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Distribution of Gamma-ray Burst Ejecta Energy with Lorentz Factor

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ABSTRACT

The early X-ray afterglow for a significant number of gamma-ray bursts detected by the *Swift* satellite is observed to have a phase of very slow flux decline with time ($F_\nu \propto t^{-\alpha}$ with $0.2 \lesssim \alpha \lesssim 0.8$) for $10^{2.5}$ s $\lesssim t \lesssim 10^4$ s, while the subsequent decline is the usual $1 \lesssim \alpha_3 \lesssim 1.5$ behavior, that was seen in the pre-*Swift* era. We show that this behavior is a natural consequence of a small spread in the Lorentz factor of the ejecta, by a factor of $\sim 2-4$, where the slower ejecta gradually catch-up with the shocked external medium, thus increasing the energy of forward shock and delaying its deceleration. The end of the "shallow" flux decay stage marks the beginning of the Blandford-McKee self similar external shock evolution. This suggests that most of the energy in the relativistic outflow is in material with a Lorentz factor of $\sim 30-50$.

Subject headings: gamma-rays: bursts — shock waves — hydrodynamics

1. Introduction

Among the discoveries made by the *Swift* satellite within a few months of its launch is the observation that a fraction of long duration gamma-ray bursts (GRBs) go through an early phase of relatively slow decline in the X-ray afterglow flux that typically starts at a few minutes after the burst and lasts for about an hour (Nousek et al. 2005). This phase is followed by a somewhat faster and more typical flux decay that satisfies the expected relation between the temporal decline index α and the spectral index β, where $F_\nu \propto \nu^{-\beta} t^{-\alpha}$, similar to what was observed before the *Swift* era when the monitoring of the afterglow light curves started at least several hours after the GRB. The spectral index does not seem to undergo any change when the light-curve transitions (at $t_{\rm break,2} \sim 10^4$ s) from a shallow decline (α_2) to the "regular" decline (α_3). It has been argued convincingly by a number of authors that the more slowly declining lightcurve, like the "regular" flux decay rate that follows it, are both produced by the shock heated circum-burst medium (Nousek et al. 2005; Panaitescu et al. 2005; Zhang et al. 2005). The shallow flux decay is widely attributed to energy injection into the afterglow shock, which may be caused by either a long lived activity of

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the central source, or a short lived central explosion that produces ejecta with some distribution of Lorentz factor (LF). In either of these scenarios the deceleration of the afterglow shock is reduced due to the energy being added to it, and this in turn produces a slowly declining light curve.

A long lived activity of the central source is not very appealing since it would require the source to be active up to several hours after the GRB, with a very smooth temporal behavior, where most of the energy is in the outflow that is ejected around $t_{\rm break,2} \sim 10^4$ s; this makes the problem of the observed high efficiency for converting kinetic energy to gamma-ray radiation much worse (Nousek et al. 2005). Another interesting way to produce an early flat phase in the afterglow light curve (Eichler & Granot 2005) is by a line of sight that is slightly outside the (sharp) edge of a roughly uniform jet (Granot et al. 2002; Granot, Ramirez-Ruiz & Perna 2005). This would, however, naturally be accompanied by a weaker and softer prompt emission, perhaps resulting in an X-ray flash or X-ray rich GRB rather than a classical GRB; the more pronounced this effect is the flatter and longer lived the slow X-ray afterglow decay phase should be. Initial inspection of the data does not show such a correlation, suggesting that viewing angle effects are probably not the predominant cause of the early slow decay phase in the X-ray afterglows, at least under the simplest assumptions.[a]

