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Life of the dust macro-particles in storage rings*

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Abstract

The sudden drop of the beam life time and bursts of background radiation were detected in many machines and associated with microscopic dust particles. We present the model of the dust particle dynamics explaining the long time of the dust events observed in the PEP-II B-factory and BEPC-II machines.

1 Introduction

The sudden drop of the beam life time and bursts of background radiation were detected in many machines and associated with microscopic dust particles. The dust events could last several tens of second. Dust events were observed in many machines including such different machines as PEP-II B-factory and Beijing Electron-Positron Collider (BEPC) where the revolution frequency and the beam current are different by an order of magnitude. Some relevant parameters of the machines and notations are given in the Table 1. In PEP-II, the life time in the dust events was reduced by approximately 10%. The results of observation at BEPC can be summarized as following. BEPC could operate in dual mode as a collider and the synchrotron radiation (SR) light source. In the first case, both e^- and e^+ beams are stored in the same beam pipe using electro-static separators.

In the single-beam operations, reduction of the beam life was observed from the normal life time $\tau \simeq 8-10$ hours to 2 hours or even to few minutes. These events happened randomly, at wide range of beam current, both in single-bunch and multi-bunch cases,

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Table 1: **BEPC parameters.**

	PEP-II	BEPC
Beam energy, E_B, GeV	9.1	2.2
Frequency rf, f_{rf}, MHz,	500.	200.0
Beam current, I_B, mA	1000	65
Bunch population N_b,	$2.\,10^{10}$	$0.81\,10^{10}$
RF voltage/cavity, V_{rf}, MV,	0.8	0.8
Circumference $2\pi R$, m	2200	240
Bunch spacing s_b, m	1.2	6.0
V/H tune, $\nu_{y/x}$	22.6/23.6	4.75/8.72
rms $\sigma_{x/y}$, μm	150/800	28/497
rms σ_z, cm	1.0	3.0
Beam pipe radius, b, cm	3.5	4.0

were not reproducible with the same machine parameters, and never happened with the positron beam. The life time can recover by itself or, sometimes, it could be recovered by applying kicks to the beam (using the strip-line shaker normally used for beam excitation in betatron tune measurements or using the injection kicker). The shift of the beam orbit does not affect the events.

Study of the effect turning the localized pumps (LP) and distributed ion pumps (DIPs) on and off had shown that with all LP turned off no sudden life time loss had happened although some systematic life time reduction was observed due to lower vacuum. It was concluded that DIPs are the main source of the dust.

Although the dust events had been known for many years, the phenomenon eluded explanation. The basic processes defining dust trapping, the life time of the dust particles, and the dust-beam interaction were described by F. Zimmermann [1] and in [2]. For the PEP-II B-factory the model [2] predicts that most of the dust particles should burn down in 50 μs, by many orders of magnitude faster than in reality, and only microscopic silica needles may have chance to survive longer.

We have found such explanation of the long time dust events possible but exceptional. One can expect that a needle with sufficiently large positive charge Q (as in examples we give below) in the field of the electron beam would be electrostatically aligned along the beam line. The balance of ionization heating and radiation cooling in this case would be the same as for a spherical dust particle.

We found that the large amplitude oscillations can provide another explanation of the dust long life time. In the next section we discuss formulas and equations used later for calculations. The formulas refined in several aspects describe the main processes and are, basically, the same as in the previous analysis [1]. Results of simulations of one dimensional (1D) vertical motion of the dust particles in the field of a flat beam are discussed in the third section. The life time for large amplitude of oscillations is, as we expected, substantially longer than it was obtained before [2]. However, especially for 1A PEP-II beam current, the dust life time is still too short and the simulations were modified to describe 2D dust motion in Section 4. With such modification, we were able to get the time of the dust event comparable with that in observations. Effect on the beam life time is discussed in Section 5 where we corrected the estimate of the beam life time used in [1].

Elsewhere numerical examples are given for the Ti dust particles.

2 Main processes

In most cases, the velocity of the dust is relatively low and the dust collides with several bunches while crossing the beam line. Even at 1 A PEP-II current, the typical time of crossing the beam is by an order of magnitude larger than the time duration of the ion gap in the bunch train. For BEPC parameters, that is true even in a single-bunch operation. That allows us to consider coasting uniformly charged beam with the linear density N_b/s_b.

A dust particles is ejected, probably, from the ion pumps. It may have initial charge or acquires some charge by the photo-effect due to direct or scattered SR. The charge of the dust particle approaching the beam grows rapidly due to ionization by the beam electrons. The process starts at several transverse rms $\sigma_\perp$ and, as a result, the dust particle is trapped in the field of the electron beam. The motion of the dust particle, its radius a, charge Q, and temperature T has to be defined self-consistently by the beam ionization losses, radiative cooling, and field emission of positive ions. In this section we summarize formulas used below in our calculations.

2.1 Dust dynamics

The dust particle moves in the potential $U(x, y)$ of the Gaussian coasting beam with rms σ_x and σ_y. The potential in the dimensionless coordinates x, y in units of σ_x and σ_y, respectively, can be written as

$$U(x,y) = -\frac{N_b Q e^2}{s_b}\int_0^\infty \frac{d\lambda}{\lambda\sqrt{1+2\lambda\sigma_x^2}\sqrt{1+2\lambda\sigma_y^2}}\exp\left(-\frac{\lambda\sigma_x^2 x^2}{1+2\lambda\sigma_x^2}-\frac{\lambda\sigma_y^2 y^2}{1+2\lambda\sigma_y^2}\right). \quad (1)$$

Potential $U(x, y)$ is known as Bassetti-Erskin potential and can be expressed in terms of the error function of complex argument [3]. For small amplitude, the motion is oscillations with the frequency Ω proportional to the beam current. For the BEPC averaged beam current I_B, Ω_0 is low compared with the revolution frequency f_{rev}. That is true for BEPC parameters for any filling pattern, even for a single bunch in the ring. Therefore, the filling pattern does not affect the dust events as it was observed in the experiments.

