# Scaling properties of high $p_T$ inclusive hadron production<sup>\*</sup>

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Submitted to Physics Letters B

<sup>\*</sup>Work supported in part by the Department of Energy under contract number DE–AC02–76SF00515.

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We analyze the scaling properties of inclusive hadron production in proton-proton and in heavy ion collisions from fixed target to collider energies. At large transverse momentum  $p_T$ , the invariant cross section exhibits a power-like behavior  $Ed^3\sigma/d^3p \propto p_T^{-n}$  at fixed transverse x,  $x_T = \frac{2|\vec{p}_T|}{\sqrt{s}}$ , and fixed center-of-mass scattering angle  $\theta_{cm}$ . Knowledge of the exponent n allows one to draw conclusions about the production mechanisms of hadrons, which are poorly known, even at high  $p_T$ . We find that high- $p_T$  hadrons are produced by different mechanisms at fixed-target and collider energies. For pions, higher-twist subprocesses where the pion is produced directly dominate at fixed target energy, while leading-twist partonic scattering plus fragmentation is the most important mechanism at collider energies. High- $p_T$  baryons on the other hand appear to be produced by higher-twist mechanisms at all available energies. The higher-twist mechanism of direct proton production can be verified experimentally by testing whether high  $p_T$  protons are produced as single hadrons without accompanying secondaries. In addition, we find that medium-induced gluon radiation in heavy ion collisions can violate scaling.

PACS: 13.85.Ni, 24.85.+p

#### I. INTRODUCTION

A fundamental feature of quantum chromodynamics (QCD) and asymptotic freedom is the nearly scale-invariant behavior of quark and gluon two-particle hard-scattering processes. If these pointlike hard-scattering subprocesses are convoluted with the structure functions of the incident hadrons and the fragmentation functions which produce final state interactions, the resulting inclusive cross section  $E^{\frac{d^3\sigma(h_ah_b\to hX)}{d^3p}}$  scales nominally as  $p_T^{-4}$  at fixed  $x_R=\frac{2|\vec{p}_{cm}|}{\sqrt{s}}$  and  $\theta_{cm}$  (We denote the center-of-mass (cm.) scattering angle by  $\theta_{cm}$  and the three momentum of the produced hadron in the cm. frame by  $\vec{p}_{cm}$ . The cm. energy is  $\sqrt{s}$ ). In order to validate this scaling at  $\theta_{cm}=90^\circ$ , one needs measurements at various cm. energies and corresponding values of  $\vec{p}_{cm}$ . The conformal scaling prediction is modified by the logarithmic running of the QCD coupling and the logarithmic corrections to the scale-free parton model arising from the evolution of the structure functions and fragmentation functions. These results are rigorous consequence of perturbative QCD factorization theorems for inclusive hadron reactions at large  $p_T^2$ .

In the same way as Bjorken scaling provides evidence for quarks in deep inelastic lepton scattering, the scaling behavior of the cross sections for the production of high  $p_T$  particles in hadronic collisions can be used to test the scaling of the underlying QCD subprocesses as well as the existence of point-like constituents each carrying a finite fraction of the hadron's momentum [1, 2]. However, in some cases such as hard exclusive reactions, the underlying hard subprocess cannot be the simple  $2 \to 2$  reactions; since all of the valence quarks of the interacting hadrons are involved in the scattering process. Even in inclusive reactions, higher parton number processes can contribute, particularly at high  $x_R$  where there is a trade-off between the fall-off at high  $p_T$  which favors minimal number of scattering partons and the fall-off at  $(1 - x_R) \to 0$  which favors more valence partons entering the hard subprocess.

It is clearly important to carefully analyze the scaling behavior of inclusive reactions in order to confirm the validity and applicable kinematic range of the leading-twist  $2 \to 2$  subprocesses. Our aim is to determine the mass dimension of the microscopic matrix element from dimensional and spectator counting rules [3–5], which have recently been derived nonperturbatively for nearly-conformal theories using AdS/CFT duality [6, 7].

Since the dimension of this matrix element is related to the number of participating elementary fields, such an analysis provides detailed information about the specific microscopic hard process underlying high  $p_T$  hadron production. This is particularly important for the interpretation of hard processes in nuclear collisions.

