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TeV-Scale Black Holes in Warped Higher-Curvature Gravity * †

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Abstract

We examine the properties of TeV-scale extra dimensional black holes (BH) in Randall-Sundrum-like models with Gauss-Bonnet higher-curvature terms present in the action. These theories naturally lead to a mass threshold for BH production in TeV particle collisions which could be observable at LHC/ILC. The lifetimes of such BH are examined and, in particular, we focus on the predicted lifetime differences between the canonical and microcanonical thermodynamical descriptions of BH decaying to Standard Model brane fields and the possibility of long-lived relics. The sensitivity of these results to the particular mix of fermions and bosons present in the Standard Model spectrum is also briefly examined.

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1 Introduction

The Randall-Sundrum(RS) model[1] provides a geometric solution to the hierarchy problem through an exponential warp factor whose size is controlled by the separation, πr_c, of two 3-branes, situated at the S^1/Z_2 orbifold fixed points, which are embedded in 5-d Anti-deSitter space, AdS_5. It has been shown that this interbrane distance can be naturally stabilized at a value necessary to produced the experimentally observed ratio of the weak and Planck scales[2]. In the original RS model, Standard Model(SM) matter is confined to one of the 3-branes while gravity is allowed to propagate in the bulk. A generic signature of this kind of scenario is the existence of TeV-scale Kaluza-Klein(KK) excitations of the graviton. These states would then appear as a series of spin-2 resonances[3] that might be observable in a number of processes at both hadron and e^+e^- colliders which probe the TeV-scale. Another possible RS signature is the copious production of TeV scale black holes, though this is not a qualitatively unique feature of the RS model as they also appear in non-warped scenarios[4].

One can easily imagine that this simple RS scenario is incomplete from either a top-down or bottom-up perspective. Apart from the placement of the SM matter fields, we would generally expect some other 'soft' modifications to the details of the scenario presented above, hopefully without disturbing the nice qualitative features of the model in the gravity sector. It is reasonable to expect that at least some aspects of the full UV theory may leak down into these collider measurements and may lead to potentially significant quantitative and/or qualitative modifications of simple RS expectations that can be experimentally probed.

One such extension of the basic RS model is the existence of higher curvature invariants in the action which might be expected on general grounds from string theory[5, 6] or other possible high-scale completions. A certain string-motivated class of such terms with interesting properties was first generally described by Lovelock[7] and, hence, are termed Lovelock invariants. The Lovelock invariants come in fixed order, m, which we denote as $\mathcal{L}_m$, that describes the number of powers of the curvature tensor, contracted in various ways, out of which they are constructed, *i.e.*,

$$\mathcal{L}_m \sim \delta^{A_1B_1...A_mB_m}_{C_1D_1...C_mD_m}\ R_{A_1B_1}{}^{C_1D_1}.....R_{A_mB_m}{}^{C_mD_m}\,, \tag{1}$$

where $\delta^{A_1B_1...A_mB_m}_{C_1D_1...C_mD_m}$ is the totally antisymmetric product of Kronecker deltas and $R_{AB}{}^{CD}$ is the D-dimensional curvature tensor. The resulting Einstein equations of motion are ghost free and lead to a unitary perturbation theory due to the absence of derivatives of the metric higher than second. In addition, the new Lovelock contributions to the Einstein tensor are symmetric and have vanishing covariant derivatives. For a fixed number of dimensions the number of the Lovelock invariants is quite restricted, *e.g.*, for $D = 4$, only $\mathcal{L}_0 = 1$ and $\mathcal{L}_1 = R$, the ordinary Ricci scalar, can be present in the action; all higher order invariants can be shown to vanish. Adjusting the numerical coefficients of these terms we see that the resulting action is just the ordinary Einstein-Hilbert(EH) action of General Relativity with a cosmological constant. When D=5, as in the RS model, $\mathcal{L}_2 = R^2 - 4R_{MN}R^{MN} + R_{MNAB}R^{MNAB}$, the Gauss-Bonnet(GB) invariant, can also be present in the action with an arbitrary coefficient, α, with all other $\mathcal{L}_{m>2} = 0$.

Some of the modifications of the RS model due the presence of GB terms have been discussed by other authors (*e.g.*, Refs. [8, 9, 10, 11, 12, 13, 14, 15]). Recently, we have begun a phenomenological examination of the effect of the presence of GB invariants in the action on the predictions of the RS model[16]. In that work we concentrated the modifications of graviton KK properties due to the GB term and how the value of the parameter α can be extracted from this collider data. In other work[17] we have shown that the presence in the action of Lovelock invariants can lead to TeV-scale BH in ADD-like models with properties that can differ significantly from the usual EH expectations including the possibility that BH may be stable in n-odd dimensions. The purpose of the present paper is to examine the effects of GB terms in the action on TeV scale Schwartzschild-AdS BH in the RS model. The possible lifetimes of these BH will be examined and, in particular, we focus on the predicted differences between canonical and microcanonical thermodynamical descriptions of BH decays and the possibility of long-lived relics.

In the next section we provide a brief overview of the essential aspects of the GB-extended RS model necessary for our analysis. In Section 3, we discuss the basic properties of the BH in this model and calculate their corresponding production cross sections for TeV colliders. We also discuss the differences between the use of the canonical ensemble and microcanonical ensemble descriptions for BH Hawking radiation when the BH mass is comparable to the 5D effective Planck scale as might be expected at future colliders. In Section 4, we present the results of a numerical study of BH decay rates in the GB-extended RS model. We perform a detailed comparison of the two possible statistical descriptions for BH decay and show the sensitivity of these results to variations in the model parameters. The sensitivity of BH mass loss to the statistics of the final state particles is also discussed. The last section of the paper contains a summary and our conclusions.

