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Black holes in many dimensions at the LHC: testing critical string theory

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Critical string theory requires that the number of extra dimensions is $n = 6(7)$. We show that if the large extra dimensions scheme of Arkani-Hamed, Dimopoulos, and Dvali (ADD) is realized, this prediction can be tested through black hole production properties at the LHC. In particular, we demonstrate that if n is significantly larger than 6(7), as required if ADD is to truly solve the hierarchy problem, then the critical string theory prediction can be excluded at high confidence.

One of the most difficult questions facing theoretical high-energy physics for many years has been how to consistently combine General Relativity with Quantum Mechanics, as naive quantization produces unrenormalizable divergences. This issue is exacerbated by the hierarchy problem, which asks why the electroweak scale, $M_{wk} \sim$ TeV, is so small compared with the (reduced) Planck scale, $\overline{M}_{Pl} \sim$ a few 10^{18} GeV, which is associated with the energy at which non-renormalizable Einstein gravity becomes strong. It appears that resolution of these puzzles may require a complete theory of quantum gravity.

As of now, the best candidate for such a possibility is (critical) string theory (CST), which reduces to Einstein gravity at low energies and allows for the computation of finite $S-$matrix amplitudes. For CST to be a consistent theory there are three essential ingredients: (i) the fundamental objects of the theory are no longer point-like and must have a finite size of order M_s, the string scale; (ii) supersymmetry must be a good symmetry, at least at scales $\gtrsim M_s$; (iii) space-time must be ten or eleven dimensional, (*i.e.*, $D = 4 + n = 10$, if the string coupling is perturbative, $D = 11$ if it is non-perturbative), with the additional dimensions being compactified at a radius $R_c \gtrsim 1/M_s$. Most research in string theory so far has focused on critical string theories, where the world-sheet anomalies are automatically canceled. It is precisely this anomaly cancelation that requires $D = 10$. However, there are consistent string theories, known as non-critical, in arbitrary numbers of dimensions, where the anomalies are canceled by a dilaton background with nontrivial spacetime dependence [1]. In either case, the common expectation is that M_s is slightly below or equal to $\overline{M}_{Pl}$ which would imply that the predictions of CST are difficult to test directly. Currently there is no evidence for any of these basic assumptions. If indeed $M_s \sim \overline{M}_{Pl}$ it may be that CST can never be directly tested in laboratory experiments. Furthermore, even if supersymmetry and/or extra dimensions were discovered in future experiments, this would be no guarantee that CST represents the correct theory of nature.

In recent years it has been proposed that the fundamental scale of gravity might not be $\overline{M}_{Pl}$, but rather $M_* \sim$ TeV [2, 3]. There is then no large hierarchy between the gravitational and electroweak scales. In this scenario, the observed weakness of gravity results from the presence of extra dimensions with large radii. In the simplest picture, gravity is able to propagate in all D dimensions, but the SM fields are restricted to a $3 + 1$ dimensional "brane". The strength of gravity at long distances is then diluted by the volume of the extra dimensions. If string theory is correct, the string scale must then be near a TeV in this scenario. Signals of string theory, such as string resonances would then be visible at future colliders, such as the Large Hadron Collider (LHC) at CERN [4]. However, the interpretation of these signals is likely to be ambiguous, especially if the string coupling is non-perturbative. In the large extra dimensions picture of Arkani-Hamed, Dimopoulos and Dvali (ADD) [3], M_* and $\overline{M}_{Pl}$ are related by $\overline{M}^2_{Pl} = V_n M_*^{n+2}$, where V_n is the volume of the n compactified large dimensions. If CST is correct $D = 10(11)$, thus we must have $n \leq 6(7)$ (note that all of the extra dimensions need not be large). It is *impossible* to have $n > 6(7)$ in the CST realization of ADD; an experimental determination that $n > 6(7)$ would thus *exclude* CST.