2.2 Evaporation

The life time of the dust particle is limited by evaporation of neutral atoms and their thermal dissociation. Evaporation of ions may be important as well at large temperature. The number of nuclei A in the spherical dust particle is defined by the radius a,

$$A = \frac{4\pi a^3}{3}\frac{\rho}{m_N}A_a. \tag{2}$$

Here m_N is mass of a nucleon, A_a is atomic weight, and ρ is the dust density ($a^3\rho/A_a$ is dimensionless). The rate of evaporation is proportional [4] to the vapor pressure $P_v(T)$,

$$\frac{dA}{dt} = -\frac{4\pi c a^2}{\sqrt{2\pi k_B T M_a c^2}}P(T) \tag{3}$$

A_a is atomic number, M_a is the mass of an atom, c and k_B are the velocity of light and the Boltzmann constant, respectively, and we neglect reflectivity (for most metals it is very small). In our simulations we use $P(T)$ interpolating experimental data on the temperature dependence of pressure. Pressure in Pascal for Ti is [5]

$$P(T) = 1986.71 T^{2.2898} e^{-\frac{\epsilon_v}{kBT}}, \quad \text{for Ti}, \tag{4}$$

where the enthalpy of evaporation per atom $\epsilon_v = 4.412$ eV. Eq. (4) gives better parametrization of the experimental data than parametrization quoted in [1]. Comparison is shown in Fig. (1)

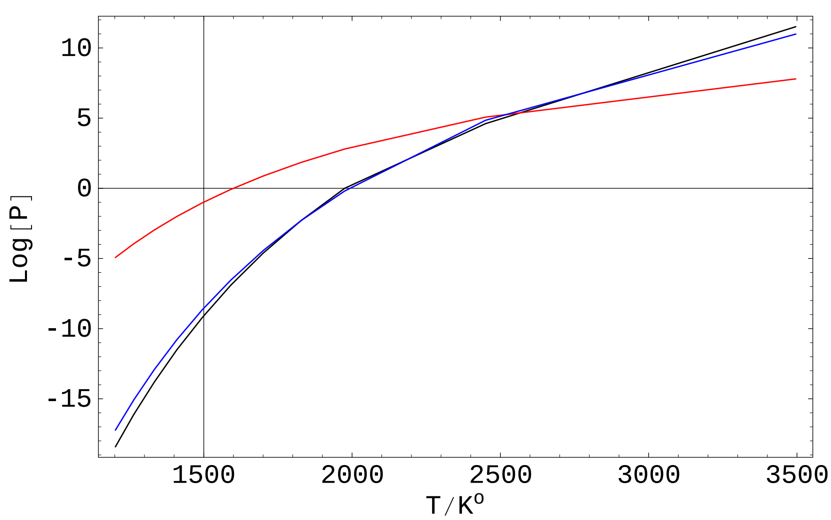

Figure 1: Interpolating function $P(T)$ for Ti, see Eq. (4) (blue line) and experimental data in Pascal (black line). The red line is pressure parametrization according to reference [1].

The total pressure $P_{tot} = P(T) + \Delta P$ for a dust particle has to be corrected by the additional term due to surface tension,

$$\Delta P = \frac{2\gamma_s}{a}\frac{\rho_v}{\rho} \tag{5}$$

where the last factor is the ratio of the density of vapor ρ_v to the density ρ of the dust particle.

The life time of large particles due to evaporation can be quite large at a moderate temperature. For $1\,\mu$ Ti particle, $A \simeq 10^{13}$ and $dA/dt \simeq 3.4\,10^8$ at $T = 1500\,K$ but grows rapidly to $dA/dt \simeq 1.2\,10^{12}$ at the melting point $T_M = 1941\,K$.

However, the actual life time may be defined by stability to fission and loss of dust particles hitting the beam pipe. The dust stability to fission, and the dynamics of the dust particle depend on its charge Qe and temperate T.

2.3 Heating

For a particle with the transverse offset x, y (in units of transverse rms σ_x and σ_y, respectively), the heating is due to deposited energy

$$\Delta E_i = \Delta E_T N_b \,(\frac{\pi a^2}{2\pi\sigma_x\sigma_y})\, e^{-\frac{1}{2}(x^2+y^2)}, \tag{6}$$

where ΔE_T is ionization energy loss per single electron of the ultra-relativistic beam colliding with the dust particle [6],

$$\Delta E_T = \Delta E_Q \,\ln[2(\frac{m_e c^2}{\hbar\omega_p})^2\,(\frac{Qr_e}{a} + \frac{1}{2}\,(\frac{\hbar\omega_p}{mc^2})^2)]. \tag{7}$$

The plasmon energy [6]

$$\hbar\omega_p = 28.816\sqrt{\frac{Z_a}{A_a}\rho}, \tag{8}$$

and

$$\Delta E_Q = 2\pi r_e^2 mc^2 N_{Av}\frac{4a}{3}\frac{Z_a}{A_a}\rho. \tag{9}$$

Here r_e and mc^2 are the classical radius and the rest energy of an electron, ρ is the density in g/cm^3, and $N_{Av} = 6.02\,10^{23}$ is the Avogadro constant. Other parameters are explained in tables 1 and 2.

