Current Observational Constraints on Cosmic Doomsday

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Abstract

In a broad class of dark energy models, the universe may collapse within a finite time t_c. Here we study a representative model of dark energy with a linear potential, $V(\phi) = V_0(1 + \alpha\phi)$. This model is the simplest doomsday model, in which the universe collapses rather quickly after it stops expanding. Observational data from type Ia supernovae (SNe Ia), cosmic microwave background anisotropy (CMB), and large scale structure (LSS) are complementary in constraining dark energy models. Using the new SN Ia data (Riess sample), the CMB data from WMAP, and the LSS data from 2dF, we find that the collapse time of the universe is $t_c \gtrsim 42$ (24) gigayears from today at 68% (95%) confidence.

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I. INTRODUCTION

The existence of dark energy [1, 2] is one of the most significant cosmological discoveries ever made. The nature of this dark energy has remained a mystery. Viable models of dark energy include a cosmological constant, quintessence [3, 4, 5], or modified Friedmann equations [6]. The only available model of dark energy based on string theory describes a metastable cosmological constant with an exponentially large decay time [7]. See [8] for reviews with more complete lists of references.

Although current observational data seem to favor a cosmological constant [9, 10, 11, 12], there are two good reasons to continue dark energy studies. First, the measured dark energy density[1] as an arbitrary function of time, $X(z) \equiv \rho_X(z)/\rho_X(0)$, has large uncertainties [10, 14, 15], hence it does not rule out dark energy models with $\rho_X(z)$ that do not vary greatly with time [16]. Second, even if $\rho_X(z)$ were measured to be a constant at high accuracy in future observations, we still need to find physically well motivated models that give an effective cosmological constant with an incredibly small value $\Lambda \sim 10^{-29}$ g/cm^3.

One of the first (and by far the simplest) models of dark energy, which we are going to discuss in our paper, was proposed in [4] in order to address the cosmological constant problem. In this model the cosmological constant was replaced by the energy density of a slowly varying scalar field ϕ with the linear effective potential

$$V(\phi) = V_0(1 + \alpha\phi). \tag{1}$$

Here we use the units $M_p = (8\pi G)^{-1/2} = 1$. If the slope of the potential is sufficiently small, $\alpha V_0 \lesssim 10^{-120}$ in Planck units, the field ϕ practically does not change during the last 10^{10} years, its kinetic energy is very small, so its total potential energy $V(\phi)$ acts nearly like a cosmological constant. The anomalous flatness of the effective potential in this scenario is a standard feature of most of the models of dark energy [3, 4, 5].

Even though the energy density of the field ϕ in this model practically does not change at the present time, it changed substantially during inflation. Since ϕ is a massless field, it experienced quantum jumps with the amplitude $H/2\pi$ during each time interval H^{-1}. These jumps moved the field ϕ in all possible directions. As a result, the universe became divided into an infinitely large number of exponentially large 'universes' containing all possible values of the field ϕ, and, consequently, all possible values of the effective cosmological constant $\Lambda(\phi) = V(\phi)$. This quantity may range from $-M_p^4$ to $+M_p^4$ in different parts of the universe, but we can live only in the 'universes' with $|\Lambda| \lesssim 10^{-28}$ g/cm^3.

Indeed, if $\Lambda \lesssim -10^{-28}$ g/cm^3, the universe collapses within a time much smaller than the present age of the universe $\sim 10^{10}$ years [4, 17, 18]. On the other hand, if $\Lambda \gg 10^{-28}$ g/cm^3, the universe at present would expand exponentially fast, energy density of matter would be exponentially small, and life as we know it would be impossible [4, 17]. This means that we can live only in those parts of the universe where the cosmological constant does not differ too much from its presently observed value $|\Lambda| \sim 10^{-29}$ g/cm^3. This approach constituted the basis for many subsequent attempts to solve the cosmological constant problem using the anthropic principle in inflationary cosmology [18, 19, 20, 21, 22].

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[1] The dark energy density is better constrained by data than the dark energy equation of state, since it is more closely related to data.[13]

There are several other models of a slowly rolling scalar field which may allow one to solve the cosmological constant problem. For example, one may consider models with quadratic potential $m^2\phi^2/2 - \Lambda$ [3, 20], with potentials $V_0 e^{\alpha\phi} - \Lambda$ [23], potentials unbounded from below in $N = 8$ supergravity [22], or potentials depending on two different fields $\alpha V_0\phi + \beta\xi^n$ [24]. In all of these models the anthropic solution of the cosmological constant problem is based on the assumption that the effective potential continuously changes in a sufficiently broad interval near $V = 0$. In all of these models the scalar fields eventually roll down to negative values of V, and the universe experiences global collapse, just as the universe with a negative cosmological constant. A detailed description of this process can be found in [25].

Thus, in all of the models mentioned above, *the anthropic solution of the cosmological constant problem goes hand in hand with the prediction of the global collapse of the universe.*[2] This makes it especially interesting whether there could be any observational evidence in favor of these models, and what kind of bounds on the lifetime of the universe one could obtain by cosmological observations.