It is natural to expect that matter ejected in any explosion will have a range of velocities or LFs. After a while (on a time scale, in the observer frame, of order a few times the duration of the central engine activity) the ejecta will rearrange themselves such that the fastest moving plasma is at the head of the outflow and the slowest at the tail end. This can occur either through internal shocks within the outflow, or by a smooth decrease in the LF of the outflow toward the end of the central source activity. If the ejecta have a finite range of LFs, the slower ejecta would gradually catch up with the shocked external medium, injecting energy into the forward shock. If the slower ejecta carry more energy than the faster ejecta, then this added energy would gradually increase the energy of the afterglow shock, causing it to decelerate more gradually. Once the energy in the lower LF ejecta becomes small compared to the energy already in the afterglow shock, the blast wave evolution becomes impulsive (i.e. the subsequent small amount of energy injection hardly effects the evolution of the forward shock), and if radiative losses are unimportant then it approaches the adiabatic Blandford & McKee (1976) self-similar solution. This occurs when the LF of the afterglow shock drops slightly below $\Gamma_{\rm peak}$, the LF where $dE/d\ln\Gamma$ peaks and where most of the energy in the outflow resides.

In this paper we use the *Swift* data to determine the time dependence of the blast wave LF.

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[a]Eichler & Granot (2005) point out that viewing angle effects might still be the dominant cause of the flat early decay of the afterglow light curves if along some lines of sight the kinetic energy in the afterglow shock is very low while the energy in gamma-rays remains high.

We find that the LF typically drops by a factor of $\sim 2-4$ during the shallow decline phase. This is consistent with the basic picture suggested above, where a finite LF distribution for the ejecta causes a more gradual decline of the forward-shock LF, which gives rise to a shallow light-curve, and is an intermediate transition stage before the onset of the adiabatic Blandford-McKee solution.

2. Dependence of Burst Kinetic Energy on Lorentz Factor

The emission from an external shock can be described in terms of the shock front LF (Γ) and the density profile of the circum-stellar medium (CSM). For a uniform CSM the synchrotron characteristic frequency (ν_m), the cooling frequency (ν_c) and the flux at the peak of the spectrum ($F_{\nu,\max}$), in the observer frame, are proportional to Γ^4, $\Gamma^{-4}t^{-2}$ and $t^3\Gamma^8$ respectively, where t is the observed time. The flux at a frequency between the ν_m and ν_c is proportional to $t^3\Gamma^{6+2p}$ and for the observed band above ν_m and ν_c the flux scales as $t^2\Gamma^{4+2p}$. The observed flux is strongly dependent on Γ and therefore even a small deviation from the $\Gamma \propto t^{-3/8}$ scaling has a very large effect on the observed light-curve. The observed flux has a weaker dependence on Γ for a wind like density stratification of the CSM; the flux in the two regimes considered above scales roughly as $\Gamma^{1+p}t^{(1-p)/2}$ and $\Gamma^{2+p}t^{-(p-2)/2}$, respectively.

More generally, for a power law external density profile, $\rho_{\rm ext} = Ar^{-k}$, we have $F_{\nu,\max} \propto \Gamma B R^{3-k} \propto \Gamma^2 R^{3-3k/2} \propto \Gamma^{8-3k} t^{3-3k/2}$, $\nu_m \propto \Gamma B \gamma_m^2 \propto \Gamma^4 R^{-k/2} \propto \Gamma^{4-k} t^{-k/2}$, $\gamma_c \propto 1/\Gamma B^2 t$ and $\nu_c \propto \Gamma B \gamma_c^2 \propto \Gamma^{-1} B^{-3} t^{-2} \propto R^{3k/2}\Gamma^{-4}t^{-2} \propto \Gamma^{3k-4} t^{-2+3k/2}$. Therefore,

$$F_\nu \approx \begin{cases} F_{\nu,\max}(\nu/\nu_c)^{-1/2} & \propto & \Gamma^{6-3k/2}t^{2-3k/4} & \nu_c < \nu < \nu_m \ , \\ F_{\nu,\max}(\nu/\nu_m)^{(p-1)/2} & \propto & \Gamma t^{3-k(p+5)/4} & \nu_m < \nu < \nu_c \ , \\ F_{\nu,\max}(\nu_c/\nu_m)^{(p-1)/2}(\nu/\nu_c)^{-p/2} & \propto & \Gamma^{4-k+p(4-k)/2}t^{2-k(2+p)/4} & \nu > \max(\nu_m,\nu_c) \ . \end{cases} \quad (1)$$

