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Light-cone sum rules: A SCET-based formulation

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We describe the construction of light-cone sum rules (LCSRs) for exclusive B-meson decays into light energetic hadrons from correlation functions within soft-collinear effective theory (SCET). As an example, we consider the SCET sum rule for the $B \to \pi$ transition form factor at large recoil, including radiative corrections from hard-collinear loop diagrams at first order in the strong coupling constant.

1. INTRODUCTION

Form factors parameterizing hadronic matrix elements defining B decays to a light pseudoscalar (P) or vector meson (V) play an important role in several respects. For example, they enter in the determination of $|V_{ub}|$ from exclusive modes. However, since they are non-perturbative objects, their determination is a difficult task.

Let us consider for definiteness the decay of a B meson in its rest frame into a highly energetic pion. Several energy scales are involved: i) $\Lambda = \text{few} \times \Lambda_{\rm QCD}$, the *soft* scale set by the typical energies and momenta of the light degrees of freedom in the hadronic bound states; ii) m_b, the *hard* scale set by the b-quark mass; iii) the hard-collinear scale $\mu_{\rm hc} = \sqrt{m_b\Lambda}$ appearing via interactions between soft and energetic modes. The dynamics of hard and hard-collinear modes can be described perturbatively in the heavy-quark limit. The separation of the two perturbative scales from the non-perturbative hadron dynamics is formalized within the framework of soft-collinear effective theory (SCET) [1,2]. The small expansion parameter in SCET is given by $\lambda = \sqrt{\Lambda/m_b}$, such that $\lambda^2 m_b \ll \mu_{\rm hc} \sim \lambda m_b \ll m_b$.

SCET describes B decays to light hadrons with energies much larger than their masses, assuming that their constituents have momenta almost collinear to the hadron momentum p^μ. Introducing two light-like vectors $n_+^\mu = (1,0,0,-1)$, $n_-^\mu = (1,0,0,1)$ one can generically write: $p^\mu = p_+^\mu + p_-^\mu + p_\perp^\mu$, with $p_+^\mu = (n_-p)/2n_+^\mu$, $p_-^\mu = (n_+p)/2n_-^\mu$; momenta are then classified according to the scaling of their light-cone coordinates $(p_+, p_-, p_\perp)$.

In order to see how SCET can be exploited in the case of heavy-to-light B decays, we have to recall some general features of the form factors relevant to these kind of transitions.

In the large energy limit of the final state, $B \to P, V$ form factors obey spin symmetry relations [3], broken by hard gluon corrections to the weak vertex and hard spectator interactions. In the heavy-quark limit one can write [4] (see also [5,6]):

$$\langle \pi|\bar\psi\,\Gamma_i\,b|B\rangle = C_i(E,\mu_I)\,\xi_\pi(\mu_I,E) \quad + \qquad (1)$$
$$T_i(E,u,\omega,\mu_{\rm II}) \otimes \phi_+^B(\omega,\mu_{\rm II}) \otimes \phi_\pi(u,\mu_{\rm II}) \quad + \quad \ldots,$$

where Γ is a generic Dirac structure and the dots stand for sub-leading terms in Λ/m_b. The matrix elements in (1) get therefore two contributions. The first one contains the short-distance functions C_i, arising from integrating out hard modes: $\mu_I < m_b$, and a “soft” form factor ξ_π which does not depend on the Dirac structure of the decay current. In this contribution, the hard-collinear interactions are *not* factorizable, so

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that the "soft" form factor is in general a non-perturbative object of order $(\alpha_s)^0$. The second term in (1) factorizes into a hard-scattering kernel T_i and the light-cone distribution amplitudes ϕ_B and ϕ_π. T_i contains the effect of both hard and hard-collinear dynamics: $\mu_{\rm II} < \mu_{\rm hc}$. Both C_i and T_i can be computed as perturbative series in α_s, and potentially large logarithms $\ln m_b/\mu_{\rm I}$ and $\ln \mu_{\rm hc}/\mu_{\rm II}$ can be resummed by renormalization-group techniques (the effective theories for the two short-distance regimes are known as $\rm SCET_I$ and $\rm SCET_{II}$, respectively). A still controversial question is to what extent the first contribution is numerically suppressed by Sudakov effects.