Talking about the future observations, even if the results to be obtained by JDEM and Planck satellites will favor the simplest cosmological constant model, they will not necessarily imply that the universe is going to expand forever. They will only place some model-dependent limits on the possible time of the doomsday. For example, in the context of the simplest linear model (1), such data would only imply that the collapse of the universe cannot happen earlier than $t_c \sim 40$ gigayears from now, at the 95% confidence level [29].

The actual data to be obtained many years from now may alter these estimates. It is important to understand what we can say about the possible doomsday time on the basis of the presently available data. There were several attempts to study this question [9, 10, 23, 29, 30], but the results of these investigations appear to be strongly model-dependent. Our approach will be similar to that of Ref. [9], where the constraint on the doomsday time $t_c \gtrsim 30$ Gyrs was obtained for models with the potential $V_0 e^{\phi/2} - \Lambda$. However, this constraint would change significantly if one considered a more general function $V_0 e^{\alpha\phi} - \Lambda$ depending on three parameters, α, V_0, and Λ, in addition to the initial value of the field ϕ.

In this paper we are going to study the observational constraints on the lifetime of the universe in the simplest but rather general class of models where the effective potential with respect to the slowly rolling field ϕ can be represented by the linear function (1) in the cosmologically interesting interval $|V(\phi)| \lesssim 10^{-28}$ g/cm^3 [4]. An advantage of this model (see also [31]) is that it depends only on one parameter αV_0. This can be made manifest by a shift $\phi \to \phi - V_0/\alpha$ which reduces the potential (1) to an even simpler expression $\alpha V_0\phi$. This will allow us to obtain more definite constraints on the doomsday t_c.

We will use the Markov Chain Monte Carlo (MCMC) technique, which will allow us to compare observational data with the model (1) with different values of α. Even for this simple model, we still need an additional input from the theory. The simplest and perhaps the most reasonable approach is to say that since we do not know anything about the origin of

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[2] One can avoid this conclusion in the models where the cosmological constant can take an exponentially large number of *discrete* values near $V = 0$ [26, 27, 28]. In the string theory based models of this type, instead of the global collapse one faces another catastrophe: a spontaneous decompactification of the four-dimensional space [7].

the parameter α, we will consider all values of α being equally probable, and use flat prior probability distribution $p(\alpha) = const$ as an input in the MCMC calculations. But various theoretical arguments may give different prior probability distributions $p(\alpha)$, which may affect the results. For example, one may argue that if there is some reason why the parameter αV_0 in (1) is smaller than 10^{-120}, in Planck units, then perhaps it is many orders of magnitude smaller than 10^{-120} [32]. This is a very strong argument which suggests that $p(\alpha)$ may be strongly peaked at small α, in which case the theory will have almost exactly the same observational consequences as the simple cosmological constant model. On the other hand, instead of the model (1) one may consider models with potentials $\alpha V_0 \phi + \beta \xi^n$. During inflation, each of these two fields will fluctuate, so that in different parts of the universe with $|V| \lesssim 10^{-28}$ g/cm^3 one will have different combinations of the fields ϕ and ξ, and the slope of the effective potential in the steepest descent direction will also take different values. Such models in the observationally important region $|V| \lesssim 10^{-28}$ g/cm^3 behave just as the linear model (1), but the prior probability distribution to live in a part of the universe with a given effective slope αV_0 will depend on the parameter n [24].

That is why in our calculations we will examine the linear model (1) using the simplest prior probability $p(\alpha) = const$, as well as two other possibilities, $p(\alpha) \propto \alpha$ and $p(\alpha) \propto \alpha^{-1/2}$. Whereas the resulting constraints on the lower bound on the doomsday time t_c appears not to be very sensitive to the choice of $p(\alpha)$, one should keep in mind that, in general, predictions of the fate of the universe depend not only on the choice of the potential, but also on the prior probability for the parameters of the model.

In Sec. II of our paper we will describe our analysis technique and the data used. We present our results in Sec. III. In Sec. IV we will make some methodological comments, comparing the results of the MCMC calculations with the Fisher matrix approach. Sec. V contains a critical discussion of the possibility to obtain model-independent constraints on the doomsday time. We summarize our results and discuss their implications in Sec. VI.

II. METHOD

A. Evolving the scalar field

Following Ref.[29], we assume a flat universe, $\Omega_{tot} = \Omega_m + \Omega_\phi = 1$, which is consistent with current observational data [33, 34]. Note that since the expansion rate of the universe today, H_0, depends on Ω_m, V_0 and α, the only independent parameters in this model are (Ω_m, α, H_0).

The usual equations of motion are

$$\ddot{\phi} + 3H(a)\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \tag{2}$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left[V(\phi) - \dot{\phi}^2 - \frac{1}{2}\frac{\rho_m^0}{a^3}\right], \tag{3}$$

where a is the cosmic scale factor, ρ_m^0 is the matter density today, and the dots denote derivatives

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with respect to the cosmic time t. The Hubble parameter is

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\frac{\rho_m^0}{a^3} + \frac{\dot{\phi}^2}{2} + V_0(1 + \alpha\phi)\right], \tag{4}$$

It is not straightforward to solve the usual equations of motion for a given set of (Ω_m, α, H_0), since Ω_m depends on H_0, which in turn is only known once we have solved the equations for $a(t)$ and found its derivative at $a = 1$.