2 RS Background

The basic ansatz of the RS scenario is the existence of a slice of warped, Anti-deSitter space bounded by two 'branes' which we assume are fixed at the S^1/Z_2 orbifold fixed points, $y = 0, \pi r_c$, termed the Planck and TeV branes, respectively[1]. The 5-d metric describing this setup is given by the conventional expression

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,. \tag{2}$$

As usual, due to the S^1/Z_2 orbifold symmetry one requires $\sigma = \sigma(|y|)$ and, in keeping with the RS solution, we have $\sigma = k|y|$ with k a dimensionful constant of order the fundamental Planck scale. As first shown in Ref.[8] the inclusion of GB terms does not alter this basic setup. Based on the above discussion, the action for the model we consider takes the form

$$S = S_{bulk} + S_{branes}\,, \tag{3}$$

where

$$S_{bulk} = \int d^5x\,\sqrt{-g}\left[\frac{M^3}{2}R - \Lambda_b + \frac{\alpha M}{2}\Big(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\Big)\right], \tag{4}$$

describes the bulk with M being the $D = 5$ fundamental Planck scale, Λ_b the bulk cosmological constant and α the above mentioned dimensionless constant which measures the relative strength of the GB interaction. If we anticipate that the GB terms be (mainly) subdominant to the EH ones as might arise in some sort of perturbative expansion then one can argue[17, 18] that the natural size of this parameter is $|\alpha| \sim 1/D^2 \sim$ O(0.01) which we will assume in what follows. In order to avoid both ghosts and tachyons[19] in the perturbative graviton and radion[20] sectors, α must be chosen negative. Similarly

$$S_{branes} = \sum_{i=1}^{2} \int d^4x \, \sqrt{-g_i}\Big(\mathcal{L}_i - \Lambda_i\Big) \,, \tag{5}$$

describes the two branes with g_i being the determinant of the induced metric and Λ_i the associated brane tensions; the $\mathcal{L}_i$ describe possible SM fields on the branes. In what follows we will assume as usual for simplicity that the SM fields are all localized on the TeV brane at $y = \pi r_c$. The (Einstein) equations of motion then lead to the following parametric relations:

$$\begin{aligned} \Lambda_{Planck} = -\Lambda_{TeV} &= 6kM^3\Big(1 - \frac{4\alpha k^2}{3M^2}\Big) \\ 6k^2\Big(1 - \frac{2\alpha}{M^2}k^2\Big) &= -\frac{\Lambda_b}{M^3} \,, \end{aligned} \tag{6}$$

where we explicitly see the α-dependent modifications to the conventional RS results. In the classic RS case, one ordinarily assumes that the ratio k^2/M^2 is small to *avoid* higher curvature effects. Here such terms are put in from the beginning and we need no longer make this *a priori* assumption; we will allow a wide range of k^2/M^2 values in the analysis that follows. For later convenience we will define the parameter $M_* = M\epsilon = Me^{-\pi k r_c}$, which is the warped-down fundamental scale, $\sim$ TeV. This parameter in many ways acts in a manner similar to the fundamental $\sim$ TeV scale in ADD models when discussing BH production on the RS TeV brane as will be seen in the following analysis.

3 Black Hole Properties

How do we describe a Schwartzschild-like BH in AdS_5 that is created on the TeV brane through SM particle collisions when GB terms are present in the action? In the usual BH analysis within the ADD framework[21], one employs a modified form of the conventional D-dimensional Schwartzschild solution[22, 23] assuming all dimensions are infinitely large. Under the assumption that the size of the BH, given by its Schwartzschild radius, R_s, is far smaller than the size of the compactified dimension, πR_c, this is a valid approximate description. This approximation is seen to hold to a very high degree in the usual ADD models with low values of D. The analogous requirement in the RS case for a TeV brane BH is that $(\epsilon R_s/\pi r_c)^2 << 1$ which is also well satisfied as we will see below. A further requirement in the RS case is that the bulk is now AdS_5 and not Minkowski-like as in ADD so that asymptotically flat $D = 5$ Schwartzschild-like solutions are not directly applicable here. This becomes immediately obvious when we think of the curvature of the space-time, $\sim k\epsilon$ as measured on the TeV brane, becoming comparable to R_s^{-1}. Fortunately, the solutions for this asymptotic AdS-Schwartzschild BH were found long ago[23] and such BH have had their properties discussed in some detail in the literature; we will make use of these results in the analysis that follows. Clearly, RS BH in models with large values of k^2/M^2 may differ significantly from their ADD cousins and this is particularly true when higher order curvature terms are present in the action as they are here.

The first step in our analysis is to determine the relationship between R_s, M_* and the BH mass M_{BH}. Following, *e.g.*, Cai[23], and employing the definitions above as well as the relationship between k and Λ_b implied by the Einstein equations of motion in Eq.(6), we find that

$$m = c\Big[x^2 + 2\alpha + \gamma x^4\Big] \,, \tag{7}$$

where we employ the dimensionless quantities $x = M_* R_s$, $m = M_{BH}/M_*$,

$$\gamma = \frac{k^2}{M^2}\Big(1 - 2\alpha\frac{k^2}{M^2}\Big) \,, \tag{8}$$

and the numerical factor $c = 3\pi^2$. Note that γ is a measure of the curvature of the AdS_5 space; when $\gamma \to 0$ the bulk becomes 'flat', *i.e.*, Minkowski-like and we recover the ADD-like result for $D = 5$ with GB terms in the action. To cover all eventualities we will consider the range $10^{-4} \leq \gamma \leq 1$ in our subsequent numerical analysis. Since we generally want $x(m)$ and not $m(x)$ as above we simply invert the expression in Eq.(7) to give

$$x^2 = (2\gamma)^{-1}\Bigg[-1 + \Big[1 + 4\gamma(m/c - 2\alpha)\Big]^{1/2}\Bigg], \tag{9}$$

where the sign has been chosen to insure that $x^2 \geq 0$. In $D = 5$ ADD-like models, to insure that $x^2 \geq 0$ one requires that $m \geq 2\alpha c$ (since there α is positive) which would indicate the existence of a mass threshold for BH production. Here there is no *apparent* threshold of this type arising from Eq.(9) since α is *negative*. Note that when $m \to 0$, x remains *finite*; conversely, note that $x = 0$ corresponds to a *negative* value of m since $\alpha < 0$. This behavior is quite distinct from that in ADD-like models even with Lovelock terms present.

Given the mass-radius relationship we can now ask if the requirement $(\epsilon R_s/\pi r_c)^2 << 1$ discussed above is satisfied. Recall that we do not want our BH to 'see' the fact that it is living in a bounded space which would invalidate our solution. Using the notation above we can re-write this condition as $(k^2/M^2)(\pi k r_c)^{-2}x^2 << 1$; if we take typical values of these quantities: $k^2/M^2 \lesssim$ O(1), $x \sim$O(1) and $kr_c \sim 11$, we see that this constraint is very easily satisfied for our range of parameters.