### II. DIMENSIONAL COUNTING RULES

The partonic S matrix element is related to the partonic amplitude by  $S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^{(4)}(\sum p_{in} - \sum p_{out}) A_{fi}$ . With single-particle states normalized to  $\langle p|p'\rangle = 2E_p(2\pi)^3 \delta^{(3)}(\vec{p}-\vec{p'})$ , the amplitude  $A_{fi}$  on the microscopic level has dimension  $mass^{4-n_a}$ . Here  $n_a$  is the number of active fields, *i.e.* the number of elementary fields entering  $A_{fi}$ . The simplest example is  $2 \to 2$  scattering between quarks and gluons. In this case,  $n_a = 4$  and the partonic matrix element is dimensionless, as is natural for a scale invariant theory. However, because of color confinement the partonic subprocess cannot be observed directly, and one needs to find a way to connect to the hadronic reaction.

The kinematics of an inclusive reaction  $h_a h_b \to hX$  is described by 3 Lorentz invariants. These are e.g. the center-of-mass energy squared  $s = (P_a + P_b)^2$ , the transverse momentum transfer (squared)  $t = (P_h - P_a)^2$  and the missing mass  $M_X$ . It is common to introduce the dimensionless variables  $(u = M_X^2 - s - t)$ ,

$$x_1 = -\frac{u}{s}, \quad x_2 = -\frac{t}{s}.$$
 (1)

These variables are related to the rapidity y and radial  $x_R$  of the observed hadron by

$$y = \frac{1}{2} \log \left( \frac{x_1}{x_2} \right), \tag{2}$$

$$x_R = \frac{2|\vec{p}_{cm}|}{\sqrt{s}} = 1 - M_X^2/s \approx x_1 + x_2,$$
 (3)

Since most of existing data are at y=0 where  $x_R=x_T=2p_T/\sqrt{s}$ , one often refers to the scaling of the invariant cross section as " $x_T$  scaling". For  $y\neq 0$ , we find the variable  $x_R$  more useful than  $x_T$ , since  $x_R$  allows a smooth matching of inclusive and exclusive reactions in the limit  $x_R\to 1$ .

We shall assume that at high  $p_T$ , the inclusive cross section takes a factorized form, even if the microscopic mechanism is higher twist,

$$d\sigma(h_a h_b \to hX) = \sum_{abc} G_{a/h_a}(x_a) G_{b/h_b}(x_b) dx_a dx_b \frac{1}{2\hat{s}} |A_{fi}|^2 dX_f D_{h/c}(z_c) dz_c. \tag{4}$$

The dimensionless functions  $G_{a/h_a}(x_a)$  describe the momentum distributions of partons of type a in hadron  $h_a$ , where a may stand for quarks and gluons as well as for composite degrees of freedom, such as diquarks and intrinsic hadrons. These functions cannot be calculated perturbatively, except in the limits  $x_{a,b} \to 1$ . For quarks and gluons, the scale dependence of the distribution functions is described by the DGLAP evolution equations, but the evolution of color-neutral degrees of freedom is suppressed by at least one power of the hard scale, since gluon radiation off color neutral objects is suppressed. Similar observations can be made for the fragmentation function  $D_{h/c}(z_c)$ , which accounts for the transition of a parton c into a hadron h with momentum fraction  $z_c = x_1/x_a + x_2/x_b$ . The amplitude of the hard subprocess  $A_{fi}$  is assumed to be calculable in perturbative QCD. Integration and summation over all unobserved variables, such as the phase space  $dX_f$  of the final state, is understood.

By keeping all ratios of Mandelstam variables fixed, the x dependence of the distribution functions does not affect the scaling behavior of the hadronic cross section. The factorization hypothesis Eq. (4) then yields the power law

$$E\frac{d^3\sigma(h_ah_b \to hX)}{d^3p} = \frac{f(t/s, u/s)}{s^{n_a-2}},\tag{5}$$

which reflects the mass dimension of the microscopic amplitude. Hence, the inclusive cross section multiplied by  $p_T^n$  with

$$n = 2n_a - 4, (6)$$

is a function of the dimensionless variables y and  $x_R$  only,

$$E\frac{d^3\sigma(h_ah_b \to hX)}{d^3p} = \frac{F(y, x_R)}{p_T^{n(y, x_R)}}.$$
 (7)