In this paper we propose a 'null' test of CST; this test can *only* reveal whether CST is excluded and cannot tell us if it is the correct theory of nature. We will do this by examining the properties of black hole (BH) production and decay at the LHC with different numbers of extra dimensions. From this, we will show that the number of compactified large dimensions can be determined; if $n > 6(7)$ is measured with high confidence then CST is excluded, however if $n \leq 6(7)$ is measured then very little information about CST is obtained. We will show that if n is sufficiently large, then the region $n \leq 6(7)$ as required by CST will be highly disfavored by many standard deviations. There are, of course, more general reasons for determining the number of extra dimensions from BH production: (i) It allows one to differentiate a BH in the ADD 'flat' extra-dimensions scenario from

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one in a Randall-Sundrum-like model [5] with warped geometry and very weakly coupled graviton resonances [6]. (*ii*) If string resonances are observed at colliders this test is the only way to determine whether or not they arise from CST. (*iii*) Whatever the theory of quantum gravity, the energy region near M_* is likely to be fairly complex; measuring BH properties may be one of the few handles on this physics. As a proof of principle for our proposal, we will show that there exists a region in the (n, M_*) parameter space of the ADD model where we can experimentally exclude the case $n \leq 6(7)$ at 5σ significance.

For simplicity in what follows, we will assume that this $n-$dimensional space is compactified on a torus of equal radii so that $V_n = (2\pi R_c)^n$, where R_c is the compactification radius. Given $\overline{M}_{Pl}$ and $M_* \sim$ a few TeV, R_c becomes completely fixed by the relation above. Note that the case $n = 1$ is excluded while $n = 2$ with low M_* is disfavored by current data [7]. For the case of a torus, the graviton has Kaluza-Klein(KK) excitations $h_{\mu\nu}^{(n)}$, with masses given by $M_n^2 = \mathbf{n}^2/R_c^2$, where $\mathbf{n}$ labels a set of occupation numbers. The KK graviton couplings to the Standard Model (SM) fields are described by the stress-energy tensor $T^{\mu\nu}$, given in D dimensions by $\mathcal{L} = -\sum_n h_{\mu\nu}^{(n)} T^{\mu\nu}/M_*^{1+n/2}$. The ADD scenario has three distinct experimental signatures which have been studied in some detail in the literature: (*i*) missing energy events associated with KK graviton emission in the collisions of SM fields; (*ii*) new contact interactions associated with spin-2 KK exchanges between SM fields [8]; (*iii*) black hole production in particle collisions [9, 10].

Is there any guide as to what values of $n > 6(7)$ we should consider? For ADD with $n \leq 6$ it is well known that the hierarchy problem is *not* truly solved. Although we have reduced M_* to a few TeV, $M_* R_c \gg 1$, as seen in Fig 1. By contrast, with n large we could have $M_* R_c \lesssim 10$. Note that, if $M_* R_c < 1$ the theory would lose its predictive power since the compactification scale is above the cutoff. To obtain the interesting range of compactification radii, $1 \lesssim M_* R_c \lesssim 10$, requires $17 \lesssim n \lesssim 39$, hence we will focus on this set of values in what follows. If the compactification topology is a sphere, rather than a torus, this changes to $n \geq 30$, as seen in Fig 1. It is important to notice that this model does solve the hierarchy problem for large n, but that would lie outside the realm of CST. For such large values of n the Kaluza-Klein masses are at the TeV scale. Since each KK state is coupled with 4 dimensional Planck strength, $\overline{M}_{Pl}$, to the SM fields, it is clear that this sufficiently weakens the KK contributions to the processes (*i*) and (*ii*) above, such that no meaningful constraints are obtainable. For example, with $n = 2$, precision measurements at the International Linear Collider at $\sqrt{s} = 1$ TeV will be sensitive to $M_* \lesssim 10$ TeV, while with $n = 21$, this drops to $M_* \lesssim 1$ TeV. Thus for reasonable values of M_* the *only* signal for large n in ADD is black hole production.