The BH Hawking temperature can be obtained from the derivative of the metric tensor in the usual manner; following, *e.g.*, Cai[23] we obtain

$$T = \frac{1}{2\pi}\frac{x + 2\gamma x^3}{x^2 + 4\alpha}\,. \tag{10}$$

Since α is negative while γ is positive it is clear that x^2 must be bounded from below *if* we demand that the BH Hawking temperature is to remain positive, *i.e.*, $x^2 \geq -4\alpha$. Through the mass-radius relationship this implies that there is a corresponding critical lower bound on the BH mass, m_{crit}:

$$m_{crit} = -6\pi^2\alpha(1-8\gamma\alpha)\,, \tag{11}$$

which is of O(1) when $|\alpha|$ is O(0.01); smaller BH masses lead to negative temperatures. Note that the BH temperature is infinite precisely at $m = m_{crit}$. This is unlike the ADD-like models where a minimum BH mass also arises from the presence of Lovelock terms in the action in odd numbers of extra dimensions. In that case both $x, T \to 0$ at a fixed value of m[17] producing the mass threshold. Here, as a function of the BH radius, x, the temperature starts off infinite at $x^2 = -4\alpha$, goes through a minimum at some fixed radius and then grows rapidly again as $x \to \infty$ due to the finite value of the curvature factor γ. The threshold temperature behavior in GB augmented RS is thus more like that obtained from the traditional D-dimensional EH action where the temperature diverges as $x \to 0$.

Given the BH mass-radius relationship we can calculate the cross section for BH at colliders. The leading approximation for the subprocess cross-section for the production of a BH of mass M_{BH} is just its geometric size[21]:

$$\hat{\sigma} = f\pi R_s^2(\sqrt{s} = M_{BH})\,, \tag{12}$$

where R_s is the 5-dimensional Schwarzschild radius corresponding to the mass M_{BH} and f is some factor of order unity. The specific range of values for this factor has been discussed extensively in the literature[24]. For our numerical purposes we will assume $f = 1$ in the analysis that follows. Fig. 1 shows the numerical results we obtain for the BH cross section as a function of m/m_{crit} for different choices of the α, γ parameters. Several features are immediately apparent: (i) BH do not form for masses below m_{crit} as this leads to negative temperatures. However, the Schwartzschild radius for a BH of mass m_{crit} is non-zero which implies a step-like threshold behavior for the cross section. This is unlike the case of the ADD-like models where Lovelock terms produce a very smooth threshold behavior for both odd and even numbers of dimensions[17]. In a more realistic UV completed theory it is likely that this threshold is smoothed out to some degree by quantum corrections. (*ii*) The cross section above threshold has an approximately linear m dependence; the deviations from linearity are related to the amount of curvature, *i.e.*, the size of γ. For the range of parameters we consider the qualitative sensitivity to γ is generally rather weak; the numerical scale of the cross section is set by M_*^{-2}. (*iii*) The overall magnitude of the cross section is approximately linear in $-\alpha$. Properties (*i*)-(*iii*) imply that experimental measurements of the BH cross section at colliders can be used to determine the basic GB-extended RS model parameters.

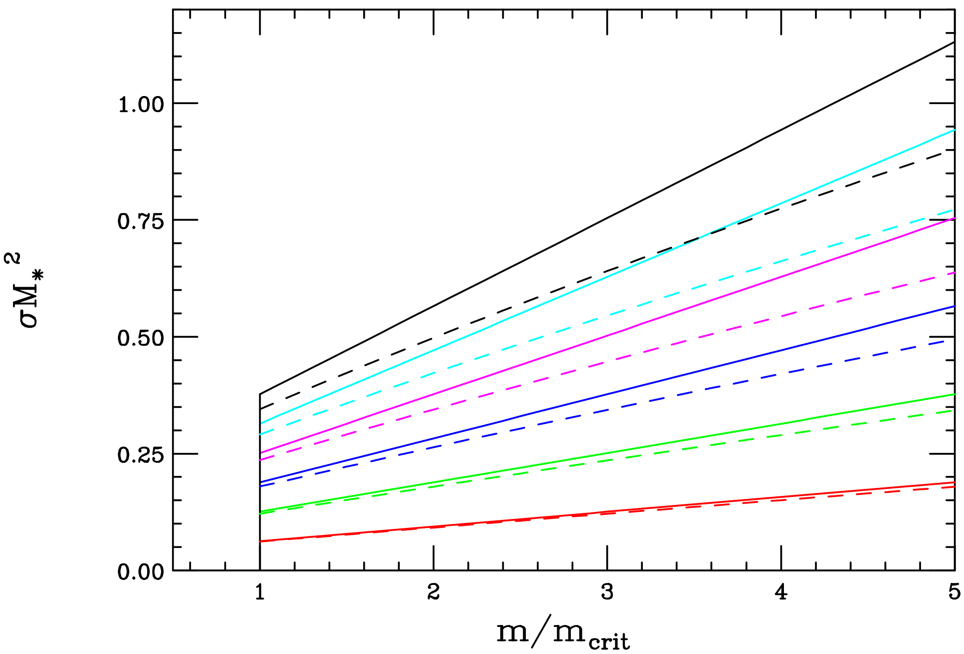

Figure 1: The scaled BH production cross section as a function of m/m_{crit} for $\gamma = 10^{-4}$(solid) and 1(dashed) for different values of α. From bottom to top the curves correspond to $\alpha = -0.005, -0.010...$ increasing in magnitude by 0.005 for each subsequent curve.

We now turn to the issue of BH decays through Hawking radiation; for simplicity we will ignore the influence of possible grey-body[21] factors that may be present. The standard thermodynamical description of TeV scale ADD-like and RS BH decays in the literature is via the canonical ensemble(CE)[21]. However, several sets of authors[25] have pointed out that while applicable to very massive BH, this approach does not strictly apply when $m = M_{BH}/M_*$ is not much greater than unity due to the back-reaction of the emitted particles on the BH. In the decay of TeV-scale BH, the energy of the emitted particles may be comparable to both M_* as well as the mass of the BH itself. In addition, in the CE approach the BH is treated as a heat bath whose temperature is not (much) influenced by the emission of an individual particle. This is an excellent approximation for reasonably heavy BH but becomes worse as the TeV scale is approached. It has been suggested that these issues can be dealt with simultaneously if we instead use the microcanonical ensemble(MCE) as the correct statistical mechanics treatment for BH decays in this mass range. As m grows larger the predictions of these two treatments will agree since one then recovers the CE results from the MCE treatment, *i.e.*, the BH becomes more and more like a heat bath. The two approaches differ in the region which is of most interest to us, *i.e.*, where m is not so far above unity. For the ADD scenario within the framework of the EH action it has been emphasized by several sets of authors[25] that TeV-scale BH lifetimes can be increased by many orders of magnitude when the MCE approach is employed in comparison to the CE expectations. Here we must address these CE vs. MCE differences in the case of the RS model with GB terms present. It is important to remember that the values of the various thermodynamic quantities themselves , *e.g.*, the BH temperature, T, are the same in both approaches.