This is the desired relation: the  $p_T$  dependence of the inclusive cross section is directly related to the number of participants  $n_a$  in the microscopic matrix element. The large- $p_T$  behavior of Eq. (7) is determined by the minimum number of active partons associated with the reaction  $h_a + h_b \rightarrow h + X$ . Of course, n will depend on y and  $x_R$ . Let us illustrate the scaling behavior by the following two examples:

- For  $pp \to pX$ , one can have the subprocess  $uu \to uud\bar{d}$ , which has  $n_a = 6$  rather than 4 active elementary fields. Because of proton compositeness, the inclusive cross section  $E \frac{d^3\sigma(pp\to pX)}{d^3p}$  scales nominally as  $\frac{f_N^2}{p_T^3}$  at fixed  $x_R$  and y where the dimensional factor  $f_N$  reflects the physics of the proton distribution amplitude. In this process, the proton is made directly in the short distance reaction rather than from quark fragmentation or resonance decay. Because no energy is wasted in the fragmentation process, the cross section falls off relatively slowly  $\sim (1-x_R)^7$  at large  $x_R$ . In general, one finds the power law behavior  $(1-x_R)^{2n_s-1}$  of the inclusive cross section at large  $x_R$ , where  $n_s$  is the total minimum number of spectators in  $h_a, h_b$  and h, which do not participate in the hard scattering reaction. In this example  $n_s = 2 + 2 = 4$ . [We ignore here complications from parton spin.]
- In the limit  $x_R \to x_R^{max} \approx 1$ , the missing mass  $M_X$  reaches its minimum allowed value and the reaction becomes exclusive. In this case, there are no spectator fields, so that the number of active participants  $n_a$  attains its maximum value, e.g.  $n = 2n_a 4 = 20$  for  $pp \to pp$ . Thus, the leading-twist pQCD approach to high  $p_T$  hadron production must fail as  $x_R$  increases toward unity and the process becomes exclusive. From the definition of  $x_R$  in Eq. (3) it becomes clear that this is the case at large  $x_1$  and small  $x_2$  (or vice versa), i.e. for high  $p_T$  hadron production at large |y|.

Finally, we remark that  $x_{1,2}$  are not momentum fractions. The latter are denoted by  $x_{a,b}$ . The momentum fractions are defined only within a given parton model of hadron production, and cannot be related to hadronic invariants. Instead, the factorization ansatz for the cross section Eq. (4) leads to integrals over  $x_{a,b}$ . In the case of  $2 \to 2$  hard scattering, the lower integration limits are

$$x_a^{min} = \frac{x_2}{1 - x_1}, \qquad x_b^{min} = \frac{x_2 x_a}{x_a - x_1}.$$
 (8)

In the exclusive limit  $x_{1,2} \to (1 \pm x_F)/2$  ( $x_F$  is Feynman x), both momentum fraction approach unity, so that only valence partons are important. Hence, inclusive hadron production at very large rapidity is unaffected by gluon saturation. Such coherence effects disappear at the largest  $x_F$ . (The authors of Ref. [8] come to a similar conclusion from a different viewpoint.) This shows that the high energy limit of QCD cannot be completely described by the

color glass condensate [9]. However, (nearly) exclusive reactions at  $\Lambda_{QCD} \ll p_T \ll \sqrt{s}$  still allow one to study perturbative QCD processes in a kinematic regime where Regge theory applies.

#### III. PHENOMENOLOGICAL APPLICATIONS

In real QCD the nominal power laws discussed in the previous section receive corrections from the breaking of scale invariance in QCD, *i.e.* from the running coupling and the scale breaking of structure functions and fragmentation functions. These corrections have been discussed a long time ago in Ref. [10] but have not yet been studied quantitatively.

Including scaling violations, the inclusive cross section of Eq. (7) changes to

$$E\frac{d^3\sigma(h_ah_b \to hX)}{d^3p} = \left[\frac{\alpha_s(p_T^2)}{p_T^2}\right]^{n_a-2} \frac{(1-x_R)^{2n_s-1+3\xi(p_T)}}{x_R^{\lambda(p_T)}} \alpha_s^{2n_s}(k_{x_R}^2)f(y). \tag{9}$$

The threshold behavior of the cross section follows from spectator counting rules [10]. We ignore here an extra contribution to this power which arises from helicity mismatch in the fragmentation process. The strong coupling constant  $\alpha_s^{2n_s}(k_{x_R}^2)$  ( $n_s$  is the number of spectator fields) arises at large momentum fraction, since all spectators must combine their momentum to produce one high-x quark. This quark is far off-shell with virtuality  $k_x^2 = -\frac{k_T^2 + \tilde{m}_q^2}{1-x}$ , so that the high-x tail of the structure function is calculable in perturbative QCD. Here,  $k_T$  is the transverse momentum of the quark and  $\tilde{m}_q$  is related to the quark mass, see Ref. [10] for details.