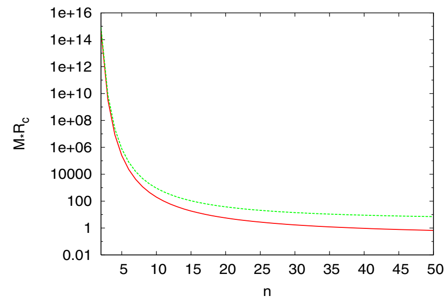

FIG. 1: $M_* R_c$ as a function of n for $M_* = 1$ TeV for a torus (solid) and sphere (dashed) compactifications.

We now investigate BH production at the LHC in detail; for previous studies see [11]. When $\sqrt{s} \gtrsim M_*$ BHs are produced with a geometric (subprocess) cross section, $\hat{\sigma} \simeq \pi R_s^2$. Here R_s is the Schwarzschild radius corresponding to a BH of mass $M_{BH} \simeq \sqrt{\hat{s}}$. R_s is given by [12]

$$M_* R_s = \left[\frac{\Gamma(\frac{n+3}{2})}{(n+2)\pi^{(n+3)/2}} \frac{M_{BH}}{M_*} \right]^{1/(n+1)}. \quad (1)$$

Note that $\hat{\sigma} \sim n$ for large n. Numerical simulations and detailed arguments have shown that the geometric cross section estimate is good to within factors of a few [13]. The total number of BH events at the LHC with invariant mass above an arbitrary value $M_{\mathrm{BH,min}}$ is shown in Fig. 2. The scale of the total inclusive BH cross-section, ~ 100 pb, is huge compared to that which is typical of new physics processes, $\lesssim 1$ pb. Thus, over much of the parameter space the LHC will be producing over a million BH events per year. This high rate means that there will be tremendous statistical power, and essentially all measurements will be systematics limited.

The semiclassical treatment, used here and in all previous studies [12], may recieve potentially large corrections from two sources: (*i*) distortions from the finite compactification scale as R_s approaches R_c, and (*ii*) quantum gravity. Case (*i*) is easily controllable. We know that in 5 dimensions the critical point for instabilities due to finite compactification is $(R_s/R_c)^2 \approx 0.1$ [14]. For LHC energies we always have $(R_s/R_c)^2 \ll 0.1$, so these corrections are negligible. In more dimensions the ratio of the volume of a BH with fixed R_s to the volume of the torus with fixed R_c drops rapidly with n, so we expect the corrections to be even smaller. Case (*ii*) is more problematic; we estimate the quantum gravity effects by looking at the corrections from higher curvature terms in the action, *e.g.*

$$S = \frac{M_*^{D-2}}{2} \int d^D x \, \left(R + \frac{\alpha_1}{M_*^2} \mathcal{L}_2 + \frac{\alpha_2}{M_*^4} \mathcal{L}_3 + \ldots \right). \quad (2)$$

Here R is the Ricci scalar, and $\mathcal{L}_i$ is the ith order Lovelock invariant, with $\mathcal{L}_2$ being the Gauss-Bonnet term [15].