4 Numerical Results: Mass Loss Rates and Lifetimes

In order to be definitive in our calculation of the BH Hawking decay rates we follow the formalism of Hossenfelder[25]; to simplify our presentation and to focus on the GB modifications to RS, as well as potential MCE and CE differences, we remind the reader that we will ignore the effects due to grey-body factors[21] in the present analysis. Since bulk decays are expected to be generally sub-dominant and since the only bulk modes are gravitons which have KK excitations that are quite heavy in the RS model, $\sim$ TeV, we will here assume that the only (numerically) important BH decays are into SM brane fields. In this approximation the rate for BH mass loss (time here being measured in units of M_*^{-1}) due to decay into brane fields employing the MCE approach is given by

$$\left[\frac{dm}{dt}\right]_{brane} = \frac{\Omega_3^2}{(2\pi)^3}\ \zeta(4)x^2\sum_i\ \int_{m_{crit}}^{m}\ dy\ (m-y)^3 N_i\Big[e^{S(m)-S(y)}+s_i\Big]^{-1}\,, \tag{13}$$

where, $x = M_* R_s$ as above, S in the corresponding entropy of the BH, i labels various particle species which live on the brane and obey Fermi-Dirac(FD), Boltzmann(B), or Bose-Einstein(BE) statistics, corresponding to the choices of $s_i = 1, 0, -1$, respectively, with the corresponding number of degrees of freedom N_i, ζ is the Riemann zeta function with $\zeta(4) = \pi^4/90$, and as usual

$$\Omega_{d+3} = \frac{2\pi^{(d+3)/2}}{\Gamma((d+3)/2))}\,, \tag{14}$$

so that $\Omega_3 = 4\pi$. To proceed further we need to know the BH entropy S; this entropy can be calculated using the familiar thermodynamical relation

$$S = \int dx\ T^{-1}\frac{\partial m}{\partial x}\,, \tag{15}$$

which yields

$$S = \frac{4\pi}{3}c\Big(x^3 + 12\alpha x + K\Big)\,, \tag{16}$$

where K is an integration constant. Since only the entropy difference $S(m) - S(m-y)$ enters into the expression above this constant can be chosen arbitrarily within our present analysis. However, perhaps the most convenient choice is to choose $K = 16(-\alpha)^{3/2}$ such that $S(m_{crit}) = 0$. In this special case S starts at zero when $x^2 = -4\alpha$ and monotonically increases as x increases since $\partial S/\partial x$ is always positive. The free energy of these BH, $F = m - TS$, is also always positive in this case.

How sensitive is the BH mass loss rate and lifetime to the statistics of the particles in the final state? In the CE analysis, to be discussed below, this sensitivity is only at the level of $5-10\%$ and is due to a simple overall multiplicative factor. To address this issue for the MCE approach we display in Fig. 2 the results obtained by assuming BH decays to the pure BE, B and FD statistical final states taking $N = 60$, $\alpha = -0.01$ and $\gamma = 10^{-4}$ for purposes of demonstration. While the B and FD statistics cases are rather close numerically over the entire mass range, the BE choice leads to a more rapid decay and a substantially shorter lifetime as can be seen in Fig. 2. Here we see that BH decaying only to fermions may have a lifetime which differs from one decaying only into bosons by a few orders of magnitude, $\sim 10^3 - 10^4$. Note that at larger values of m, the MCE $\rightarrow$ CE limit, all three curves become rather close, at the level of $\sim 10\%$ as expected, while still differing at smaller m values. Though the detailed meaning of these these curves will not be discussed until later in this section it is clear than these differing statistics can in general be quite important when performing the BH mass loss analyses using the MCE approach.

In the CE case, the expression above simplifies significantly as there are no non-trivial integrals remaining. The reason for this is that in the CE treatment, the factors $S(m)$ and $S(m-y)$ appearing in the exponential factor above are considered nearly the same since backreaction is neglected, *i.e.*, one replaces this difference in the MCE expression above by the leading term in the Taylor series expansion $S(m) - S(m-y) \simeq y\partial_m S = y/T$[25]. Taking the limit $m \rightarrow \infty$, *i.e.*, no recoil, and integrating over y then produces the familiar CE result:

$$\left[\frac{dm}{dt}\right]_{brane} = Q\ \frac{\Omega_3^2}{(2\pi)^3}\ \zeta(4)x^2\Gamma(4)T^4\,, \tag{17}$$

where Q takes the value $\pi^4/90(1,\ 7\pi^4/720)$ for BE(B, FD) statistics. Note that the difference in statistics here in the CE case is essentially trivial: just a simple multiplicative factor which is close to unity unlike in the MCE approach as seen above where there is a functional difference at low m values. In practical calculations, especially since we are concerned with decays to SM brane fields where the numbers of fermionic degrees of freedom (48, assuming only light Dirac neutrinos) is somewhat larger than the number for bosons (14), the value of s_i does not play much of an important role. In our numerical analysis that follows we will for simplicity take $s_i = 0$ and assume that the number of SM fields is 60. The reason why this is a good approximation is that (*i*) the SM is mostly fermionic and the results for Boltzmann and FD statistics lie rather close to one another