Eq. (9) matches smoothly onto the exclusive limit  $x_R \to 1$ . This is still true in the presence of scaling violations: the correction to the simple power  $2n_s - 1$  due to gluon radiation is contained in the function

$$\xi(p_T) = \frac{C_R}{\pi} \int_{k_{x_R}^2}^{p_T^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \alpha_s(k_{\perp}^2) = \frac{4C_R}{\beta_0} \ln \frac{\ln(p_T^2/\Lambda_{QCD}^2)}{\ln(k_{x_R}^2/\Lambda_{QCD}^2)}.$$
 (10)

Here,  $\beta_0 = 11 - 2N_f/3$  is the QCD  $\beta$ -function,  $C_R = C_F = 4/3$  for quarks and  $C_R = C_A = 3$  for gluons. Note the lower integration limit  $k_{x_R}^2$ : at large  $x_R$ , the phase space for gluon radiation vanishes and QCD scaling violations disappear. Hence, the simple spectator counting rules become exact at the exclusive boundary.

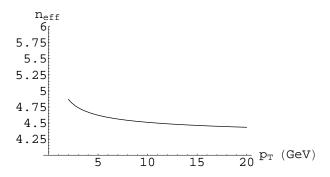


FIG. 1: The effective power  $n_{eff}$  according to Eq. (11). The lower curve assumes 4 active fields and asymptotically approaches  $n_{eff}(n_a=4, p_T \to \infty)=4$ . Calculations were performed at  $x_T=0.03$  and y=0, which are typical values for RHIC.

We shall now investigate, how QCD scaling violations affect  $x_R$  scaling. For that purpose, we define an effective power  $n_{eff}(p_T)$  by taking the logarithmic derivative

$$n_{eff}(p_T) = -\frac{d \ln E \frac{d^3 \sigma(h_a h_b \to hX)}{d^3 p}}{d \ln(p_T)}$$
(11)

of the cross section.

We first concentrate on RHIC kinematics at y=0, where rather low values of  $x_T \sim 0.03$  can be reached. Therefore, we drop all factors describing the large  $x_R$  behavior of the cross section and determine  $n_{eff}$  from the running coupling only. Different choices of the hard scale change numerical results by only few percent. We also checked that the  $(1-x_R)$  term is numerically irrelevant. Results are shown in Fig. 1. For the lowest order process  $2 \to 2$  process we find that the effective power  $n_{eff}$  approximately increases by unity. This is close to what is seen in direct photon production at RHIC  $(n_{eff} \approx 5)$  [11].

Following the suggestion of one of us (SJB), the PHENIX collaboration has analyzed the scaling properties of data [12]. For neutral pions, a value of  $n_{eff} = 6.33 \pm 0.54$  has been determined, which is somewhat larger than the leading-twist values shown in Fig. 1. Strictly speaking, the analysis of Ref. [12] was done for the invariant hadron yields rather than the cross sections. Taking into account the variation of the inelastic cross section between  $\sqrt{s} = 130$  GeV and 200 GeV, the value of  $n_{eff}$  turns out to be about 6.16, which

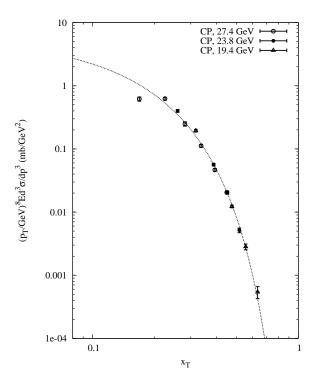


FIG. 2: Invariant cross sections for  $pp \to (\pi^+ + \pi^-)/2 + X$  at three different energies ( $\sqrt{s} = 19.4$  GeV, 23.8 GeV and 27.4 GeV) multiplied by  $p_T^8$ . The power  $n_{eff} = 8$  indicates a higher-twist mechanism. The curve shows the  $(1 - x_R)^9$  threshold behavior.

is still within error bars. However, since next-to-leading (NLO) order perturbative QCD is able to reproduce RHIC data on pion production, using fragmentation functions from  $e^+e^-$  annihilation as input [13], we conclude that pion production is dominated by leading-twist processes.