The cut-off energy in Eq. (7) is taken equal to the potential of the dust particle Qe^2/a. Another limitation on the cut-off energy is set by the free path $l(\varepsilon)$ of the secondary electrons. The experimental energy dependence can be fitted by

$$l(\varepsilon) = \frac{0.412}{\rho}\varepsilon^{n(\varepsilon)}, \quad n(\varepsilon) = 1.265 - 0.0954\ln(\varepsilon), \tag{10}$$

where l is in cm, ε is in MeV, and ρ in g/cm^3, we can write the condition on the energy of the secondary electron $l(\varepsilon) < a$ in the form $\varepsilon < Q_\delta e^2/a$, where Q_δ defined in this way depends on the radius a. Then, Q in Eq. (7) has to be replaced by $Q_{eff} = \max\{Q, Q_\delta\}$ where $Q_\delta(a)$ for Ti found numerically is shown in Fig. (2). More careful consideration has to to define $l(\varepsilon)$ as the distance to the surface of the dust particle from the point (x_s, y_s) where the secondary electron was generated averaged over (x_s, y_s). This correction is, however, small.

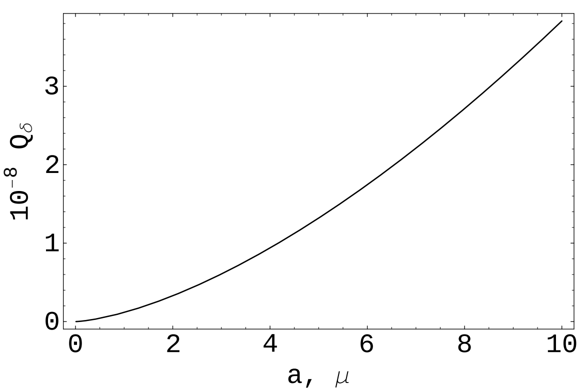

Figure 2: $Q_\delta(a)$ given by the limit on the free path length of secondary electrons, see text.

The heating due to ionization loss competes with cooling by the thermal radiation,

$$(\frac{dE}{dt})_{rad} = 4\pi a^2 \sigma_{SB} T^4 \Phi(\mu), \tag{11}$$

where $\sigma_{SB} = 3.53\,10^7\ eV(sec\,cm^2)^{-1}\,K^{-4}$ is the constant of the black body radiation. $\Phi(\mu)$ is the form factor taking into account suppression of the long wave length radiation with $\omega < \omega_c$, where $\omega_c a/c \simeq 1$. The suppression in the Mie theory is described by the factor [7], [1]

$$\Phi(\mu) = \frac{1}{1 + C_w/(aT)}, \tag{12}$$

where a is in cm and T in K.

Additional cooling is provided by the evaporation of neutral atoms,

$$(\frac{dE}{dt})_n = -\epsilon_v \frac{dA}{dt}. \tag{13}$$

The temperature variation per collision with a bunch is described by the following equation:

$$\frac{dT}{dt} = [\frac{c}{s_b}\Delta E_i - (\frac{dE}{dt})_{rad} - (\frac{dE}{dt})_n]\,(\frac{3A_a}{4\pi a^3 C_T \rho}), \tag{14}$$

where C_T is the heat capacity per mole.

Table 2: **Table 2. Some relevant parameters.**

Elements,	{Al, Cu, Ti, Si}
Atomic weight, A_a	{27,64,48,28.1}
Atomic number Z_a,	{13,29,22,14}
Density, ρ, g/cm^3,	{2.7, 8.96, 4.54, 2.5}
Melting point, T_M, K,	{933,1352,1941,1983}
Heat capacity, C_T, $J/(molK)$,	{29.3,35.6,33.7,72.47}
Surface tension, γ_S, mN/m,	{840,1150,1427,10000}
Ionization potential I_P, eV,	{5.98,7.72,6.82,8.15}
Enthalpy of fission ϵ_f, kJ/mol,	{10.7, 13.1, 15.45, 50.2}
Enthalpy of vaporization ϵ_v, kJ/mol,	{293., 300., 425.,359.}
Conversion	eV=1.6 10^{-19} J
Coefficients	eV/cm^2=1.6 10^{-12} mN/m
	Torr=0.83 10^{15} eV/cm^3
	10^5 Pascal=760 Torr

2.4 Charge variation

Eq. (7) describes only the part of the ionization losses which go to the dust heating. Another part, with the energy loss larger than $Q_{eff}e^2/a + (\hbar\omega_p)^2/2mc^2$, is taken away by the secondary electrons which escapes the dust particle. That increases the charge Q of the dust particle with the rate

$$(\frac{dQ}{dt})_i = (\frac{c}{s_b})\,\Delta E_Q\,(\frac{N_B}{Q_{eff}e^2/a + (1/2)(\hbar\omega_p/mc^2)^2})\,(\frac{\pi a^2}{2\pi\sigma_x\sigma_y})\,e^{-\frac{1}{2}(x^2+y^2)}. \tag{15}$$

The thermal dissociation of evaporated neutral atoms at the surface of the charged dust particle can stop further ionization. Electrons are captured by the positively charged dust reducing Q. The rate of the process is given by the Richardson's formula [8]

$$(\frac{dQ}{dt})_d = \frac{2(2\pi a k_B T)^2 M_i}{(2\pi\hbar)^3}\,e^{-\frac{1}{k_B T}(I - W + \epsilon_v - \frac{e^2}{a}\sqrt{Q}) + \frac{\Delta C_p}{k_B}}. \tag{16}$$

The Schottky term $\propto \sqrt{Q}$ takes into account the electric field of the charged particle. Here I is the ionization potential, W is the work function for Ti (and affinity for Si), M_i is the ion mass, and

$$\Delta C_P = \int_0^T \frac{d\tau}{\tau^2} \int_0^\tau C_P(T')dT'. \tag{17}$$

Interpolating experimental data for the heat capacity C_P, we get

$$\Delta C_P = \frac{A_{int}}{N_{Av}} \frac{\ln[1 + B_{int}T]}{B_{int}} [1 + \frac{1}{B_{int}T}] \frac{eV}{K}, \tag{18}$$

where parameters $A_{int} = 0.2265\ J/K$, $B_{int} = 0.00608\ 1/K$ for Ti.