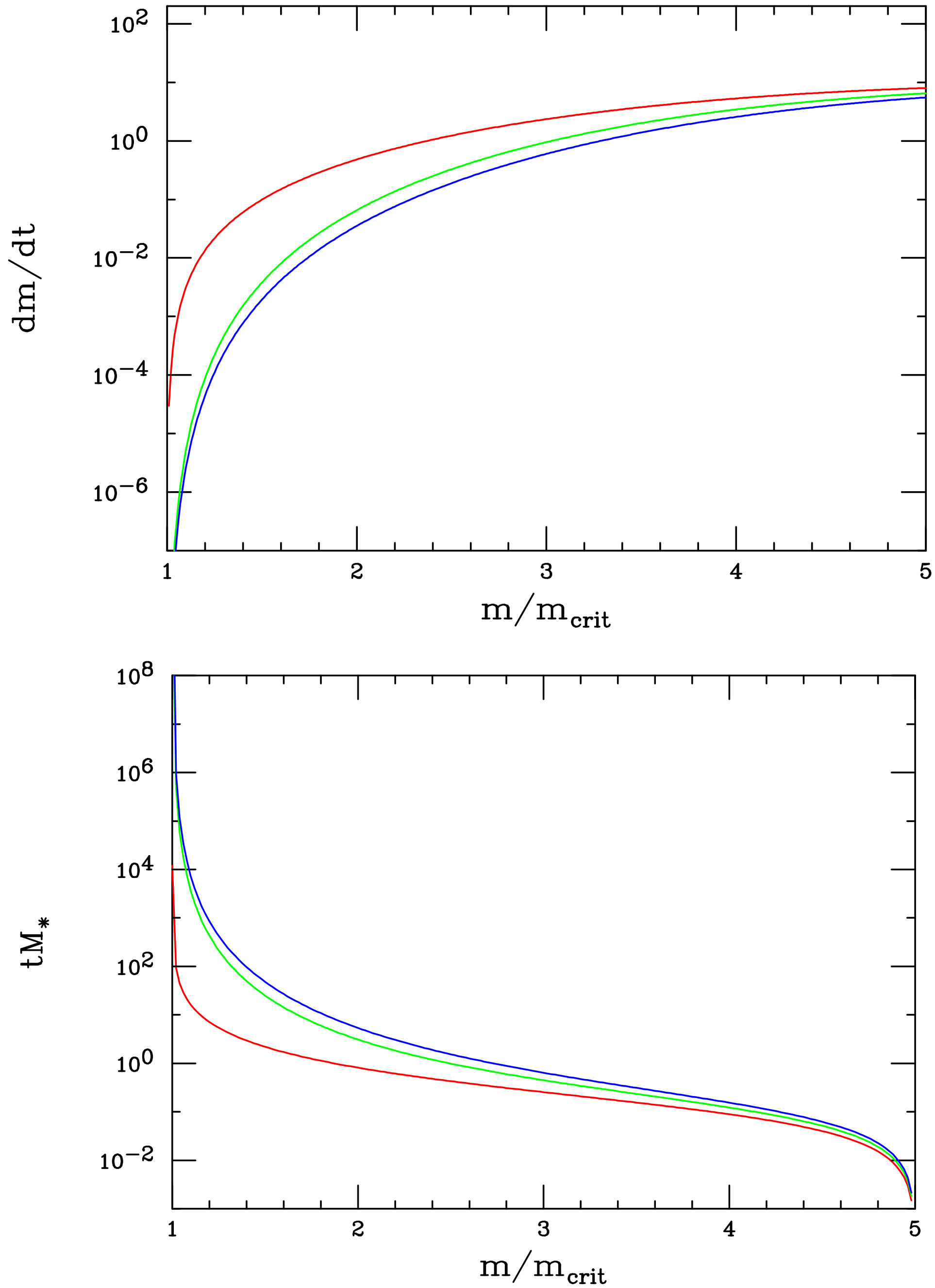

Figure 2: The BH mass loss rate(top) and lifetime(bottom) in the MCE approach as a function of m/m_{crit} assuming an initial value of $m = 5m_{crit}$, with $\alpha = -0.01$ and $\gamma = 10^{-4}$. In the top panel, from top to bottom, the curves correspond to the choice of BE, B or FD statistics, respectively. In the bottom panel, the order for the statistics choice is reversed.

and (*ii*) the B distribution lies between the BE and FD ones. It would be interesting to know how this approximation fares for BH decays in the ADD-like case when several Lovelock terms are present in the action simultaneously.

Given the results above we can now address a number of issues, in particular, (*i*) how do BH decay rates and lifetimes depend on the values of the parameters α and γ and (*ii*) how sensitive are these results to the choice of the MCE or CE analysis approach. To be specific we first perform our analysis by following the usual CE approach; Figs. 3 and 4 show the results of these specific calculations which were performed for two widely different values of $\gamma = 10^{-4}$ and $\gamma = 1$, respectively. Any realistic value for this parameter must lie within this range. Assuming a BH with an initial mass of $m = 5m_{crit}$, as might be expected at TeV colliders, the top panel in both figures shows the rate of mass loss, dm/dt, for this BH. It is important to note that the mass loss rate is predicted to increase dramatically using the CE approach as $m \rightarrow m_{crit}$ making the BH radiate faster and faster. This is to be expected as in the CE approach the mass loss rate is proportional to T^4 and T increases dramatically as the BH looses mass and gets smaller. This implies that in the CE analysis the final state remnant is reached in a reasonably short amount of time. The bottom panel for both γ cases shows the time taken for the initial BH to radiate down to a smaller mass; when $m = m_{crit}$ this is the BH radiative 'lifetime', *i.e.*, the time taken to decay down to m_{crit}. In this CE case this time is short and finite, $\sim 0.1M_*^{-1}$, as might be naively expected; it is not quite clear what happens to the remnant without a more complete theory but within the present framework we are left with a stable remnant. This differs from the case of the ADD-like model with Lovelock terms in odd dimensions where the decay down to the finite mass remnant takes essentially an infinite amount of time. We further note that the BH specific heat, $C = \partial m/\partial T$, here remains negative, as is typical for EH BH, over the relevant RS parameter space. This again differs from the ADD-like model with Lovelock terms where C can have either sign depending on the values of the Lovelock parameters and the BH mass.

For fixed γ, Figs. 3 and 4 show that both the BH mass loss rate and lifetime have only

