarrow V'' \rightarrow 0 \quad (3.19)$$

while the second direct summand in (3.18) parameterizes extensions of the form

$$0 \rightarrow V'' \rightarrow V \rightarrow V'(-D) \rightarrow 0. \quad (3.20)$$

In particular this shows that the bundle V is a deformation of the direct sum $V'(-D) \oplus V''$. For future reference note that

$$V'(-D) \simeq \pi^*(K_S^{-a} \otimes \mathcal{M} \otimes \mathcal{O}_S(-F)). \quad (3.21)$$

Summarizing this discussion, it follows that we can construct a more general class of bundles on Y associated to reducible spectral covers by taking extensions. In order for this construction to be useful in physical applications, we would like to have a stability criterion for bundles of this form. Fortunately, such a criterion has been formulated in [47,46,48,52]. Given an extension of the form

$$0 \rightarrow E' \rightarrow E \rightarrow E'' \rightarrow 0 \quad (3.22)$$

where E', E'' are stable bundles corresponding to irreducible spectral covers, E is stable if

- a) the extension (3.22) is not split, and
- b) $\mu_J(E') < \mu_J(E)$, where the slope μ_J of a bundle E is defined by

$$\mu_J(E) = \frac{c_1(E) \cdot J^2}{\mathrm{rk}(E)}.$$

Here by stability we mean stability with respect to a polarization of the form (3.13) with sufficiently small $\epsilon > 0$.

This is all the formal machinery we will need, so we can turn to the explicit construction of bundles.

3.6. The GUT bundle

Let us first construct the GUT $SU(5)$ bundle V_2 . The spectral cover construction suffices in this case. From now on we take the base S to be the del Pezzo surface dP_8 . Pick a spectral cover \mathcal{C} in the linear system

$$|5\sigma - 6\pi^*K_S|$$

which implies $\eta = 6c_1(S)$. Take V_2 to be a bundle of the form $V_{5,1}[K_S^{-3}]$. Then, using formulas (3.11) a straightforward computation shows that

$$\text{ch}_1(V_2) = 0, \quad \text{ch}_3(V_2) = -3 \quad (3.23)$$

in agreement with (3.4). Note that the stability criteria (i'), (ii) formulated below (3.13) are automatically satisfied because $\eta = 6c_1(S)$ is very ample on S . Therefore V_2 is stable and has three generations.

3.7. The hidden sector bundle

In this case we have not been able to find an irreducible spectral cover construction satisfying conditions (I) – (III). We will however construct an $SU(5)$ bundle V_1 with the required properties using a reducible spectral cover and extensions as explained in §3.5.

Let us consider a reducible spectral cover of the form (3.14) with $\mathcal{C}' = \sigma$ and \mathcal{C}'' a smooth irreducible divisor in the linear system

$$|4\sigma - \pi^*(6K_S + \Gamma)|$$

where Γ is a smooth irreducible (-1) curve on S . For example we can take Γ to be any generator of the Mori cone of S . Recall [36] that the Mori cone of $S = dP_8$ is generated by the 240 (-1) curve classes

$$\begin{aligned} &e_i, \quad h - e_i - e_j, \quad 2h - e_i - e_j - e_k - e_l - e_m, \\ &3h - 2e_i - e_j - e_k - e_l - e_m - e_n - E_o, \\ &4h - 2(e_i + e_j + e_k) - \sum_{s=1}^5 e_{m_s}, \\ &5h - 2 \sum_{s=1}^6 e_{m_s} - e_k - e_l, \quad 6h - 3e_i - 2 \sum_{s=1}^7 e_{m_s}, \end{aligned} \quad (3.24)$$

where h is the hyperplane class and e_i , $i = 1, \dots, 8$ are the exceptional curve classes. The indices i, j, k, l, m, n, o, m_s in (3.24) are pairwise distinct and take values from 1 to 8. We will denote by $\eta'' = -6K_S - \Gamma = 6c_1(S) - \Gamma$.

One can check that that \mathcal{C}'' is indeed irreducible using the criteria (i'), (ii) below equation (3.13). We have to check that $\eta'' \cdot \zeta \geq 0$ for any of the generators ζ listed in (3.24). This follows by direct computations. Moreover, if Γ is any of the Mori cone generators, one can check that the class $\eta'' - 4c_1(S) = 2c_1(S) - \Gamma$ is again a generator. Therefore $\eta'' - 4c_1(S) = F$ is an effective (-1) curve class on S .

We pick the spectral line bundles $\mathcal{N}', \mathcal{N}''$ to be the restrictions of a line bundle of the form

$$\mathcal{N} = \mathcal{O}_Y((1-a)\sigma) \otimes \pi^*(K_S^{-1} \otimes \mathcal{M}) \quad (3.25)$$

to \mathcal{C}' and respectively \mathcal{C}'' . Then we find

$$\mathcal{N}' = K_S^{-a} \otimes \mathcal{M}, \quad \mathcal{N}'' = \mathcal{O}_{\mathcal{C}''}((1-a)F) \otimes \pi^*(K_S^{-1} \otimes \mathcal{M}). \quad (3.26)$$

The bundles V', V'' determined by the spectral data $(\mathcal{C}', \mathcal{N}')$, $(\mathcal{C}'', \mathcal{N}'')$ are

$$V' \simeq \pi^*(K_S^{-a} \otimes \mathcal{M}), \quad V'' \simeq V_{4,a}[\mathcal{M}] \quad (3.27)$$

where the bundles $V_{n,a}[\mathcal{M}]$ have been defined below equation (3.9). Note that both V', V'' are stable with respect to a polarization of the form (3.13) since $\mathcal{C}', \mathcal{C}''$ are irreducible.

The rank 5 bundle V associated to the reducible spectral data $(\mathcal{C}, \mathcal{N})$ is determined by an elementary modification of the form (3.17). Let us first compute the Chern classes of $\text{ch}_1(V), \text{ch}_3(V)$ using formulas (3.11). For the purpose of this computation, we may assume that V_5 is a direct sum $V'(-D) \oplus V''$ since the Chern classes do not change under deformations. Then we obtain

$$\begin{aligned} \text{ch}_1(V) &= \pi^* [(5a+6)c_1(S) - (a+3)\eta'' + 5c_1(\mathcal{M}) - F] \\ \text{ch}_3(V) &= \frac{1}{2}\eta''(\eta'' - c_1(S) - 2c_1(\mathcal{M})), \end{aligned} \quad (3.28)$$

where we have used the isomorphism $\mathcal{O}_Y(-D) \simeq \pi^*\mathcal{O}_S(-F)$. Substituting $\eta'' = 6c_1(S) - \Gamma$ and $F = 2c_1(S) - \Gamma$ in (3.28), we obtain

$$\begin{aligned} \text{ch}_1(V) &= 5c_1(\mathcal{M}) - (a+14)c_1(S) + (a+4)\Gamma \\ \text{ch}_3(V) &= \frac{1}{2}(18 - 12c_1(S) \cdot c_1(\mathcal{M}) + 2\Gamma \cdot c_1(\mathcal{M})). \end{aligned} \quad (3.29)$$

Note that choosing

$$a = -4, \quad \mathcal{M} \simeq K_S^{-2}, \quad (3.30)$$

we obtain

$$\text{ch}_1(V) = 0, \quad \text{ch}_3(V) = -1 \quad (3.31)$$

in agreement with the condition (I).

Next we have to check stability using criteria (a), (b) in the previous subsection. Let us first compute the extension moduli (3.18). We start with

$$\text{Ext}^1(V'', V'(-D)) = H^1((V'')^\vee \otimes V'(-D)).$$

This cohomology group can be computed using the Leray spectral sequence

$$H^p(S, R^q \pi_*((V'')^\vee \otimes V'(-D))) \Rightarrow H^{p+q}((V'')^\vee \otimes V'(-D)). \quad (3.32)$$

Since $V'(-D)$ is pulled back from S according to equation (3.27), the left hand side of (3.32) can be simplified to

$$H^p(S, R^q \pi_*((V'')^\vee \otimes K_S^{-a} \otimes \mathcal{M} \otimes \mathcal{O}_S(-F))). \quad (3.33)$$

Note that only terms with $(p, q) = (0, 1), (1, 0)$ can occur in the computation of H^1 . The direct images $R^q \pi_*((V'')^\vee)$ for a stable spectral cover bundle have been computed for example in [54] (§2.1) We have

$$R^0 \pi_*((V'')^\vee) = 0, \quad R^1 \pi_*((V'')^\vee) \simeq K_F \otimes \mathcal{N}^{-1}|_F \otimes K_S^{-1}|_F. \quad (3.34)$$

Note that $R^1 \pi_*((V'')^\vee)$ is a torsion sheaf on S supported on F . Therefore we are left with

$$H^1((V'')^\vee \otimes V'(-D)) \simeq H^0(F, K_F \otimes \mathcal{N}^{-1}|_F \otimes K_S^{-1-a}|_F \otimes \mathcal{M}|_F \otimes \mathcal{O}_F(-F)). \quad (3.35)$$

Equation (3.25) implies

$$\mathcal{N}^{-1}|_F \simeq K_S^a|_F \otimes \mathcal{M}^{-1}|_F.$$

Substituting this relation in (3.35) we obtain

$$H^1((V'')^\vee \otimes V'(-D)) \simeq H^0(F, K_F \otimes K_S^{-1}|_F \otimes \mathcal{O}_F(-F)). \quad (3.36)$$

