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Semi-classical geometry of charged black holes

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At the classical level, two-dimensional dilaton gravity coupled to an abelian gauge field has charged black hole solutions, which have much in common with four-dimensional Reissner-Nordström black holes, including multiple asymptotic regions, timelike curvature singularities, and Cauchy horizons. The black hole spacetime is, however, significantly modified by quantum effects, which can be systematically studied in this two-dimensional context. In particular, the back-reaction on the geometry due to pair-creation of charged fermions destabilizes the inner horizon and replaces it with a spacelike curvature singularity. The semi-classical geometry has the same global topology as an electrically neutral black hole.

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The maximally extended Reissner-Nordström geometry, describing a static electrically charged black hole, has intriguing global structure [1, 2]. There are multiple asymptotic regions and Cauchy horizons associated with timelike singularities, which makes an initial value formulation problematic. Similar difficulties arise in the context of the Kerr spacetime of a rotating black hole. The physical relevance of much of the extended structure is questionable, however [3, 4, 5, 6, 7]. At the classical level, a dynamical instability, referred to as mass inflation, manifests itself when in- and outgoing energy fluxes cross near the inner horizon, replacing it by an initially null singularity which turns spacelike deep inside the black hole [8, 9, 10, 11, 12]. This null singularity is relatively weak, however, with finite integrated tidal effects acting on extended timelike observers [9], leaving open the possibility of extending the physical spacetime through it.

It is natural to ask how this classical picture is modified by quantum effects. These include the pair-creation of charged particles by the Schwinger effect [13] in the background electric field of the charged black hole. In the Reissner-Nordström solution the electric field diverges as the curvature singularity is approached, leading to copious production of electron-positron pairs. At the quantum level, the black hole charge is screened and the singularity surrounded by a charged matter fluid. This fluid is a source of electric field and also modifies the black hole geometry. The combined electromagnetic and gravitational back-reaction can potentially alter the global structure of the spacetime [14, 15]. The dynamical instability affecting the Cauchy horizon may also be enhanced by the production of charged pairs [16].

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The full back-reaction problem is non-trivial and to our knowledge the geometry has not been fully elucidated. On the basis of a simple model of static electrically charged black holes [15], it has been argued that the effect of pair-production on the interior geometry of a black hole can be quite dramatic, in some cases eliminating the Cauchy horizon altogether and rendering the singularity spacelike. The classical mass inflation instability has been observed in numerical work involving the dynamical formation of charged black holes [11]. In subsequent work [17], the quantum effect of pair-creation was modeled by introducing a non-linear dielectric response that prevents the electric field from getting any stronger than the critical field for pair-creation.

In the following we study the internal geometry of electrically charged black holes in the simplified context of a 1+1-dimensional model of dilaton gravity, coupled to a gauge field and charged Dirac fermions. This model has classical charged black hole solutions, whose Penrose diagram is identical to that of 3+1-dimensional Reissner-Nordström black holes [18, 19]. Closely related two-dimensional models, with different matter content, have been shown to exhibit mass inflation at the classical level [20, 21, 22] and quantum effects due to an electrically neutral scalar field were considered in [20]. The main new feature of the present work is to include quantum effects due to *charged* matter, which turn out to significantly modify the internal structure of a charged black hole. These effects are physically important since charged matter fields will be present in any model where charged black holes can be formed by gravitational collapse. In our two-dimensional model, fermion pair-creation is conveniently described via bosonization and the resulting effective action allows us to study the back-reaction on the geometry in a systematic way. Our approach is semi-classical in that it only involves quantum effects of the matter field but the dilaton gravity sector remains classical throughout. Gravitational quantum effects could also be included along the lines of [23, 24] but we prefer

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to postpone that until the effect of charged pair-creation has been mapped out. We are primarily interested in this 1+1-dimensional theory on its own merits as a simplified model of gravity coupled to matter but we note that it can be obtained by spherical reduction from 3+1-dimensional dilaton gravity in the background of an extremal magnetically charged black hole [23, 25, 26].

The classical action of 1+1-dimensional dilaton gravity coupled to an abelian gauge field is given by

$$S_{dg} = \int d^2x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2 - \frac{1}{4}F^2\right]. \tag{1}$$

Here λ is a dimensionful parameter, inversely related to the charge of the 3+1-dimensional extremal dilaton black hole, which sets a mass scale for the theory. In the following we use units where $\lambda = 1$.