We rewrite the equations of motion as

$$\frac{d^2\overline{\phi}}{d\tau^2} + 3E_*(z)\frac{d\overline{\phi}}{d\tau} + \overline{\alpha} = 0, \tag{5}$$

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = 1 + \overline{\alpha}\overline{\phi} - \left(\frac{d\overline{\phi}}{d\tau}\right)^2 - \frac{1}{2}\frac{\Omega_m^*}{a^3}, \tag{6}$$

$$\left(\frac{1}{a}\frac{da}{a\tau}\right)^2 = E_*^2(z) \equiv \frac{\Omega_m^*}{a^3} + \frac{1}{2}\left(\frac{d\overline{\phi}}{d\tau}\right)^2 + 1 + \overline{\alpha}\overline{\phi}, \tag{7}$$

where

$$\tau \equiv \sqrt{\frac{8\pi G V_0}{3}}\,t, \qquad \overline{\phi} \equiv \sqrt{\frac{8\pi G}{3}}\,\phi, \qquad \overline{\alpha} \equiv \sqrt{\frac{3}{8\pi G}}\,\alpha, \qquad \Omega_m^* \equiv \frac{\rho_m^0}{V_0}. \tag{8}$$

Note that the equations of motion (Eqs.[5]-[7]) only depend on $(\Omega_m^*, \overline{\alpha})$.

At $\tau \to 0$, $a \to 0$, we take $\phi = d\phi/dt = 0$ [29]. Evolving the equations of motion to $a = 1$ ($z = 0$) gives us

$$H_0 = \sqrt{\frac{8\pi G V_0}{3}}\,E_*(z = 0), \qquad \Omega_m = \frac{\Omega_m^*}{E_*^2(z = 0)}. \tag{9}$$

B.   Prior distributions of parameters

In the linear model of Eq.(1), the expansion rate of the universe depends on the parameters (ρ_m^0, V_0, α) (see Eq.(4)). We assume uniform priors for ρ_m^0 and V_0. It can be shown that choosing uniform prior for V_0 is equivalent to choosing uniform prior for the initial value of the scalar field ϕ after inflation, which is the standard assumption made in [4, 24]. For α, we consider three different priors (see Ref.[24] for justifications):

$$P(\alpha) \propto 1; \qquad P(\alpha) \propto \alpha^{-0.5}; \qquad P(\alpha) \propto \alpha \tag{10}$$

which correspond to $n = 2, 3, 3/2$ in Eq.(29) of Ref.[24].

Using the Bayes' Theorem, we can take the prior in α into consideration by replacing the usual χ^2 with

$$\tilde{\chi}^2 = \chi^2 - 2\ln P(\alpha). \tag{11}$$

C. Data used

The measured distance-redshift relations of type Ia supernovae (SNe Ia) provide the foundation for testing dark energy models. In a flat Universe, the dimensionless luminosity distance

$$d_L(z) = cH_0^{-1}(1+z)\Gamma(z), \qquad \Gamma(z) = \int_0^z dz'/E(z'), \qquad (12)$$

where

$$E(z) = E_*(z)/E_*(z=0) \qquad (13)$$

with $E_*(z)$ given by Eq.(7).

The observational data we use in this paper are the same as in [10]. We use the "gold" set of 157 SNe Ia (the Riess sample) published in [9] and analyze it using flux-averaging statistics [14, 35] to reduce the bias due to weak gravitational lensing by intervening matter [36]. A Fortran code that uses flux-averaging statistics to compute the likelihood of an arbitrary dark energy model (given the SN Ia data from [9]) can be found at *http://www.nhn.ou.edu/~wang/SNcode/* [10, 14, 35].

We only use CMB and LSS data that are not sensitive to the assumptions made about dark energy [10, 37].

The only CMB data we use is the measurement of the CMB shift parameter [38],

$$R \equiv \Omega_m^{1/2}\Gamma(z_{CMB}) = 1.716 \pm 0.062 \qquad (14)$$

from CMB (WMAP, CBI, ACBAR) [33, 39], where $z_{CMB} = 1089$.

The only large-scale structure information we use is the linear growth rate $f(z_{2dF}) = 0.51 \pm 0.11$ measured by the 2dF galaxy redshift survey (2dFGRS) [40], where $z_{2dF} \simeq 0.15$ is the effective redshift of this survey and $f \equiv (d\ln D/d\ln a)$ is determined by solving the equation for the linear growth rate D,

$$D''(\tau) + 2E(z)D'(\tau) - \frac{3}{2}\Omega_m(1+z)^3 D = 0, \qquad (15)$$

where primes denote $d/d(H_0 t)$.

We use the Markov Chain Monte Carlo (MCMC) technique in the likelihood analysis (based on the MCMC engine of [41]), and obtain a few million samples of $(\Omega_m^*, \overline{\alpha}, H_0)$. This method samples from the full posterior distribution of the parameters, and from these samples the marginalized posterior distributions of the parameters (i.e., their probability distribution functions [pdf]) can be estimated.