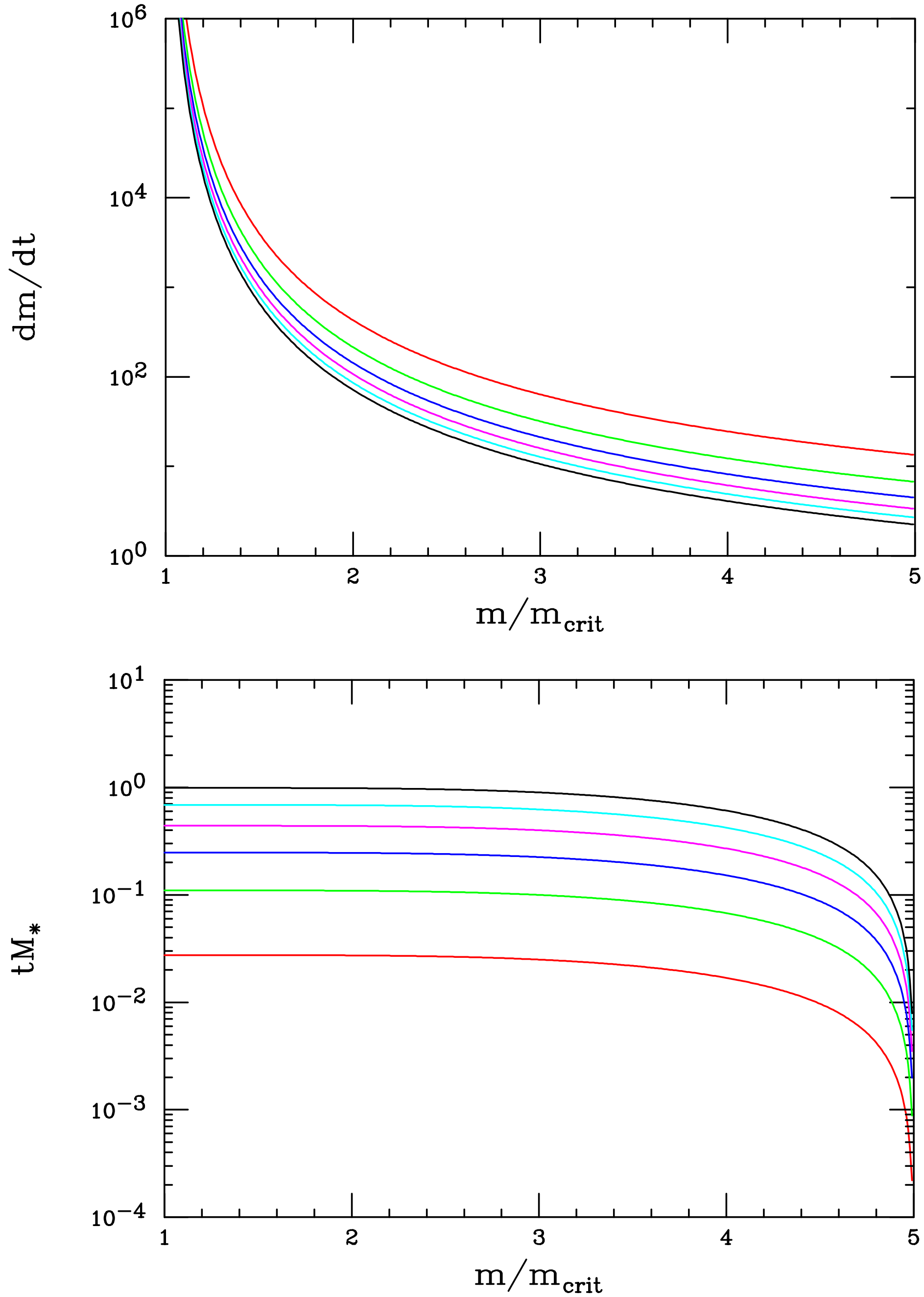

Figure 3: The BH mass loss rate(top) and lifetime(bottom) as a function of m/m_{crit} assuming $\gamma = 10^{-4}$ employing the CE scheme and assuming an initial mass $m = 5m_{crit}$. In the top panel, from top to bottom, the curves correspond to the choices $\alpha = -0.005, -0.010, ...$ In the bottom panel, the order is reversed.

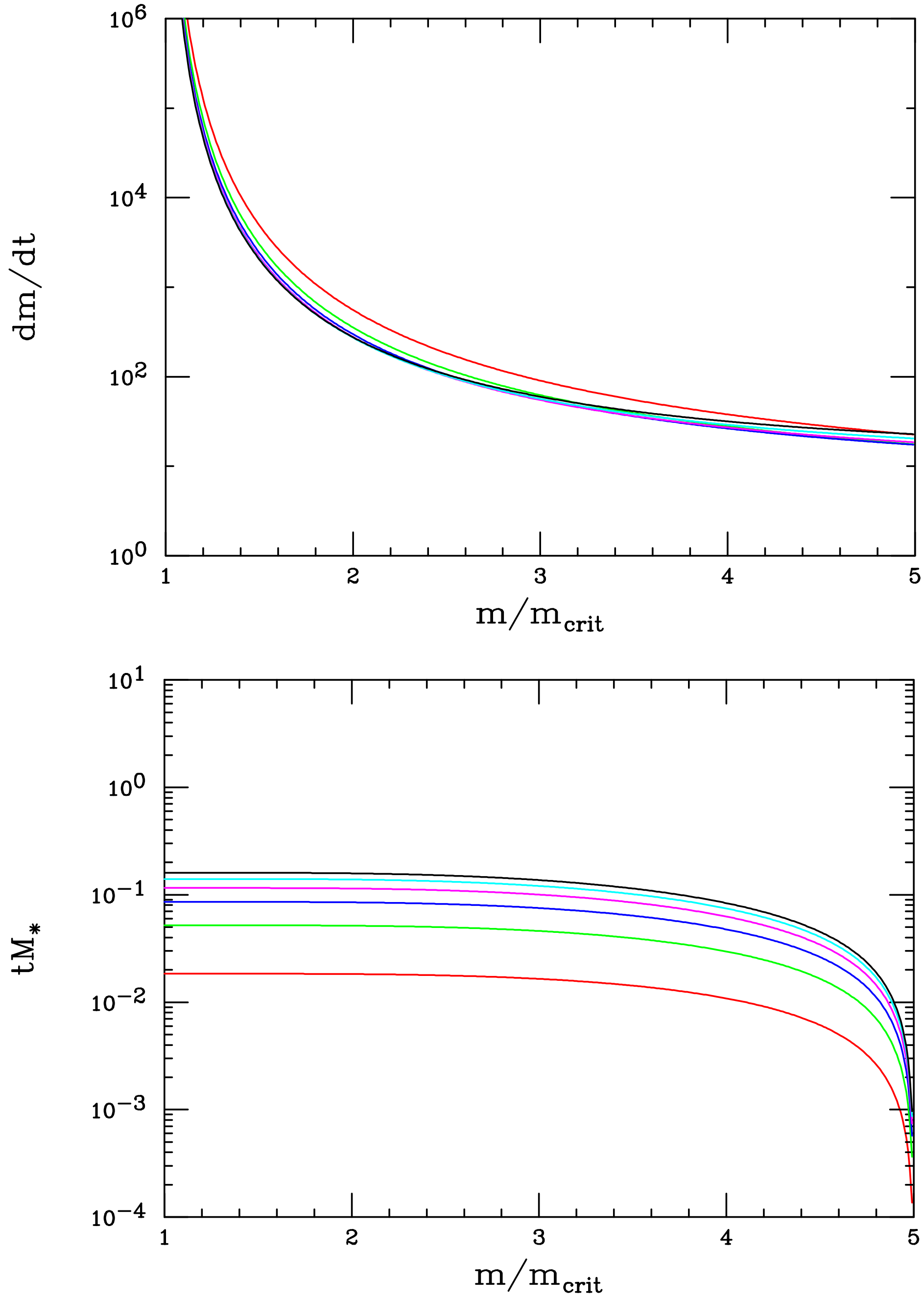

Figure 4: Same as the previous figure but now assuming $\gamma = 1$.

modest dependence on the value of α (for the parameter range considered here). The exact α sensitivity is itself, however, dependent on the specific value of γ as a comparison of these two figures show. Generally the BH decay rate decreases (and the corresponding lifetime increases) as the magnitude of α is raised.

