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The computer code NOVO for the calculation of wake potentials of the very short ultra-relativistic bunches*

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Abstract

The problem of electromagnetic interaction of a beam and accelerator elements is very important for linear colliders, electron-positron factories, and free electron lasers. Precise calculation of wake fields is required for beam dynamics study in these machines. We describe a method which allows computation of wake fields of the very short bunches. Computer code NOVO was developed based on this method. This method is free of unphysical solutions like "self-acceleration" of a bunch head, which is common to well known wake field codes. Code NOVO was used for the wake fields study for many accelerator projects all over the world.

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The problem of electromagnetic interaction of a beam and accelerator elements is very important for linear colliders, electron-positron factories, and free electron lasers. Precise calculation of wake fields is required for beam dynamics study in these machines. We describe a method which allows computation of wake fields of the very short bunches. Computer code NOVO was developed based on this method. This method is free of unphysical solutions like "self-acceleration" of a bunch head, which is common to well known wake field codes. Code NOVO was used for the wake fields study for many accelerator projects all over the world.

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I. INTRODUCTION

Twenty years ago, we developed a computer algorithm to calculate wake fields and wake potentials of a very short bunch in a very long accelerator structure. Based on this algorithm we designed a computer code NOVO. For some reasons we did not formally publish it. However, we presented the algorithm at the Workshop on "Advance in Electromagnetic Research" in September 1998, held in Kleinwalsertal, Austria [1] and Collective Effects Group Meeting at SLAC in October 2001[2].

We used code NOVO for the wake field study for many accelerator projects all over the world: electron-positron injector of the Novosibirsk B-factory [3]; TESLA Linear Collider and TESLA Free Electron Laser [4-11]; SOLEIL light source [12, 13]; Next Linear Collider (NLC) [14]; PEP-II B-factory [14-16] and Super B-factory [17-19]. We did many calculations in order to understand the nature of the wake fields excited because of the surface roughness of beam pipes [20-24]. We carried out these studies at BINP, Novosibirsk (Russia), CEA, Saclay (France), Darmstadt University (Germany), and SLAC, Stanford (USA).

As demand for the calculations of wake potentials of the very short bunches is still high, we decided to make this publication. One can definitely apply code NOVO for the beam dynamics studies for International Linear Collider (ILC), Linac Coherent Light Source (LCLS) including Sub-Picosecond Pulsed Source (SPPS).

II. THE ALGORITHM

Main equation

We consider here the case of azimuthally symmetrical electromagnetic fields. Naturally, main equation comes from the Maxwell equations:

$$\begin{aligned}
&\frac{\partial \varepsilon_0 E_z}{\partial t} = \frac{1}{r}\frac{\partial (rH_\varphi)}{\partial r} - j_z \\
&\frac{1}{r}\frac{\partial (r\varepsilon_0 E_r)}{\partial r} + \frac{\partial \varepsilon_0 E_z}{\partial z} = \rho \\
&\frac{\partial \mu_0 H_\varphi}{\partial t} = -\frac{\partial E_r}{\partial z} + \frac{\partial E_z}{\partial r} \\
&\frac{\partial \varepsilon_0 E_r}{\partial t} = -\frac{\partial (H_\varphi)}{\partial z}
\end{aligned} \tag{1}$$

For the wake potential calculations, we usually consider the case when charged particles travel with a constant velocity $\vec{V} = V_z$ in z-direction. Then we can separate variables for the charge density $\rho(r,z,t)$ and current density $j_z(r,z,t)$

$$\begin{aligned}
&\rho(r,z,t) = \rho(r, V_z t - z) \\
&j_z(r,z,t) = V_z \bullet \rho(r,z,t) \cdot
\end{aligned} \tag{2}$$

Let us use the flux of electric field

$$\Phi(r,z,t) = 2\pi \int_0^r \varepsilon_0 E_z(r',z,t)\, r' dr' \tag{3}$$

and the bunch linear density

$$\lambda = 2\pi \int_0^r \rho(r', V_z t - z)\, r' dr' \tag{4}$$

to describe the electric and magnetic field components:

$$\begin{aligned}
&E_z = \frac{1}{2\pi\varepsilon_0 r}\frac{\partial}{\partial r}\Phi \qquad E_r = \frac{1}{2\pi\varepsilon_0 r}\left(\lambda - \frac{\partial}{\partial z}\Phi\right) \\
&H_\varphi = \frac{c}{2\pi r}\left(\lambda\frac{V_z}{c} + \frac{\partial}{c\partial t}\Phi\right)
\end{aligned} \tag{5}$$

Substitution (5) into the third equation of (1) yields a second order equation for the flux $\Phi(r,z,t)$

$$\frac{\partial^2\Phi}{c^2\partial t^2} - \frac{\partial^2\Phi}{\partial z^2} - r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi}{\partial r}\right) = \left(\frac{V_z^2}{c^2} - 1\right)\frac{\partial\lambda}{\partial z} \tag{6}$$

We used the following relation for the speed of light $c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}$; where ε_0 is permittivity and μ_0 is permeability of the vacuum. The right-side term in (6) vanishes for ultra-relativistic particle ($V_z = c$). In this case, equation (6) takes very simple form

$$\frac{\partial^2 \Phi}{c^2 \partial t^2} - \frac{\partial^2 \Phi}{\partial z^2} - r \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial r}\right) = 0 \qquad (7)$$