III. RESULTS

In models that lead to a Big Crunch, the Hubble parameter $H(z) \propto E(z)$ will decrease with time until $E(z) = 0$ at $t = t_{turn}$, when the universe stops expanding and starts to collapse. Fig.1 of Ref.[29] shows the cosmic scale factor in five models (the linear potential model with five different parameter choices). Clearly, the universe collapses rather quickly after it stops expanding.
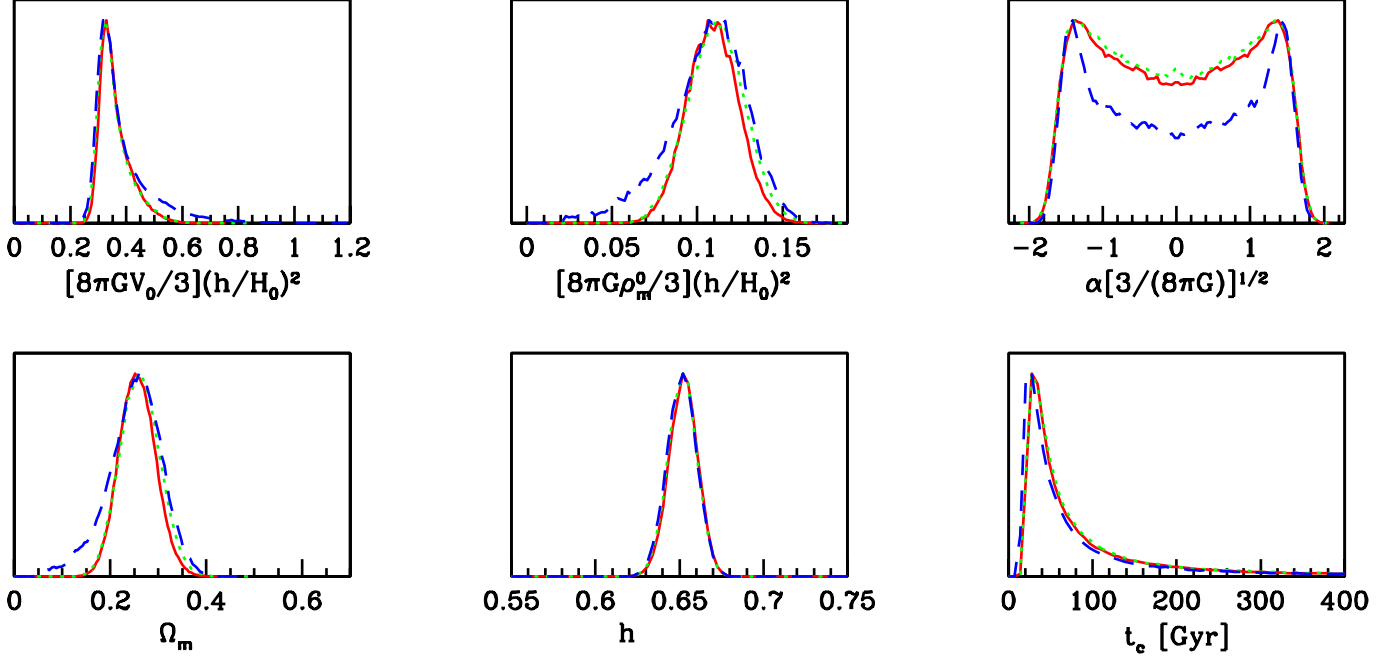
FIG. 1: The constraints on the linear model parameters from current SN data only (Riess sample gold set, flux-averaged with $\Delta z = 0.05$) (dashed lines), SN together with CMB data (dotted lines), SN together with CMB and LSS data (solid lines). Uniform priors on (ρ_m^0, V_0, α) are assumed. The first row shows the probability distribution functions of the set of independent parameters V_0, ρ_m^0, and α The second row shows derived parameters Ω_m, h, and the time to collapse from today t_c.

Fig.1 shows the constraints on the linear model from current observational data, assuming uniform priors on (ρ_m^0, V_0, α). The first row of figures in Fig.1 shows the probability distribution functions of the set of independent parameters V_0, ρ_m^0, and α. The second row of figures in Fig.1 shows derived parameters Ω_m, h, and the time to collapse from today t_c. The dashed lines denote results using SN data only, the dotted lines denote results using SN together with CMB data, the solid lines denote results using SN together with CMB and LSS data.

Table 1 lists the mean and the 68% and 95% confidence ranges of Ω_m, h, α, and t_c from Fig.1.

