

# VIRTUAL ACCELERATOR FOR ACCELERATOR OPTICS IMPROVEMENT

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## Abstract

Through determination of all quadrupole strengths and sextupole feed-downs by fitting quantities derivable from precision orbit measurement, one can establish a virtual accelerator that matches the real accelerator optics. These quantities (the phase advances, the Green's functions, and the coupling ellipses tilt angles and axis ratios) are obtained by analyzing turn-by-turn Beam Position Monitor (BPM) data with a model-independent analysis (MIA). Instead of trying to identify magnet errors, a limited number of quadrupoles are chosen for optimized strength adjustment to improve the virtual accelerator optics and then applied to the real accelerator accordingly. These processes have been successfully applied to PEP-II rings for beta beating fixes, phase and working tune adjustments, and coupling reduction to improve PEP-II luminosity.

## INTRODUCTION

We consider all quadrupole strengths and sextupole feed-downs as well as all BPM gains and BPM cross-plane couplings as variables to fit the Local Green's functions [1] [2] and the phase advances [3] calculated from a lattice model to those derived from orbit measurement using a model-independent analysis (MIA) [4]. The fitting process is with an SVD-enhanced Least Square fitting technique [5] that is efficient enough for a system of tens of thousand constraints with thousands of variables. Once the lattice model is fitted to the orbit measurement, we would confirm if it matches the real accelerator in linear optics by checking the coupling eigen ellipses [6] between those calculated from the fitted model and those derived from the orbit measurement to see if they are automatically matched at the double-view BPM locations before we call this fitted lattice model the computer virtual accelerator.

Once the virtual accelerator is obtained, instead of comparing it to the ideal lattice model for finding and adjusting one or two magnets with noticeable differences, we would go on with this virtual accelerator to search for an easily-approachable better-optics model by pre-selecting and fitting a group of limited number of quadrupole strengths and then create a machine operation knob for practicing the corresponding quadrupole strengths adjustment in the real accelerator. These procedure has been successfully applied for PEP-II optics improvement. Noticeable achievement has been that MIA has helped PEP-II achieve its breaking record peak luminosity above  $6.5 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  in 2003 by bringing the LER working tune to near half integer and

simultaneously fixing the beta beat and improve the linear coupling, which would, otherwise, be difficult without MIA because of the strong LER coupling effect.

We now briefly describe the procedures.

## A COMPLETE SET OF DATA ACQUISITION

The linear geometric optics is determined if one gets 4 independent linear orbits. This can be clearly shown by the obtainable linear mapping,  $Z^b = R^{ab} Z^a$ , and so  $R^{ab} = Z^b Z^{a-1}$ , where the 4-by-4 matrix,  $Z^a = [\bar{z}_1^a, \bar{z}_2^a, \bar{z}_3^a, \bar{z}_4^a]$ , represents 4 independent linear orbits at location  $a$ , and  $R^{ab}$  is the linear map from location  $a$  to location  $b$ . Therefore, a complete set of data must be able to provide the extraction of 4 independent orbits.

Unlike linacs where there is often enough incoming jitter in the beam to measure and identify betatron modes, in the rings, to offset radiation damping, the most economic process for such data acquisition would be through two orthogonal resonance excitations, one at the horizontal (eigen-plane 1) and the other at the vertical (eigen-plane 2) betatron tunes, and then take and store buffered BPM data. Since a betatron motion has two degrees of freedom (the phase and the amplitude), each excitation would generate a pair of conjugate (cosine- and sine-like) betatron motion orbits from zoomed FFT after removing non-physical BPM data. Therefore, a complete 4 independent linear (X and Y) orbits can be extracted from the two eigen-mode excitations. A typical set of such orbits for PEP-II LER is shown in Figure 1. One can clearly see the linear coupling in the IR region but not at IP where BPM sequence numbers are around 160.