The equations of motion of (1) have a two-parameter family of static solutions that are analogous to four-dimensional Reissner-Nordström black holes. In the so-called 'linear dilaton' gauge, they take the form

$$\phi = -x, \qquad F_{01} = Qe^{-2x}, \tag{2}$$

$$ds^2 = -a(x)dt^2 + \frac{1}{a(x)}dx^2, \tag{3}$$

with $a(x) = 1 - Me^{-2x} + \frac{1}{8}Q^2e^{-4x}$. In the asymptotic region $x \to \infty$, the metric approaches the two-dimensional Minkowski metric and the gravitational coupling strength e^{ϕ} goes to zero. The constants M and Q are the mass and charge of the classical geometry. Horizons occur at zeroes of the metric function $a(x)$. We will consider non-extremal black holes with $M > |Q|/\sqrt{2}$, for which $a(x)$ has two zeroes at $x = x_\pm$,

$$e^{2x_\pm} = \frac{1}{2}\left(M \pm \sqrt{M^2 - \frac{1}{2}Q^2}\right) \equiv \psi_\pm. \tag{4}$$

The analogy with Reissner-Nordström black holes can be developed in detail. The metric (3) is singular at $x = x_+$ but the spacetime curvature is finite there. The singularity signals the breakdown of the linear dilaton coordinate system. It is straightforward to find coordinates which extend into the region where $\phi > -x_+$. The new coordinate system, in turn, breaks down at the inner horizon where $\phi = -x_-$. The solution can once again be extended but inside the inner horizon it eventually runs into a curvature singularity, where the gravitational coupling e^{ϕ} diverges. The maximally extended spacetime is covered by an infinite number of coordinate patches that occur in a repeated pattern. The associated Penrose diagram is identical to that of 3+1-dimensional Reissner-Nordström black hole.

Our goal is to determine how the global structure of the black hole spacetime is modified when Schwinger pair-production is taken into account. To this end, we add matter in the form of a 1+1-dimensional Dirac fermion to the theory,

$$S_m = \int d^2x\sqrt{-g}\left[i\bar{\psi}\gamma^\mu(\mathcal{D}_\mu + iA_\mu)\psi - m\bar{\psi}\psi\right]. \tag{5}$$

With this matter sector, our model can be viewed as a generalization to include gravitational effects in the 'linear dilaton electrodynamics' developed in [27, 28, 29].

The quantum equivalence between the Schwinger model in 1+1 dimensions and a bosonic theory with a Sine-Gordon interaction [30] provides an efficient way to include pair-creation effects. The identification between the fermion field and composite operators of a real boson field Z [30, 31] carries over to curved spacetime, except for regions of extreme gravity where the curvature gets large on the microscopic length scale of the quantum theory. The matter current is given by

$$j^\mu = \bar{\psi}\gamma^\mu\psi = \frac{1}{\sqrt{\pi}}\varepsilon^{\mu\nu}\nabla_\nu Z, \tag{6}$$

where $\varepsilon^{\mu\nu}$ is an antisymmetric tensor, related to the Levi-Civita tensor density by $\varepsilon^{\mu\nu} = \epsilon^{\mu\nu}/\sqrt{-g}$. The covariant effective action for the boson field is

$$S_b = \int d^2x\sqrt{-g}\left[-\frac{1}{2}(\nabla Z)^2 - V(Z) - \frac{1}{\sqrt{4\pi}}\varepsilon^{\mu\nu}F_{\mu\nu}Z\right], \tag{7}$$

where $V(Z) = cm^2(1 - \cos(\sqrt{4\pi}Z))$, with c a numerical constant whose precise value does not affect our conclusions. When the charge-to-mass ratio of the original fermions is large, *i.e.* when $m \ll 1$ in our units, the semi-classical geometry of a charged black hole, including the back-reaction due to pair-creation, is reliably described by classical solutions of the combined boson and dilaton gravity system, (1) and (7).

We work in conformal gauge $ds^2 = -e^{2\rho}d\sigma^+d\sigma^-$ and write $F^{\mu\nu} = f\varepsilon^{\mu\nu}$ with $f(\sigma^+, \sigma^-)$ a scalar. The Maxwell equations then reduce to

$$\partial_\pm(e^{-2\phi}f + \frac{1}{\sqrt{\pi}}Z) = 0, \tag{8}$$

and it follows that the gauge field can be eliminated in favor of the bosonized matter field

$$f = -(\frac{1}{\sqrt{\pi}}Z + q)e^{2\phi}. \tag{9}$$

The constant of integration q represents a background charge located at the strong coupling end of our one-dimensional space. In the following we will set $q = 0$. This is natural when we consider gravitational collapse of charged matter into an initial vacuum configuration.