Table 1

The constraints on the linear potential dark energy model from current data

	SN only[a]	SN plus CMB[b]	SN plus CMB and LSS[c]
Ω_m	.25 [.19, .30][.12, .35]	.26 [.22, .30][.18, .35]	.26 [.22, .30][.19, .33]
h^d	.651 [.642, .660][.634, .668]	.651 [.643, .660][.635, .668]	.652 [.644, .661][.635, .668]
$\alpha \left(\frac{3}{8\pi G}\right)^{1/2}$	0.00 [−1.30, 1.30][−1.60, 1.60]	0.00 [−1.20, 1.20][−1.60, 1.60]	0.00 [−1.20, 1.20][−1.60, 1.60]
$t_c/[\text{Gyr}]^e$	56.01 [35.75, —] [19.18, —] [—, 108.18] [—, 2704.01]	67.48 [42.95, —] [23.61, —] [—, 134.34] [—, 4095.18]	65.66 [41.92, —] [23.74, —] [—, 130.52] [—, 3652.01]
χ^2_{min}/N^f_{dof}	25.98/23	26.04 /24	26.16/25

[a] Riess sample gold set (157 SNe Ia), flux-averaged with $\Delta z = .05$.

[b] CMB shift parameter $R \equiv \Omega_m^{1/2}\Gamma(z_{CMB}) = 1.716 \pm 0.062$.

[c] The linear growth rate $f(z_{2dF}) = 0.51 \pm 0.11$.

[d] Statistical error only, not including the contribution from the much larger SN Ia absolute magnitude error of $\sigma_h^{int} \simeq 0.05$ [14].

[e] For t_c, we list median, the 68% and 95% lower bounds and the 68% and 95% upper bounds.
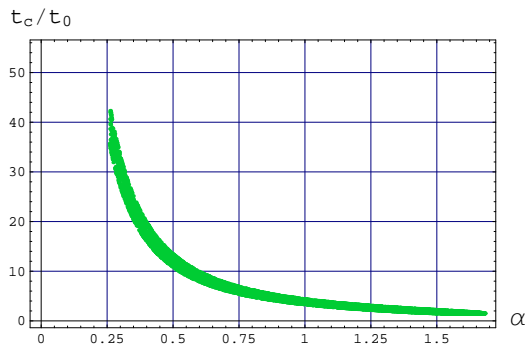
[f] The number of degrees of freedom.

FIG. 2: The ratio of the collapse time from today, t_c, and the age of the universe today, t_0, as a function of the linear potential parameter $|\alpha|$. Fig.4.

Note that only the median, 68% and 95% confidence lower and upper limits of the collapse time from today t_c are given, since $\alpha = 0$ is the cosmological constant model with $t_c \to \infty$ (the mean of t_c is not well defined for this reason). Computationally, we have to make a cutoff in t_c for all models that are longer lived than the computation limit. The dependence of t_c/t_0 on α is shown in Fig.2, where t_0 is the age of the universe today.
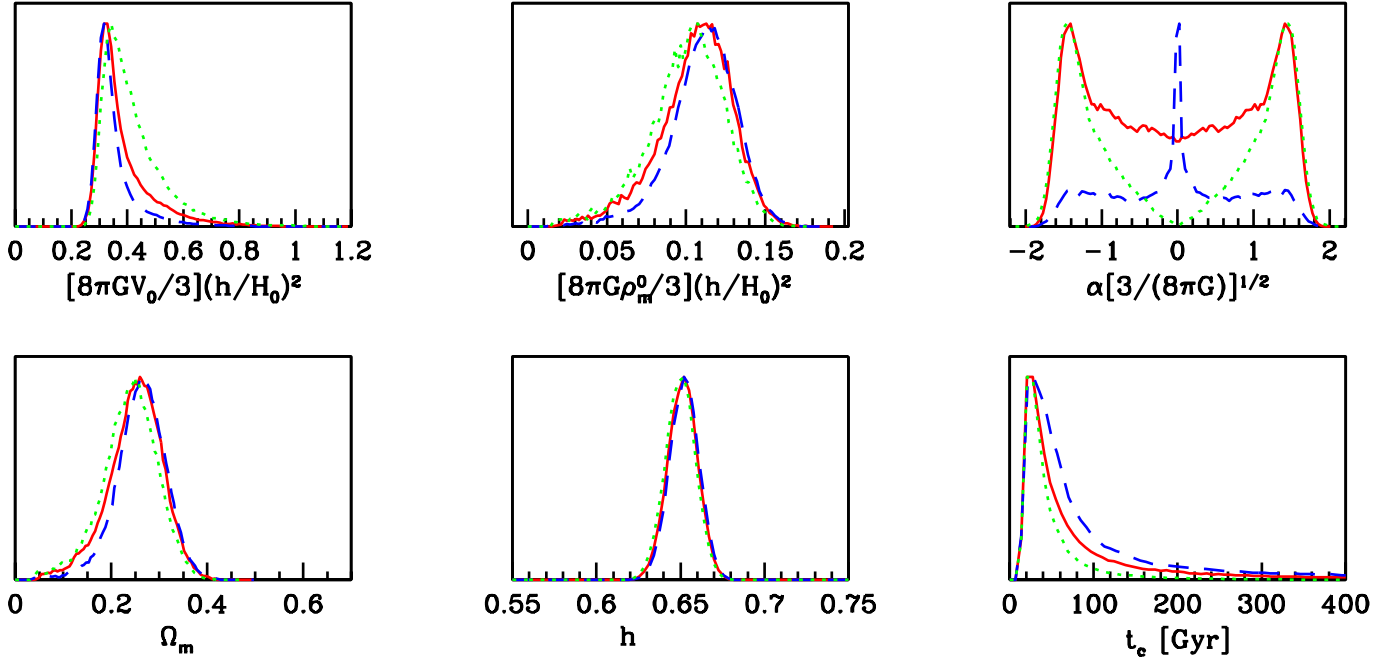
Fig.3 shows the effect of assuming different priors for α, using only SN data (Riess sample gold set, flux-averaged with $\Delta z = 0.05$). The parameters are the same as in Fig.1. The solid, dashed, and dotted lines correspond to priors of $p(\alpha) \propto 1$, $\alpha^{-0.5}$, and α respectively. Table 2 shows how assuming different priors for α changes the median, 68% and 95% lower and upper bounds on the collapse time from today t_c.