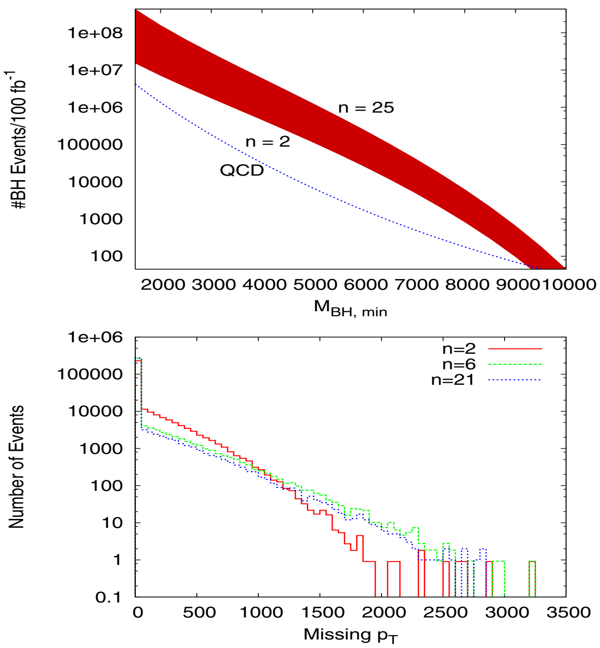

FIG. 2: Top panel: Cross-section for production of black holes with mass $M > M_{BH,min}$ with $M_* = 1.5$ TeV, for $n = 2$(bottom) to 25(top) of the band. Also shown is the QCD dijet cross-section for dijet invariant mass $M \geq M_{BH,min}$, and $|\eta| < 1$. Bottom panel: $\not{p}_T$ distribution of BH events passing cuts described in the text for $M_* = 1$ TeV and $n = 2, 6, 21$.

This equation also defines our convention for the fundamental scale M_*.[1] Schwartzchild solutions are known for arbitrary values of the α_i [16]. If we assume that the higher curvature terms are radiatively generated, and hence each α_i is the ith power of an expansion parameter α (as occurs in string models [17]), we find that $\alpha n^2 \leq 1$. For α of this size we find that the corrections are always less severe as n increases, with a $\sim 20\%$ correction to R_s for $n = 20$. This does not qualitatively affect our conclusions here; for a more detailed study of these corrections, see [18].

We now come to the crucial question, is there any property of the produced black holes that can resolve the number of dimensions? The cross-section is n-dependent, but the overall scale is set by $1/M_*^2$, so one would first have to measure M_* independently to good accuracy to obtain any resolution on n. Cross section ratios at different BH masses could be used, however, the range of energies that are clearly in the geometric regime and accessible to the LHC is not likely to be large. This leads us to the decay properties of black holes. One generically expects that black holes produced at colliders are formed in highly asymmetric states, with high angular momentum, and possibly a non-zero charge. However, they quickly shed their charge and angular momentum by emitting bulk graviton modes and soft brane modes, and relax to a simple Schwartzchild state; their decay then proceeds primarily by thermal emission of Hawking radiation [12] until $M_{BH} \sim M_*$, where quantum gravity effects will mediate the final decay. The Hawking temperature is given by

$$T_H = \frac{(n+1)M_*}{4\pi}\left[\frac{\Gamma(\frac{n+3}{2})}{(n+2)\pi^{(n+3)/2}}\frac{M_{BH}}{M_*}\right]^{-1/(n+1)}. \quad (3)$$

From this we can see that, at fixed M_{BH}, higher dimensional BHs are hotter. Since the average multiplicity goes inversely with the temperature, a low dimensional BH will emit many quanta before losing all of it's energy. By contrast, the decay of a high dimension BH will have fewer final state particles, and each emitted quanta will carry a larger fraction of the BH energy. We will use this difference to obtain experimental resolution on n.

The previous argument suggests we examine the final state multiplicity, or the individual particle p_T distributions as a probe of n. The multiplicity is affected by two major sources of uncertainty: (a) contributions from initial and final state radiation that produce additional jets, and (b) the details of the final quantum gravity decay of the BH are unknown. In what follows we will assume that this decay is primarily 2-body. However, this is clearly model-dependent; we prefer observables that are independent of this assumption, disfavoring the multiplicity. By contrast, the p_T spectra of individual particles, particularly at high-p_T, will be mostly sensitive to the *initial* temperature of the BHs. There are many such distributions that one could consider. In particular, one would like to examine all possible distributions and see that the candidate BH states are coupling equally to each SM degree of freedom, verifying that these are gravitational phenomena [18]. For illustration we will focus here on the $\not{p}_T$ and individual jet p_T distributions for the BH final state.

To calculate these distributions, we have simulated BH events using a modified version of CHARYBDIS [19], linked to PYTHIA [20]. First, a large sample of BHs with masses above a critical value $M_{\rm min} = M_*$ is generated. From these we select events by cutting on the reconstructed invariant mass, $M_{\rm inv}$ of the event, defined by summing over all visible final state particles or jets with rapidity $|\eta| < 3$, and with $p_T \geq 50$ GeV. We would like to select events where the BH mass is large enough that the event is in the geometrical regime, and quantum gravity corrections are small. To do this, one would need to extract from the data an estimate of the size of M_*. While we have no fundamental model for the quantum gravity effects near threshold, we can assume that there will be a turn-on for BH production near M_*, and the cross-section will then asymptote to the geometric value.