Let us consider eq. (1) for $\Gamma_i = \gamma_\mu$, i.e. for the QCD vector current. This can be matched onto $\rm SCET_I$ currents as follows [1]:

$$\bar q \, \gamma_\mu \, b \to \left(C_4 n_-^\mu + C_5 v^\mu\right) \bar\xi_{\rm hc} W_{\rm hc} \, Y_s^\dagger h_v + \ldots \quad (2)$$

where dots represent subleading terms and $C_4 = 1 + \mathcal{O}(\alpha_s)$, $C_5 = \mathcal{O}(\alpha_s)$. v^μ is the heavy-quark velocity with $n_\pm v = 1$. The direction of the momentum of the (massless) pion is given by $p_\pi^\mu = (n_+ p_\pi) \, n_-^\mu/2$. Besides, $\xi_{\rm hc}(x) = \frac{\not n_- \not n_+}{4} \, \psi_{\rm hc}(x)$ is a hard-collinear light-quark field in $\rm SCET_I$ and h_v is the usual heavy quark field in HQET. The hard-collinear and soft Wilson lines $W_{\rm hc}$ and Y_s appear to render the definition gauge-invariant.

The soft form factor in (1) can be defined as [4]

$$\begin{aligned} &\langle \pi(p') | (\bar\xi_{\rm hc} W_{\rm hc})(0) \, (Y_s^\dagger h_v)(0) | B(m_B v)\rangle = \\ &(n_+ p') \, \xi_\pi(n_+ p', \, \mu_{\rm I}) \, , \end{aligned} \quad (3)$$

Neglecting $\mathcal{O}(\alpha_s)$ effects the approximate symmetry relations mentioned above between the vector and tensor form factors for $B \to \pi$ transitions read [3,7]:

$$\begin{aligned} f_+(q^2) \; &\simeq \; \frac{m_B}{n_+ p_\pi} \, f_0(q^2) \simeq \frac{m_B}{m_B + m_\pi} \, f_T(q^2) \\ &\simeq \; \xi_\pi(q^2) \, . \end{aligned} \quad (4)$$

SCET thus provides a field-theoretical framework to achieve the factorization of short- and long-distance physics, and to calculate the former in renormalization-group-improved perturbation theory. However, non-perturbative quantities such as the soft form factors remain undetermined without further phenomenological or theoretical input. A theoretical approach for this purpose is represented by QCD/light-cone sum rules (see for instance [8,9,10]). In [11] we have shown that it is possible to formulate light-cone sum rules *within* SCET, in a different way with respect to the traditional method. We summarize below the main features of this new formulation.

2. SUM RULES IN SCET: THE CASE OF $B \to \pi$ DECAY

In contrast to the traditional approach where the B meson is represented by an interpolating current, we treat it as an external field and not as a propagating particle in the correlation function (see also [12]). Actually, the heavy quark is nearly on-shell in the end-point region. In $\rm SCET_I$ this is reflected by the fact that hard sub-processes (virtualities of order m_b^2) are already integrated out and appear in coefficient functions multiplying J_0. Instead, the short-distance (off-shell) modes in $\rm SCET_I$ are the hard-collinear quark and gluon fields. Hence, our starting point is the correlator

$$\Pi(p') = i \int d^4x \, e^{ip'x} \, \langle 0 | T[J_\pi(x) J_0(0)] | B(p_B)\rangle \, , \quad (5)$$

where $p_B^\mu = m_B v^\mu$, and

$$\begin{aligned} J_0(0) \; &= \; \bar\xi_{\rm hc}(0) W_{\rm hc}(0) Y_s^\dagger(0) h_v(0) \, , && (6) \\ J_\pi(x) \; &\equiv \; -i \, \bar\psi(x) \not n_+ \gamma_5 \, \psi(x) \\ &= \; -i \, \bar\xi_{\rm hc}(x) \not n_+ \gamma_5 \, \xi_{\rm hc}(x) && (7) \\ &\quad -i \, \left(\bar\xi_{\rm hc} W_{\rm hc}(x) \not n_+ \gamma_5 Y_s^\dagger q_s(x) + h.c.\right) \, , \end{aligned}$$

