SLAC-PUB-10702
hep-ph/0408240
September 2004

The Higgs Boson Mass at 2 loops in the Finely Tuned Split Supersymmetric Standard Model

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Abstract

The mass of the Higgs boson in the finely tuned Split Supersymmetric Standard Model is calculated. All 1 loop threshold effects are included, in addition to the full RG running of the Higgs quartic coupling through 2 loops. The 2 loop corrections are very small, typically less than 1GeV. The 1 loop threshold corrections to the top yukawa coupling and the Higgs mass generally push the Higgs mass down a few GeV.

Work supported by the Department of Energy contract DE-AC02-76SF00515.

1 Introduction

The so-called gauge hierarchy problem of the standard model (SM) of particle physics has been a fruitful source of inspiration for beyond the SM physics. Most notably, a main reason for the prominence of supersymmetry was its natural solution to this problem. In recent years, additional pieces of evidence for the existence of supersymmetry have arisen from gauge coupling unification and from dark matter, although these successes have been partially offset by difficulties with FCNC and CP violation which arise from light SUSY scalars. Thus, it may be reasonable to abandon the original motivation for SUSY and consider the implications of a theory which maintains all of the successes of the MSSM, except the hierarchy problem, and does away with many of the difficulties. This proposal, called finely tuned, or split SUSY, has recently appeared [1, 2], and some phenomenology has been discussed [3, 4, 5, 6, 7]. In split supersymmetry, a single Higgs scalar is fine tuned to be light, with the understanding that the fine tuning will be resolved by some anthropic-like selection effects. This approach may have a natural realization within inflation and string theory[8]. Basically, an almost infinite landscape of vacua may contain a small percentage with the desired fine tuned parameters, which are necessary for life and the properties of our universe.

2 The Higgs Mass

The starting point for our analysis is the split SUSY lagrangian

$$\begin{aligned}\mathcal{L} &= m^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 + F_u \hat{H}^\dagger Q\overline{u} + F_d H^\dagger Q\overline{d} + F_e H^\dagger L\overline{e} + \text{h.c.} \\ &- \frac{M_1}{2}\widetilde{B}\widetilde{B} - \frac{M_2}{2}\widetilde{W}^A\widetilde{W}^A - \frac{M_3}{2}\widetilde{g}^a\widetilde{g}^a - \mu\widetilde{H}_u^T\epsilon\widetilde{H}_d \\ &- \frac{H^\dagger}{\sqrt{2}}\Big(\kappa_u\sigma^a\widetilde{W}^a + \kappa'_u\widetilde{B}\Big)\widetilde{H}_u + \frac{\widetilde{H}^\dagger}{\sqrt{2}}\Big(-\kappa_d\sigma^a\widetilde{W}^a + \kappa'_d\widetilde{B}\Big)\widetilde{H}_d + \text{h.c.},\end{aligned} \tag{1}$$

where $\hat{H} = -i\sigma_2 H^*$ and $H = (H^+, H^0)^T$.

The following discussion will be similar to the work of [9][10]. After electroweak symmetry breaking, the bare Higgs mass is $M_{h_0}(\mu) = \sqrt{\lambda_0(\mu)}v_0(\mu)$ where $v_0 = \sqrt{2}\langle H^0\rangle$. The pole mass is related to the bare mass through

$$M_h^2 = M_{h_0}^2 + \text{Re}\ \Sigma_h(M_h,\mu) + 3\frac{T_h}{v}, \tag{2}$$

where Σ_h is the self energy and T_h is the tadpole. The bare vev is related to the renormalized vev via muon decay :

$$v_0^2(\mu) = v_F^2\left(1 + \Pi_{WW}(0) + E - 2\frac{T_h}{M_H^2 v_F}\right), \tag{3}$$

where $v_F = 1/\sqrt{\sqrt{2}G_F} \approx 246.22\mathrm{GeV}$, μ is the renormalization scale, and E represents universal vertex and box corrections to muon decay in the standard model. Finally the bare coupling is related to the $\overline{MS}$ coupling by

$$\lambda_0(\mu) = \lambda(\mu) + \frac{\beta_\lambda}{2\lambda} C_{UV}, \tag{4}$$

where $C_{UV} = \frac{1}{\epsilon} - \gamma_E + \log 4\pi$. Putting these together, one finds

$$\begin{aligned} M_h &= \sqrt{\lambda(\mu)} v_F (1 + \delta_h(\mu)) \\ \delta_h(\mu) &= \frac{1}{2}\Big(\Pi_h(M_h) + \frac{T_h}{M_h^2 v_F} + E + \Pi_{WW}(0) + \frac{\beta_\lambda}{2\lambda} C_{UV}\Big). \end{aligned} \tag{5}$$