Boundary condition at the high-conductivity metal wall $\vec{n} \times \vec{E} = 0$ now takes the following form

$$\vec{n} \cdot \left(\vec{\nabla}\Phi - \frac{\vec{V}}{c}\lambda\right) = 0\,. \qquad (8)$$

Explicit algorithm

Well-known wake field codes MAFIA [25] and ABCI [26] use explicit finite-difference algorithm. Let us check the implicit algorithm for our equation (7)

$$\Phi_k^{n+1} - 2\Phi_k^n + \Phi_k^{n-1} = \left(\frac{c\Delta t}{\Delta z}\right)^2 \left(\Phi_{k+1}^n - 2\Phi_k^n + \Phi_{k-1}^n\right) + \left(\frac{c\Delta t}{\Delta r}\right)^2 \times r\Delta\frac{1}{r}\Delta\Phi_k^n \qquad (9)$$

Here we denote index *n* is for time and index *k* is for z coordinate. Stability condition easily comes from the Fourier analyses $\Phi_k^n \sim e^{i\omega t + i\beta z + i\alpha r}$. In the case, when $r >> \Delta r$ relation between frequency ω and wave vector with longitudinal component β and radial component α is

$$\sin^2\frac{\omega\Delta t}{2} = \left(\frac{c\,\Delta t}{\Delta z}\right)^2 \times \sin^2\frac{\beta\,\Delta z}{2} + \left(\frac{c\,\Delta t}{\Delta r}\right)^2 \times \sin^2\frac{\alpha\,\Delta r}{2} \qquad (10)$$

For stability reason we need ω to be real. It happens when right part of (10) is less than one. To have stability for any β and α we need

$$\left(\frac{c\Delta t}{\Delta z}\right)^2 + \left(\frac{c\Delta t}{\Delta r}\right)^2 \le 1 \qquad (11)$$

On the other hand equation (10) is the dispersion relation of the free "finite-difference domain". It shows that phase velocities of the free electromagnetic waves are different for different frequencies in the "finite-difference domain". In the case when $\alpha = 0$ phase velocity is

$$V_{ph} = \frac{\omega}{\beta} = c\frac{2}{\beta c\Delta t} \times \arcsin\left(\frac{c\Delta t}{\Delta z}\sin\frac{\beta\Delta z}{2}\right) \qquad (12)$$

We show the plot of the phase velocity at Fig. 1 as a function of frequency for different ratio of coordinate step to time step.

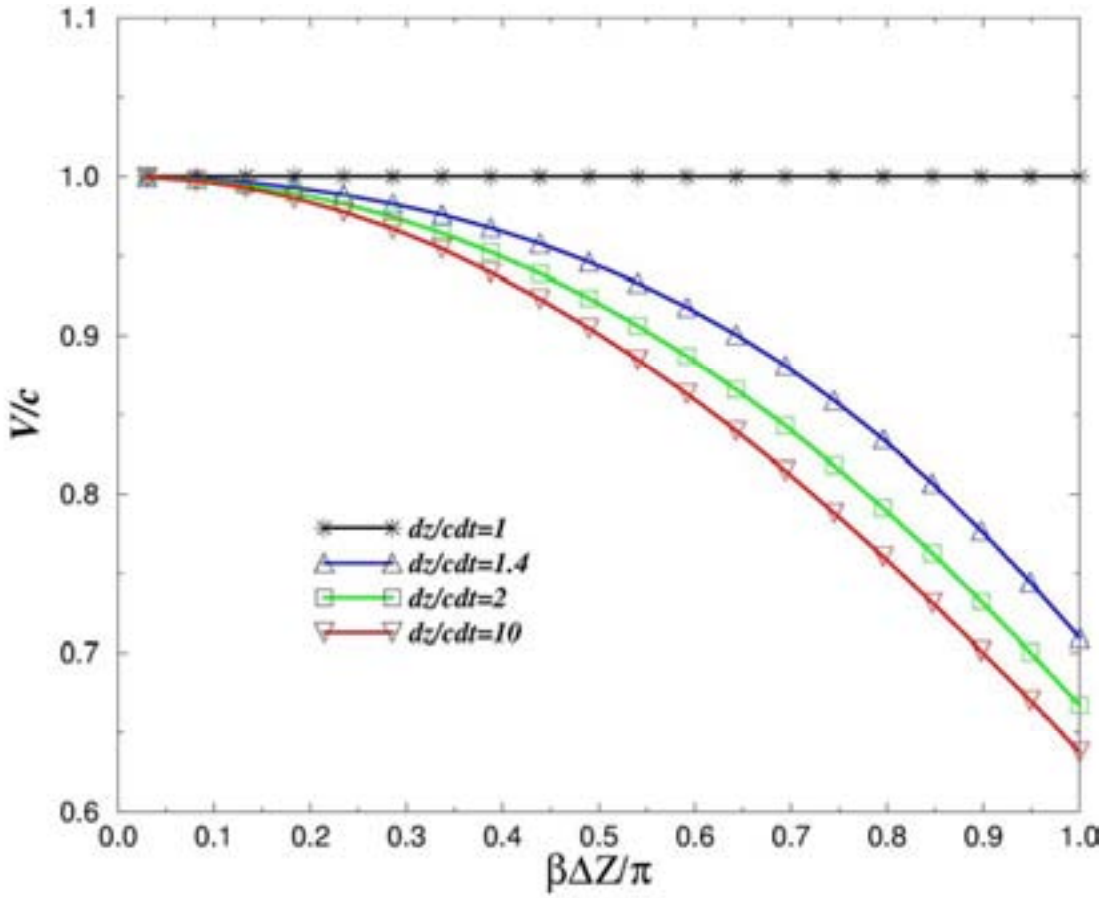

Figure 1: Frequency dependence of the phase velocity in the "finite-difference domain" for different ratio of coordinate step to time step: 1, 1.4, 2, and 10.