Assuming that the LF distribution for the ejecta is $E(>\Gamma) \propto \Gamma^{-a}$, we find $g \equiv -d\log\Gamma/d\log t$ is smaller by an amount δ compared to the standard value of $(3-k)/2(4-k)$, i.e. 3/8 (1/4) for a uniform (wind) CSM, where

$$\delta = \frac{(3-k)a}{2(4-k)[2(4-k)+a]} = \begin{cases} 3a/[8(8+a)] & k=0 \ , \\ a/[4(4+a)] & k=2 \ . \end{cases} \quad (2)$$

The deviation to the LC temporal power-law index ($\Delta\alpha$) from the standard case of Blandford-

McKee self-similar solution (α) is easily related to δ. For $\nu_m < \nu < \nu_c$ we have

$$\Delta\alpha = \left[6 - \frac{p(4-k)-5k}{2}\right]\delta = \begin{cases} 3(3+p)a/[4(8+a)] & k=0\,, \\ (1+p)a/[4(4+a)] & k=2\,, \end{cases} \tag{3}$$

while for $\nu > \max(\nu_m, \nu_c)$,

$$\Delta\alpha = \left[4 - k + \frac{p(4-k)}{2}\right]\delta = \begin{cases} 3(2+p)a/[4(8+a)] & k=0\,, \\ (2+p)a/[4(4+a)] & k=2\,. \end{cases} \tag{4}$$

We next calculate δ for a number of Swift detected GRBs with a shallow LC using the observed spectral index and the change in the temporal power-law index for the X-ray lightcurve ($\Delta\alpha$) between the shallow and the "regular" parts of the LC. The results for δ, and the change to the LF during the shallow LC are shown in Table 1.

We note that the change to Γ during the shallow phase of the LC was calculated using the appropriate dependence of Γ on t; for a uniform CSM this is $t^{\delta-3/8}$. It can be seen in Table 1 that Γ changes by a factor $\sim 2-4$ for all the bursts, with a uniform CSM, during the shallow LC phase; these numbers change only by a small amount even if we take the forward shock emission, and the shallow decline, to begin at the end of the GRB.

The function $dE/d\ln\Gamma$ peaks at $\Gamma_{\rm peak} \sim \Gamma(t_{\rm break,2})$, the LF of the forward shock at the end of the shallow decline phase of the X-ray lightcurve. For $\Gamma > \Gamma_{\rm peak}$, $dE/d\ln\Gamma \propto \Gamma^{-a}$. The power-law index a is given in table 1 for a number of bursts detected by Swift and lies between ~ 1 and ~ 2.5 if the CSM has uniform density (Nousek et al. 2005, report similar values – $s-1$ in their notation); $a \gtrsim 5$ if the medium in the vicinity of GRB is taken to be a wind-CSM or alternatively the central source has to be active for several hours with little variability and a roughly constant rate of energy output in relativistic outflow – neither of these possibilities seem very plausible and so the case of a wind-CSM is not considered any further in this paper. For $\Gamma < \Gamma_{\rm peak}$ the function $dE/d\ln\Gamma \propto \Gamma^b$ should decrease with decreasing Γ (i.e. $b > 0$) as otherwise slower moving ejecta will continue to add substantial amount of energy to the forward shock thereby retarding its deceleration and slowing down the decline of the lightcurve. Since the spectral index and the lightcurve power-law decay index after the end of the shallow decline phase obey the relationship expected for an adiabatic forward shock evolution we conclude that indeed $b > 0$, but its exact value is otherwise unconstrained. Radio calorimetry for a number of GRBs has concluded that there is not a whole lot of energy in GRBs in the form of mildly relativistic ejecta with $\Gamma \sim 2$ (e.g. Berger et al. 2004; Frail et al. 2005). This further strengthens our conclusion that $b > 0$, and that this scaling might extends to $\Gamma \sim 2$. We note that for a given total energy in the explosion of order 10^{52} erg the relation $dE/d\ln\Gamma \propto \Gamma^{-a}$, with $a \sim 1.5$, must turnover at some Γ of order 10 or so otherwise the energy in the relativistic ejecta will exceed the total available energy (energy in relativistic ejecta with $\Gamma > \Gamma_{\rm peak} \sim 50$ is of order 10^{51} erg). We have now considerable body of evidence that long duration GRBs are accompanied by a supernova of Type Ic, which expels a few solar masses of material at velocities of order 10^4 km s^{-1}. Thus, $dE/d\ln u$, where $u = \beta\Gamma = (\Gamma^2-1)^{1/2}$, must again turnover over and have a peak at $u \sim 0.05$. Putting all these together we show a schematic behavior of $E(\beta\Gamma)$ (i.e. $dE/d\ln u$) in Figure 1.