This formula includes all 1-loop threshold and RG corrections, and the can be improved to include the leading RG 2-loop corrections. The scale μ should be chosen near M_h, although at one loop the μ (and C_{UV}) dependence exactly cancels from Eq.(5). The result for $\delta_h(\mu)$ in the SM has been given in [9]. The corrections for split SUSY will be given below.

The algorithm used will be as follows. In the next section, the chargino and neutralino mass eigenstates and mixing matrices will be defined, since the results are written in terms of them. Next, the top yukawa coupling will carefully be extracted from the top mass, including all one loop threshold corrections. The bottom and tau yukawas are also included, although their effect is slight. The 2 loop running of the Higgs quartic coupling will be given. Solving the coupled RG equations for $g_1, g_2, g_3, F_U, F_D, F_L, \kappa_u, \kappa_d, \kappa'_u, \kappa'_d$ yields the required inputs to solve the λ RGE. Once this is done, all that remains is to include the finite threshold corrections in Eq.(5).

2.1 The Chargino and Neutralino mass spectrum

The chargino and neutralino mass matrix diagonalization proceeds similar to the MSSM[11].

The mass matrices are given by

$$X = \begin{pmatrix} M_2 & \frac{\kappa_u v}{\sqrt{2}} \\ \frac{\kappa_d v}{\sqrt{2}} & \mu \end{pmatrix} \tag{6}$$

and

$$Y = \begin{pmatrix} M_1 & 0 & -\frac{\kappa'_d v}{\sqrt{2}} & \frac{\kappa'_u v}{\sqrt{2}} \\ 0 & M_2 & \frac{\kappa_d v}{\sqrt{2}} & -\frac{\kappa_u v}{\sqrt{2}} \\ -\frac{\kappa'_d v}{\sqrt{2}} & \frac{\kappa_d v}{\sqrt{2}} & 0 & -\mu \\ \frac{\kappa'_u v}{\sqrt{2}} & -\frac{\kappa_u v}{\sqrt{2}} & -\mu & 0 \end{pmatrix}. \tag{7}$$

These are diagonalized by matrices U, V in the chargino sector ($\chi_i^+ \chi_i^-$) and N in the neutralino sector (χ_i^0):

$$\chi_i^+ = V_{ij} \psi_j^+ \quad \chi_i^- = U_{ij} \psi_j^- \quad \chi_i^0 = N_{ij} \psi_j^0, \tag{8}$$

where the gauge eigenstates are

$$\psi_j^+ = \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix} \quad \psi_j^- = \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix} \quad \psi_j^0 = (\tilde{B}, \widetilde{W}_3, \widetilde{H}_d^0, \widetilde{H}_u^0)^T. \tag{9}$$

The matrices U, V, N are specified by

$$\begin{aligned} N^* Y N^{-1} = M^{(N)} &= \mathrm{diag}\{M_1^{(N)}, M_2^{(N)}, M_3^{(N)}, M_4^{(N)}\} \\ N Y^\dagger Y N^{-1} &= (M^{(N)})^2 \\ U^* X V^{-1} &= M^{(C)} = \mathrm{diag}\{M_1^{(C)}, M_2^{(C)}\} \\ V X^\dagger X V^{-1} &= (M^{(C)})^2 = U^* X X^\dagger U^T \end{aligned} \tag{10}$$