## FITTING CONSTRAINTS

For linear optics fitting, while it is clear that all quadrupole strengths and sextupole feed-downs as well as the BPM linear gains and cross couplings should be used as a complete set of orthogonal variables, it is somewhat flexible to choose the fitting constraints as far as all degrees of freedom are implicitly contained and the degeneracies can be well taken care of.

A linear symplectic map (matrix) contains 10 degrees of freedom [7] as one can also count that there are 10 coefficients in a second-order homogeneous polynomial in its Lie transformation. Therefore one must ultimately find 10 non-parallel (not necessarily orthogonal) quantities for each BPM location, which are derivable from the 4 independent orbits that contains only positions without conju-

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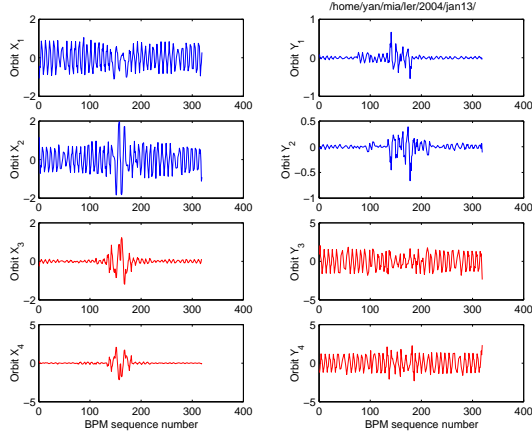


Figure 1: *Four independent orbits extracted from PEP-II LER BPM buffer data taken on January 13, 2004. The first two orbits ( $x_1, y_1$ ) and ( $x_2, y_2$ ) are extracted from beam orbit excitation at the horizontal tune while the other two orbits ( $x_3, y_3$ ) and ( $x_4, y_4$ ) are from excitation at the vertical tune.*

gate momenta. What we have for these 10 degrees of freedom (DOF) at each BPM are phase advances (2 DOFs), one-turn Green's functions (4 DOFs), and eigen-plane ellipse tilt angles (2DOFs) and axis ratios (2 DOFs) [6]. Note that the last 4 quantities are also a complete subset for the linear coupling.

However, for better convergent process, we would consider more constraints by taking much more Green's functions between two BPMs. Note that the number of Green's functions between two BPMs are basically unlimited. This is the key leading to success of this program. For faster fitting speed without losing accuracy, we usually use more Green's functions but leave the eigen-plane ellipse tilt angles and axis ratios out of the fitting process. They are used for after-fitting check as they have to be automatically matched between the fitting model and the direct measurement to guarantee that the fitted result is all right.

Given all quadrupole strengths and sextupole feed-downs, calculations for the above quantities from the lattice model are trivial. Their corresponding derivations directly from orbit measurement are discussed below.

### Phase advances

One can derive the orbit betatron phase at each BPM location by simply taking the arctangent of the ratio of the imaginary part to the real part of the resonance excitation FFT mode [3]. Phase advances between adjacent BPMs can then be calculated by subtraction. Note that the ratio of the imaginary part to the real part of the FFT will cancel the linear BPM gains but not the BPM cross couplings. Therefore the phase advances among BPMs are repeatedly calculated during the Least Square fitting process as the BPM cross couplings and BPM gains are updated to correct the linear orbits.

### Linear Green's functions

The linear Green's function are simply the  $R_{12}^{ab}, R_{34}^{ab}, R_{14}^{ab}, R_{32}^{ab}$  of the linear transfer matrix between any two BPMs labeled as  $a$  and  $b$ . They are given in the data measurement space as [1]

$$(x_1^a x_2^b - x_2^a x_1^b)/Q_{12} + (x_3^a x_4^b - x_4^a x_3^b)/Q_{34} = \mathcal{R}_{12}^{ab}, \quad (1)$$

$$(x_1^a y_2^b - x_2^a y_1^b)/Q_{12} + (x_3^a y_4^b - x_4^a y_3^b)/Q_{34} = \mathcal{R}_{32}^{ab}, \quad (2)$$