3. Conclusion

We have pieced together the distribution of energy in gamma-ray burst ejecta as a function of the four-velocity $u = \beta\Gamma = (\Gamma^2-1)^{1/2}$ for $0.1 \lesssim u \lesssim 10^2$. The distribution function, $dE/d\ln u$, has two peaks: one at $u \sim 0.1$ and another at $u \sim 30-50$. For $u \gtrsim 50$, it falls off as $dE/d\ln u \propto u^{-a}$ with $a \sim 1-2$, as is determined from the shallow decline of the X-ray lightcurve at early times ($10^{2.5}$ s $\lesssim t \lesssim 10^4$ s) observed for a good fraction of bursts detected by the Swift satellite. The distribution at low $u \sim 0.1$ is obtained by observations of supernovae Ic that are associated with GRBs. In the intermediate regime of $1 \lesssim u \lesssim 30$ the shape of the distribution function $dE/d\ln u$ is very uncertain, but we argue that it is likely to be at least flat or slowly rising in this range.

The challenge posed for GRB/SNe models is to understand what physical processes give rise to $a \sim 2$ and why the LF distribution of the ejecta peaks at a value roughly $\Gamma_{\rm peak} \sim 30-50$. Understanding these results should help illuminate the processes operating during the period in which the central engine of gamma-ray burst is active and the interaction of the relativistic outflow with the collapsing star and its immediate surroundings.

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Table 1. The Change in the Lorentz Factor During the Energy Injection Episode

GRB #	T_{90}/s	$t_{\mathrm{break},1}/\mathrm{s}$	$t_{\mathrm{break},2}/\mathrm{s}$	a	δ	$\xi_{\min}$ †	$\xi_{\max}$	Γ_{peak} ⋆	Γ_0 ♣
050128	13.8	< 230	1720^{+940}_{-570}	1.1 ± 0.2	0.045 ± 0.07	2.0	4.9	—	—
050315	96.0	400 ± 20	12000 ± 400	2.4 ± 0.1	0.094 ± 0.03	2.6	3.9	30	117
050319	10.0	370 ± 15	40000 ± 300	1.6 ± 0.5	0.063 ± 0.016	4.3	13.3	21	279
050401	33.0	< 127	5500^{+1150}_{-1050}	1.7 ± 0.1	0.066 ± 0.003	3.2	4.9	58	284
050416a	2.4	< 80	1350^{+2070}_{-620}	1.1 ± 0.1	0.043 ± 0.004	2.6	8.2	28	230
050607	26.5	510 ± 50	6400 ± 900	1.5 ± 0.1	0.059 ± 0.004	2.2	5.7	—	—