On the other hand, the Chicago-Princeton data exhibit a strikingly different power law and could never be described in the conventional parton model [14]. An analysis of early data on inclusive  $\pi^+$  production yields  $n=8.2\pm0.5$  for  $x_R=x_T\geq0.35$ , i.e.  $p_T\gtrsim3.5$  GeV at  $\sqrt{s}\approx20$  GeV [15]. Similar results are obtained for  $\pi^-$ , see Fig. 2. The power law  $Ed^3\sigma/d^3p(pp\to\pi^+X)\propto p_T^{-8.2}$  giving  $n_a=6$  may indicate a quark-quark scattering process which produces in addition to the incoming quarks a  $q\bar{q}$  pair, which becomes the observed pion with high transverse momentum. This process has been analyzed within the Constituent Interchange Model (CIM) [1], where an incoming  $q\bar{q}$  pair collides with a quark

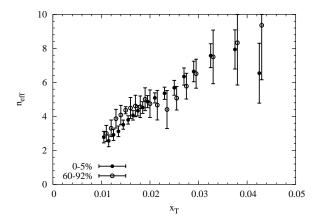


FIG. 3: Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available  $p_T$  range. Shown are data for central (0-5%) and for peripheral (60-90%) collisions.

by interchanging a quark and antiquark. The CIM is motivated by the inclusive to exclusive transition mentioned above and is in good agreement with the Chicago-Princeton (CP) data [15]. The model even can reproduce the absolute normalization of the inclusive cross section.

Obviously, the production mechanism for high  $p_T$  hadrons changes from  $\sqrt{s} = 20$  GeV to  $\sqrt{s} = 200$  GeV. For constituent interchange longitudinal momenta of O(1 GeV) can still be accommodated in the wave function of the proton. When the relevant longitudinal momenta are about O(10 GeV) at higher energies, interchange is no longer possible which the different reaction mechanisms with increasing energy.

Moreover, for proton production the  $p_T$  dependence at Chicago-Princeton energies is also explained by CIM. A value of n=12 is a strong indication that higher twists from wave function effects dominate high  $p_T$  hadron production around  $\sqrt{s}=20$  GeV. Here the produced proton is the result of proton scattering on a quark. If protons and pions were both produced by fragmentation as in the Feynman-Field-Fox parton model, it is hard to understand how a dimensionless fragmentation function could change n from 8 for pions to 12 for protons.

Since high- $p_T$  protons are produced by higher-twist mechanisms at fixed target energies, we also investigate the scaling properties of proton production at RHIC. The points in Fig. 3 were obtained from the 130 GeV data of Ref. [16] and the 200 GeV data of Ref. [17]. Unfortunately, the data do not extend out to large enough  $p_T$  and error bars become too large

at high  $p_T$  to establish  $x_T$  scaling. It is important to measure inclusive proton production out to larger  $p_T$  for at least two values of  $\sqrt{s}$ . From these data one could find out whether proton production is leading or higher twist. If protons are produced in nuclear collisions by parton recombination (see e.g. [18]), the cross section should fall off exponentially, i.e. there would be no  $x_T$  scaling.

### IV. NUCLEAR EFFECTS

It is interesting to investigate nuclear effects on the observed scaling laws, *i.e* to compare the scaling properties of  $Ed^3\sigma/d^3p(pp\to hX)$  to  $Ed^3\sigma/d^3p(AA\to hX)$ . Since pions appear to be produced by a leading-twist mechanism, the quenching of pion spectra may be due to medium induced gluon radiation. In the following, we shall adopt the formalism of Baier et al. (BDMPS-Z formalism, see Ref. [19] for a review).