We can notice that the phase velocity decreases strongly with frequency when the ratio between coordinate step and time step is more than one. Strong frequency dependence of the phase velocity may destroy the time structure of "finite-difference" fields. It may develop a strong diffusion of an initially smooth field distribution and reveal high frequency oscillations. These all lead to unphysical results like "self-acceleration" of a bunch head. Explicit scheme is not good for the calculation of the wake fields of the very short bunches.

Implicit scheme

Equation (10) shows that under the condition of equivalent step of time and coordinate we will not have dispersion. We can realize this condition by using the stable implicit scheme. We really need only "partially" implicit scheme, which for our equation is the following:

$$\Phi_k^{n+1} - 2\Phi_k^n + \Phi_k^{n-1} = \left(\frac{c\Delta t}{\Delta z}\right)^2 \left(\Phi_{k+1}^n - 2\Phi_k^n + \Phi_{k-1}^n\right) + \frac{1}{2}\left(\frac{c\Delta t}{\Delta r}\right)^2 \times r\Delta\frac{1}{r}\Delta\left(\Phi_k^{n+1} + \Phi_k^{n-1}\right) \qquad (13)$$

Dispersion relation for this scheme is:

$$\sin^2\frac{\omega\Delta t}{2} = \frac{\left(\frac{c\Delta t}{\Delta z}\right)^2 \sin^2\beta\frac{\Delta z}{2} + \left(\frac{c\Delta t}{\Delta r}\right)^2 \sin^2\alpha\frac{\Delta r}{2}}{1 + 2\left(\frac{c\Delta t}{\Delta r}\right)^2 \sin^2\alpha\frac{\Delta r}{2}} \qquad (14)$$

Equation (14) shows that this scheme is stable in the case of equal coordinate and time step $c\Delta t = \Delta z$ and can be definitely used for calculations of the wake fields of the very shot short bunches.

Numerical solution

We will integrate (13) using "round trip" method in radial direction (index m)

$$a_m\Phi_{k,m+1}^{n+1} + b_m\Phi_{k,m}^{n+1} + c_m\Phi_{k,m-1}^{n+1} = d_m \qquad (15)$$

One can find coefficients a_m, b_m, c_m and d_m from (13). Coefficients d_m include longitudinal and radial derivatives at the two previous time steps.

To solve (15) we calculate coefficients α_m ,γ_m

$$\alpha_m = -\frac{a_m}{b_m + c_m\alpha_{m-1}} \quad \gamma_m = -\frac{\gamma_{m-1}c_m + d_m}{b_m + c_m\alpha_{m-1}} \qquad (16)$$

and then we calculate the flux

$$\Phi_{k,m}^{n+1} = \alpha_m\Phi_{k,m}^{n+1} + \gamma_m \qquad (17)$$

Boundary conditions (8) will give us equations for values of coefficients at boundary points.

It is important to note that implicit scheme does not need more computer time than explicit one.

III. EFFICIENCY OF THE METHOD AND COMPARISON WITH OTHER METHODS

Comparison of the results attained with different size of the mesh can show the efficiency of our method. Fig. 2 shows the results of calculations of the wake potential of 1 mm bunch passing the SOLEIL [12] accelerating structure. This accelerating structure consists of two cavities and two tapers. Total structure length is 5 m. To make comparison we did calculations using different mesh sizes: 2, 4 and 10 times smaller than the bunch length. Grey circles, green line, and red line show correspondent wake potentials. Bunch has the Gaussian shape and is shown by the black dotted line.

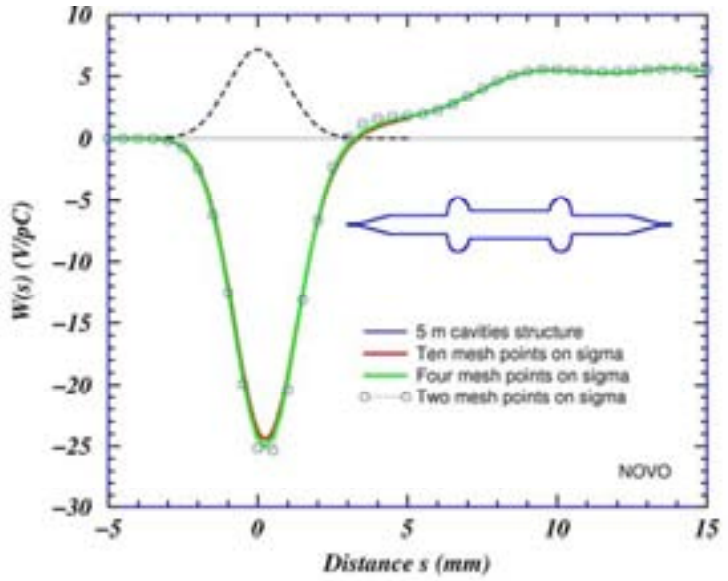

Figure 2: Comparison of the wake potentials, calculated with different mesh sizes.

We can see very good agreement between calculated wake potentials. Even two mesh points on the bunch length are enough to get good accuracy of the potential. Wake potentials do not have high frequency modulation and diffusion.