2.2 The Top Quark Yukawa coupling and pole mass

Since the Higgs mass is most sensitive to the top yukawa coupling, it is important to carefully extract this from the pole mass. In the canonical work of Hempfling and Kniehl [10], the authors found the following relation between the $\overline{MS}$ yukawa coupling in the Standard Model (SM) and the pole mass :

$$y_t(\mu) = \sqrt{2} \frac{M_t}{v_F} (1 - \delta_t(\mu)). \tag{11}$$

The correction term, which comes from relating the bare vev to the Fermi constant in muon decay, is given by

$$\delta_t(\mu) = \mathrm{Re}\Pi_t(M_t) + \frac{\Pi_{WW}(0)}{2M_W^2} + \frac{E}{2} + \frac{\beta_{y_t}}{2y_t} C_{UV}. \tag{12}$$

In this formula, Π_t represents the top self energy and E is the vertex and box corrections to muon decay, neither of which receive new contributions in split SUSY. However, the W boson self energy, Π_{WW}, does receive corrections, which we calculate below. The UV divergence, C_{UV}, multiplying the top yukawa beta function β_{y_t} comes from the relation between the bare and $\overline{MS}$ yukawa coupling and is cancelled by the divergent parts of Π_t, Π_{WW}, and E.

It turns out that in the range of Higgs masses of interest, the SM contribution is approximately constant with respect to M_t and M_h and is well approximated by $\delta_t^{SM}(M_t, M_h) = -0.044 - (M_h(\mathrm{GeV}) - 125)0.00002 + (M_t(\mathrm{GeV}) - 178)0.0008$.

Now we turn to the split SUSY corrections.

The $W^{\pm}-\chi_i^0-\chi_j^{\pm}$ vertex is given by $ig\gamma_\mu(L_{ij}P_L+R_{ij}P_R)$, with $L_{ij} = -\frac{1}{\sqrt{2}}N_{i4}V_{j2}^* + N_{i2}V_{j1}^*$ and $R_{ij} = \frac{1}{\sqrt{2}}N_{i3}^*U_{j2} + N_{i2}^*U_{j1}$. From this we derive the split SUSY corrections involving charginos and neutralinos :

$$\begin{aligned}
16\pi^2\Pi_{WW}^{(C,N)}(0) &= -2M_W^2 C_{UV} X_2(SS) + g^2\sum_{i=1}^{4}\sum_{j=1}^{2}\Bigg((L_{ij}L_{ij}^* + R_{ij}R_{ij}^*)\Big[a^2(\log\frac{a^2}{\mu^2} - 1/2) \\
&+ b^2(\log\frac{b^2}{\mu^2} - 1/2) + \frac{a^2b^2}{a^2-b^2}\log\frac{a^2}{b^2}\Big] \\
&+ 2(L_{ij}R_{ij}^* + R_{ij}L_{ij}^*)\frac{ab}{a^2-b^2}\Big[-a^2(\log\frac{a^2}{\mu^2} - 1) + b^2(\log\frac{b^2}{\mu^2} - 1)\Big]\Bigg), \quad (13)
\end{aligned}$$

where we used the shorthand $a = M_j^{(C)}, b = M_i^{(N)}$ and $X_2(SS)$ is given in Eq.(14). The resulting correction term $\delta_t^{SS}(M_t) = \frac{\Pi_{WW}^{(C,N)}(0)}{2M_W^2}$ depends on the soft gaugino mass terms, $\tan\beta$, and the scalar mass scale M_S. Generically however $|\delta_t^{SS}(M_t)| \lesssim 0.01$.

Analogous corrections to the bottom and tau yukawa couplings are numerically unimportant.

2.3 The 2-loop running of the Higgs Quartic Coupling

It is useful to define the following invariants involving the standard model yukawas and the new split susy yukayas :