FIG. 3: The effect of assuming different priors for α, using only SN data (Riess sample gold set, flux-averaged with $\Delta z = 0.05$). The parameters are the same as in Fig.1. The solid, dashed, and dotted lines correspond to priors of $p(\alpha) \propto 1$, $\alpha^{-0.5}$, and α respectively.

Table 2

Effect of assuming different priors for α on the collapse time from today t_c (in Gyrs)

$P(\alpha) \propto$	1	$\alpha^{-0.5}$	α
median	56.01	96.92	35.04
68% lower bound	35.75	38.77	27.41
95% lower bound	19.18	19.38	17.91
68% upper bound	108.18	523.35	48.75
95% upper bound	2704.01	714260.81	186.42

IV.   MCMC VERSUS FISHER MATRIX

In this section we compare the joint confidence regions from our MCMC calculations to that of the popular Fisher matrix estimates. The Fisher matrix approach is an easy way to estimate the confidence regions in an n-dimensional parameter space p_i, $(i = 1, 2, ..., n)$, and it is especially powerful in the case when the probability distributions are close to the Gaussian distribution. The Fisher matrix is the inverse of the covariance matrix in this n-dimensional parameter space, see [42]:
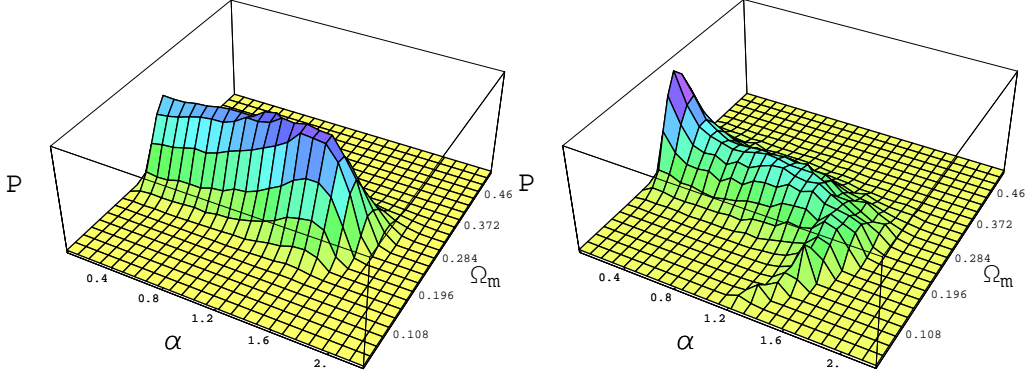
$$F \equiv C^{-1}.$$

FIG. 4: The probability distribution function for the set of parameters $|\alpha|$ and Ω_m with uniform prior on α (left figure) and with the prior proportional to $\alpha^{-0.5}$ (right figure).

It is easy to calculate the covariance matrix C using the MCMC data and marginalizing over additional parameters. We will consider the distribution and confidence regions for α and Ω_m. In our case, the distribution of the parameter α (slope of the potential) is significantly non-Gaussian (see Fig.1). There are a few facts that we have to take into account. First, the symmetry of the picture is dictated by the fact that the $-\alpha$ and α slopes in Eq.(1) are indistinguishable. It also leads to a paradoxical statement that Ω_m and α parameters become uncorrelated (the corresponding off-diagonal elements of the Fisher matrix are 0). Second, the actual parameter that we are trying to measure, and, hopefully, distinguish from 0 (cosmological constant case) is actually $|\alpha|$. This means that in our estimations for joint confidence regions we have to consider absolute value $|\alpha|$ rather than α. In Fig.4 the two-dimensional probability distribution functions for the set of parameters $|\alpha|$ and Ω_m are presented for two different cases - SN plus CMB and LSS data with uniform priors on (ρ_m^0, V_0, α) (right plot) and SN data with $p(\alpha) \propto \alpha^{-0.5}$ (left plot). In both cases the probability distributions are non-Gaussian.