In 1997, B. Zotter (CERN) did comparison of the results of the wake field calculations using different codes: NOVO, MAFIA/T2 and ABCI (Fig.3). He came to the conclusion that "The first code (NOVO) appears to be much more efficient and hence faster than the codes which we have been using at CERN so far" [27].

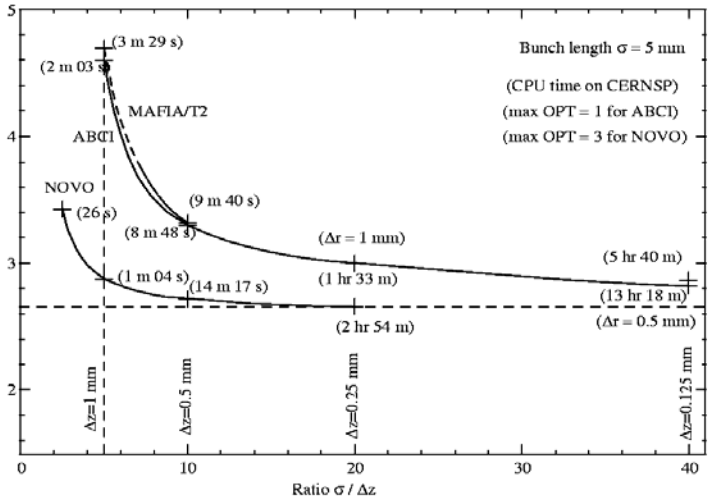

Figure 3: Loss factor comparison (from the reference [27])

VI. SOME APPLICATIONS

It is easier to understand the behavior of the wake fields when we see the plot of electric force lines. It is very simple to make such a plot in our case, because electric force line is just a line where the flux of electric field is constant $\Phi(z,r) = const$. Fig. 4 shows the plot of electric force lines of the wake field left by a short bunch in the NLC accelerating section. The position of the bunch is at the exit of the section. The bunch length is 100 micron. We can clearly see the diffraction behavior of the wake field and excitation of the high frequency modes.

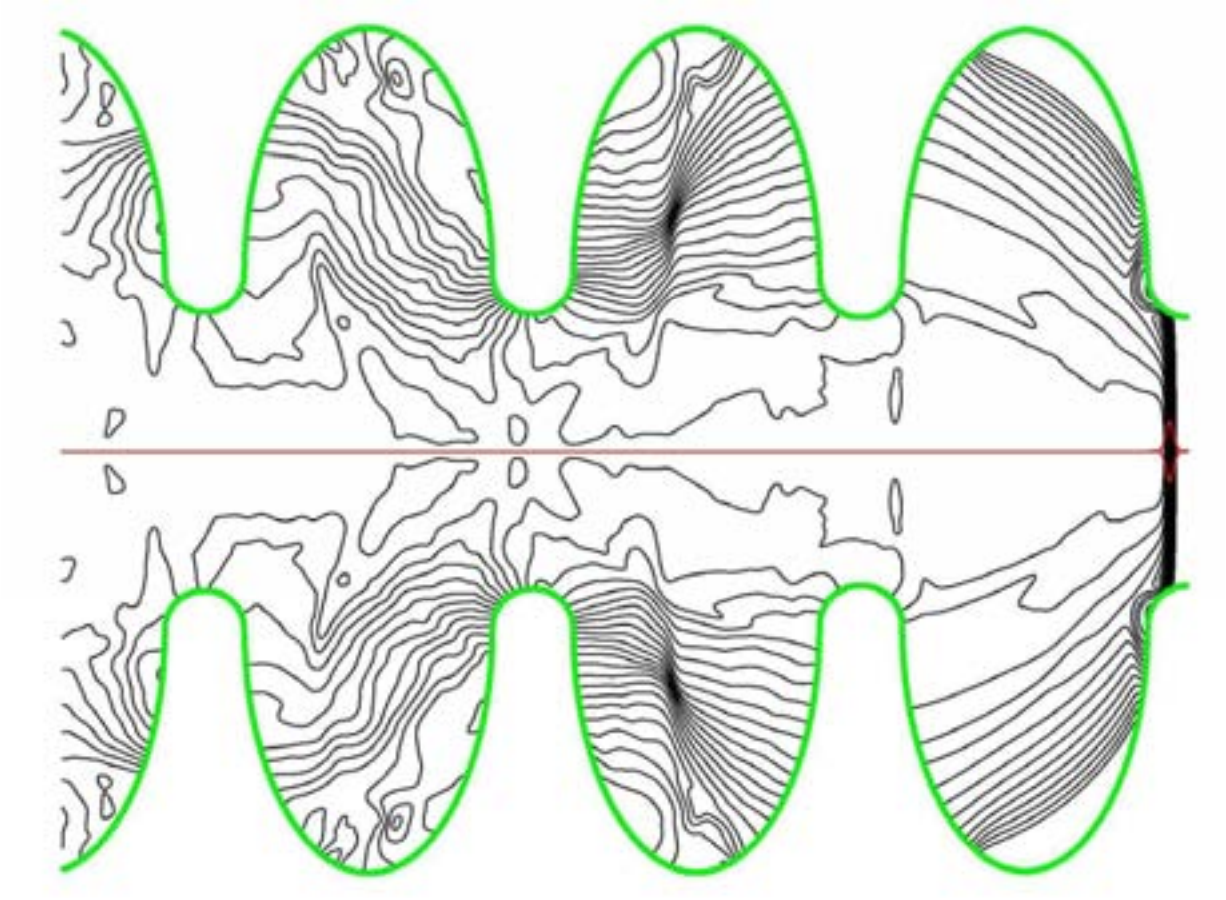

Figure 4: Electric force lines of the wake fields left by a 100-micron bunch in the NLC section.