where q_s is the soft quark field in SCET and $\langle 0 | J_\pi | \pi(p')\rangle = (n_+ p') \, f_\pi$. In the following we will consider a reference frame where $p'_\perp = v_\perp = 0$ and $n_+ v = n_- v = 1$. In this frame the two independent kinematic variables are $(n_+ p') \simeq 2E_\pi = \mathcal{O}(m_b)$, $0 > (n_- p') = \mathcal{O}(\Lambda)$, with $|n_- p'| \gg m_\pi^2/(n_+ p')$. The dispersive analysis will be performed with respect to $(n_- p')$ for fixed values of $(n_+ p')$.

As with all QCD sum rule calculations, the procedure consists in writing the correlator (5) in two different ways: we will refer to them as the *hadronic* side and the *SCET* side. On the hadronic side, one can write:

$$\Pi^{\rm HAD}(n_- p') = \Pi(n_- p')\Big|_{\rm res.} + \Pi(n_- p')\Big|_{\rm cont.} \; ; \quad (8)$$

the first term represents the contribution of the pion, while the second takes into account the role of higher states and continuum above an effective threshold $\omega_s = \mathcal{O}(\Lambda^2/n_+p')$. One has

$$\Pi(n_-p')\Big|_{\text{res.}} = \frac{\langle 0|J_\pi|\pi(p')\rangle\langle\pi(p')|J_0|B(p_B)\rangle}{m_\pi^2 - p'^2} = -\frac{(n_+p')\xi_\pi(n_+p')f_\pi}{n_-p'}\,, \quad (9)$$

obtained in the chosen frame where $p'_\perp = 0$ and neglecting the pion mass. At tree level, the SCET

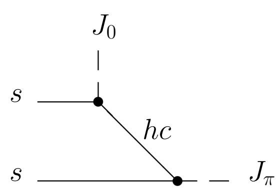

Figure 1. Leading contribution to the correlation function for the SCET current J_0.

side stems from calculating the diagram in Fig. 1, with the result:

$$\Pi(n_-p') = f_B m_B \int_0^\infty d\omega\, \frac{\phi_-^B(\omega)}{\omega - n_-p' - i\eta}\,, \quad (10)$$

where $\omega = n_-\cdot k$, k^μ being the momentum of the soft light quark that ends up as spectator in the B. In (10) we used the momentum-space representation of LCDAs for B mesons as in [13,7],

$$\mathcal{M}^B_{\beta\alpha} = -\frac{if_Bm_B}{4}\times \left[\frac{1+\not v}{2}\left\{\phi_+^B(\omega)\not n_+ + \phi_-^B(\omega)\not n_- + \ldots\right\}\gamma_5\right]_{\beta\alpha}.$$

Notice that (10) has already the form of a dispersion relation in the variable n_-p':

$$\Pi(n_-p') = \frac{1}{\pi}\int_0^\infty d\omega'\, \frac{\mathrm{Im}[\Pi(\omega')]}{\omega' - n_-p' - i\eta}\,, \quad (11)$$

with $\frac{1}{\pi}\mathrm{Im}[\Pi(\omega')] = f_Bm_B\phi_-^B(\omega')$. The final sum rule is obtained by writing also $\Pi(n_-p')\Big|_{\text{cont.}}$ according to a dispersion relation in which the spectral function is identified with the one computed in SCET. Finally, a Borel transformation with parameter ω_M is applied to both sides, giving the following sum rule at tree level:

$$\xi_\pi(n_+p') = \frac{f_Bm_B}{f_\pi(n_+p')}\int_0^{\omega_s} d\omega\, e^{-\omega/\omega_M}\,\phi_-^B(\omega)\,. \quad (12)$$

The inclusion of radiative corrections to the correlation function (5) comes from hard-collinear loops, as shown in Fig. 2 for the leading order in α_s. The explicit calculation shows that the scale-dependence of the correlation function cancels with that of the $C_i(\mu)$ at the considered leading logarithmic order (involving double logs). As for