Since $F \simeq \mathbb{P}^1$ is a smooth (-1) rational curve on S we have

$$K_S^{-1}|_F \simeq \mathcal{O}_{\mathbb{P}^1}(1), \quad \mathcal{O}_F(-F) \simeq \mathcal{O}_{\mathbb{P}^1}(1), \quad K_F \simeq \mathcal{O}_{\mathbb{P}^1}(-2).$$

Therefore (3.36) reduces to

$$H^1((V'')^\vee \otimes V'(-D)) \simeq H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}) = \mathbb{C}. \quad (3.37)$$

This shows that up to isomorphism we have a unique nontrivial extension of the form

$$0 \rightarrow V'(-D) \rightarrow V \rightarrow V'' \rightarrow 0. \quad (3.38)$$

One can similarly compute

$$\text{Ext}^1(V'(-D), V'') \simeq H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(-2)) = 0,$$

therefore there are no nontrivial extensions of the form

$$0 \rightarrow V'' \rightarrow V \rightarrow V'(-D) \rightarrow 0.$$

In conclusion, taking into account (3.30), we are left with a unique bundle V_2 defined by the unique nontrivial extension

$$0 \rightarrow \pi^*(K_S^2 \otimes \mathcal{O}_S(-F)) \rightarrow V_2 \rightarrow V_{4,-4}[K_S^2] \rightarrow 0. \quad (3.39)$$

According to criterion (b) stated below (3.22), in order to show that V is stable, it suffices to check that

$$\mu_J(\pi^*(K_S^2 \otimes \mathcal{O}_S(-F))) < \mu_J(V).$$

Since $\text{ch}_1(V_2) = 0$, we have $\mu_J(V_2) = 0$. For a polarization of the form (3.13) we have

$$\mu_J(\pi^*(K_S^2 \otimes \mathcal{O}_S(-F))) = \pi^*(-4c_1(S) + \Gamma) \cdot (\epsilon^2 J_0 + 2\epsilon\pi^*H).$$

Note that $(4c_1(S) - \Gamma)$ is an effective curve class on S , and H is ample on S , therefore

$$(-4c_1(S) + \Gamma) \cdot H < 0.$$

Then we can satisfy the criterion by taking $\epsilon > 0$ sufficiently small.

3.8. Heterotic fivebranes

In order to complete the description of the model, we have to compute the heterotic fivebrane class Λ and check that it is effective. Using formula (3.10), it is straightforward to compute

$$\begin{aligned}\mathrm{ch}_2(V_1) &= \pi^* \mathrm{ch}_2(K_S^2 \otimes \mathcal{O}_S(-F)) + \mathrm{ch}_2(V_{4,-4}[K_S^2]) \\ &= 5E - \sigma\pi^*(6c_1(S) - \Gamma), \\ \mathrm{ch}_2(V_2) &= 5E - 6\sigma\pi^*c_1(S),\end{aligned}$$

where E is the class of the elliptic fiber on Y . Therefore we obtain

$$c_2(V_1) + c_2(V_2) = \sigma\pi^*(12c_1(S) - \Gamma) - 10E.$$

The second Chern class of Y is

$$\begin{aligned}c_2(Y) &= 12\sigma\pi^*c_1(S) + (c_2(S) + 11c_1(S)^2)E \\ &= 12\sigma\pi^*c_1(S) + 22E.\end{aligned}$$

Therefore equation (3.5) yields

$$\Lambda = \sigma\pi^*\Gamma + 32E, \tag{3.40}$$

which is effective. Note that in addition to vertical fivebranes wrapping the elliptic fiber, we have a horizontal fivebrane wrapping the (-1) curve Γ in the base.

3.9. F-theory interpretation

We conclude this section with a brief discussion of the F-theory dual models. Let us first recall some aspects of heterotic F-theory duality following [55,35,56] and [27] (§6.1). Suppose we have a heterotic model given by two stable bundles V_1, V_2 on a smooth Weierstrass model $\pi : Y \rightarrow S$ corresponding to smooth irreducible spectral covers $\mathcal{C}_1, \mathcal{C}_2$. First we assume that the anomaly cancellation condition (3.5) is satisfied in the absence of horizontal fivebranes, that is the class Ξ introduced below (3.5) vanishes. This means that the two classes η_1, η_2 associated to the spectral covers satisfy

$$\eta_1 + \eta_2 = 12c_1(S). \tag{3.41}$$

Let us write

$$\eta_1 = 6c_1(S) - t, \quad \eta_2 = 6c_1(S) + t, \tag{3.42}$$

where t is a divisor class on S . Let \mathcal{T} be a line bundle on S with $c_1(\mathcal{T}) = -t$.

For our purposes it suffices to consider take the structure groups of V_1, V_2 to be $SU(n_1)$ and respectively $SU(n_2)$. The spectral cover of an $SU(n)$ bundle is determined [27] by n sections a_k of the line bundles

$$K_S^k \otimes \mathcal{O}_S(\eta) \simeq K_S^{-6+k} \otimes \mathcal{T}, \quad k = 0, 2, \dots, n. \quad (3.43)$$

The dual F-theory model is an elliptic fibration $X \rightarrow P$ with a section over a base P , where P is a \mathbb{P}^1 bundle over S . More precisely, P is the projectivization of the rank two bundle of the form $\mathcal{O}_S \oplus \mathcal{T}$. Note that P has two canonical sections S_0, S_∞ with normal bundles

$$N_{S_0/P} \simeq \mathcal{T}, \quad N_{S_\infty/P} \simeq \mathcal{T}^{-1}. \quad (3.44)$$

The canonical class of P is

$$K_P = -S_0 - S_\infty + p^*(K_S)$$

where $p : P \rightarrow S$ denotes the canonical projection. The fourfold X is a Weierstrass model of the form

$$y^2 = x^3 - fx - g \quad (3.45)$$

where f, g are sections of K_P^{-4} and respectively K_P^{-6} . The discriminant of the elliptic fibration is given by

$$\delta = 4f^3 - 27g^2. \quad (3.46)$$

We will denote by capital letters F, G, Δ the zero divisors of f, g, δ on P .

The heterotic bundles V_1, V_2 correspond to ADE degenerations of the elliptic fibration $X \rightarrow P$ along the sections S_0, S_∞ . The discriminant Δ decomposes into three components

$$\Delta = \Delta_0 + \Delta_\infty + \Delta_n \quad (3.47)$$

where Δ_0, Δ_∞ are multiples of S_0, S_∞ and Δ_n denotes the nodal component.

The heterotic bundle moduli are encoded in the complex structure moduli of X . For concreteness, let us consider the first bundle V_1 which corresponds to an ADE degeneration along S_0 ; V_2 follows by analogy replacing S_0 with S_∞ and \mathcal{T} with \mathcal{T}^{-1} in the following. The complex structure of the elliptic fibration in a neighborhood of S_0 is captured by a hypersurface equation of the form

$$y^2 = x^3 - fx - g \quad (3.48)$$

in the total space of the rank three bundle $\mathcal{T} \oplus (\mathcal{T}^2 \otimes K_S^{-2}) \oplus (\mathcal{T}^3 \otimes K_S^{-3})$. Let s denote a linear coordinate on the total space of the line bundle $\mathcal{T} \rightarrow S$. Then f, g in (3.48) have expansions of the form

$$\begin{aligned} f &= f_0 + f_1 s + f_2 s^2 + \dots \\ g &= g_0 + g_1 s + g_2 s^2 + \dots \end{aligned} \tag{3.49}$$

where f_i are sections of $K_S^{-4} \otimes \mathcal{T}^{4-i}$, $i = 0, \dots, 4$ and g_j are sections of $K_S^{-6} \otimes \mathcal{T}^{6-j}$, $j = 0, \dots, 6$. The duality map relates the nonzero sections f_i, g_j to the sections a_k of the line bundles (3.43) which determine the heterotic spectral cover.

This picture is valid as long as the spectral cover is irreducible and there are no horizontal heterotic fivebranes. Horizontal heterotic fivebranes correspond to blow-ups in the base of the F-theory elliptic fibration [57,58]. More specifically, suppose we have a single heterotic fivebrane wrapping a smooth curve Ξ contained in the section of the Weierstrass model Y . In F-theory, this is represented by performing a blow-up of the base P along a curve isomorphic to Ξ contained in a section \mathcal{S} of the \mathbb{P}^1 fibration $p : P \rightarrow S$. We will call this curve Ξ as well since the distinction will be clear from the context. This is the four-dimensional counterpart of the more familiar six-dimensional F-theory picture for small instantons developed in [55,59,60,61]. The vector bundle degenerations associated to heterotic fivebranes on Calabi-Yau threefolds have been studied in [62,63,51].