The total charge variation is given by Eq. (15) and Eq. (16):

$$\frac{dQ_i}{dt} = (\frac{dQ}{dt})_i - (\frac{dQ}{dt})_d. \tag{19}$$

The field of the beam can cause polarization of the dust particles. The effect, however, is small and can be neglected.

2.5 Fission

The dust particle melts when temperature exceeds the melting point $T > T_M$. Then the deposited energy goes to the latent heat of transition from solid to liquid state

$$\frac{dE_M}{dt} = [\frac{c}{s_b}\Delta E_i - (\frac{dE}{dt})_{rad} - (\frac{dE}{dt})_n] \tag{20}$$

converting the dust particle to a droplet. After melting, the fission of the liquid dust particle into two smaller droplets become possible. The process has the threshold charge Q_{th} defined by the equilibrium of the electrostatic potential energy and the potential energy of the surface tension [9]. For small deformations, the threshold Q_{th} is given by

$$\frac{Q_{th}^2 r_e mc^2}{16\pi\gamma_S a^3} = 1. \tag{21}$$

(The threshold is lower for the large deformation).

The energy released in fission of a dust particle with initial radius a and charge $Q > Q_{th}$ depends on the size and the charge xQ and $(1-x)Q$ of the droplets in the final state. The gain is maximum for fission in halves, $x = 1/2$, see the upper plot in Fig. 3. The bottom plot show dependence of the threshold Q to fission in halves as function of the radius a of the dust particle.

The typical energy released in fission E_f is large and, because the beam potential at large distances is only logarithmic, the final droplets acquiring large momentum can hit the walls of the beam pipe. Therefore, in the dust events with long life time, the dust particle either has to stay below the melting point or, if it melts, should not go to fission.

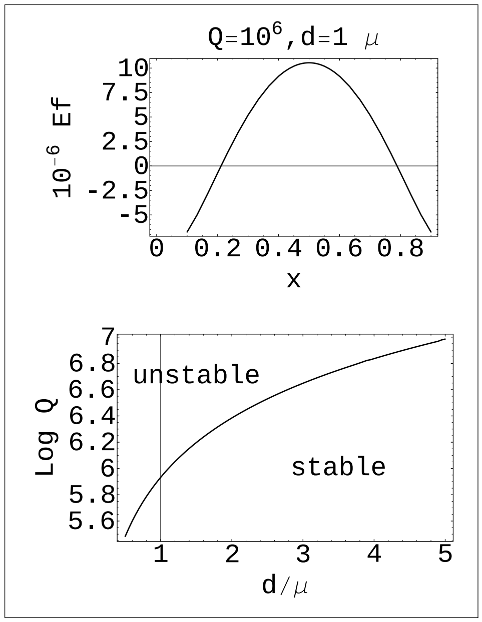

Figure 3: Upper plot: energy released in fission at $T > T_M$ to particles with masses x and $1-x$ of initial mass as function of x. Bottom: The threshold charge Q as function of the radius d of the dust particle.

Fig. 4 shows the ratio of the released energy to the depth of the potential, well. The product of fission survive provided the ratio is less than one.

Other effects such as diffusion due to multiple collisions and capture of the photo electrons may affect the dust dynamics. Estimates show that these effects are small for moderate Q and we do not include them in simulations.

3 1D Dynamics

Equations of motion, and equations (3), (19), (14), and (20) give the system of differential equations for coordinates x, y, dust radius a, charge Q, and temperature T.

For a flat beam, it seems reasonable to consider only 1D vertical oscillations of the dust particle.

Dynamics of the dust particle is quite complicated. The particle starting with low Q approaches the beam increasing Q and T in the first encounter with the beam. After the first crossing of the beam, the charge Q and velocity are large. While away from the beam, the ionization heating by the beam is negligible and continuing radiation and evaporation provides cooling of the dust particle. The particle returns to the beam with reduced T, Q, and velocity. Eventually, after several crossings, the temperature increases

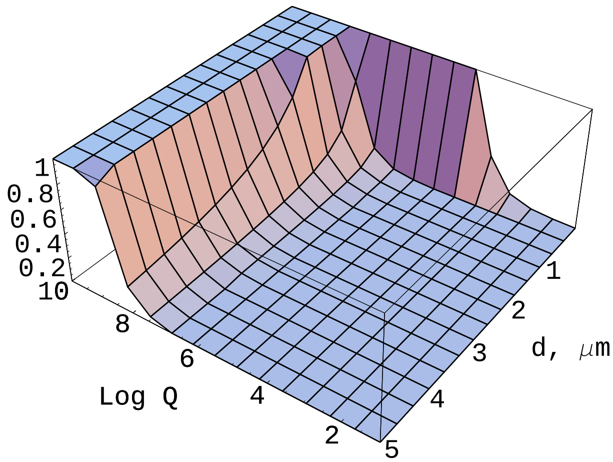

Figure 4: The ratio of the energy released in the fission to the depth of the potential well. The product of decay do not hit the beam pipe provided the depicted parameter is less than one.