Another example, presented at Fig. 5 shows a plot of the wake fields excited due to the surface roughness. Here we simulate the surface roughness as random bumps in average of 5-micron height and 50-micron width. A bunch of 10-micron length is moving in the tube of 5 mm

radius. Fig. 5 contains a full plot shown on the left and the zoomed part of the surface shown on the right.

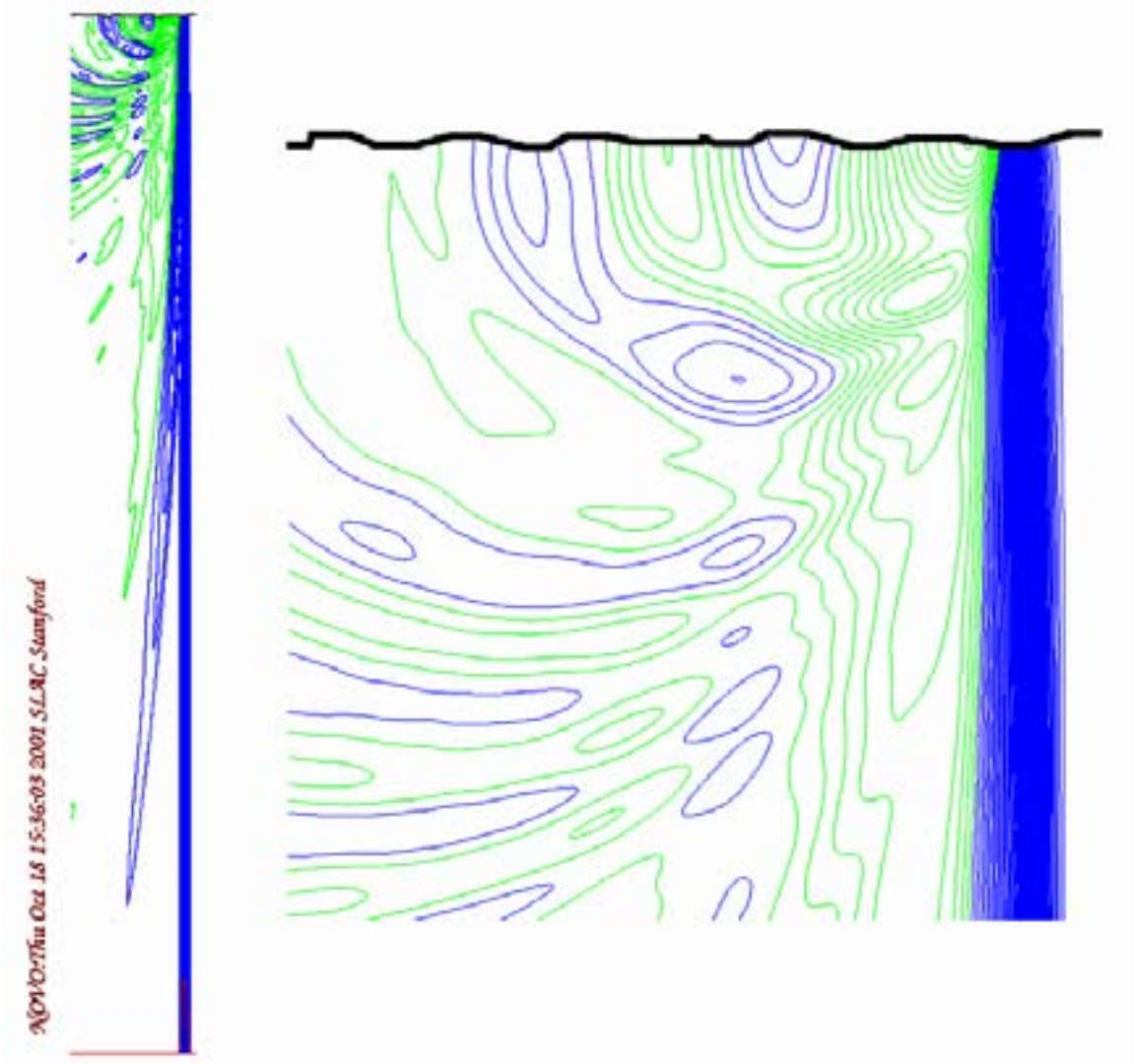

Figure 5: Electric force lines of the field excited by a 10-micron bunch in a surface roughness tube.

This is the record for such kind of calculation: simulated surface roughness thickness is less than a beam pipe radius by more than one thousand times.

In the next example, code "NOVO" was used to calculate wake fields excited in a very small gap between RF gasket and a flange wall in a pair of flanges of beam chamber. Results of calculations helped to understand the nature of temperature rise in one flange connection in the beam pipe of PEP-II low energy ring. Fig. 6 shows the results of these calculations [15]. We easily

managed to calculate wake fields even in the gap size of 40 micron of a beam tube of 89 mm in diameter.