Defining new field variables $\psi = e^{-2\phi}$ and $\theta = 2(\rho - \phi)$, the remaining equations reduce to

$$-4\partial_+\partial_-Z = \frac{Ze^\theta}{\pi\psi^2} + \frac{V'(Z)e^\theta}{\psi}, \tag{10}$$

$$-4\partial_+\partial_-\psi = (4 - \frac{Z^2}{2\pi\psi^2})e^\theta - \frac{V(Z)e^\theta}{\psi}, \tag{11}$$

$$-4\partial_+\partial_-\theta = \frac{Z^2e^\theta}{\pi\psi^3} + \frac{V(Z)e^\theta}{\psi^2}, \tag{12}$$

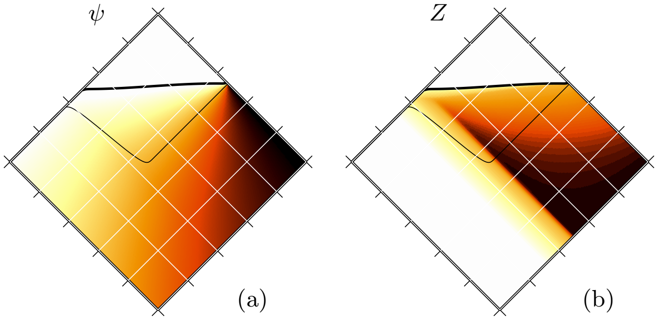

FIG. 1: (a) Density plot of the area function ψ of a black hole formed by the gravitational collapse of charged matter. The curves indicate the apparent horizon (thin) and the curvature singularity (thick). (b) Density plot of the bosonized matter field Z. Screening due to pair-creation prevents the electric charge from penetrating into the strong-coupling region.

along with two constraints

$$\partial_\pm^2\psi - \partial_\pm\theta\partial_\pm\psi = -\frac{1}{2}(\partial_\pm Z)^2. \qquad (13)$$

In the 3+1 dimensional dilaton gravity interpretation, ψ is proportional to the area of the transverse two-sphere in the Einstein frame.

We have solved equations (10)-(13) numerically on a double-null grid using a second-order, staggered leap-frog algorithm. A more detailed account will appear in [32] but the main points can be summarized as follows. The numerical evolution requires initial data on null slices at early advanced and retarded time respectively. For gravitational collapse of charged matter into vacuum we describe the incoming matter at early σ^- by a kink configuration in Z as a function of σ^+. The incoming charge is given by the height of the kink while the incoming energy is determined by its steepness. The slice is chosen to lie at early enough σ^- so that the incoming matter is initially at weak coupling and the black hole has yet to form. The other initial null slice is taken to lie at constant σ^+ in the vacuum region before the incoming matter has arrived. This is achieved by choosing the initial kink configuration such that $\partial_+ Z$ has compact support in an interval $\sigma_0^+ < \sigma^+ < \sigma_1^+$. Then the fields ψ, θ, and Z take vacuum values on any null slice at constant $\sigma^+ < \sigma_0^+$.

Coordinate transformations that act separately on σ^+ and σ^- preserve the conformal gauge and we use this freedom to fix ψ as some given monotonic function of the coordinates along our initial slices. The starting profile of the metric function θ can then be obtained by solving the constraint equations (13) on the initial slices. Numerical results for ψ and Z obtained from such initial data are shown in figure 1. In order to see the global geometry we use coordinates that bring $\sigma^+ \to \infty$ to a finite distance. The spacetime curvature diverges as $\psi \to 0$ and this occurs on a spacelike curve in figure 1. The spacelike singularity approaches the apparent horizon at future null infinity, removing all traces of the classical Cauchy horizon inside the black hole. We have obtained this behavior for a range of black hole masses and charges, and for various (small) values of the fermion mass.