$$(y_1^a x_2^b - y_2^a x_1^b)/Q_{12} + (y_3^a x_4^b - y_4^a x_3^b)/Q_{34} = \mathcal{R}_{14}^{ab}, \quad (3)$$

$$(y_1^a y_2^b - y_2^a y_1^b)/Q_{12} + (y_3^a y_4^b - y_4^a y_3^b)/Q_{34} = \mathcal{R}_{34}^{ab}, \quad (4)$$

where  $Q_{12}$  and  $Q_{34}$  are the two invariants relating to the two resonance excitation amplitude,  $(x_1, y_1)$ ,  $(x_2, y_2)$ ,  $(x_3, y_3)$ ,  $(x_4, y_4)$  are the 4 independent linear orbits, and  $\mathcal{R}_{12}, \mathcal{R}_{32}, \mathcal{R}_{14}, \mathcal{R}_{34}$  are given as

$$\mathcal{R}_{12}^{ab} = g_x^b R_{12}^{ab} g_x^a + g_x^b R_{14}^{ab} \theta_{xy}^a + \theta_{xy}^b R_{32}^{ab} g_x^a + \theta_{xy}^b R_{34}^{ab} \theta_{xy}^a,$$

$$\mathcal{R}_{32}^{ab} = g_y^b R_{32}^{ab} g_x^a + g_y^b R_{34}^{ab} \theta_{xy}^a + \theta_{yx}^b R_{12}^{ab} g_x^a + \theta_{yx}^b R_{14}^{ab} \theta_{xy}^a,$$

$$\mathcal{R}_{14}^{ab} = g_x^b R_{14}^{ab} g_y^a + g_x^b R_{12}^{ab} \theta_{yx}^a + \theta_{xy}^b R_{34}^{ab} g_y^a + \theta_{xy}^b R_{32}^{ab} \theta_{yx}^a,$$

$$\mathcal{R}_{34}^{ab} = g_y^b R_{34}^{ab} g_y^a + g_y^b R_{32}^{ab} \theta_{yx}^a + \theta_{yx}^b R_{14}^{ab} g_y^a + \theta_{yx}^b R_{12}^{ab} \theta_{yx}^a,$$

where  $g_x$ 's,  $g_y$ 's are the BPM gains, and  $\theta_{xy}$ 's and  $\theta_{yx}$ 's are the BPM cross-coupling multipliers [1].  $R_{12}^{ab}, R_{34}^{ab}, R_{32}^{ab}, R_{14}^{ab}$  are the Green's functions between any two BPMs labeled as  $a$  and  $b$  of the machine.

### Coupling ellipses

For each double-view BPM, one can trace the MIA extracted high-resolution real-space orbits to obtain a coupling ellipse in real space for each resonance (eigen) excitation. Therefore, one can calculate coupling ellipse tilt angles and axis ratios for all double-view BPMs in each of the two eigen planes [6]. The tilt angle of the coupling ellipse at IP for the horizontal eigen plane is very close to the real tilt angle of the beam at IP. One can also calculate these corresponding coupling parameters from the linear map of a lattice model. Therefore, these quantities can be used as part of the fitting parameters. However, in most cases, we do not fit for the coupling ellipses derivatives, but reserve them for after-fit check. They automatically match - a necessary condition to make sure the fitting is all right. Shown in Figure 2 are optics characteristics plots for a well-fitted LER.