[1] We note that this is related to the other definitions in the literature by $M_* = (8\pi)^{-\frac{1}{n+2}} M_{\rm DL}[9] = [2(2\pi)^n]^{-1\frac{1}{n+2}} M_{\rm GT}[10]$.

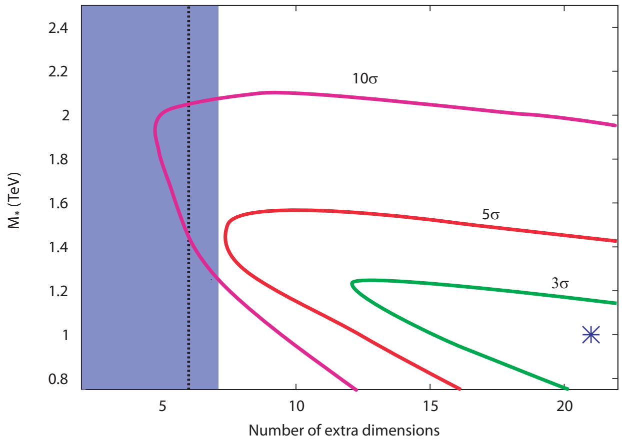

FIG. 3: Exclusion curves in the (n, M_*) plane, assuming the data lies at the point $(21, 1$ TeV$)$. Points outside the curves are excluded at 3, 5, or 10σ.

While this will not lead to a precision determination of M_*, it can clearly be used to set an optimum cut on $M_{\rm inv}$. In the context of a particular model of the threshold based on the action (2), we find that $M_{\rm inv} \geq 2M_*$ is a reasonable cut [18]. We include initial-state radiation in the simulations, since that can lead to a contamination of lower $\sqrt{\hat{s}}$ events in our sample. In the case of jets, for simplicity we turn off hadronization, and simply look at the parton-level characteristics.

To be specific, we generate a "data" set of $\sim 300k$ events with $n = 21$ and $M_* = 1$ TeV. We use this size sample as a conservative lower estimate of BH production. If the cross section is within an order of magnitude of that in Fig. 2, the LHC will collect many millions of events, giving an increase in statistical power over that presented here. These "data" events are then compared to a number of template sets of events. We then ask at what confidence the template can be excluded by performing a χ^2 test using only the resulting $\not{p}_T$ distribution (shown in Fig. 2). We examine the range $2 \leq n \leq 21$, and $0.75 \leq M_* \leq 5$ TeV. The lower bound on M_* comes from non-observation at the Tevatron, while the upper bound is set by demanding that the LHC be able to collect at least $50k$ events given the cross-section uncertainties. We then determine whether the CST region can be excluded at high confidence. For this test case, we find at least a 5σ exclusion for the entire CST region using the $\not{p}_T$ distribution alone, or $\sim 40\sigma$ using the jet-p_T spectrum. Though the statistical power in jets is much higher, it suffers from more systematic uncertainties. Fig. 3 shows the 3, 5, and 10σ exclusion contours in the (n, M_*) plane obtained using the $\not{p}_T$ distribution for this test case. If the LHC collects a few million events rather than the $300k$ sample used here, simple scaling tells us that the 5σ curve excludes $n \leq 20$, and the 10σ curve excludes $n \leq 11$.

We have shown that the CST region can be excluded if $n = 21$. What about other values of n? On changing the number of dimensions used in generating the "data", we find that for any $n \geq 15$ the CST region can be excluded by at least 5σ, with $300k$ events. We would, of course, like to know in what region of the parameter space this type of definitive test can be performed. A more detailed study of the parameter space is in progress [18].

In conclusion, we have shown that if there exist many TeV sized extra dimensions, which solves the Hierarchy Problem, then there exists an observable that can rule out critical string theory.

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