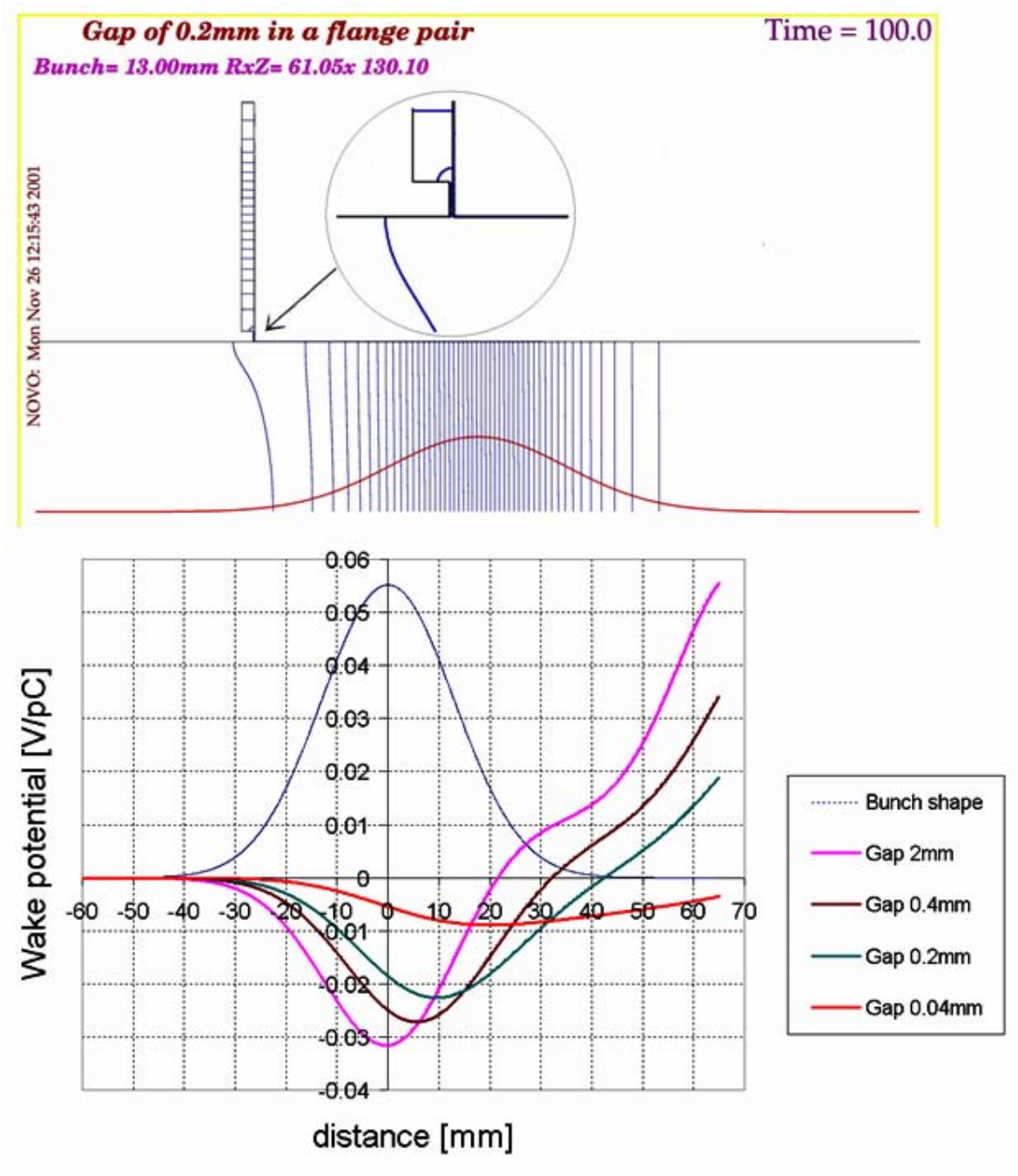

Figure 6: Wake fields excited due to small gap between a RF gasket and a flange wall. Bunch length is 13 mm. Gap size varies from 2 mm to 40 micron.

Very good stability of the algorithm allows to calculate wake potentials of almost infinite length. Next example (Fig.7) shows wake potential and frequency spectrum of the PEP-II RF cavity [16]. Upper plot shows wake fields of a short bunch (1.8 mm) in the PEP-II cavity. Middle plot shows wake potential. The length of the wake potential is 4 m. Bottom picture shows real and imaginary parts of Fourier spectrum of the wake potential. Calculated impedances and frequencies of the main and higher order modes are in very good agreement with RF measurement.