to a value larger than the melting point T_M and some energy is stored as latent heat of transition. After the crossing, the temperature remains equal T_M until the latent energy goes to zero due to radiation and evaporation. Due to high rate of dissociation of neutrals at high temperature, Q may drop to lower value, the returning force decreases, and the particle goes to large amplitudes. That may, generally speaking, result in stable particle motion with the life time limited by evaporation. However,even if fission does not take place, the particle life time can not be long. The rate of evaporation dA/dt is high. For example, for $1\,\mu$ Ti dust particle, $A = 2\,10^{12}$, and evaporation rate $dA/dt = 1.5\,10^{11}\ 1/s$ at $T = 2000\ K$. Therefore, one could expect the life time of only few seconds for $a = 1\,\mu$.

The typical case is illustrated in Fig. (5) for $a = 0.2\,\mu$ and BEPC parameters. Depicted results are obtained using MATHEMATICA by solving the system of equations described above. The charge Q and temperature T slowly rise after each crossing and are reduced when particle is away from the beam. Growing Q leads to adiabatic decrease of the amplitude of oscillations enhancing further growth of Q and T. When Q exceeds the fission threshold, the temperature already $T > T_M$, and fission takes place producing such a large kick to the product of decay that they hit the wall. The life time is only $30\,ms$.

The same happens even faster for larger particles and at larger PEP-II 1 A current. The fission repeats several times. The large number of small particles could reduce the beam life time. However, their own life time due to evaporation is too short and the ions, the final product of the process, are unstable in the train with the sufficiently long ion gap.

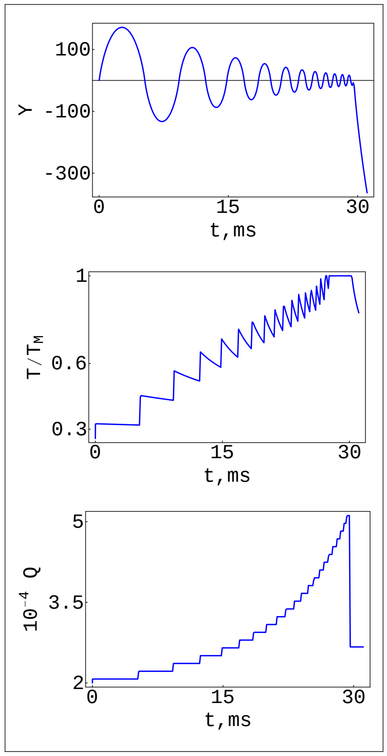

Figure 5: 1D dynamics of $a = 0.2\,\mu$ Ti dust particle for BEPC parameters. The life time of the dust particle is limited by fission due to the growing charge Q when T exceeds the melting point T_M. The vertical Y is in units of σ_y.

4 2D Dynamics

At large currents such as the PEP-II 1 A beam current, the dust crossing the beam within 1-2 σ_y is heated to temperature larger than the melting point T_M and, after melting, decays to smaller fragments which evaporate shortly. That is consistent with the conclusion in reference [1] but contradict to the observations. Thus, it is essential to consider the 2D motion.

It is instructive to consider the axially-symmetric case where the azimuthal component of the angular momentum $L_\phi = p_\phi r$ is constant while the charge Q may vary. The radial potential of the particle has minimum at the radius r_m,

$$\frac{1}{r_m^2} = \frac{Z_0 e I_B Q M_d}{2\pi L_\phi^2}, \tag{22}$$

where I_B is the average beam current and $Z_0 = 120\pi$ Ohm. Initial $p_\phi(0) \simeq p(0)$ and $L_\phi \simeq p(0)b$ are defined by the initial momentum $p(0)$ and the beam pipe radius b. The dust is trapped at $r_m < \sigma_\perp$, if the charge

$$Q > [\frac{Z_0 e I_b \sigma_\perp}{4\pi b^2}]^{-1} E_0. \tag{23}$$

Numerical calculations show that the typical Q is lower then that and the trapped particle does not approach the beam closer than several $\sigma_\perp$.

For the flat beam, the axial component L_ϕ is not an integral of motion. A trajectory of the dust particle in this case is a rotating ellipse and is confined between two circles. As in the axially-symmetric case, the particle can stay away from the beam line interacting only with the tails of the beam distribution. That means that the heating of the particle is reduced and it can live longer but the effect on the beam life time is also reduced. The trade-off of these two factors define the range of parameters where the dust events can take place.

Result of 2D simulations are shown in Figs. (6), (7) for BEPC, and Fig. (8) for PEP-II. The estimate from the slow growth of temperature in the last case shows that it would survive another 10 s. We checked accuracy of simulations calculating the azimuthal L_ϕ for a round beam where we put $\sigma_y = \sigma_x$. L_ϕ is preserved during the 2000 ms of tracking with computer accuracy.

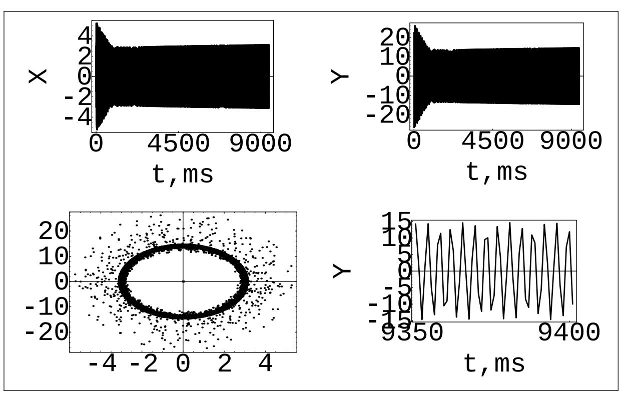

Figure 6: Trajectories $x(t)$ and $y(t)$ of $5\,\mu$ Ti particle for BEPC parameters. Initial conditions $x(0) = 3.0$, $p_x(0) = 0$, $y(0) = 0.0$, $v_y(0) = 6.56\,m/s$, $T = 1500\,K$, $Q_0 = 10^8$. The bottom row shows the trajectory in the (x, y) plane and the zoomed part of the trajectory for 5 ms in the end of tracking, at $t \simeq 10$ s.