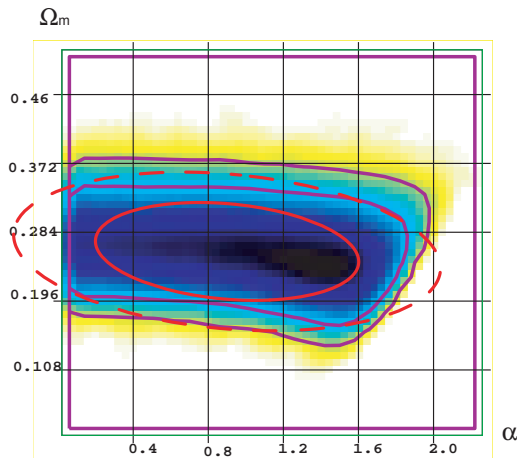The confidence regions form the Fisher matrix and MCMC method are shown in Fig.5 for

FIG. 5: The confidence region for the set of parameters $|\alpha|$ and Ω_m. The purple contours are MCMC 68% and 95% regions and red solid (dashed) ellipsoids correspond to 68% (95%) Fisher matrix estimations.

10

SN with CMB and LSS data and uniform priors on (ρ_m^0, V_0, α) (left plot in Fig.4). The MCMC confidence regions are presented by two uneven contours for 1-σ (68%) and 2-σ (95%) values. Two Fisher ellipses (red solid ellipse corresponds to 68% confidence region and red dashed ellipse corresponds to 95% region) are shown as well. It is clear that due to the non-Gaussian nature of the distribution the Fisher ellipses could be used only for very rough estimations of the confidence regions. In fact, according to Fig.5, $\alpha = 0$ (the cosmological constant case) would be excluded at 1-σ level for the Fisher matrix approach, that is clearly not the case from the MCMC results. The 2-σ regions for both cases give similar results for $\alpha = 0$ and 10% discrepancy for the upper bound on $|\alpha|$.

V. PREDICTIONS FROM MODEL-INDEPENDENT BOUNDS ON THE PAST EVOLUTION

We now derive complementary bounds on the cosmic doomsday by finding linear potential models that fit completely under the 68% and 95% confidence contours of the model-independent reconstruction of dark energy density $\rho_X(z)$ from Ref.[10].

The idea is to map out the boundaries of the past behavior of $\rho_X(z)$ in a model-independent way based on SN Ia and other observations, in this case by the means of splines. Splines have the advantage over polynomials, for instance, that they do not have the tendency to wiggle in between measurement points. While the results will depend somewhat on the type of splines used, i.e. on the number of parameters (the more parameters one uses, the more information is extracted from the data, but the weaker the constraints will become), it is important to realize how this approach differs from earlier, conventional, model-dependent studies. The splines approach is able to model any arbitrary function, not making any assumption about the functional form of $\rho_X(z)$ or $w_X(z)$, contrary to the practice prevailing in present literature of using two parameter fits.[3]

As a starting point for our study, we take the model-independent boundaries on the past evolution of $\rho_X(z)/\rho_X(0)$ obtained in [10]. Next, we solve the differential equations for the evolution history of the universe with the linear potential for all possible initial conditions for the scalar field consistent with the 95% and 68% confidence regions of Ω_m, and fit their corresponding past $\rho_X(z)/\rho_X(0)$ history into the above envelope. The model parameters yielding the shortest lifetime, but still not being in violation anywhere with this envelope, determine the minimal lifetime obtainable by this method. These limiting cases, together with the boundaries, are graphically represented in Fig.6.

Our approach is different from [10] in that we recognize that an accurate prediction of the future can only be obtained by a model-specific extrapolation. One can study the past model-independently, yet whatever the conclusion from it is, the future will always depend on the specific underlying physical model. Even in the absence of knowledge of the exact physics—as it is the case presently with dark energy—one is forced to assume a specific physical model to obtain a prediction on the future lifetime of the universe. In our case, the model is the scalar field linear potential.

We use the Hubble time H_0^{-1} as the time scale in our calculations. To translate the lifetime

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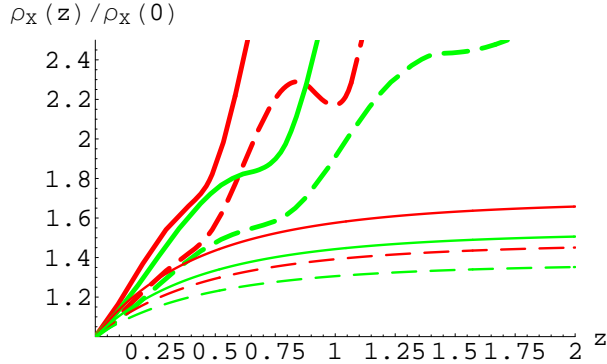[3] Refs. [10] and [43] show the perils of such practice.

FIG. 6: The model-independent constraints on the past of $\rho_X(z)/\rho_X(0)$ and how they constrain the linear model. The bounds are represented by thick lines, while the corresponding linear model is shown by thin lines with same color and same texture (same dashing). Green denotes the 3-parameter spline bound and its linear model, while red shows the 4-parameter splines. The dashed lines depict the 68% confidence level, while the solid lines stand for 95%. Since the 3-parameter constraint contours always lie inside the 4-parameter spline at a given confidence level, one can argue that the 3-spline investigation is sufficient for this analysis, yielding a minimal future lifetime of 20.6 Gyr at 95% confidence level.

into Gyr, we need the value of the Hubble constant H_0. The Hubble constant, as obtained from the model-independent analysis in [10], has a mean value of $H_0 = 63.7$km/s/Mpc, corresponding to a Hubble time $H_0^{-1} = 15.36$Gyr, for the 3-parameter splines, while $H_0 = 63.5$km/s/Mpc ($H_0^{-1} = 15.41$Gyr) for the 4-parameter spline.[4]

Even the 95% error bars of the distribution for h do not cause an uncertainty of more than 5.7%, corresponding to 0.87Gyr, thus we will neglect this uncertainty in our lifetime calculation in this section.