The semi-classical theory also has static black hole solutions. To study them we write $\xi = e^\theta$ and define a new spatial coordinate y via $dy = \xi d\sigma$. Outside the event horizon the semi-classical equations (10)-(12) reduce to

$$\xi\ddot{Z} + \dot{\xi}\dot{Z} = \frac{Z}{\pi\psi^2} + \frac{V'(Z)}{\psi}, \qquad (14)$$

$$\xi\ddot{\psi} + \dot{\xi}\dot{\psi} = 4 - \frac{Z^2}{2\pi\psi^2} - \frac{V(Z)}{\psi}, \qquad (15)$$

$$\ddot{\xi} = \frac{Z^2}{\pi\psi^3} + \frac{V(Z)}{\psi^2}, \qquad (16)$$

where the dot denotes $\frac{d}{dy}$, and the constraints (13) become

$$\ddot{\psi} + \frac{1}{2}(\dot{Z})^2 = 0. \qquad (17)$$

Inside the event horizon y becomes timelike and the derivative terms in equations (14)-(16) change sign.

The corresponding classical system is

$$\dot{\xi}\dot{\psi} = 4 - \frac{Q^2}{2\psi^2}, \qquad \ddot{\xi} = \frac{Q^2}{\psi^3}, \qquad \ddot{\psi} = 0. \qquad (18)$$

These equations describe classical black holes of charge Q. They are obtained from the semi-classical equations by dropping the matter field equation of motion (14), and replacing Z in equations (15)-(17) by a constant $\sqrt{\pi}Q$, with Q an integer. In these variables a classical black hole solution takes the form

$$\psi(y) = \psi_+ + \alpha y,$$
$$\xi(y) = \frac{4}{\alpha^2}\left|\alpha y + \frac{\psi_+\psi_-}{\psi_+ + \alpha y} - \psi_-\right|, \qquad (19)$$

where $Q^2 = 8\psi_+\psi_-$ has been used. The free parameter α sets the scale of the spatial coordinate y, with $\alpha = 2$ matching the coordinate scale in (2)-(3). The solution (19) is shown in figure 2(a). It describes all three regions: outside, inside, and between the two horizons. The area function ψ extends smoothly through both horizons. It decreases linearly as we go deeper into the black hole and reaches zero at the curvature singularity. Meanwhile ξ, which contains the conformal factor of the metric, goes to zero at both horizons and diverges at the singularity. The absolute value sign in (19) reflects the signature change of the metric between horizons.

Returning to the semi-classical equations (14)-(17) we find significant departure from the above classical behavior. Explicit analytic solutions are not available but the equations can be integrated numerically. A black hole solution with a smooth event horizon is obtained by starting the integration at the horizon $y = 0$ with $\xi(0) = 0$, and tuning the initial values $\dot{\psi}(0)$ and $\dot{Z}(0)$ so that the solution is regular there [33]. Different black holes are

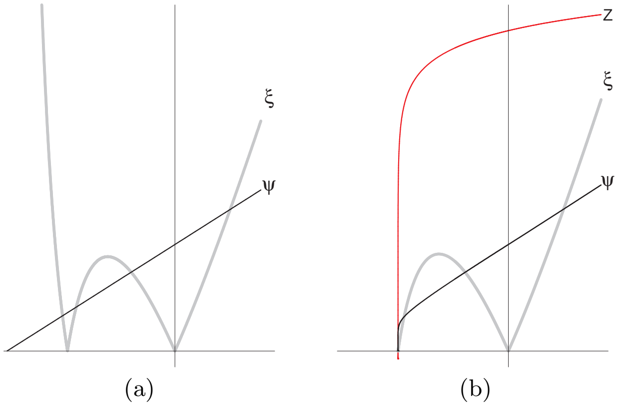

FIG. 2: (a) ψ and ξ plotted as a function of y for a classical black hole solution. The two horizons are at the zeroes of ξ and the curvature singularity is where ψ goes to zero. (b) Semi-classical black hole. The inner horizon is replaced by a singularity where ψ, ξ, and Z all approach zero.

then parametrized by $\psi(0)$ and $Z(0)$. The choice of $\dot{\xi}(0)$ sets the scale of y but does not affect the geometry.