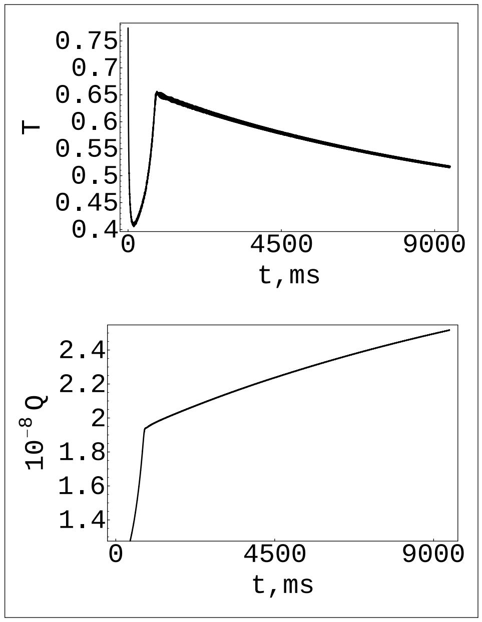

Figure 7: Variation of temperature and charge with time for the same case as in Fig. (6).

5 Beam life time

The beam life time is defined either by the energy loss $\Delta E/E_B$ of the beam electrons exceeding the energy acceptance of the ring δ_{max} or by the large angle scattering. The estimate shows that, for the typical $\delta_{max} \simeq 10^{-2}$ the first constrain is stronger.

The energy loss can be due to bremsstrahlung with the impact parameter $b_0 > a$ (the process named duststrahlung [1]) or due to bremsstrahlung on the nuclei of the dust particle with $b_0 < a$.

The life time due to duststrahlung for the 2D dynamics is exponentially long, see Appendix. The dominant process defining the beam life time in this case is the bremsstrahlung on nuclei of the dust particle. The cross-section of this process [10] is

$$d\sigma(\omega) = 4Z_a^2\alpha_0 r_e^2 \frac{d\omega}{\omega}\frac{E'}{E}(\frac{E}{E'} + \frac{E'}{E} - \frac{2}{3})[\ln\frac{2EE'}{m\omega} - \frac{1}{2}], \tag{24}$$

where $E' = E - \hbar\omega$. The life time

$$\frac{1}{\tau_B} = \int_{\omega_m}^{\infty} d\sigma(\omega) f_{rev} n\pi a^2 \frac{4a}{3}\langle df\rangle, \tag{25}$$

where $\langle df\rangle$ is the distribution functions of the beam particles crossing the dust, and $n = (\rho/A_a)N_{Av}$ is the number of nuclei per unit volume. For the dust particle moving periodically over ellipse with axes A_e and B_e (in units σ_x and σ_y, respectively), $\langle df\rangle$ should be averaged over the period:

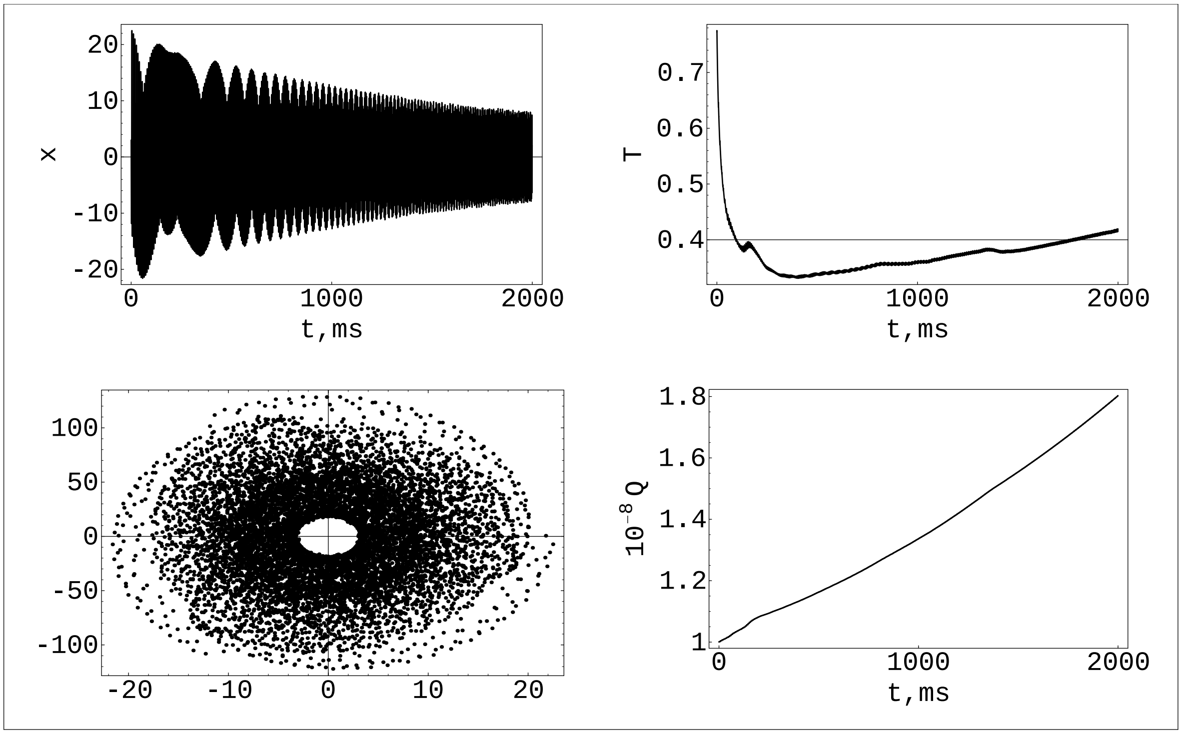

Figure 8: Tracking of $5\,\mu$ Ti particle for PEP-II parameters. Initial conditions are: $x(0) = 3.0$, $v_y(0) = 41.5$ m/s, $y(0) = v_x(0) = 0$, $T(0) = 1500\,K$, $Q(0) = 10^8$. Depicted are x and y in units of $\sigma_x = 835\,\mu$ and $\sigma_y = 145\,\mu$, respectively, Q in units of 10^8, $T(t)$ in units of the melting point $T_M = 1941\,K$. Radius a does not change with accuracy 10^{-12} during 2000 ms. Interpolation of $T(t)$ predicts that T_M will be reached in another 10 s. However, that is uncertain because Q has not reached saturation yet. Beating is due to mismatch of initial conditions.

$$
\begin{aligned}
\langle df \rangle &= \int \frac{d\phi}{2\pi} \frac{1}{2\pi\sigma_x\sigma_y}\, e^{-\frac{1}{2}(A_e^2 \cos^2\phi + B_e^2 \sin^2\phi)} \\
&= \frac{1}{2\pi\sigma_x\sigma_y}\, e^{-\frac{1}{4}(A_e^2 + B_e^2)} I_0[\frac{1}{4}(A_e^2 - B_e^2)]. \qquad (26)
\end{aligned}
$$

If $B_e >> A_e$ (see, for example, Figs. 6, 8),

$$
\langle df \rangle = \frac{1}{2\pi\sigma_x\sigma_y}\, e^{-\frac{A_e^2}{2}} \sqrt{\frac{2}{\pi B_e^2}}. \qquad (27)
$$

That means that the electrons are lost when the dust particle crosses the horizontal plane where it is relatively close to the beam. Eq. 25 gives the estimate

$$\frac{1}{\tau_B} = \frac{16}{3}\alpha_0 (Z_a r_e)^2 f_{rev} \ln(\frac{1}{\delta_m})[\ln(\frac{2\gamma\lambda_c}{Z_a r_e \delta_m}) - \frac{1}{2}]\frac{4an}{3}\frac{\pi a^2}{2\pi\sigma_x\sigma_y}\, e^{-\frac{A_e^2}{2}}\sqrt{\frac{2}{\pi(B_e^2 - A_e^2)}}. \quad (28)$$

As an example let us take $a = 5\,\mu$ Ti dust particle, acceptance $\delta_m = 0.01$, and $A_e = 4.0$, $B_e = 20.$. Then, for PEP-II parameters ($E = 9$ GeV, $2\pi R = 2.2$ km), Eq. (28) gives $1/\tau_B = 0.79\,10^{-6}\ s^{-1}$, consistent with the experimental $1/\tau_B = 1.4\,10^{-6}$ corresponding to 10% drop of the nominal life time (20 h). For BEPC and $5\,\mu$ Ti particle, Eq. (28) gives $1/\tau_B = 0.06\,10^{-3}\ 1/s$ with $A_e = 4.0$, $B_e = 15$, and $1/\tau_B = 0.4\,10^{-3}\ 1/s$ with $A_e = 3.5$, $B_e = 15$, what has to be compared with $1/\tau_B = 0.13\,10^{-3}$ in observations.

6 Conclusion

The dust events were observed during the last 30 years in many machines. Although main relevant processes are well understood [1] the present models predict the life time of dust particles by many order of magnitude shorter than in the experiments. In the special case of long silica needles the life time can be longer. In this case, it is defined by the time to build-up charge sufficient to align to needle along the beam. In our study we have found that the life time of a dust particles trapped and oscillating with amplitudes less or comparable to the beam transverse rms is, indeed, very short [2]. The life time of such particle is limited both by fission and evaporation. Typically, the melting point is reached and the fission takes place much faster then dust evaporates. After fission, the fragments of decay obtain large momentum and hit the beam pipe wall or go to fission again. Eventually, fission and evaporation produce ions. The estimates show that even heavy ions do not survive long in the bunch trains with a sufficiently large ion gap. Although the islands of stability do exist for any bunch train, they occupy a small fraction of the ring, especially for rings with small revolution period. The life time of ions in the islands of stability is also limited by the secondary ionization [1] and by the space charge of accumulated ions [11].

The situation may be different for the dust oscillating with large amplitudes. Such particles spend most of the time outside of the beam and may have enough time to be cooled by radiation. The simulations show that, with BEPC parameters, such particles can have life time of the order of $100\,ms$, long but still much shorter than in observations. Our attempts to prolong the dust life time by varying within the reasonable limits the input parameters (ionization potential, enthalpy, chemical composition and the initial size of the particles, etc.) were unsuccessful. We came to conclusion that it is essential to consider 2D motion where particles can remain at large distances from the beam all the time. The life time of a dust particle in this case can be large. However, the interaction of such dust particles with the beam is reduced. We show that the effect of the dust-strahlung in this case is small and the beam life time is defined by the bremsstrahlung on the dust nuclei. The beam life time in our examples is in reasonable agreement with the experiment. Although we were unable to explore the whole range of parameters (beam currents, geometry, initial charge and velocity of the dust particles, the size and composition of the particle, etc.) limiting the dust events, we give realistic examples where both life time of the dust and the beam life time are comparable with observations.