## APPLICATION TO PEP-II, AN EXAMPLE

PEP-II LER is a strong coupled accelerator. Without dealing with the linear coupling, it is difficult to fix the LER beta beat. Before MIA was fully developed, without MIA, we have several unsuccessful attempts trying to bring the LER horizontal working tune to near half integers

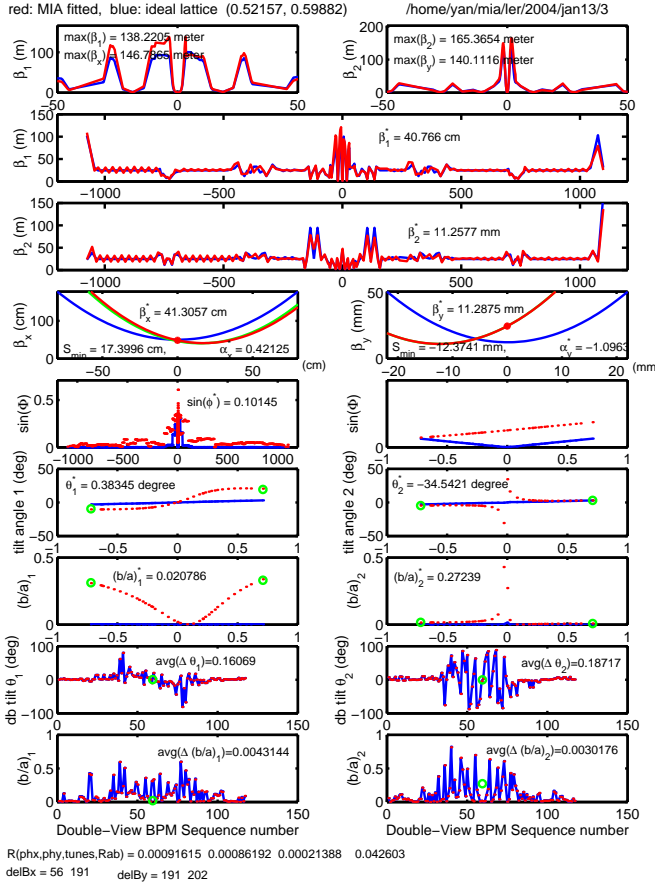


Figure 2: A typical plot to show a virtual accelerator linear optics characteristics (red color) compared with those of the designed lattice (blue color), except the last 4 plots which compare the coupling ellipse parameters calculated from the fitted virtual accelerator to those from measurement. In this case, it is PEP-II LER on January 13, 2004. The top two plots show the two eigen beta functions in the vicinity of IP followed by two plots that show the beta functions for the whole machine and then the beta function plots at IP, which show the  $\beta^*$ 's and the waist. The next two plots show the phase-space coupling angles followed by 4 plots that show the coupling eigen-plane ellipse tilt angles and axis ratios at IP. The coupling ellipse parameters, the tilt angles and the axis ratios, for all double-view BPMs are compared in the last (bottom) 4 plots for those from measurement and those calculated from the virtual accelerator, which show a very good match - a necessary condition for a good fitting.

because of strong beta beat. Once MIA was fully developed in 2003, we successfully use MIA to bring the LER to a near half integer working tune and simultaneously fixed the beta beat by dialing in a MIA generated knob that involved adjustment of the linear trombone quadrupoles and the 4 global skews. The reason MIA was successful is because MIA takes care of the complete set of linear optics characteristics. The linear coupling is fully treated with-

out any discount. Success of bringing PEP-II horizontal working tune to near half integer improves the beam-beam effects and subsequently, we have a breaking-record PEP-II luminosity above  $6.5 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  after getting LER and HER matched at near half integer working tunes.

## SUMMARY

MIA has been developed to a mature practical stage for offering countable computer virtual accelerator that matches the real accelerator linear optics. It has been successfully applied to PEP-II LER and HER for fixing the beta beats, adjusting the working tunes to near half integers, and improving the linear couplings. Noticeable contribution to PEP-II machine development has been that MIA has helped PEP-II achieve its breaking record peak luminosity above  $6.5 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  in 2003 by bringing the LER working tune to near half integer and simultaneously fixing the beta beat and reducing the linear couplings, which would, otherwise, be difficult without MIA because of the strong LER coupling effect.

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