Let \tilde{P} denote the total space of the blow-up, and let $\tilde{\mathcal{S}}$ denote the proper transform of \mathcal{S} in \tilde{P} . Note that both \mathcal{S} and $\tilde{\mathcal{S}}$ are naturally isomorphic to S . Then the normal bundle of $\tilde{\mathcal{S}}$ in \tilde{P} is determined by

$$N_{\tilde{\mathcal{S}}/\tilde{P}} \simeq N_{\mathcal{S}/P} \otimes \mathcal{O}_S(-\Xi). \tag{3.50}$$

Let us now construct the F-theory dual of our model. The line bundle \mathcal{T} is isomorphic to \mathcal{O}_S in our case, and we have a horizontal heterotic fivebrane wrapping the (-1) curve Γ . Therefore the F-theory base is a blow-up of

$$P = \mathbb{P}^1 \times S$$

along a curve isomorphic to Γ contained in a section \mathcal{S} . According to equation (3.50), $N_{\tilde{\mathcal{S}}/\tilde{P}} \simeq \mathcal{O}_S(-\Gamma)$, hence $\tilde{\mathcal{S}}$ is rigid in \tilde{P} . We will take $\mathcal{S} = S_0$ in the following.

We have two $SU(5)$ bundles V_1, V_2 on Y . Recall that the bundle V_2 admits an irreducible spectral cover description with $\eta_2 = 6c_1(S)$. According to the duality map reviewed above, V_2 corresponds to an A_4 degeneration of the fourfold X along a section Σ of the

fibration $\tilde{P} \rightarrow S$ with trivial normal bundle. Such a section Σ is in fact the proper transform of any section of P over S distinct from S_0 . Note that Σ moves in a one dimensional linear system, and it can degenerate to a reducible divisor of the form $\tilde{S}_0 + E$, where E is the exceptional divisor of the blow-up map $\tilde{P} \rightarrow P$.

There is one more condition on the A_4 degeneration along Σ , namely it has to be split [35]. This means that the degeneration has no monodromy along curves in Σ . The conditions for an A_4 singularity to be split have been derived in [35]. The expansions (3.49) have are truncated to

$$\begin{aligned} f &= s^2 f_2 + \dots + s^4 f_4 \\ g &= s^3 g_3 + \dots + s^6 g_6 \end{aligned} \tag{3.51}$$

where f_i are sections of K_S^{-4} , $i = 2, \dots, 4$, and g_j are sections of K_S^{-6} , $j = 3, \dots, 6$ since $\mathcal{T} \simeq \mathcal{O}_S$. The split condition requires f_2, f_4 and g_3, g_4, g_6 to be written as polynomial functions of a smaller set of sections

$$h \in H^0(K_S^{-1}), \quad H \in H^0(K_S^{-2}), \quad q \in H^0(K_S^{-3}).$$

Therefore the complex structure moduli of a split A_4 degeneration along the section Σ are controlled by the coefficients

$$h \in H^0(K_S^{-1}), \quad H \in H^0(K_S^{-2}), \quad q \in H^0(K_S^{-3}), \quad f_3 \in H^0(K_S^{-4}), \quad g_6 \in H^0(K_S^{-6}).$$

These sections are in one-to-one correspondence with the sections (3.43) which determine the spectral cover \mathcal{C}_2 .

The duality map for bundles with reducible spectral covers, or more generally bundles constructed by extensions, is more subtle and not completely understood. Here we are only interested in reducible spectral covers of the form $\mathcal{C} = \sigma + \mathcal{C}''$, and extensions of the form (3.39). According to [53] (§5.1) in this case, the moduli of the rank four bundle V'' corresponding to \mathcal{C}'' are mapped to complex structure deformations of the fourfold. The extension moduli are related to expectation values of D7-D7 fields which span nongeometric branches in F-theory. This correspondence is not understood in detail at the present stage, but the current understanding suffices for our purposes.

Recall that \mathcal{C}'' is an irreducible spectral cover with $\eta'' = 6c_1(S) - \Gamma$, therefore $\mathcal{T} \simeq \mathcal{O}_S(-\Gamma)$. This is the normal bundle of the proper transform \tilde{S}_0 in \tilde{P} , hence we will have

a fourfold degeneration along the rigid section S_0 . The spectral cover moduli (3.43) are parameterized in this case by

$$\begin{aligned} a_0 &\in H^0(K_S^{-6}(-\Gamma)), & a_2 &\in H^0(K_S^{-4}(-\Gamma)) \\ a_3 &\in H^0(K_S^{-3}(-\Gamma)), & a_4 &\in H^0(K_S^{-2}(-\Gamma)). \end{aligned} \tag{3.52}$$

Using the results of [35] (§4.5) one can check that these are precisely the complex structure moduli of a split D_5 singularity along the section \tilde{S}_0 [35]. So naively we seem to obtain a $SO(10)$ gauge group in F-theory. However, this is not true, since we have not taken into account the D7-D7 strings localized on the intersection between \tilde{S}_0 and the nodal component of the discriminant. The expectation values of these fields should be related to extension moduli in the heterotic model [53]. In our case, we have no extension moduli, as shown at the end of §3.6. We can construct only a direct sum bundle which is unstable or a non-split extension, which is stable. Note that this behavior is not solely determined by the spectral cover \mathcal{C}'' . The line bundle \mathcal{N} also plays a crucial role in the extension moduli computation. The choice of line bundle is expected to be related to the background flux G in F-theory [62,64,65].

Given all this data, we propose the following interpretation of the F-theory dual. Naively, the gauge symmetry seems to be $SO(10)$, but with the present combination of geometry and flux, the D7-D7 strings localized on the intersection are tachyonic. Therefore they must condense spontaneously breaking the $SO(10)$ gauge symmetry on the D7-brane wrapping \tilde{S}_0 to $SU(5)$. The tachyonic nature of the D7-D7 strings is related to the fact that the direct sum bundle is unstable, therefore breaks supersymmetry at tree level in the heterotic model. It would be very interesting to understand this correspondence in more detail, but we leave this for future work.

Summarizing this discussion, we conclude that one can construct an F-theory dual to the previous heterotic model. The geometry of the F-theory fourfold is an elliptic fibration over a blow-up \tilde{P} of $\mathbb{P}^1 \times S$ and the GUT $SU(5)$ bundle corresponds to a split A_4 degeneration along a movable section of \tilde{P} over S . The hidden $SU(5)$ bundle corresponds to a split D_5 degeneration along a rigid section, and the interplay of geometry and flux leads to tachyon condensation in F-theory, breaking $SO(10)$ to $SU(5)$.

An important point for us is that the resulting F-theory fourfold has complex structure deformations which bring the movable section supporting the GUT $SU(5)$ arbitrarily close to the fixed section supporting the hidden $SU(5)$. This allows us to tune the messenger masses to be small enough to arrange for gauge mediation as the dominant source of SUSY

breaking in the observable sector: different values of the fluxes which stabilize the complex structure moduli lead to a wide range of possibilities for M .

Since this point is slightly subtle, let us provide more details. First note that this question reduces to the analogous problem in eight dimensions⁹ since the F-theory elliptic fibration is a (blow-up of a) direct product $P = \mathbb{P}^1 \times S$. This means it suffices to consider an eight dimensional F-theory compactification on an elliptically fibered K3 surface with a section. We will be interested in a subspace of the moduli space where the K3 surface has one Kodaira fiber of type I_5 (corresponding to an A_4 singularity) and another Kodaira fiber of type I_1^* (corresponding to a D_5 singularity) in addition to 12 Kodaira fibers of type I_1 . In this case one can construct a one parameter family of K3 surfaces so that the generic surface has singular fibers $I_5 + I_1^* + 12I_1$ as above and the central fiber has singular fibers of type $I_3^* + 15I_1$. (The singular fiber I_3^* corresponds to a D_7 singularity.) Locally, this is a collision of I_5 and I_1^* singular fibers; it is constructed explicitly in §11 of [66].

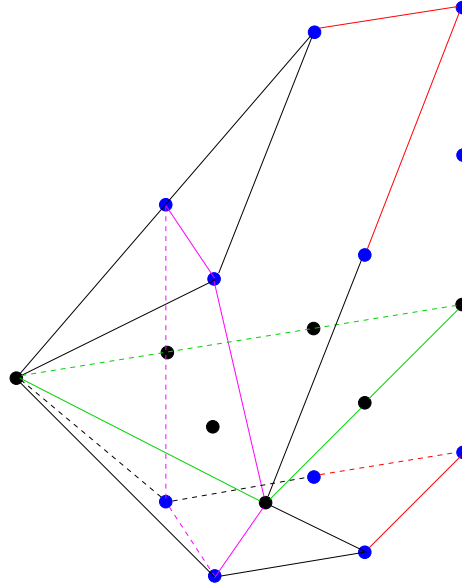


Fig. 1: Reflexive polyhedron describing the family of K3 surfaces. The color code is as follows: magenta is associated with the elliptic fiber of the $Spin(32)/\mathbb{Z}_2$ fibration, while red corresponds to the associated extended Dynkin diagram; green is associated with the elliptic fiber of the $E_8 \times E_8$ fibration, while blue corresponds to the respective extended Dynkin diagrams.

⁹ We thank T. Pantev for clarifying discussions on these points.