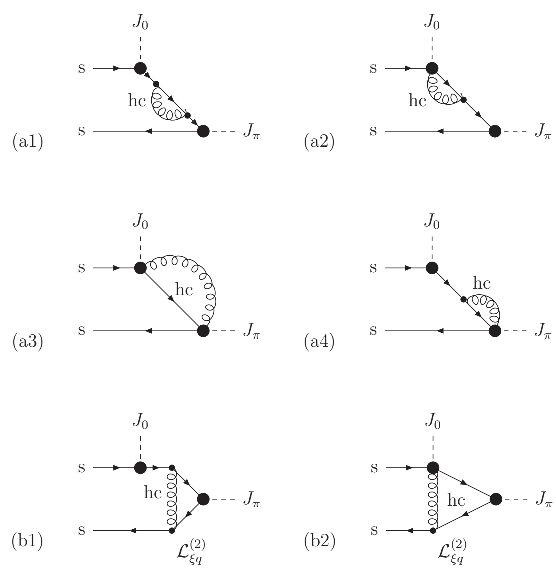

Figure 2. Diagrams contributing to the sum rule for ξ_π to order α_s with hard-collinear loops and no external soft gluons.

the numerical analysis, we fix the sum rule parameters ω_S and ω_M from the sum rule for f_π, which provides us with the *default values* $\omega_M \simeq \omega_S \simeq 0.2$ GeV. For $\phi_-^B(\omega)$, we use the parametrization proposed in [13]: $\phi_-^B(\omega) = e^{-\omega/\omega_0}/\omega_0$ with $1/\omega_0 = \phi_-^B(0) = 2.15$ GeV^{-1}. Fixing one of the two parameters to its default value and varying the other, we may investigate the dependence on such quantities. It turns out that going from LO to NLO such a dependence becomes moderate (see ref. [11] for details). Taking into account the various uncertainties, we obtain:

$$\frac{C_i(\mu)}{C_i(m_b)} \cdot \xi_\pi(m_B, \mu) = 0.27^{+0.09}_{-0.11}\,, \qquad (13)$$

which compares well with other estimates for the $B \to \pi$ form factor in *full* QCD.

Our approach can also be applied to calculate the factorizable form factor contribution, which comes from spectator scattering terms. This can be obtained starting from the correlator:

$$\Pi_1(p') = i \int d^4x\, e^{ip'x}\, \langle 0|T[J_\pi(x) J_1(0)]|B(p_B)\rangle\,,$$

where $J_1 = \bar{\xi}_{\rm hc}\, g\, \not{A}^\perp_{\rm hc}\, h_v$ in the light-cone gauge.

The remarkable result of the SCET-sum-rule for the $B \to \pi$ form factor is that the ratio of factorizable and non-factorizable contributions is independent of the B-meson wave function to first approximation and amounts numerically to about $\approx 6\%$, which is in line with the power counting used in QCD factorization [14,7], but contradicts the assumptions of the pQCD approach [15] and the results of a recent study in [16].

3. CONCLUSIONS

We have described the approach derived in [11] consisting in the derivation of light-cone sum rules for exclusive B-decay amplitudes at large recoil within soft-collinear effective theory (SCET). This formalism defines a consistent scheme to calculate both factorizable and non-factorizable contributions to exclusive B decays as a power expansion in Λ/m_b. The non-perturbative information is encoded in the light-cone wave functions of the B meson, and in the sum-rule parameters.

An explicit example is provided by the study of the factorizable and non-factorizable contributions to the $B \to \pi$ form factor at leading power in Λ/m_b. The result for the central value of the "soft"/non-factorizable $B \to \pi$ form factor is consistent with corresponding estimates in full QCD. In particular, to first approximation, the *ratio* of factorizable and non-factorizable contributions is independent of the B-meson wave function and small (formally of order α_s at the hard-collinear scale, numerically of the order of 5-10%), thus confirming the power-counting adopted in the QCD-factorization approach.

The improvement of the SCET sum rule for the $B \to \pi$ form factor and the extension to other decays requires a better understanding of both, the size and the renormalization-group behaviour, of the light-cone wave functions for higher Fock states in the B meson. These issues are left for future investigations.

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