In this manner, we obtain for the future lifetime of the universe:

Table 3

The lower bound on future lifetime of the universe

Spline Type	Confidence Level	Future Lifetime
4-parameter	95% lower bound	18.1 Gyr
	68% lower bound	22.2 Gyr
3-parameter	95% lower bound	20.6 Gyr
	68% lower bound	25.3 Gyr

These results turn out to be a little weaker than our model-specific analysis above. This is not unexpected; it will be very difficult, if not impossible, to close the gap and raise the

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[4] These Hubble constant values are fully consistent with other *direct* measurements. SN Ia data typically give a H_0 value consisitent with $H_0 = 60 \pm 10 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ [44]. The Hubble Key Project (based on Cepheid distances) gives $H_0 = 72 \pm 8 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ [45]. Note that CMB data do *not* give a direct measurement of H_0.

model-independent analysis to the same accuracy level as the model-dependent one. After all, no assumption about the model in the past is made.

The model-independent analysis also depends on the number of free parameters used. The difference in results between the 3-parameter and the 4-parameter spline is roughly 10%. As usual, stronger constraints are achieved by fewer parameters at the cost of less flexibility. Since the 3-parameter spline contour lies completely within the 4-parameter one, one can argue that for the studied sample of supernovae using 3-parameter splines is sufficient.

We conclude that the results obtained by the model-independent study of the past are somewhat less stringent than our results obtained in Sec. III. Indeed, constraining the parameters of a given model directly is the most precise way to proceed. However, it is quite convenient to have a *model-independent* analysis of the past, and then apply it to any particular model for a quick estimate of the lifetime of the universe. This method provides a useful shortcut, which lets one to obtain approximate results without having to employ the whole machinery of MCMC that we have used to obtain our results in Sec. III.

VI. DISCUSSION AND SUMMARY

We have studied a representative dark energy model with a linear potential, $V(\phi) = V_0(1 + \alpha\phi)$, assuming a flat universe. This model has the interesting property of dooming the universe to a Big Crunch for $\alpha \neq 0$. It is the simplest doomsday model, in which the universe collapses rather quickly after it stops expanding.

Since this model has only one free parameter, αV_0, it is well constrained even by SN Ia data alone (Riess sample), see Fig.1.

We studied this model using current SN Ia (Riess sample), CMB (WMAP, CBI, and ACBAR), and LSS (2dF) data. We have used flux-averaging to minimize the effect of weak lensing [14, 35], and included CMB and galaxy survey data in a consistent manner [10]. We found that in the context of the model with $V(\phi) = V_0(1 + \alpha\phi)$, observations imply that the cosmic doomsday is more than about 42 billion years from today at 68% confidence, and more than about 24 billion years from today at at the 95% confidence level.

These results represent a stronger bound on the doomsday time than the estimate of $t_c \gtrsim 11$ Gyr based on the previously available observational data [29]. When the data from SNAP and Planck satellites become available, it will be possible to further strengthen our bound up to $t_c \gtrsim 40$ Gyr at 95% confidence [29].

Note that these timescales have been derived assuming a uniform prior on α (the parameter of the linear model). Changing the prior on α can change the bounds on the cosmic doomsday. We found that taking different priors, $p(\alpha) \sim \alpha$ and $p(\alpha) \sim \alpha^{-0.5}$, altered the value of t_c obtained for $p(\alpha) = const$ only by about 10%. If one assumes that α must be exponentially small, we would conclude that the universe will live for an exponentially long time [20, 24]. This means that when one evaluates the cosmological consequences of a given model, one should specify not only its Lagrangian and the cosmological initial conditions, but also the theoretical expectations for the values of the parameters in the model.

Interestingly, we were able to obtain not only the lower bound on the doomsday time, but the upper bound as well. For the uniform prior on α the full bound on the doomsday time

13

is 24 Gyr $< t_c <$ 3652 Gyr at the 95% confidence level (SN Ia plus CMB and LSS data, see Table 1). In other words, if one believes that the dark energy is described by the simplest linear model, and the parameter α can take any value with equal probability, then our universe most probably will not collapse earlier than in 24 billion years, but it will most probably live no longer than 3.65×10^3 billion years. Note that this doomsday prediction is a consequence of our assumption that dark energy is described by the linear model (1), and that all values of the parameter α are equally probable. On the other hand, as we already mentioned, this model is rather generic; it is the simplest representative of a broad class of the dark energy models where the effective cosmological constant can change continuously and the cosmological constant problem can be solved using anthropic considerations [4, 20, 22, 24]. To improve our constraints on the cosmic doomsday time in this class of models one would need further cosmological observations and a deeper understanding of the theory of dark energy.

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