The presence of a new dimensionful scale in nuclear collisions, namely the BDMPS transport coefficient  $\hat{q}$ , gives rise to the possibility that  $x_T$  scaling is modified or violated. With the quenching weight  $Q(p_T)$  defined according to

$$\frac{1}{N_{coll}} \frac{d^2 \sigma(AB \to hX)}{dy dp_T^2} = \frac{d^2 \sigma(pp \to hX)}{dy dp_T^2} Q(p_T), \tag{12}$$

medium effects may modify  $n_{eff}$  as

$$n^{med} = n^{vac} - \frac{d \ln Q(p_T)}{d \ln p_T} \tag{13}$$

The calculation of  $Q(p_T)$  has been performed within the BDMPS-Z formalism in Ref. [20]. These are the basic steps: Let  $P(\Delta E)$  be the probability that a fast parton loses energy  $\Delta E$  due to gluon radiation. The medium modified cross section can then be written as the convolution

$$\frac{d\sigma^{med}}{dp_T^2} = \int d(\Delta E)P(\Delta E)\frac{d\sigma^{vac}}{dp_T^2}(p_T + \Delta E). \tag{14}$$

Since the partonic cross section is a steeply falling function of  $p_T$ , a small value of  $\Delta E$  produces a large suppression. For  $\Delta E \ll p_T$ , one can expand the logarithm of  $d\sigma^{vac}/dp_T^2(p_T + \Delta E)$  in Eq. (14) and obtains [20]

$$Q(p_T) \approx \int d(\Delta E) P(\Delta E) \exp\left(-\frac{\Delta E}{p_T} n^{vac}\right) = \exp\left(-\int_0^{\omega_{max}} d\omega (1 - e^{-\frac{n^{vac}\omega}{p_T}}) \frac{dI}{d\omega}\right). \quad (15)$$

The advantage of this approximation is that one obtains a particularly simple relation between  $Q(p_T)$  and the gluon multiplicity  $dI/d\omega$ , provided one also assumes a Poissonian probability distribution  $P(\Delta E)$ . The upper integration limit in Eq. (15) is given by  $\omega_{max} = \min(\omega_{LPM}, E)$ , where  $E = p_T$  is the energy of the fast parton. Omitting overall numerical constants, the Landau-Pomeranchuk effect in QCD has the gluon spectrum

$$\frac{dI}{d\omega} \propto \alpha_s \frac{\sqrt{\omega_{LPM}}}{\omega^{3/2}},\tag{16}$$

where  $\omega_{LPM} = \frac{1}{2}\hat{q}L^2$  (L is the length of the traversed medium) will be estimated below.

Parametrically, we obtain the following result: depending on whether the energy  $p_T$  of the fast parton exceeds the critical value  $\omega_{LPM}$ , one distinguishes the two regimes,

$$\Delta n = -\frac{d \ln Q(p_T)}{d \ln p_T} \sim \begin{cases} \alpha_s n^{vac} \sqrt{\frac{\omega_{LPM}}{p_T}} & p_T \ll \omega_{LPM} \\ \alpha_s n^{vac} \frac{\omega_{LPM}}{p_T} & p_T \gg \omega_{LPM}. \end{cases}$$
(17)

Hence,  $x_T$  scaling should be strongly violated in nuclear collisions, in contradiction to what is seen in experiment. Data are available from the PHENIX collaboration at RHIC, see Fig. 18 of Ref. [12]. Remarkably, despite the strong nuclear suppression of pion spectra,  $n_{eff}$  for neutral pions has almost no centrality dependence, *i.e.*  $n_{eff} = 6.33 \pm 0.54$  for peripheral and  $n_{eff} = 6.41 \pm 0.55$  for central collisions. We conclude that radiation of a large number of soft gluons is not the dominant mechanism behind jet quenching.

However, including all charged hadrons, the power n increases with centrality from  $n=6.12\pm0.49$  in  $pp\to(h^++h^-)X$  to  $n=7.53\pm0.44$  in  $AuAu\to(h^++h^-)X$ . The reasons for this difference may be in baryon production, since the inclusive baryon cross section has a steeper  $p_T$  dependence. In heavy ion collisions, pions are strongly suppressed while proton production at not too large  $p_T$  has almost no centrality dependence. We therefore argue that  $n_{eff}=7.5$  reflects the scaling behavior of baryon production at RHIC. The underlying mechanism could be the process  $uu\to p\bar{d}$  with  $n_{eff}=8$  as explained above.