In fact we can give an alternative argument based on a presentation of these K3 surfaces as hypersurfaces in toric varieties; this is based on a beautiful observation due to Candelas and Skarke [67]. We represent the corresponding toric polyhedron below. Note that the K3 surfaces develop a D_7 singularity along a certain subspace of the moduli space. To see this, note that the polyhedron contains a reflexive subpolyhedron drawn with magenta in fig. 1; the corresponding torus fibration admits one singular I_3^* fiber. The red edges of the ∇ polyhedron correspond to the nontrivial pairwise intersections of the exceptional divisors obtained by resolving the D_7 singularity. Complex structure deformations in the normal directions to this subspace will split the D_7 singularity in $A_4 + D_5$. To see this, note that the polyhedron contains another reflexive subpolyhedron, which is the elliptic curve in $\mathbb{P}_{[1,2,3]}^2$; this is represented with green in fig. 1. The corresponding elliptic fibration has one D_5 and one A_4 singularity; the exceptional divisors associated with the resolution of the singularities and the affine components are represented with blue in fig. 1.

For completeness, note also that the elliptic fibrations with D_7 singularities admit two sections. This suggests that our model can be equally well constructed using the $Spin(32)/\mathbb{Z}_2$ heterotic string, according to [60]. This is not surprising since it is well known that the two heterotic models are equivalent when compactified on a two-torus.

In order to put this discussion in proper perspective, note that the picture developed here does not contradict the more familiar parameterization of complex structure moduli in F-theory. Usually one fixes the locations of the A_4 and respectively D_5 singularities at ∞ and respectively 0 using the $PSL(2, \mathbb{C})$ automorphism group of the base \mathbb{P}^1 . Then the moduli of the K3 surface are parameterized by deformations of the K3 surface preserving these singularities. Note however that in this manner one obtains only a parameterization of an open subset of the moduli space where the two singular points are away from each other. The construction sketched above yields a parameterization of a different open subset of the moduli space, centered on the subspace of K3 surfaces with D_7 singularities.

Note also that the parameterization commonly used in the literature covers a neighborhood of the $E_8 \times E_8$ semi-stable degeneration locus in the moduli space [60,27]. In this region, the F-theory model admits an alternative description in terms of heterotic M-theory. The region we are interested in is not near the $E_8 \times E_8$ semi-stable degeneration locus, and the model does not admit a heterotic M-theory interpretation.

3.10. Kähler moduli stabilization

The typical Kähler moduli stabilization mechanism in F-theory models relies on D3-brane instanton effects [68]. The D3-brane instantons which contribute to the nonperturbative superpotential are classified by arithmetic genus one divisors D in the resolution of X which project to a surface in the base P . If the base is a \mathbb{P}^1 bundle over a surface S , one can show [69] that the inverse image of any (-1) curve C in S is such a divisor D which contributes to the superpotential. In our case, the base is $S = dP_8$, and we can find 240 (-1) curves which generate the Mori cone (3.24). Moreover, the base is in fact a blow-up of $P = \mathbb{P}^1 \times S$ along a curve Ξ lying in a section. According to [57], the inverse image of the exceptional divisor $E \subset \tilde{P}$ contributes to the superpotential if Ξ is a (-1) curve as well. This is precisely condition (III) formulated in section §3.3, which is satisfied in our model.

Finally, the vertical divisors obtained by ruling the exceptional components of the D_5 fiber over the section S_0 are rigid and have arithmetic genus 1. Therefore, according to [68], they contribute to the nonperturbative superpotential. We conclude that this model admits sufficient contributions to the superpotential such that it is possible to find vacua with all the Kähler moduli fixed.

4. Dynamical Supersymmetry Breaking from Quivers

D3-branes placed at a smooth point in a Calabi-Yau manifold realize a world-volume gauge theory which, at low energies, flows to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. To get theories with less supersymmetry, one can place the branes at singular points in the Calabi-Yau: simple examples include orbifolds [70,71,72] and conifolds [73]. Suitable classes of singularities may also include collapsed curves; in such cases, one can sometimes make supersymmetric configurations which include some number of D5-branes wrapping the collapsed curve (often called “fractional branes”). For instance, the famous Klebanov-Strassler solution [74] arises in this way, by placing D3-branes and wrapped D5-branes at a conifold singularity.

By now, quite a bit has been learned about the classes of so-called quiver gauge theories which arise from general configurations of fractional branes at the singularities in Calabi-Yau moduli space where a divisor collapses to zero size. A nice review with references can be found in [28]. One very interesting insight which has recently emerged is

that quiver theories which preserve SUSY to all orders in perturbation theory, but break it non-perturbatively, are easy to find (and may even be generic) [26].

This suggests that an easy way to make a model of gauge mediation, may be to realize the Standard Model on one stack of D-branes, and a quiver theory which dynamically breaks SUSY on another stack. By now there are many papers which realize variants of the SM on different kinds of brane stacks; we will avail ourselves of two different kinds of constructions. On the one hand, we can realize the Standard Model as in [28], which realizes the SM using fractional branes in a (partially) collapsed dP_8 . As long as the DSB quiver is sufficiently close to the SM branes, the interbrane strings connecting the two stacks will have a mass $M \ll M_s$; they will serve as the messengers of gauge mediated supersymmetry breaking. On the other hand, we can also arrange for our quiver theory to arise “close to” a GUT D7-brane stack, which is the F-theory dual to a standard heterotic GUT.

One of the simplest quiver theories which exhibits DSB [26] involves an $U(3M) \times U(2M) \times U(M)$ gauge group with matter fields in the

$$(\overline{\mathbf{3M}}, \mathbf{2M}, \mathbf{1}), 2 \times (\mathbf{3M}, \mathbf{1}, \overline{\mathbf{M}}), 3 \times (\mathbf{1}, \overline{\mathbf{2M}}, \mathbf{M}) \quad (4.1)$$

representations (and a suitable tree-level superpotential). It can be obtained from fractional branes at a collapsed dP_1 as in [26].¹⁰

Therefore, in the next sections, we engineer appropriate collapsed surface singularities close to: 1) F-theory duals of heterotic GUT models and, 2) avatars of the D3-brane MSSM-like model of [28]. Before doing so, however, we discuss a very minor modification of the example given in [26].

4.1. DSB from the dP_8 quiver

We will be mostly interested in compact Calabi-Yau manifolds that admit dP_8 and dP_5 singularities (instead of dP_1). Here we briefly explain the elementary point, that this does not hamper us in using the construction of [26]. This is in keeping with their statements about the genericity of the phenomena they discuss.

¹⁰ We note that the del Pezzo quiver gauge theories may actually have several branches, with the dynamical supersymmetry breaking vacua being the result of local analysis on some subset of the branches. Simpler gauge theories like the Klebanov-Strassler theory [74] already exhibit a very rich branch structure [75]. We thank N. Seiberg for emphasizing this point to us.

A three-block exceptional collection for the dP_8 singularity is provided in §3 of [28]. Recall that the middle cohomology of dP_8 is spanned by the hyperplane class h and the exceptional curves e_i , $i = 1, \dots, 8$ with

$$h \cdot h = 1, \quad e_i \cdot e_j = -\delta_{ij}, \quad h \cdot e_i = 0. \quad (4.2)$$

The canonical class is given by

$$K_{dP_8} = -3h + \sum_{i=1}^8 e_i. \quad (4.3)$$

A given member F of the exceptional collection can be specified by a charge vector $(\text{rank}(F), c_1(F), \text{ch}_2(F))$ giving the Chern classes of the sheaf it represents (the D7, D5 and D3 charge respectively). If one takes n copies of a given member, one finds a $U(|n|)$ gauge theory with no adjoint matter. Given two such sheaves F_i, F_j , the spectrum of bifundamentals may be computed in terms of the Euler character $\chi(F_i, F_j)$.

The results for dP_8 are that there exists an exceptional collection with

$$\begin{aligned} \text{ch}(F_i) &= (1, h - e_i, 0), \quad i = 1, \dots, 4, \quad \text{ch}(F_j) = (1, -K_{dP_8} + e_i, 0), \quad j = 5, \dots, 8, \\ \text{ch}(F_9) &= (1, 2h - \sum_{i=1}^4 e_i, 0), \quad \text{ch}(F_{10}) = (3, -K_{dP_8} + \sum_{i=5}^8 e_i, -1/2), \\ \text{ch}(F_{11}) &= (6, -3K_{dP_8} + 2 \sum_{i=5}^8 e_i, 1/2). \end{aligned}$$

The spectrum of bifundamentals is given by

$$\chi(F_{10}, F_i) = 1, \quad \chi(F_{11}, F_i) = 1 \text{ for } i = 1, \dots, 9, \quad \chi(F_{10}, F_{11}) = 3. \quad (4.4)$$

It was observed in [28] that this quiver allows for a simple construction of a pseudo-realistic MSSM. To get this toy model, the multiplicities at the nodes should be chosen to be

$$n_i = 1, \quad i = 1, \dots, 9, \quad n_{10} = 3, \quad n_{11} = -3. \quad (4.5)$$

In this case, the charges of the fractional brane add up to the charge of a single D3-brane.