$$\begin{aligned}
Y_2(SM) &= \mathrm{Tr}\Big[3F_U^\dagger F_U + 3F_D^\dagger F_D + F_L^\dagger F_L\Big] \\
Y_4(SM) &= \mathrm{Tr}\Big[3(F_U^\dagger F_U)^2 + 3(F_D^\dagger F_D)^2 + (F_L^\dagger F_L)^2\Big] \\
Y_6(SM) &= \mathrm{Tr}\Big[3(F_U^\dagger F_U)^3 + 3(F_D^\dagger F_D)^3 + (F_L^\dagger F_L)^3\Big] \\
Y_G(SM) &= \Big(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\Big)\mathrm{Tr}(F_U^\dagger F_U) \\
&+ \Big(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\Big)\mathrm{Tr}(F_D^\dagger F_D) + \frac{3}{4}(g_1^2+g_2^2)\mathrm{Tr}(F_L^\dagger F_L) \\
X_2(SS) &= 3(\kappa_u^2 + \kappa_d^2) + \kappa_u'^2 + \kappa_d'^2 \\
X_4(SS) &= 5(\kappa_u^4 + \kappa_d^4) + 2\kappa_u^2\kappa_d^2 + 2(\kappa_u\kappa_u' + \kappa_d\kappa_d')^2 + (\kappa_u'^2 + \kappa_d'^2)^2. \quad (14)
\end{aligned}$$

The running of the quartic coupling λ of the Higgs boson is governed by

$$\beta_\lambda \equiv \frac{\partial\lambda}{\partial\mu} = \frac{1}{16\pi^2}\beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2}\beta_\lambda^{(2)} + \cdots \quad (15)$$

The 1 loop beta function is given by [2]

$$\begin{aligned}
\beta_\lambda^{(1)} &= 12\lambda^2 - 9\lambda\Big(\frac{1}{5}g_1^2 + g_2^2\Big) + \Big(\frac{27}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{4}g_2^4\Big) + 4\lambda Y_2(SM) - 4Y_4(SM) \\
&+ 2\lambda X_2(SS) - X_4(SS). \quad (16)
\end{aligned}$$

The 2 loop result is conveniently divided into two terms,

$$\beta_\lambda^{(2)} = \beta_\lambda^{(2)}(SM') + \beta_\lambda^{(2)}(SS), \tag{17}$$

where SS is the new split SUSY contribution and SM' denotes the standard model result modified to include gauginos and higgsinos in gauge boson self energies. This is given by replacing the number of generations in the SM result :

$$N_g(1) = 3 + 3/10 = 33/10 \qquad N_g(2) = 3 + 3/2 = 9/2 \tag{18}$$

$$\begin{aligned}
\beta_\lambda^{(2)}(SM') &= -78\lambda^3 - 24\lambda^2 Y_2(SM) - \lambda Y_4(SM) - 42\lambda \mathrm{Tr}(F_U^\dagger F_U F_D^\dagger F_D) + 20 Y_6(SM) \\
&- 12\mathrm{Tr}[F_U^\dagger F_U (F_U^\dagger F_U + F_D^\dagger F_D) F_D^\dagger F_D] + 10\lambda Y_G(SM) + 54\lambda(g_2^2 + g_1^2/5) \\
&- \lambda\Big[\Big(\frac{313}{8} - 10N_g(2)\Big)g_2^4 - \Big(\frac{687}{200} + 2N_g(1)\Big)g_1^4 - \frac{117}{20}g_2^2 g_1^2\Big] \\
&- 64 g_3^2 \mathrm{Tr}[(F_U^\dagger F_U)^2 + (F_D^\dagger F_D)^2] - \frac{8}{5}g_1^2 \mathrm{Tr}[2(F_U^\dagger F_U)^2 - (F_D^\dagger F_D)^2 + 3(F_L^\dagger F_L)^2] \\
&- \frac{3}{2}g_2^4 Y_2(SM) + g_1^2\Bigg[\Big(\frac{63}{5}g_2^2 - \frac{171}{50}g_1^2\Big)\mathrm{Tr}(F_U^\dagger F_U) + \Big(\frac{27}{5}g_2^2 + \frac{9}{10}g_1^2\Big)\mathrm{Tr}(F_D^\dagger F_D) \\
&+ \Big(\frac{33}{5}g_2^2 - \frac{9}{2}g_1^2\Big)\mathrm{Tr}(F_L^\dagger F_L)\Bigg] + \Big(\frac{497}{8} - 8N_g(2)\Big)g_2^6 - \Big(\frac{97}{40} + \frac{8}{5}N_g(2)\Big)g_2^4 g_1^2 \\
&- \Big(\frac{717}{200} + \frac{8}{5}N_g(1)\Big)g_2^2 g_1^4 - \Big(\frac{531}{1000} + \frac{24}{25}N_g(1)\Big)g_1^6
\end{aligned} \tag{19}$$