Numerical results for massless matter are shown in figure 2(b). A small fermion mass does not change the qualitative behavior. The scalar fields ψ and Z extend smoothly through the horizon at $y = 0$, while ξ goes to zero there and $\dot{\xi}$ changes sign, as in the classical solution (19). The new features emerge when we follow the semi-classical solution inwards from $y = 0$. The amplitude of the matter field decreases as we go into the black hole. This is due to pair-creation and it causes the area function ψ to decrease more rapidly than the linear behavior of the classical solution. In fact the constraint equation (17) implies that ψ is a concave function and any variation in the matter field will focus it towards zero. By equation (16) the metric function ξ is also concave and approaches zero. At a smooth inner horizon the fields ψ and Z, along with their derivatives, would remain finite as ξ goes to zero. If, on the other hand, ψ goes to zero the gravitational sector becomes infinitely strongly coupled and we expect a curvature singularity. In our numerical evaluation all the fields ξ, ψ and Z are simultaneously driven to zero while $\dot{\xi}$, $\dot{\psi}$, and $\dot{Z}$ become large. The Ricci scalar increases rapidly as the singular point is approached. This strongly suggests that the classical inner horizon is replaced by a curvature singularity at the semi-classical level. Unfortunately the numerical solutions are not detailed enough to describe the final approach to the singularity, but the numerical evidence indicates that it is spacelike, which is also what we found in the gravitational collapse solutions.

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[1] J. C. Graves and D. R. Brill, Phys. Rev. **120**, 1507 (1960).
[2] B. Carter, Phys. Lett. **21**, 423 (1966).
[3] R. Penrose (1968), in *Battelle Recontres* (ed. by C.M. DeWitt and J.A. Wheeler), W.A. Benjamin, New York (1968), 121-235.
[4] M. Simpson and R. Penrose, Int. J. Theor. Phys **7**, 183 (1973).
[5] J. M. McNamara, Proc. R. Soc. London **A358**, 499 (1978).
[6] S. Chandrasekhar and J. B. Hartle, Proc. R. Soc. London **A384**, 301 (1982).
[7] N. Zamorano, Phys. Rev. **D26**, 2564 (1982).
[8] E. Poisson and W. Israel, Phys. Rev. **D41**, 1796 (1990).
[9] A. Ori, Phys. Rev. Lett. **67**, 798 (1991).
[10] P. R. Brady and J. D. Smith, Phys. Rev. Lett. **75**, 1256 (1995), gr-qc/9506067.
[11] S. Hod and T. Piran, Phys. Rev. Lett. **81**, 1554 (1998), gr-qc/9803004.
[12] M. Dafermos (2003), gr-qc/0307013.
[13] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).
[14] I. D. Novikov and A. A. Starobinsky, Sov. Phys. JETP **51**, 1 (1980).
[15] R. Herman and W. A. Hiscock, Phys. Rev. **D49**, 3946 (1994).
[16] W. G. Anderson, P. R. Brady, and R. Camporesi, Class. Quant. Grav. **10**, 497 (1993), gr-qc/9211016.
[17] E. Sorkin and T. Piran, Phys. Rev. **D63**, 084006 (2001), gr-qc/0009095.
[18] M. D. McGuigan, C. R. Nappi, and S. A. Yost, Nucl. Phys. **B375**, 421 (1992), hep-th/9111038.
[19] V. P. Frolov, Phys. Rev. **D46**, 5383 (1992).
[20] R. Balbinot and P. R. Brady, Class. Quant. Grav. **11**, 1763 (1994).
[21] J. S. F. Chan and R. B. Mann, Phys. Rev. **D50**, 7376 (1994), gr-qc/9406021.
[22] S. Droz, Phys. Lett. **A191**, 211 (1994).
[23] C. G. J. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. **D45**, 1005 (1992), hep-th/9111056.
[24] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. **D46**, 3444 (1992), hep-th/9206070.
[25] S. B. Giddings and A. Strominger, Phys. Rev. **D46**, 627 (1992), hep-th/9202004.
[26] T. Banks, A. Dabholkar, M. R. Douglas, and M. O'Loughlin, Phys. Rev. **D45**, 3607 (1992).
[27] L. Susskind and L. Thorlacius, Nucl. Phys. **B382**, 123 (1992), hep-th/9203054.
[28] A. W. Peet, L. Susskind, and L. Thorlacius, Phys. Rev. **D46**, 3435 (1992), hep-th/9205114.
[29] M. G. Alford and A. Strominger, Phys. Rev. Lett. **69**, 563 (1992), hep-th/9202075.
[30] S. R. Coleman, R. Jackiw, and L. Susskind, Ann. Phys. **93**, 267 (1975).
[31] S. R. Coleman, Ann. Phys. **101**, 239 (1976).
[32] A. V. Frolov, K. R. Kristjansson, and L. Thorlacius (2005), in preparation.
[33] B. Birnir, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. **D46**, 638 (1992), hep-th/9203042.