Note. — The relevant data were taken from Nousek et al. (2005). All of the calculated quantities reported in this table – a, δ, ξ and Γ_0 – assume a uniform density medium in the vicinity of these bursts. † Here ξ is the ratio of the Lorentz factor of the afterglow shock at the start and at the end of the "shallow part" of the X-ray light-curve, and its value is estimated to be between $\xi_{\min} = (t_{\mathrm{break},2}/t_{\mathrm{break},1})^{3/(8+a)}$ and $\xi_{\max} = (t_{\mathrm{break},2}/T_{90})^{3/(8+a)}$; ⋆ These values of $\Gamma_{\mathrm{peak}} = \Gamma(t_{\mathrm{break},2})$ were estimated only for the GRBs with known redshifts, by using equation 9 of Nousek et al. (2005) where the isotropic equivalent kinetic energy at $t_{\mathrm{break},2}$ was taken to be equal to $E_{\gamma,\mathrm{iso}}$, and the external density was taken to be $n = 1\ \mathrm{cm}^{-3}$; ♣ The initial Lorentz factor is simply estimated by $\Gamma_0 = \xi_{\max}\Gamma_{\mathrm{peak}}$.

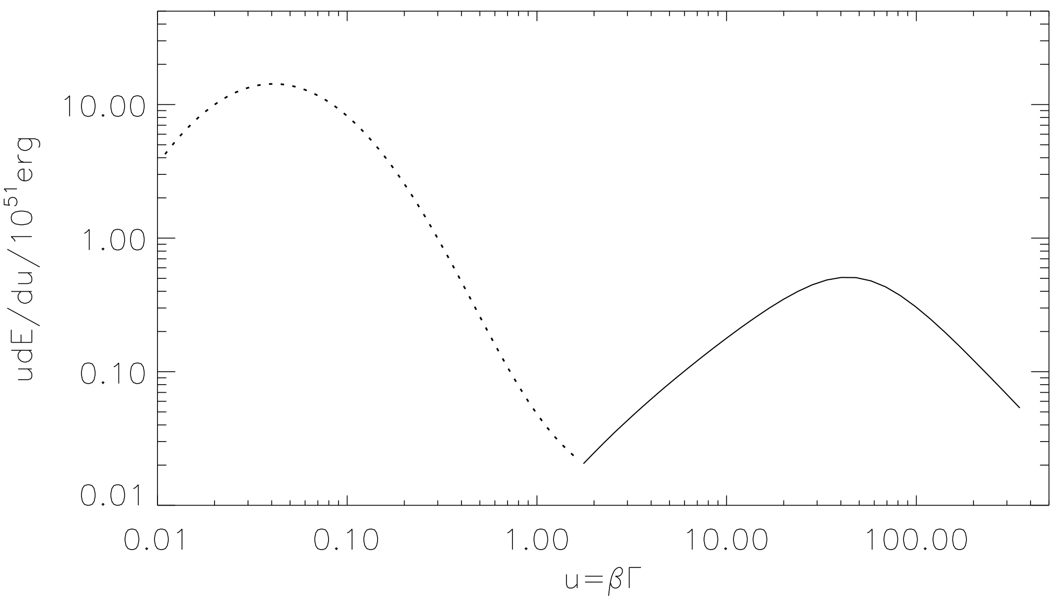

Fig. 1.— Schematic figure showing $dE/d\ln u$, in units of 10^{51} erg, as a function of $u \equiv \Gamma\beta = (\Gamma^2-1)^{1/2}$. It has one relativistic component (solid line) with total energy $\sim 10^{51}$ erg and peak at $u \sim 30-50$ that produces the gamma-ray burst and the afterglow radiations. The power-law index above the peak for this component is well constrained by the X-ray data (the shallow part of the light-curve) and is ~ -1.5 (see table 1). The slope below the peak is not constrained and is taken to be 1; in reality it can be close to zero, as the only constraint we have is from late time radio afterglow observations which suggests that there is not a lot of extra energy in material moving with Lorentz factor of order 2. The second component (dashed curve) shows schematically the kinetic energy in non-relativistic ejecta in the supernova accompanying the GRB; the peak for this component is taken to be $\sim 10^4$ km s^{-1}, the typical velocity for SNe Ic ejecta, and the energy is $\sim 10^{52}$ erg.