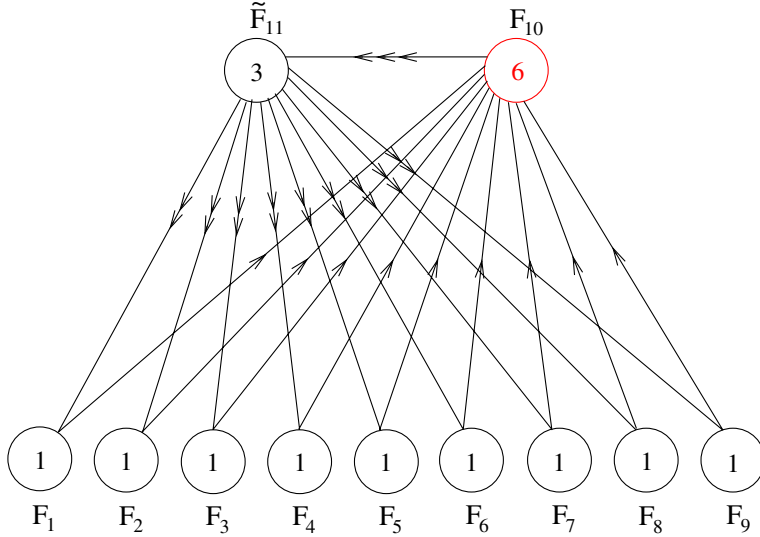


Fig. 2: The dP_8 quiver associated with the mutated exceptional collection.

Next, it is possible to perform a Seiberg duality on the node F_{10} , as in [28]. This modifies the quiver gauge theory as follows: F_{11} is replaced by a node \tilde{F}_{11} with

$$\text{ch}(\tilde{F}_{11}) = (3, \sum_{i=5}^8 e_i, -2) \quad (4.6)$$

and the multiplicities at the nodes (for the toy MSSM) are now given by

$$\tilde{n}_i = 1, \quad i = 1, \dots, 9, \quad \tilde{n}_{10} = -6, \quad \tilde{n}_{11} = 3. \quad (4.7)$$

The spectrum of bifundamentals is now determined by

$$\chi(F_{10}, F_i) = 1, \quad \chi(\tilde{F}_{11}, F_i) = 2 \text{ for } i = 1, \dots, 9, \quad \chi(\tilde{F}_{11}, F_{10}) = 3. \quad (4.8)$$

The end result is a quiver of the form given in fig. 6 of [28] which we reproduce here for convenience.

This quiver can easily accommodate the supersymmetry breaking model of [26]. For instance, the multiplicities

$$n_1 = 3M, \quad n_{10} = 2M, \quad n_{11} = M \quad (4.9)$$

with all other n_i vanishing, yield the family of theories of interest. Replacing n_1 with any of the $n_{2, \dots, 9}$ would work equally well. We would like to avoid proliferation of messengers, so for practical purposes the most interesting case is $M = 1$.

As was correctly described in the papers [26], these theories have several FI terms associated with the $U(1)$ factors in the gauge group. The supersymmetry is broken for any finite value of these terms, in the field theory analysis. The question of whether the supersymmetry is broken once they become dynamical is a more detailed and subtle one, depending both on details of the gauge theory which are hard to compute, and details of the global embedding. We discuss this issue in §5.3; it is worth pointing out that there is no analogous issue with the models of §3.

4.2. DSB from the dP_5 quiver

We are also interested in a compact geometry containing a collapsing del Pezzo dP_5 surface S . We can determine the corresponding quiver gauge theory starting again with a three-block strongly exceptional collection on S . Such a collection is given by [76]

$$\begin{aligned}
 G_1^a &= \mathcal{O}_S(e_4), & G_2^a &= \mathcal{O}_S(e_5), & G_3^b &= \mathcal{O}_S(h), & G_4^b &= \mathcal{O}_S(2h - e_1 - e_2 - e_3), \\
 G_5^c &= \mathcal{O}_S(3h - e_1 - e_2 - e_3 - e_4 - e_5), & G_6^c &= \mathcal{O}_S(2h - e_1 - e_2), & G_7^c &= \mathcal{O}_S(2h - e_2 - e_3), \\
 G_8^c &= \mathcal{O}_S(2h - e_1 - e_3),
 \end{aligned}
 \tag{4.10}$$

where as before h denotes the hyperplane class and $e_i, i = 1, \dots, 5$ are the exceptional curve classes. Then, the associated quiver gauge theory is described by the following diagram

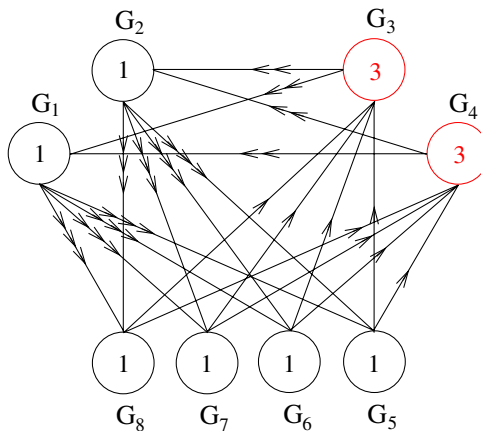


Fig. 3: The quiver gauge theory associated with a collapsing dP_5 surface.

It is easy to see that dP_5 quiver gauge theory also accommodates the SUSY breaking model of [26]. One possible choice of multiplicities is as follows:

$$n_1 = M, \quad n_3 = 3M, \quad n_5 = 2M
 \tag{4.11}$$

with all the other n_i vanishing. Again, the most interesting case is $M = 1$.

5. Fractional Brane Supersymmetry Breaking and GUT Models

Motivated by the previous discussion, in this section we construct examples of IIB Calabi-Yau orientifolds including fractional branes at del Pezzo singularities. Since we will focus on examples with fixed O7-planes, our models can also be regarded as limiting cases of F-theory compactifications [77,78].

Consider a IIB compactification on a Calabi-Yau threefold Z equipped with a holomorphic involution $\sigma : Z \rightarrow Z$ which flips the sign of the global holomorphic three-form

$$\sigma^* \Omega_Z = -\Omega_Z.$$

Furthermore, let us assume that the fixed locus of σ is a smooth complex surface $R \subset Z$ so that the quotient Z/σ is a smooth threefold P and the projection map $\rho : Z \rightarrow P$ is a double cover with ramification divisor R . We will denote by $B \subset P$ the branch divisor of the double cover.

We construct an orientifold theory by gauging the discrete symmetry $(-1)^{F_L} \Omega \sigma$ where Ω is world-sheet parity. According to [77,78], the resulting model is equivalent to a F-theory compactification on $X = (Z \times T^2)/\mathbb{Z}_2$, where \mathbb{Z}_2 acts as σ on Z and simultaneously as (-1) on T^2 . It is easy to check that X is an elliptically fibered Calabi-Yau fourfold over P with D_4 singular fiber along the branch divisor B . This elliptic fibration admits complex structure deformations which will modify the singular fibers and the discriminant. Such deformations correspond to more general F-theory compactifications away from the orientifold limit.

In the following we will be interested in models in which the base P develops collapsing del Pezzo singularities away from the branch locus of $p : Z \rightarrow B$. More precisely, we would like to find a collection S_1, \dots, S_k of del Pezzo surfaces on P which do not meet the branch locus B , nor each other, and a map $p : P \rightarrow \hat{P}$ which contracts S_1, \dots, S_k to singular points p_1, \dots, p_k on \hat{P} .

Assuming that these conditions are met, note that the inverse image of each surface S_i via the double cover $\rho : Z \rightarrow P$ is a pair of disjoint del Pezzo surfaces S'_i, S''_i in Z . The involution $\sigma : Z \rightarrow Z$ maps S'_i isomorphically to S''_i . Moreover, the contraction map p projects the branch locus B onto a smooth divisor $\hat{B} \subset \hat{P}$ which is isomorphic to B and supported away from the singular points of \hat{P} . Let $\hat{\rho} : \hat{Z} \rightarrow \hat{P}$ be the double cover of

\widehat{P} branched along \widehat{B} . Then Z is isomorphic to the fiber product $\widehat{Z} \times_{\widehat{\rho}} P$ and we have a commutative diagram

$$\begin{array}{ccc} Z & \xrightarrow{\rho} & P \\ \downarrow z & & \downarrow p \\ \widehat{Z} & \xrightarrow{\widehat{\rho}} & \widehat{P} \end{array} \quad (5.1)$$

The map $z : Z \rightarrow \widehat{Z}$ contracts S'_i, S''_i to singular points p'_i, p''_i on \widehat{Z} which project to the singular points $p_i \in \widehat{P}$ under the map $\widehat{\rho}$.

Since S_i are disjoint from B , it follows that the infinitesimal neighborhood of S_i in P is isomorphic to the infinitesimal neighborhood of S'_i (or, equivalently, S''_i) in Z . Therefore the S_i must be locally Calabi-Yau surfaces on P , even though P is not a Calabi-Yau manifold. In particular, the normal bundle to S_i in P must be isomorphic to the canonical bundle K_{S_i} , and the restriction $c_1(P)|_{S_i}$ of the first Chern class of P to S_i is trivial.

Under these circumstances, the local physics at the singularities of \widehat{P} is identical to the local physics of typical del Pezzo singularities in Calabi-Yau threefolds. In particular we can introduce fractional branes wrapping collapsing cycles in S_i and study their dynamics as if P were globally Calabi-Yau. Since the local physics is not sensitive to complex structure deformations which preserve the singularities, this conclusion will continue to hold when we deform X away from the orientifold limit.