Do these conclusions continue to be valid when we follow the MCE prescription? This question can be answered by examining the results presented in Figs. 5 and 6. In the MCE case, the BH mass loss rate rapidly *decreases* as m approaches m_{crit}; this is the complete opposite behavior from what happens for the CE approach as discussed above. Generally we see that there is not a very large γ dependence, as we saw before in the CE analysis, but now the BH with the smaller (in magnitude) value of α leads to the smallest mass loss rates, dm/dt. This is again the completely opposite behavior to that seen in the CE analysis. We note that for $m = 5m_{crit}$, the largest value shown, dm/dt is not so different in the CE and MCE cases. For much large values of $m \gtrsim 10 - 20$ one can check that the two sets of calculations yield essentially identical numerical results. The fact that dm/dt here decreases as $m \to 0$ implies that following the MCE approach a BH lives far longer than if one employs the CE analysis. The bottom panels of Figs. 5 and 6 clearly demonstrate this result where we see the vastly longer BH lifetimes obtained for the MCE approach. In comparison to CE analysis these BH lifetimes are observed to be greater by factors of order $\sim 10^{10}$ or so. This result is certainly in line with what might have been expected based on the previously performed MCE versus CE analyses performed in the ADD model assuming the EH action[25].

The differences in BH mass loss and lifetimes we have obtained here in the RS model due to the choice of the MCE vs.CE thermodynamical description demonstrates how much we can learn from BH if they are produced at future colliders.

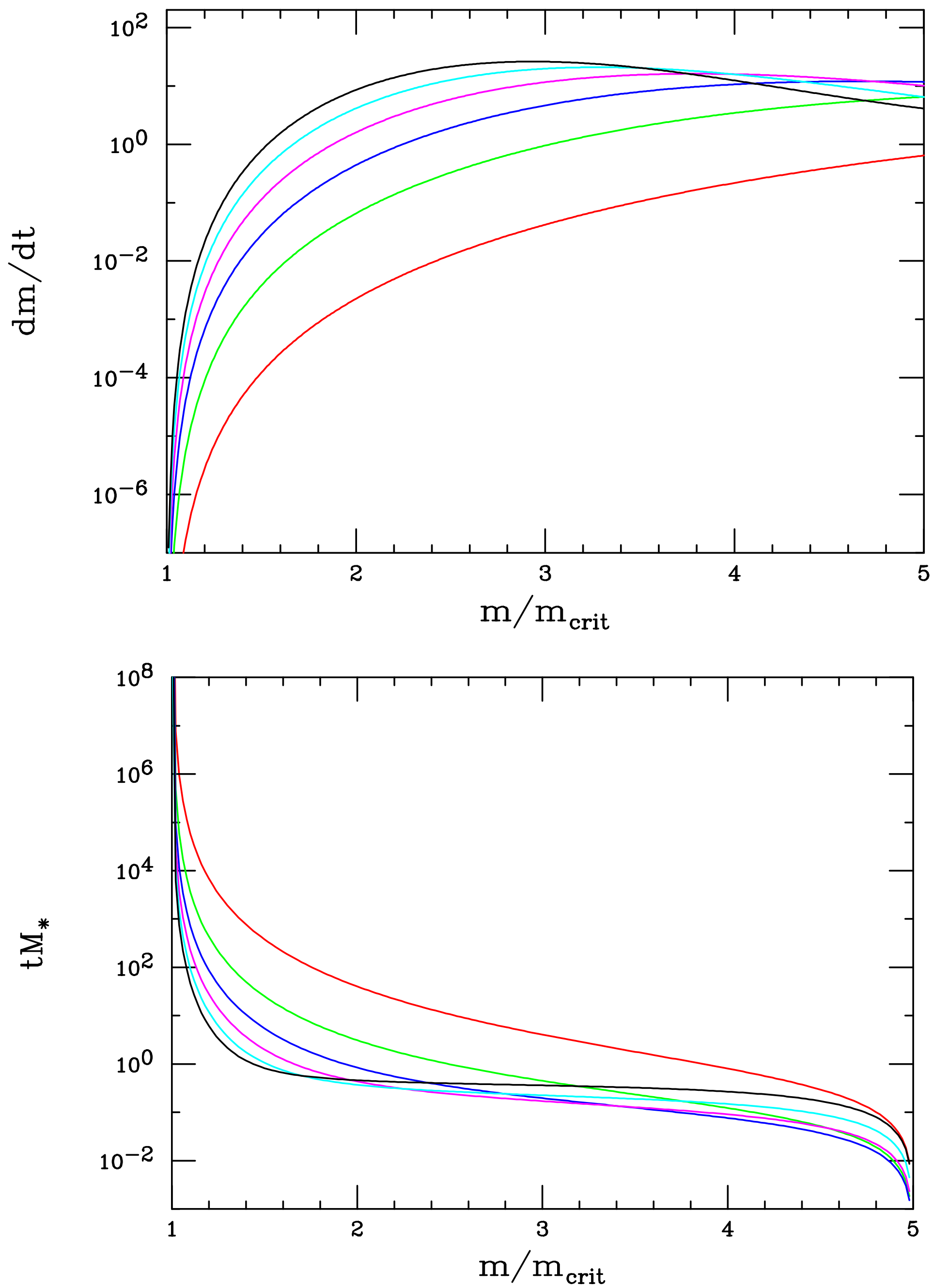

Figure 5: Same as Fig. 3, with the opposite ordering of the α dependence of the curves, but now using the MCE approach.