Finally, in order to estimate  $\omega_{LPM}$  (or equivalently  $\hat{q}$ ), we shall rely only on data that are not related to jet quenching. Using Bjorken's estimate of the initial energy density, one obtains

$$\epsilon_{Bj} = \frac{\langle m_T \rangle}{\pi R_A^2 \tau_0} \left( \frac{dN}{dy} \right)_{y=0} \approx 10 \,\text{GeV} / \,\text{fm}^3 \approx 60 \epsilon_{cold}$$
(18)

at initial time  $\tau_0 = 0.5$  fm. We account for the longitudinal expansion of the medium by employing the dynamical scaling law of Salgado and Wiedemann [21], which relates the expanding medium to an equivalent static scenario,

$$\hat{q}^{hot} = \frac{2\hat{q}(\tau_0)}{L^2} \int_{\tau_0}^{\tau_0 + L} d\tau (\tau - \tau_0) \frac{\tau_0}{\tau} \approx 10 \hat{q}^{cold} \approx 2 \frac{\text{GeV}}{\text{fm}^2}.$$
 (19)

For L=5 fm we obtain  $\omega_{LPM}=25$  GeV. The value of  $\hat{q}^{cold}$  has been estimated in [22]. Even though our estimate of  $\omega_{LPM}$  may be uncertain by a factor 3 in each direction, we are quite sure that RHIC high  $p_T$  data lie in the regime where the BDMPS-Z medium induced gluon radiation would yield an energy loss  $dE/dz \propto -\alpha_s \sqrt{\hat{q}E}$ , similar to the QED Landau-Pomeranchuk effect. A mean energy loss proportional to the projectile energy may be more consistent with the data [23].

#### V. SUMMARY

We have reviewed dimensional counting rules at  $x_T$  scaling laws for high  $p_T$  inclusive hadron production [1, 3]. At leading twist, the inclusive cross section  $Ed^3\sigma/d^3p$  scales nominally as  $p_T^{-n}$  with n=4 at fixed ratios of invariants. Scaling violations in QCD, in particular the running coupling constant slightly increase the value of n. The experimental value  $n_{eff}=6.33\pm0.54$  for neutral pion production in peripheral heavy ion collisions at RHIC is somewhat larger than the expectation from leading twist. Nevertheless, we think that leading-twist partonic scattering is dominant, since NLO perturbation theory describes the data reasonably well [13]. At fixed target energies however ( $\sqrt{s} \approx 200 \text{ GeV}$ ), an effective power of  $n_{eff} \approx 8$  for pions and  $n_{eff} \approx 12$  for protons is a strong indication for higher-twist mechanisms in the SPS energy range. The fixed target data can be reproduced in the Constituent Interchange Model of Ref. [1].

We have also investigated how medium induced gluon radiation changes the scaling properties of high- $p_T$  hadron production within the BDMPS-Z formalism. We find that radiation of an infinite number of soft gluons would violate  $x_T$  scaling. This is however not what is seen in experiment: the scaling law of neutral pions is unaffected by the nuclear medium (within error bars) [12]. We conclude that medium-induced gluon radiation is not the mechanism

responsible for pion quenching at RHIC. This is further supported by the fact that charm production appears to be strongly suppressed in nuclear collisions [24], *i.e.* there is no dead cone effect as one would expect if quenching were due to bremsstrahlung [25].

Including all charged hadrons, the effective power law changes from  $n_{eff} = 6.12 \pm 0.49$  in peripheral collision (60-80%)to  $n_{eff} = 7.53 \pm 0.44$  in central collisions (0-10%) at RHIC. We argue that the larger value of  $n_{eff}$  in central collisions is due to a large baryon contribution to the charged hadron yield. If protons are produced by a higher-twist mechanism such as  $uu \rightarrow p\bar{d}$  at not too high  $p_T$ , then there would be an intermediate  $p_T$  range in which protons scale with  $n_{eff} \approx 8$ . This higher-twist mechanism is different from parton recombination models [18], which lead to an exponential fall off. Unfortunately, existing data on inclusive proton production at RHIC do not allow one to draw definite conclusions at this time. Data at different energies will be required to separate the contributing mechanisms. In addition, the higher-twist mechanism of direct proton production which we propose can be verified experimentally by testing whether high  $p_T$  protons are produced as single hadrons without accompanying secondaries. It is clearly essential for the correct interpretation of the heavy-ion collision data that the role of the higher-twist processes be definitively determined.

**Acknowledgments**: This work was supported by the German BMBF under contract 06 HD 158, by the U.S. Department of Energy under contract number DE-AC03-76SF00515 and by the Alexander von Humboldt Foundation.

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