As explained in the previous section, dynamical supersymmetry breaking can be realized if we place certain configurations of fractional branes at del Pezzo singularities in Calabi-Yau threefolds. More specifically, one has to consider fractional branes which in the large radius limit correspond to D5-branes wrapping holomorphic cycles in the exceptional del Pezzo surface. This construction seems to be at odds with our set-up since typically O7 orientifold planes do not preserve the same fraction of supersymmetry as D5-branes. In fact the orientifold projection considered above maps a D5-brane wrapping a holomorphic curve C in the del Pezzo surface $S'_i \subset Z$ to a anti-D5-brane wrapping the image curve $\sigma(C)$ in S''_i . Obviously, for generic values of the Kähler parameters this configuration would break supersymmetry at tree level. However, we know that in the singular limit fractional D5-branes preserve the same fraction of supersymmetry as D3-branes. This is clear from the construction of quiver gauge theories associated to branes at del Pezzo singularities in which configurations of D3-branes and D5-branes give rise to supersymmetric field theories. In fact there is nothing mysterious about this phenomenon. It is well established by now that the fraction of supersymmetry preserved by holomorphic D-brane

configurations is determined by the phase of the associated central charge [79,80,81]. As we move in the Kähler moduli space, the phases of D5-branes and D3-branes change until they eventually become aligned along a wall of marginal stability.

Since configurations of O7-planes and D3-branes are supersymmetric, it follows that along such a wall of marginal stability we can also add fractional D5-branes at the singularities without breaking tree level supersymmetry. More precisely, suppose we have N_i D3-branes and M_i D5-branes at the singularity in \widehat{Z} obtained by collapsing S'_i .

The orientifold projection will map this configuration to a system of N_i D3-branes and M_i anti-D5-branes at the conjugate singularity obtained by collapsing S''_i . The central charges of the two configurations are

$$Z'_i = N_i Z_{D3} + M_i Z_{D5}, \quad Z''_i = N_i Z_{D3} - M_i Z_{D5}, \quad (5.2)$$

since orientifold projection flips the sign of the D5-brane central charge and preserves the D3-brane central charge. Since Z_{D3} and Z_{D5} are collinear, it follows that Z_i, Z''_i will also be collinear as long as the number N_i of D3-branes is sufficiently large.

As explained in section four, we will be in fact interested in a special point on the marginal stability wall where the low energy effective theory of the fractional brane system is a quiver gauge theory. This is the point where the central charges of all objects of the exceptional collection on S are collinear. In the following we will refer to this special point in the Kähler moduli space as the quiver point.

Note that tree level supersymmetry is easily achieved in typical F-theory compactifications. At the quiver point, all fractional branes corresponding to the exceptional collection preserve the same supersymmetry as the D3-brane, and they are permuted by monodromy transformations in the complexified Kähler moduli space. Therefore their central charges must be all equal and aligned to the central charge of the D3-brane. Since the sum of all fractional branes is a D3-brane, their central charges must be equal to $\frac{1}{p}Z_{D3}$, where p is the total number of fractional branes. Fractional D5-branes are linear combinations of some number $q < p$ of fractional branes. Therefore, $Z_{D5} = \frac{q}{p}Z_{D3}$, with $0 < q/p < 1$. Therefore the above conditions are easily satisfied in F-theory models with a large tadpole for three-brane charge. This number is controlled by $\frac{\chi}{24}$ where χ is the Euler character of the fourfold [82], and in simple examples $\frac{\chi}{24} \sim 10^2 - 10^3$. We will be able to compute χ in one of our examples and confirm that the above assertion is justified.

It is worth noting that similar configuration of branes have been considered before in toroidal orientifold models [83,84,85,12,13,14]. In that case one typically considers D9- $\overline{D9}$

pairs with magnetic fluxes in orientifold theories with O3/O7 planes. Such configurations are related by T-duality to D6-branes intersecting at angles, and preserve tree level supersymmetry for special values of the Kähler parameters. Here we encounter the Calabi-Yau counterpart of this construction. Other applications of magnetized branes on Calabi-Yau manifolds will be discussed elsewhere [86].

5.1. Concrete examples

Let us now present a concrete class of models. In addition to the fractional branes which cause DSB, we would also like our models to exhibit a three generation GUT sector on the background D7-branes.

We take the base P to be the projective bundle $\mathbb{P}(\mathcal{O}_S \oplus K_S)$ where $S = dP_k$ is a del Pezzo surface. Note that P has two canonical sections S_0, S_∞ with normal bundles

$$N_{S_0/P} \simeq K_S, \quad N_{S_\infty/P} \simeq -K_S. \quad (5.3)$$

The canonical class of P is

$$K_P = -2S_\infty.$$

Pick the branch locus B to be a generic smooth divisor in the linear system $|-2K_P| = |4S_\infty|$. Note that B does not intersect S_0 , since S_∞ and S_0 are disjoint. Then the double cover $\rho : Z \rightarrow P$ of P branched along B is a smooth Calabi-Yau threefold containing two disjoint surfaces S'_0, S''_0 isomorphic to dP_k , which cover S_0 .

Note that the section S_0 is locally Calabi-Yau in P according to (5.3). Moreover P is toric, and one can find a toric contraction map $p : P \rightarrow \hat{P}$ which contracts S_0 to a point. Then one can complete the diagram (5.1) by taking the double cover of the cone \hat{P} branched along the image \hat{B} of B through p .

Next, we deform the elliptic fibration $\pi : X \rightarrow P$ preserving the fibration structure and the singularity in the base. At generic points in the moduli space, we have a smooth elliptic fibration $\pi : X \rightarrow P$ which can be written in standard Weierstrass form

$$y^2 = x^3 - fx - g \quad (5.4)$$

where f, g are sections of $-4K_P$ and respectively $-6K_P$. The discriminant is given by

$$\delta = 4f^3 - 27g^2. \quad (5.5)$$

We will denote with capital letters F, G, Δ the zero divisors of f, g, δ on P . Note that Δ, F, G do not intersect S_0 , therefore the elliptic fiber is constant along S_0 . This will allow us to contract the section S_0 on X , obtaining a singular fourfold \widehat{X} with an elliptic curve of local dP_k singularities.

In order to obtain a GUT gauge group on the F-theory seven branes, we have to choose the complex structure moduli so that Δ decomposes into two irreducible components

$$\Delta = \Delta' + \Delta'' \tag{5.6}$$

where $\Delta' = 5\Sigma$, where Σ is a section of P linearly equivalent to S_∞ . Moreover, f, g should not vanish identically along Σ . Note that Σ does not intersect the section S_0 , but it can be brought arbitrarily close to S_0 by complex structure deformations. This means that the mass of the open strings between the fractional branes at the dP_k singularity and the GUT D7-branes is controlled by a complex structure modulus of the fourfold.

The chiral matter of the low energy theory is determined in principle by the open string spectrum between the GUT D7-branes and the D7-branes wrapping the nodal component Δ'' of the discriminant. However, at the present stage F-theory techniques are not sufficiently developed for an explicit computation of the spectrum, or at least of the net number of generations. This question can be more efficiently addressed invoking heterotic F-theory duality as in section three.

5.2. Heterotic duals

The above F-theory models are dual to heterotic four dimensional compactifications on elliptic fibrations. We have reviewed some aspects of the duality map and the spectral cover construction of heterotic bundles in sections §3.9 and §3.4.

The dual heterotic models are specified by a smooth Weierstrass model Y over S and a background bundle of the form $V \times W$ where V, W are stable $SU(5)$ and respectively E_8 bundles over Y . The E_8 bundle W corresponds to the hidden sector, while the $SU(5)$ bundle gives rise to the GUT sector. As explained in §3.9, since the F-theory base is a \mathbb{P}^1 bundle over Y without blow-ups, there are no horizontal heterotic fivebranes on Y . Note that in the notation of §3.9, we now have $\mathcal{T} \simeq K_S$. Applying the techniques explained there, it is not hard to check that one can enforce a split A_4 singularity along the section $\Sigma \subset P$, which has normal bundle $N_\Sigma \simeq K_S^{-1}$. This would correspond to an $SU(5)$ bundle V with a spectral cover \mathcal{C} in the linear system $|5\sigma + 7\pi^*c_1(S)|$. However, one can easily

check using formulas (3.11) that such a bundle can never yield a three generation spectrum. For $SU(5)$ bundles, the Chern classes (3.11) are

$$\begin{aligned} \text{ch}_1(V) &= -(a+4)\eta + 5(a+2)c_1(S) + 5c_1(\mathcal{M}) \\ \text{ch}_3(V) &= \frac{1}{2}\eta(\eta - c_1(S)) - \eta c_1(\mathcal{M}). \end{aligned} \tag{5.7}$$

It suffices to substitute $\eta = 7c_1(S)$ in the second equation in (5.7) obtaining

$$\text{ch}_3(V) = 21 - 7c_1(S) \cdot c_1(\mathcal{M}).$$

The right hand side of this equation is obviously a multiple of 7, hence it can never take the value ± 3 .

Then how can we obtain a three generation $SU(5)$ GUT in the low energy effective action? In order to solve this puzzle, we have to look for three generation models in heterotic vacua with horizontal fivebranes, that is we have to allow a nontrivial Ξ class. According to §3.9, this means that the F-theory base must be a blow-up of P along a curve isomorphic to Ξ . Let us blow-up P along the same curve Ξ embedded in the section Σ . Then the D7-branes carrying the GUT gauge group will wrap the strict transform $\tilde{\Sigma}$ of the section Σ , and the class η of the $SU(5)$ bundle must be corrected to

$$\eta = 7c_1(S) - \Xi. \tag{5.8}$$

The extra term in the right hand side of (5.8) reflects the change in the normal bundle of Σ under the blow-up, as explained in §3.8.