The new split SUSY yukawas contribute

$$\begin{aligned}
\beta_\lambda^{(2)}(SS) &= -12\lambda^2 X_2(SS) - \frac{\lambda}{4}\Big[5(\kappa_u^4 + \kappa_d^4) + 44\kappa_u^2\kappa_d^2 + 2(\kappa_u^2\kappa_u'^2 + \kappa_d^2\kappa_d'^2) + \kappa_u'^4 + \kappa_d'^4 - 12\kappa_u'^2\kappa_d'^2 \\
&- 80\kappa_u\kappa_d\kappa_u'\kappa_d'\Big] + \frac{1}{2}\Bigg[47(\kappa_u^6 + \kappa_d^6) + 5(\kappa_u'^6 + \kappa_d'^6) + 7\kappa_u^2\kappa_d^2(\kappa_u^2 + \kappa_d^2) + 11(\kappa_u^4\kappa_u'^2 + \kappa_d^4\kappa_d'^2) \\
&+ 21\kappa_u^2\kappa_d^2(\kappa_u'^2 + \kappa_d'^2) + 38\kappa_u\kappa_d\kappa_u'\kappa_d'(\kappa_u^2 + \kappa_d^2) + 42\kappa_u\kappa_d\kappa_u'\kappa_d'(\kappa_u'^2 + \kappa_d'^2) \\
&+ 17(\kappa_u^2\kappa_u'^4 + \kappa_d^2\kappa_d'^4 + \kappa_u'^2\kappa_d'^4 + \kappa_u'^4\kappa_d'^2) + 19\kappa_u'^2\kappa_d'^2(\kappa_u^2 + \kappa_d^2)\Bigg] \\
&+ \frac{15}{4}\lambda\Big[(g_2^2 + \frac{1}{5}g_1^2)X_2(SS) + 8g_2^2(\kappa_u^2 + \kappa_d^2)\Big] \\
&- 4g_2^2\Big[5(\kappa_u^4 + \kappa_d^4) + 2\kappa_u^2\kappa_d^2 + (\kappa_u\kappa_u' + \kappa_d\kappa_d')^2\Big] - g_2^4\Big[\frac{3}{4}X_2(SS) + 36(\kappa_u^2 + \kappa_d^2)\Big] \\
&+ \frac{3}{10}g_1^2 g_2^2\Big[21(\kappa_u^2 + \kappa_d^2) - (\kappa_u'^2 + \kappa_d'^2)\Big] - \frac{9}{100}g_1^4 X_2(SS).
\end{aligned} \tag{20}$$

2.4 The Higgs Self Energy and Tadpole Corrections

The Higgs tapole and self energy is written in terms of the mixing matrices which appear in the Feynman rules. The interaction Lagrangian in terms of the physical mass eigenstate Dirac and Majorana fermions are given by

$$\mathcal{L}_{int} = -\frac{h}{\sqrt{2}}\overline{\psi}_i^{\pm}(P_L L_{ij}^C + P_R R_{ij}^C)\psi_j^{\pm} + \frac{h}{2}\overline{\psi}_i^0(P_L(R_{(ij)}^N)^* + P_R R_{(ij)}^C)\psi_j^0, \tag{21}$$

and the mixing matrices are given by

$$\begin{aligned} R_{ij}^C &= (L_{ji}^C)^* = \kappa_u V_{i2}U_{j1} + \kappa_d V_{i1}U_{j2} \\ R_{ij}^N &= (\kappa_u N_{i2} - \kappa_u' N_{i1})N_{j4} - (\kappa_d N_{i2} - \kappa_d' N_{i1})N_{j3} \\ R_{(ij)}^N &= \frac{1}{2}(R_{ij}^N + R_{ji}^N). \end{aligned} \tag{22}$$