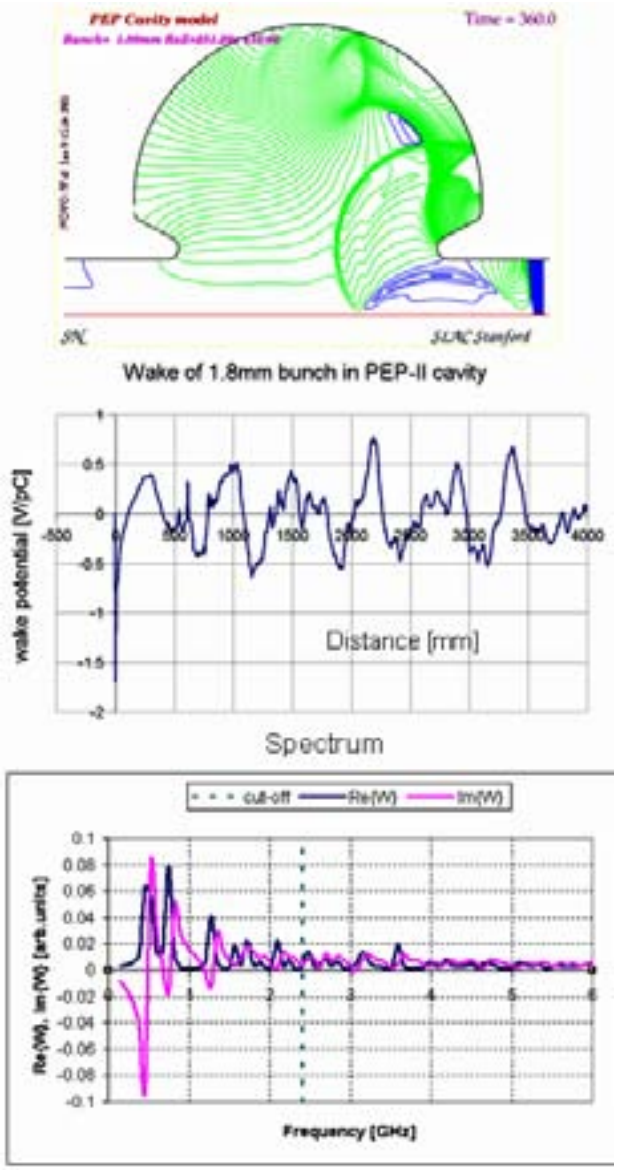

Figure 7: Wake fields in the PEP-II cavity excited by 1.8 mm bunch (upper picture). Wake potential (middle picture). Real and Imaginary parts of Fourier spectrum (bottom picture).

Finally, we present the formation of wake potential of 200-micron bunch in semi-finite periodic accelerator structure of the TESLA type cells [4]. Every picture at Fig. 8 shows variation of the wake potential in consecutive nine cells.

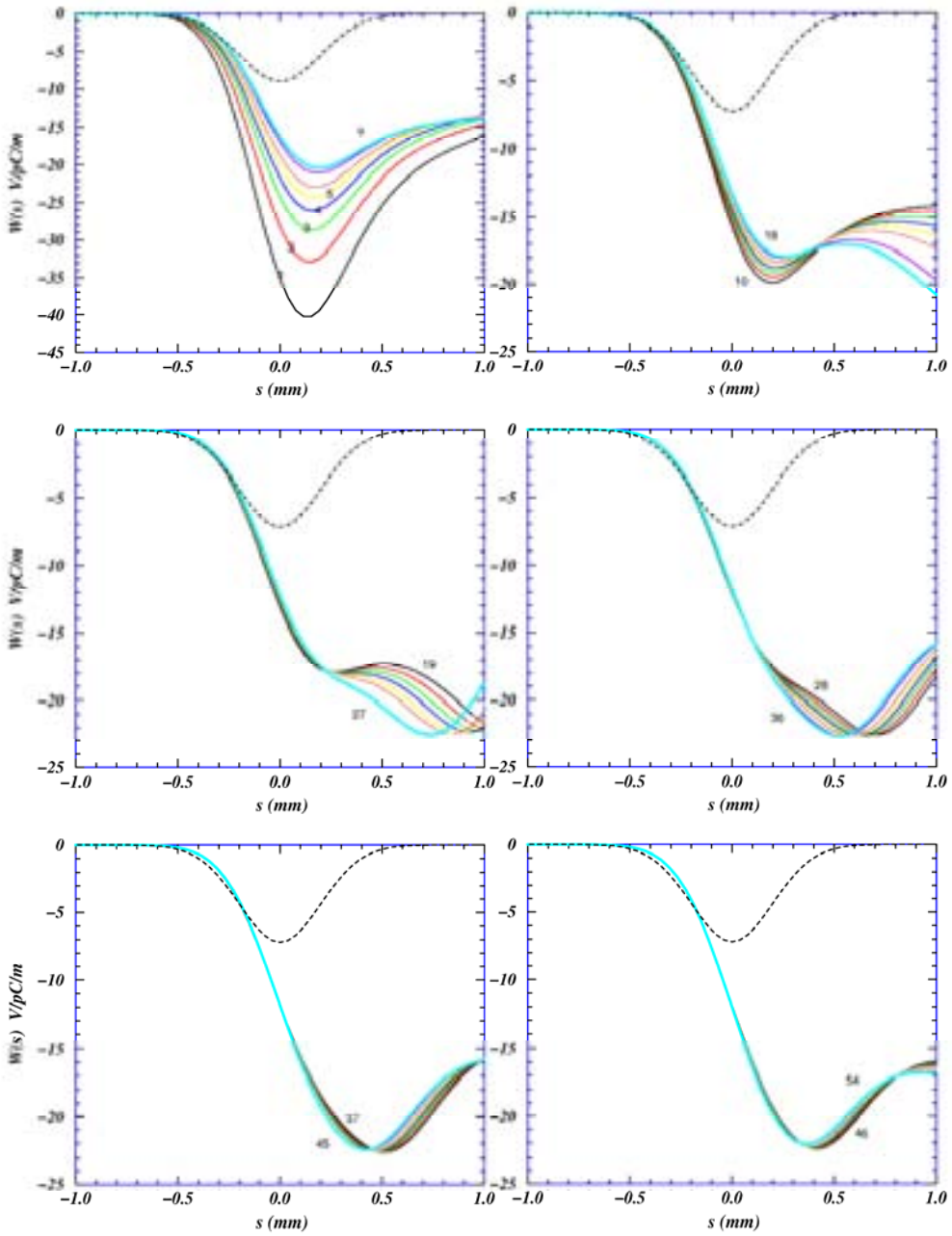

Figure 8: Formation of the wake field in semi-infinite periodic structure of TESLA type cells. Bunch length is 200 micron.