Taking into account this correction to the η class, let us try again to find three generations $SU(5)$ bundles. First we have to choose \mathcal{M} and a so that $\text{ch}_1(V) = 0$. There two obvious choices satisfying this condition

$$\begin{aligned} A) \quad a &= 1, & c_1(\mathcal{M}) &= \eta - 3c_1(S) \\ B) \quad a &= -4, & c_1(\mathcal{M}) &= 2c_1(S). \end{aligned} \tag{5.9}$$

One can probably find many more solutions, but we will focus only on these two cases in the following. Substituting (5.8) and (5.9) in the formula (5.7) for $\text{ch}_3(V)$ we obtain

$$\begin{aligned} A) \quad \text{ch}_3(V) &= -\frac{1}{2}(7c_1(S) - \Xi)(2c_1(S) - \Xi) \\ B) \quad \text{ch}_3(V) &= \frac{1}{2}(7c_1(S) - \Xi)(2c_1(S) - \Xi). \end{aligned} \tag{5.10}$$

In addition to the constraint $\text{ch}_3(V) = \pm 3$, we would also like Ξ to be a collection of disjoint (-1) curves on S . According to §3.10, this is required in order to stabilize the sizes of the exceptional divisors of the blow-up $\tilde{P} \rightarrow P$. Taking into account all these constraints, we have found two classes of solutions. Recall that we denote by h the hyperplane class on $S = dP_k$ and by e_1, \dots, e_k the exceptional curve classes.

i) Example I. Take $S = dP_5$ and

$$\Xi = \Gamma_1 + \dots + \Gamma_5$$

where $\Gamma_1, \dots, \Gamma_5$ are disjoint smooth rational curves on S . For example we can take $\Gamma_i = e_i$, $i = 1, \dots, 5$. Then one can check that

$$(7c_1(S) - \Xi)(2c_1(S) - \Xi) = 6$$

which yields $\text{ch}_3(V) = \mp 3$ in the two cases listed in (5.10). In this case we can easily compute the Euler characteristic of the fourfold X , $\chi(X) = 4128$. Therefore, the three-brane charge tadpole is large enough such that the technical condition necessary for the alignment of the D3-brane and D5-brane charge at the point in the Kähler moduli space where the negative section S_0 collapses to zero size is satisfied.

ii) Example II. Take $S = dP_8$ and

$$\Xi = \Gamma_1 + \Gamma_2$$

where Γ_1, Γ_2 are any two disjoint (-1) curves on S such as e_1, e_2 . Then we obtain again

$$(7c_1(S) - \Xi)(2c_1(S) - \Xi) = 6.$$

In each of the these examples, one can construct the SUSY-breaking quiver theory described in §4.

5.3. Comments on possible runaway behavior in Kähler moduli space

In the present construction, say for definiteness Example II above, all Kähler moduli of the dP_8 surface S_0 are present as Kähler moduli of the Calabi-Yau threefold. Therefore, one must worry about stabilizing each of them.¹¹ (The constructions of §6 are advantageous

¹¹ This also indicates that Example II would be an ideal setting for compact embeddings of the Verlinde-Wijnholt model [28]. The presence of all dP_8 moduli gives one the freedom to tune Yukawa and FI terms (at least before accounting for moduli stabilization), a necessary step in making the model realistic. For our present purposes, we wish to use the cone over dP_8 to embed a SUSY breaking sector instead.

in this respect, since as we shall see there only a single linear combination of the Kähler moduli of the dP_8 is nontrivial in $H^{1,1}$ of the Calabi-Yau space).

There are naively sufficiently many vertical arithmetic genus one divisors for this task; in the IIB picture, these project to \mathbb{P}^1 bundles over the relevant curves in the del Pezzo surface. However, the fractional brane configuration (4.9) involves one of the exceptional curves in the del Pezzo surface. It is therefore uncertain that the relevant instanton contributes. The volume of this curve plays the role of a FI parameter R . As described in e.g. §3.2.2-3.2.3 of the second reference in [26], this dynamical FI term may cause the SUSY-breaking vacuum to run away to infinity. We briefly review their discussion here, for completeness, and point out several caveats.

In a limit of the $U(3) \times U(2) \times U(1)$ gauge theory where the $U(2)$ is most strongly coupled, one can derive as in the reference above a description of the physics which is governed by only three light chiral multiplets: call them M , Z and R . M and Z are open string fields, while R is the Kähler modulus. The real part of R plays the role of an FI term in the gauge theory – by abuse of notation we shall denote this by R as well. An $SU(3)$ factor in the (Seiberg dual) gauge theory generates a nonperturbative superpotential, characterized by dynamical scale Λ . Making the very strong assumption that there are canonical kinetic terms, and expanding for small R , the theory has an F-term potential

$$V_F \sim |M|^2 + \Lambda^2 |Z + \Lambda^4 M^{-3/2}|^2 \quad (5.11)$$

and a D-term potential

$$V_D \sim (|Z|^2 - R)^2. \quad (5.12)$$

We emphasize that the assumption of canonical kinetic terms is far from justified; in such a theory, one would generically expect the Kähler potential to include complicated structure at $\mathcal{O}(\Lambda)$ in field space. Proper account of this could change the conclusions of even this heuristic discussion.

With these assumptions, it is easy to see that in the presence of the dynamical FI term, the theory has an unstable vacuum. The SUSY breaking vacuum of [26], can relax to a vacuum at $R \rightarrow \infty$, $Z \rightarrow \infty$ and $M \rightarrow 0$.

In the language of the brane configuration, one can gain some intuition for this phenomenon as follows. Classically (i.e. at $\Lambda \rightarrow 0$) the brane configuration is BPS at $R = 0$. But in fact, as mirrored in the field theory, there is a full line in Kähler moduli space along which the brane configuration remains supersymmetric. One can take $R > 0$, $Z \sim \sqrt{R}$

and $M = 0$. The non-perturbative SUSY breaking at $R = 0$ may then try to relax by moving along this (classical) flat direction, towards the larger volume classically BPS brane configuration.

It is clear from this discussion that the determination of the status of the SUSY breaking vacuum in the compact model is not amenable to a simple local analysis. Even if the full Kähler potential does not contain enough structure to meta-stabilize the DSB vacuum at $\mathcal{O}(\Lambda)$ in field space, further effects in the compactification manifold which depend on R (notably, Kähler potential corrections and possible brane instantons) can prevent the runaway behavior. This possibility is also suggested by the fact that R cannot become arbitrarily large while keeping the other Kähler moduli fixed (this runs into a wall of the Kähler cone); but the other Kähler moduli can be fixed by brane instanton effects in this model. Because the models of §3 have no such issue, we will not try to pursue the analysis of this model in further detail here. A model based on the same SUSY breaking quiver is also described in §6, where however there are fewer del Pezzo moduli and the existence of a stable vacuum seems very plausible.

6. Compactifications Incorporating the D3-Brane MSSM

In this section we construct another class of models which contain an MSSM sector realized as in [28] instead of a GUT sector on D7-branes.

We will use the same kind of geometric set-up encoded in the commutative diagram (5.1), which is reproduced below for convenience

$$\begin{array}{ccc} Z & \xrightarrow{\rho} & P \\ \downarrow z & & \downarrow p \\ \widehat{Z} & \xrightarrow{\widehat{\rho}} & \widehat{P}. \end{array} \tag{6.1}$$

Recall that Z is a Calabi-Yau threefold equipped with a holomorphic involution $\sigma : Z \rightarrow Z$ which fixes a smooth divisor R . P is the quotient Z/σ and $\rho : Z \rightarrow P$ is the canonical 2 : 1 projection map. Moreover, we assume that we can find a collection S'_i, S''_i , $i = 1, \dots, n$ of del Pezzo surfaces on Z disjoint from R so that σ maps S'_i to S''_i , $i = 1, \dots, n$. The del Pezzo surfaces must be contractible on Z , and $z : Z \rightarrow \widehat{Z}$ is a contraction map which collapses S'_i, S''_i to singular points p'_i, p''_i on \widehat{Z} .

Suppose we can find such a geometric set-up with $n = 2$. Then we can place conjugate D3-D5 systems at the singular points (p'_1, p''_1) , obtaining a supersymmetric configuration

as explained in the previous section. We can also place fractional D3-branes at the remaining singular points (p'_2, p''_2) realizing a supersymmetric standard model as in [28]. Open strings stretching between the two types of D-brane configurations mediate supersymmetry breaking. The mass of these open strings is controlled by the relative position of the points (p'_1, p'_2) (or, equivalently (p''_1, p''_2)) which is a complex modulus of Z .