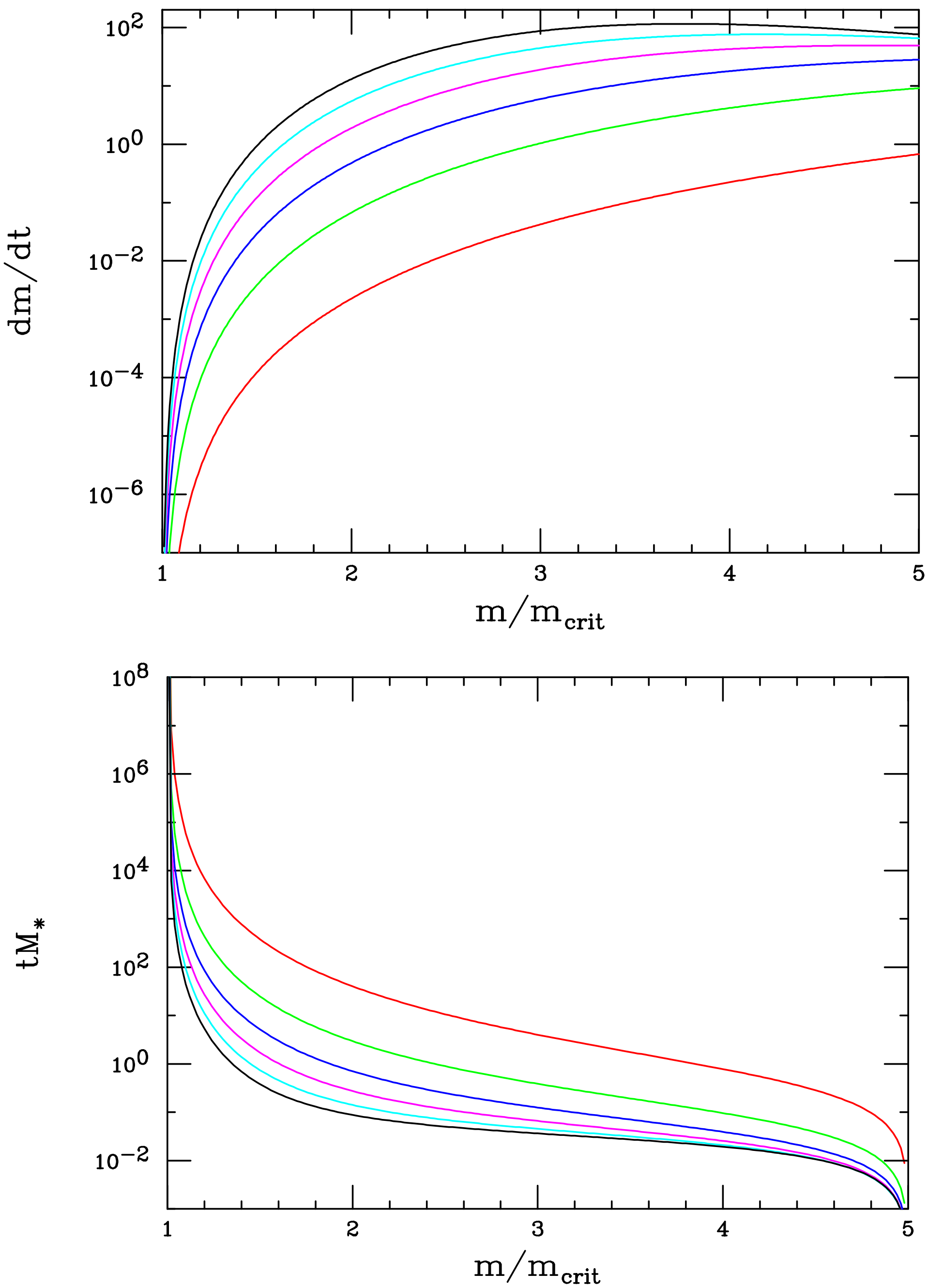

Figure 6: Same as the previous figure but now for $\gamma = 1$.

5 Summary and Conclusions

In this paper we have analyzed the properties of TeV-scale black holes in the Randall-Sundrum model with an extended action containing Gauss-Bonnet terms. In particular we performed a detailed comparison of BH mass loss rates and lifetimes obtained by analyses based on the canonical and microcanonical ensemble descriptions. In addition, we have obtained expressions for BH production cross sections in these models for future colliders.

Overall, the behavior of BH in the RS model with GB terms in the action were found to be quite different than those in ADD-like models with Lovelock terms present. Our specific results are as follows:

(i) The restriction that the Hawking temperature of a BH be positive leads to a lower bound on its radius, $R_s^2 \geq -4\alpha/M_*^2$. This implies a corresponding mass threshold, $M_{BH} \geq -6\pi^2\alpha(1 - 8\gamma\alpha)M_*$, which is $\sim M_*$ for canonical parameter values, below which BH will not be produced at colliders. The resulting production cross section is found to have a step-like behavior, since R_s is finite at threshold, and to grow approximately linearly with the BH mass and value of $|\alpha|$.

(ii) We performed a comparison of the two possible approaches to BH thermodynamics based on the canonical and microcanonical ensemble descriptions for the RS model. While yielding the same results for large BH masses, as was explicitly verified, the two differ in a number of ways when the BH mass is TeV-scale as was considered here. First, the BH mass loss rate and lifetime displayed in the CE analysis is well-known to display a very trivial dependence on the statistics of the particles into which it decays. For light BH we showed that this is not generally true in the MCE approach. For example, employing the MCE a BH decaying only to fermions may have a lifetime which differs from one decaying only into bosons by several orders of magnitude, $\sim 10^3 - 10^4$ in the specific case examined. For practical calculations involving BH decays only to SM fields the statistically weighted mass loss rate was found to be similar to that obtained for decays to particles with classical Boltzmann statistics. For light BH the CE and MCE treatments were shown to lead to drastically different lifetimes over the entire model parameter space. This can be traced to the fact that in the CE analysis the BH mass loss rate, which goes as $\sim T^4$, grows rapidly as m decreases since T is then also increasing. Given our parameter ranges, this then led to BH lifetimes which were typically $\sim 0.1/M_*^2$. On the otherhand, the mass loss rate as $m \to m_{crit}$ was found to behave in just the opposite manner when the MCE approach was followed. Since the mass loss rate decreases so rapidly the corresponding BH lifetimes were found to be enhanced by factors of order $\sim 10^{10}$ relative to those of the CE case. This greatly increases the chance of long lived relics remaining after the usual BH decay process.

The differences between ADD-like and RS model BH is rather striking as are those between the CE and MCE thermodynamical descriptions. It is clear from this discussion that the observation of BH at future TeV colliders will provide an important probe of new high scale physics.

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