The Higgs tadpole is determined by setting $iT_h = i(T_h^{(C)} + T_h^{(N)})$ equal to the tadpole diagrams involving charginos and neutralinos. The result is

$$16\pi^2 T_h^{(C)} = -2\sqrt{2}\sum_{i=1}^{2}\text{Re}\,\Big[R_{ii}^C(M_i^C)^3\Big(C_{UV} - \log\frac{(M_i^C)^2}{\mu^2} + 1\Big)\Big] \tag{23}$$

$$16\pi^2 T_h^{(N)} = 2\sum_{i=1}^{4}\text{Re}\,\Big[R_{(ii)}^N(M_i^N)^3\Big(C_{UV} - \log\frac{(M_i^C)^2}{\mu^2} + 1\Big)\Big]. \tag{24}$$

The Higgs self energies are easily written in terms of the canonical 1 loop basis functions $A(m), B_0(k^2, M_1, M_2)$[12].

$$\begin{aligned} 16\pi^2\Sigma_h^{(C)}(p^2) &= \sum_{i,j=1}^{2}\Big[\frac{1}{2}(|L_{ij}^C|^2 + |R_{ij}^C|^2)\Big(A(M_i) + A(M_j) + (M_i^2 + M_j^2 - p^2)B_0(p^2, M_i, M_j)\Big) \\ &+ 2\text{Re}\; M_i M_j R_{ij}^C(L_{ij}^C)^* B_0(p^2, M_i, M_j)\Big] \end{aligned} \tag{25}$$

$$\begin{aligned} 16\pi^2\Sigma_h^{(N)}(p^2) &= \sum_{i,j=1}^{4}\Big[|R_{(ij)}^N|^2\Big(A(M_i) + A(M_j) + (M_i^2 + M_j^2 - p^2)B_0(p^2, M_i, M_j)\Big) \\ &+ 2\text{Re}\; M_i M_j R_{(ij)}^N(R_{(ij)}^N)^* B_0(p^2, M_i, M_j)\Big] \end{aligned} \tag{26}$$

Using $\Pi_h(M_h) = \frac{\text{Re}\;\Sigma_h(M_h)}{M_h^2}$, the above results contribute

$$\delta_h^{SS}(\mu) = \frac{1}{2}\Big(\Pi_h^{(C)}(M_h) + \Pi_h^{(N)}(M_h) + \frac{T_h^{(C)} + T_h^{(N)}}{M_h^2 v_F} + \Pi_{WW}^{(CN)}(0)\Big). \tag{27}$$

Combining this with the SM results of [9], the total threshold correction is given by

$$\delta_h(\mu) = \delta_h^{SM}(\mu) + \delta_h^{SS}(\mu). \tag{28}$$

A useful check of these results is the cancellation of divergences C_{UV} in Eq.(5). This involves repeated use of the definitions in Eq.(10), and has been explicitly verified.

3 Results for the Higgs Mass

The corrections to the Higgs mass considered in this paper are of three varieties:

- **Top Yukawa.** The 1-loop threshold corrections to the Yukawa coupling initial value given in Eq.(13) are amplified because y_t is raised to the fourth power in $\beta_\lambda^{(1)}$. The size of the correction grows with the soft gaugino and higgsino masses. For soft masses less than a TeV, the downward shift in the Higgs mass is $\lesssim 2$GeV.

- **2 loop running of λ.** The 2 loop correction to the beta function is numerically small, and the SM and split SUSY contributions tend to cancel each other in most of parameter space. The shift in the Higgs mass due to including $\beta_\lambda^{(2)}$ is less than 1GeV.

- **Threshold corrections.** The correction given in Eq.(28) typically pushes down the Higgs mass by $1-4$GeV, the larger shift ocurring for small $\tan\beta$ and small M_s. Typically, the SM contributes most of this shift, with the split SUSY corrections $\lesssim 1$GeV.

In regions of parameter space with small $\tan\beta$, $M_s \sim 10^5$GeV, and large soft masses of order 1TeV, the corrections discussed above can shift the Higgs mass down ~ 5GeV. However, generically the corrections are $\lesssim 2$GeV.

Of course, all of these small corrections are nearly wiped out by the large uncertainties in the top mass $M_t = 178 \pm 4.3$GeV, which translates into an uncertainty of about 10GeV in M_h.

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