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7 Appendix: Life time due to duststrahlung

The life time was estimated in reference [1] using formulas for beamstrahlung borrowed from the theory of beam-beam collisions. We found that for moderate $a < 10\ \mu$ the estimate has to be modified.

Effect on the life time in the ultra-relativistic case [12] depends on the parameter Qr_e/b_0 equal to the ratio of the deflection angle $\alpha \simeq Qr_e/(\gamma b_0)$ to the radiation angle

$\theta_r \simeq 1/\gamma$ where γ is relativistic factor. The spectrum of radiation for $Qr_e/b_0 >> 1$ is described by the formulas for the synchrotron radiation and was assumed in the previous analysis [1]. The simulations show that more realistic is another limit $Qr_e/b_0 << 1$ where the spectrum of radiated photons is [12]

$$\frac{dE_\omega}{d\omega} = \frac{e^2\omega}{2\pi c^3}\int_{\omega/(2\gamma^2)} \frac{d\omega'}{(\omega')^2}|w(\omega')|^2[1 - \frac{\omega}{\omega'\gamma^2} + \frac{1}{2}(\frac{\omega}{\omega'\gamma^2})^2]. \tag{29}$$

Here $w(\omega)$ is the Fourier transform of the transverse acceleration $w(t)$,

$$w(t) = \frac{Qe^2}{m\gamma}\frac{b_0}{(b_0^2 + c^2t^2)^{3/2}}. \tag{30}$$

Explicitly, $w(\omega)$ is given in terms of the modified Bessel function K_1,

$$w(\omega) = \frac{2Qr_ec}{\gamma b_0}\frac{\omega b_0}{c}K_1(\frac{\omega b_0}{c}). \tag{31}$$

The average energy loss is small. The loss of particles in this case is due to rare but strong fluctuations where a photon is radiated with the frequency $\omega > \omega_{min}$, $\hbar\omega_{min}/E_B = \delta_{max}$.

Eq. (29) defines the number of photons $dN_\gamma(b_0) = d\omega dE_\omega/(\hbar\omega)$. The life time $\tau(b_0)$ of a particle with coordinates x, y is

$$\frac{1}{\tau(b_0)} = f_{rev}\int_{\omega_{min}}^{\infty} dN_\gamma(b_0), \tag{32}$$

where $b_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2}$ and x_0, y_0 are coordinates of the dust center.
Eq. (32) can be written in the form

$$\frac{1}{\tau(b_0)} = \frac{\alpha_0 f_0}{\pi}(\frac{2Qr_e}{b_0})^2 x_m^2 \int_1^\infty dz K_1^2(x_m z)[\frac{2z}{3} - 1 + \frac{1}{z} - \frac{2}{3z^2}], \tag{33}$$

where $x_m = \delta_m b_0/(2\gamma\lambda_c)$, $\alpha_0 = 1/137.$, and $\lambda_c = r_e/\alpha_0$ is the Compton wave length.
The integral is exponentially small,

$$\frac{1}{\tau(b_0)} \simeq \frac{\alpha_0 f_0}{\pi}(\frac{2Qr_e}{b_0})^2 \frac{\pi}{8x_m}e^{-2x_m}, \quad x_m >> 1 \tag{34}$$

for large x_m. For small x_m, it can be estimated as

$$\frac{1}{\tau(b_0)} \simeq \frac{\alpha_0 f_0}{\pi}(\frac{2Qr_e}{b_0})^2 \frac{2}{3}\ln(1/x_m), \quad x_m << 1. \tag{35}$$

The beam life τ_B can be obtained by averaging Eq. (33) over Gaussian distribution $df(x, y)$ of electrons of the beam. The accurate estimate of the beam life time due to duststrahlung can be done introducing additional integration

$$\frac{1}{\tau_B} = \int_a^\infty 2b_0 db_0 \delta[(x-x_0)^2 + (y-y_0)^2 - b_0^2] \int df[x,y] \frac{1}{\tau(x,y)} \tag{36}$$

and carrying out integration over $dxdy$ first.

If $r_0 = \sqrt{x_0^2 + y_0^2}$ is of the order of $\sigma_\perp$,

$$\frac{1}{\tau_B} \simeq \frac{2\alpha_0 f_0}{3\pi} (\frac{2Qr_e}{\sigma_y})^2 \ln(\frac{2\gamma\lambda_c}{\delta_m a}). \tag{37}$$

Dependence on a in this case is only logarithmic.

The life time given by Eq. (37) is comparable with that for PEP-II parameters at $Q \simeq 10^8$. However, the life time of the dust particle with such Q in the core of the beam is only few μs. The beam life due to duststrahlung for stable particles is too long to be noticeable in experiments.

In the opposite case of large amplitudes of oscillations $a_0 >> \sigma_\perp$ only few particles in the tails of the distribution encounter the dust particle,

$$\frac{1}{\tau_B} \simeq \frac{\alpha_0 f_0}{4} (\frac{Qr_e}{\sigma_y})^2 (\frac{a^2}{\sigma_\perp a_0}) e^{-\frac{\delta_m a_0}{\gamma\lambda_c}} e^{-\frac{a_0^2}{2\sigma_\perp^2}}. \tag{38}$$

The life time due to duststrahlung in the last case is exponentially long.