Even after passing 54 cells, the formation of the wake potential is not finished. Wake potential still does not have steady-state distribution.

Last example (Fig. 9) shows how irregularities of the beam chamber change the shape of the electric field of a very short bunch (10 micron). Holger Schlarb (DESY) suggested to do this calculation for the SPPS experiment. Fig. 9 shows the geometry of the transport beam line, which is of 1.7 m long; field distribution in the center of the line (0.8 m) after the "scrapper" (left plot) and near the crystal (right plot). Filed plots have different scales in z and r-directions.

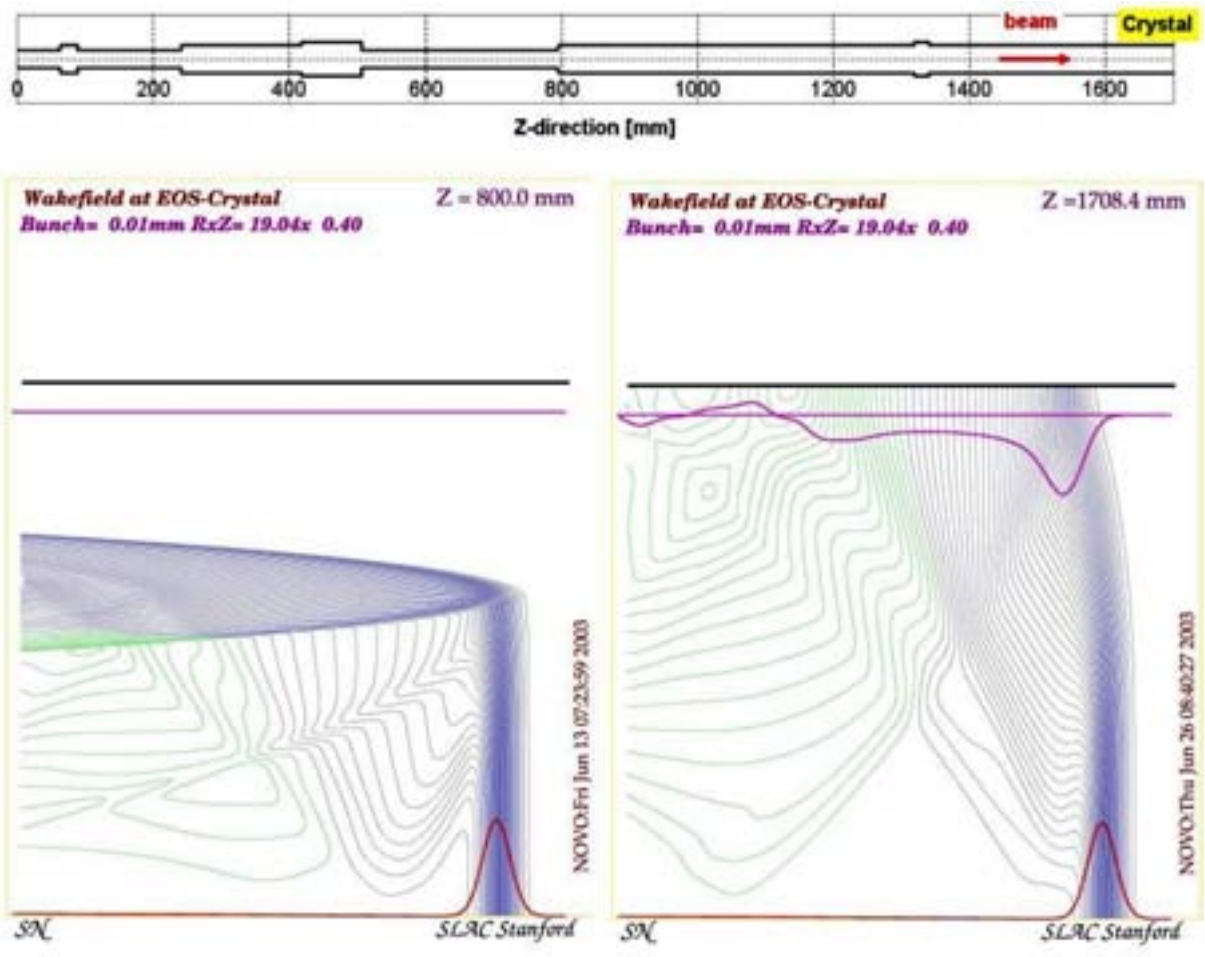

Figure 9: Wake fields of 10-micron bunch in the beam transport line: in the center (left plot) and near the crystal (right plot). Magenta line shows electric field at the chamber wall. Scales in z and r-directions are different.

In the experiment, the electric field in the crystal is used for the bunch length measurement. We can see that irregularities in geometry of the transport line changed the bunch field distribution near the

crystal. Therefore, the measured bunch length may be several times larger than the real one. Loss factor in this case is 640 V/pC that means that particles of the bunch with population of $2*10^{10}$ will lose in average 2 MeV of the kinetic energy. Additional energy spread will be of the order of 2.8 MeV.

V. CODE DESCRIPTION

The code NOVO is a 2-dimensional code for the structures with rotational symmetry. It is written in FORTRAN. The input text file contains description of the geometry by a set of points, which are to be connected by straight lines, circles, or ellipses. Additionally input data contains bunch length, mesh step and the title. The input text file contains wake potential and loss factor. If needed additional programs may be used to make PostScript plots of electric force lines and wake potentials.

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