Next let us construct a concrete example. We start with a smooth Weierstrass model Z' over a del Pezzo surface $S = dP_5$. Here we regard dP_5 as a four point blow-up of the Hirzebruch surface $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$. Let p'_i, p''_i , $i = 1, 2$ denote the centers of the blow-ups on \mathbb{F}_0 , and let e'_i, e''_i , $i = 1, 2$ denote the exceptional curves. If the fibration is generic, the restriction of the elliptic fibration to any (-1) curve C on S is isomorphic to a rational elliptic surface with 12 I_1 fibers, usually denoted by dP_9 . Therefore, taking C to be each of the four exceptional curves, we obtain four rational elliptic surfaces D'_i, D''_i . The exceptional curves e'_i, e''_i can be naturally identified with sections of the rational elliptic surface D'_i, D''_i . We can also naturally regard them as $(-1, -1)$ curves on Z' embedding S in Z' via the section of the Weierstrass model.

Threefolds Z' of this form appear quite often in F-theory, where the blow-ups in the base are associated to point-like small instantons in the dual heterotic string [55,59,60,61]. In this context, it is known that models with different number of blow-ups are related by extremal transitions which proceed as follows. One first performs a flop on the $(-1, -1)$ curves e'_i, e''_i in Z' , obtaining an elliptic fibration $Z \rightarrow \mathbb{F}_0$ with two complex dimensional components in the fiber. More precisely, the fibers over the points p'_i, p''_i have two components: a rational $(-1, -1)$ curve obtained by flopping one of the curves e'_i, e''_i and a dP_8 del Pezzo surface. We will denote the dP_8 components by S'_i, S''_i as above. Next, one can contract the del Pezzo surfaces in the fiber, obtaining a singular elliptic fibration \widehat{Z} over \mathbb{F}_0 , which can be eventually smoothed out by complex structure deformations.

Our example will be a threefold Z obtained at the intermediate stage of the extremal transition. All the required elements are in place apart from the holomorphic involution σ , which can be realized as follows. In the above construction, we restrict ourselves to a class of symmetric dP_5 surfaces S obtained by blowing-up conjugate points on \mathbb{F}_0 under a holomorphic involution $\kappa : \mathbb{F}_0 \rightarrow \mathbb{F}_0$. Using homogeneous toric coordinates

$$\begin{array}{ccccc}
& Z_1 & Z_2 & Z_3 & Z_4 \\
\mathbb{C}^* & 1 & 1 & 0 & 0 \\
\mathbb{C}^* & 0 & 0 & 1 & 1
\end{array} \tag{6.2}$$

on \mathbb{F}_0 , κ is given by

$$\kappa : (Z_1, Z_2, Z_3, Z_4) \rightarrow (Z_1, -Z_2, Z_3, Z_4). \quad (6.3)$$

Note that the fixed loci of κ are two disjoint curves on \mathbb{F}_0 determined by the equations

$$Z_1 = 0 \quad Z_2 = 0. \quad (6.4)$$

We pick (p'_1, p'_1) and (p'_2, p''_2) to be pairwise conjugate under κ and away from the fixed curves. Then a simple local computation shows that κ lifts to a holomorphic involution of S which maps e'_i isomorphically to e''_i . Abusing notation, we will also denote by κ the lift to S . The distinction should be clear from the context. Note that the fixed locus of κ on S consists of two curves C_1, C_2 which are the strict transforms of (6.4).

The elliptic fibration Z' can be written as a hypersurface in the projective bundle $\mathcal{P} = \mathbb{P}(\mathcal{O}_S \oplus K_S^{-2} \oplus K_S^{-3})$ over S . The involution $\kappa : S \rightarrow S$ constructed in the last paragraph can be trivially lifted to the total space of \mathcal{P} by taking the action on the homogeneous coordinates along the fibers to be trivial. Then we can obtain a symmetric threefold Z' by taking the defining polynomials of the Weierstrass model to be invariant under the involution $\kappa : S \rightarrow S$. Any such threefold would be preserved by the holomorphic involution on the ambient space \mathcal{P} , therefore it is equipped with an induced holomorphic involution σ' . By construction, σ' maps a rational elliptic surface D'_i to a rational elliptic surface D''_i , and the induced map is compatible with the fibration structure.

Moreover, one can check that we also have an induced holomorphic involution σ on the threefold Z obtained from Z' by flopping the curves (e'_i, e''_i) . In order to see this, let us recall the geometric description of the flop [87]. One first blows-up Z' along the $(-1, -1)$ curves (e'_i, e''_i) obtaining a non-Calabi-Yau threefold T . The exceptional divisors on T are four surfaces (Q'_i, Q''_i) , $i = 1, 2$ isomorphic to \mathbb{F}_0 . The involution σ' on Z' lifts naturally to an involution $\tau : T \rightarrow T$ which maps Q'_i isomorphically to Q''_i . Each of the surfaces Q'_i, Q''_i is equipped with two rulings which will be denoted by a'_i, b'_i , and respectively a''_i, b''_i , $i = 1, 2$. These rulings are preserved by the holomorphic involution.

Now, there are two possible contractions of T . One can find a contraction map which collapses all the a -rulings a'_i, a''_i on the surface Q'_i, Q''_i , obtaining Z' . Alternatively, one can find another contraction map which collapses the b -rulings b'_i, b''_i , obtaining the threefold Z related to Z' by a flop. Since the involution τ preserves the rulings, it follows that in both cases it induces involutions on Z' and respectively Z . In the first case, this is just the

holomorphic involution σ' we started with. In the second case, we obtain a holomorphic involution σ on Z with all the required properties.

Some final remarks are in order.

i) Note that the fixed point set of $\sigma : Z \rightarrow Z$ consists of two K3 surfaces obtained by restricting the Weierstrass model to the fixed curves C_1, C_2 of κ in the base S .

ii) By construction, the relative position of the points (p'_1, p'_2) on \mathbb{F}_0 is a complex modulus of Z which controls the mass of the open strings mediating supersymmetry breaking.

iii) Note that in the present construction the Kähler moduli of the compact threefold Z can control only the overall size of the collapsing del Pezzo surfaces $S'_i, S''_i, i = 1, 2$. One cannot control the relative size of the exceptional curves within each surface.

Point *iii)* actually means it is unlikely that one can arrange for a reasonable symmetry-breaking pattern in the embedding of [28] in this compact model – attempts to choose FI terms to make the model realistic will result in hypercharge breaking. Readers who are interested in compact embeddings of that theory where the desired Yukawa and FI terms of [28] *can* be obtained by tuning closed string parameters, are reminded that the Calabi-Yau fourfold in Example II of §5.2 can be used for this purpose.

7. Conclusion

Many features of MSSM phenomenology are governed, not so much by the dynamics of supersymmetry breaking, but rather by the mechanism by which SUSY breaking is transmitted to the Standard Model. In this work we have initiated an attempt to construct stringy models where gravity mediation is subdominant to other mediation mechanisms. There are many obvious directions for future work:

- The models of §3 have large Λ_H and messenger mass M , as well as a large messenger index. Finding examples with smaller Λ_H, M and messenger index would clearly be desirable, and may enable construction of models with all relevant dynamics taking place much closer to the TeV scale.
- Several natural questions arise in this setting that could be attacked with the statistical approach to compactifications advocated in [88]. For instance, arranging for small messenger mass $M \ll M_s$ can be regarded as a tuning of parameters. Understanding in detail what fraction of flux vacua allow complex structure stabilization with small M could give one some sense of the difficulty of arranging for gauge mediation to be the dominant mechanism of supersymmetry breaking transmission, even in cases where hidden sector

DSB dominates over flux-induced moduli F-terms. A discussion of several other relevant questions one could pursue in the statistical framework appears in [89].

- There are several issues surrounding the compactification of models using the SUSY BOG mechanism of SUSY breaking [26] that need to be clarified. The field theory itself may or may not admit metastable vacua at $\mathcal{O}(\Lambda)$ in field space once the FI terms are rendered dynamical; and global effects may stabilize the FI terms in any case. Precise circumstances in which the various possibilities occur should be specified.
- The DSB quiver theories which have been discovered so far [26], arise on D-branes at singularities of Calabi-Yau threefolds. There are singularities of F-theory fourfold compactifications where one can imagine fractional branes localized at collapsed cycles, and the normal bundle of the brane in the base of the elliptic fibration, is such that the configuration could never arise in a Calabi-Yau threefold. It is quite plausible that more interesting examples of DSB from fractional branes in F-theory, will arise by analyzing the physics of these singularities.
- We have focused on gauge mediation as a possible solution to the flavor problems of SUSY breaking in string theory. From a conceptual perspective, since many models of gauge mediation work at parametrically low energies and do not require assumptions about the UV embedding, it is less urgent to UV complete gauge mediated models than various alternatives (perhaps the most urgent issue being the need to find a plausible solution to the μ problem). Finding stringy avatars of gaugino mediation [90] and anomaly mediation [91] (including some solution to the problem of tachyonic sleptons in the latter case) would certainly be worthwhile.
- Due to the UV insensitivity of gauge mediation, it would make sense (somewhat in the spirit of [28]) to engineer non-compact brane models containing an MSSM or GUT, a SUSY breaking sector, and the messengers. The resulting gauge mediated spectrum would be largely insensitive to the global details of compactification in any compact embedding. While one can obviously infer some such models by taking limits of our compact constructions, it may be easier to identify promising classes of models in the non-compact setting first. Early work in this direction appears in [92].
- Finally, the UV properties of string theory may suggest novel new possibilities for the transmission of SUSY breaking, or realizations of gravity mediation that solve the flavor problem. Developments along these lines would be very interesting.

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