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Symplectic Treatment of a Finite Crossing Angle in the Beam-Beam Collision*

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Abstract

We introduce a symplectic method to handle a large and finite crossing angle in the beam-beam interaction. This method has been implemented in a parallel computer program to simulate three-dimensional effects in the beam-beam interaction. Our simulation results are compared with the known analytical solutions, the simulations using the Lorentz boost and experimental observations.

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1 Introduction

The beam-beam effects due to a vertical crossing angle were experimentally and theoretically investigated by Piwinski[1]. He showed that the crossing angle coupled the transverse and longitudinal oscillations and therefore excited the synchrotron-betatron resonances which lead to the degradation of luminosity. Since the horizontal beam size is naturally much larger than the vertical size in e^+e^- storage rings, the allowed crossing angle in the horizontal plane may well be much larger than the one in the vertical plane. This possibility was systematically studied by Hirata [2] who introduced a transformation called "Lorentz Boost". Using the boost, he simulated the dynamical effects due to a large crossing angle within the strong-weak approximation. His work has established the feasibility of using a crossing angle as a realistic scheme to separate the colliding beams near the interaction point (IP) in e^+e^- storage rings.

Since the birth of a new generation of high-luminosity e^+e^- colliders [3, 4, 5], the beam-beam collision with a finite crossing angle in the horizontal plane has become a reality [4, 5]. The positive and successful experience of these modern accelerators has prompted us to adopt the crossing scheme into the designs and upgrades of the e^+e^- storage rings [6] and the hadron colliders [7, 8].

Based on these recent developments, it is clear that the beam physics related to the crossing angle has become critically important. A concern regrading the Lorentz boost is: the violation of the symplecticity because of the explicit use of the Lorentz boost in its composition. It is well known that the violation of symplecticity may cause artificial growth of emittance [9]. Of course, it was pointed out by Hirata in his paper [2] and recently by Ohmi [10] that the net effect is symplectic if its inverse is used after the beam-beam kick.

In this letter, we continue along the work of Piwinski and develop a geometrical method to treat exactly a collision without use of the Lerentz boost for a finite crossing angle. The symplecticity is preserved throughout the collision process.

2 Geometrical transformations

Let's use $x, p_x, y, p_y, \delta, l$ as the canonical coordinates of a charge particle, where x, y are the transverse displacements, δ is the relative momentum deviation and l is the path length relative to the synchronous particle. When two beams collide with a horizontal crossing angle, we need a transformation that rotates the particles in a single slice ($s = 0$) to the head-on frame ($s^* = 0$) as illustrated in Fig 1. It is clear that the axis of the rotation is the y axis. It is well known [11] that, in the context of single-particle dynamics, this transformation can be generated by the Lie operator: $\mathcal{R}_y(\phi) = \exp(: xp_s : \phi)$, where $p_s = \sqrt{(1+\delta)^2 - p_x^2 - p_y^2}$

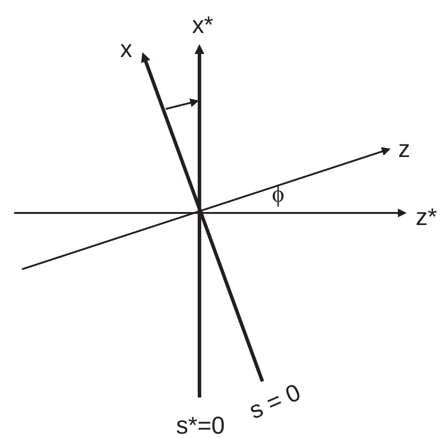

Figure 1: *A rotational transformation.*

The explicit transformation can be obtained by solving the Hamiltonian's equations with $H = -xp_s$ and ϕ as the independent variable. It can be written as follows,

$$
\begin{aligned}
x^* &= \frac{xp_s}{\cos\phi(p_s - p_x \tan\phi)}, \\
p_x^* &= p_x \cos\phi + p_s \sin\phi, \\
y^* &= y + \frac{xp_y \tan\phi}{(p_s - p_x \tan\phi)}, \\
p_y^* &= p_y, \\
\delta^* &= \delta,
\end{aligned}
$$

$$l^* \;=\; l + \frac{x(1+\delta)\tan\phi}{(p_s - p_x \tan\phi)}. \tag{1}$$

Since it is the exact solution of the Hamiltonian's equation, it is symplectic. To treat a three-dimensional beam, there are other coordinate transformation needed. They are the horizontal displacement $\mathcal{D}_x(\delta x) = \exp(: p_x : \delta x)$ and the drift operator $\mathcal{D}_z(\delta s) = \exp(: p_s : \delta s)$.

3 Collision

For every collision, the macro particles are cast into the slices according to their longitudinal positions. Since the beam distributions are dynamically evolved during the collision, the sequence of the colliding slices is identical to the time sequence.

For a given pair of colliding slices at $z^{\pm} = -l^{\pm}$, we need to compute where the collision actually occurs: $s^{\pm} = (z^{\pm} - z^{\mp})/2$ and drift the particles in the slices to the collision point by the operator $\mathcal{D}_z^{\pm}(s^{\pm}) = \mathcal{D}_z(s^{\pm})$ so that the hourglass and phase-average effects due to a finite bunch length are properly included in the simulation.

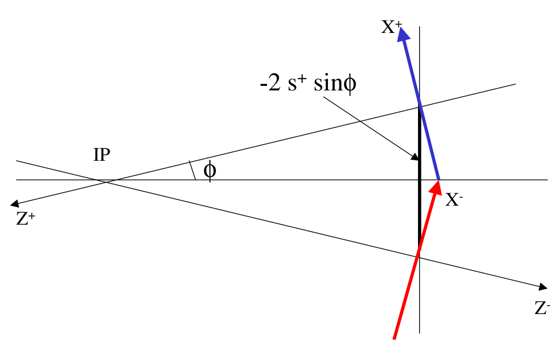

Figure 2: *Two slices of beam colliding at their actual collision point with an angle* 2ϕ*.*

As illustrated in Fig. 2, if there is a crossing angle, we need to make the transformation $\mathcal{R}_y^{\pm}(\phi) = \mathcal{R}_y(\pm\phi)$. After the rotations, there is still a displacement of two coordinate systems in the horizontal plane as shown in Fig 2; we use operator: $\mathcal{D}_x^{\pm}(s^{\pm},\phi) = \mathcal{D}_x(-2s^{\pm}\sin\phi)$ to transform the coordinates of the particles to the coordinate system in which the beam-beam force from the opposing beam is calculated so that the force can be applied to the particles. After the beam-beam kick, we applied the reverse operation in the inverted order to move the particles in the slice back inside the beam.

The whole process can be summarized and written as

$$\mathcal{T}^{\pm}(s^{\pm},\phi)\cdot\mathcal{O}^{\mp}_{BB}(x^{\pm},y^{\pm},\phi)\cdot\mathcal{T}^{\pm}(s^{\pm},\phi)^{-1}, \tag{2}$$

where

$$\mathcal{T}^{\pm}(s^{\pm},\phi)=\mathcal{D}^{\pm}_{z}(s^{\pm})\cdot\mathcal{R}^{\pm}_{y}(\phi)\cdot\mathcal{D}^{\pm}_{x}(s^{\pm},\phi), \tag{3}$$

and $\mathcal{O}^{\mp}_{BB}(x^{\pm},y^{\pm},\phi)$ represents the operator for the beam-beam kick.

Here we use the following convention in the map operation: The operator on the left acts on function of the canonical coordinates first and the dot represents the concatenation of two maps.

Using the particle distributions at the collision point, we obtain the beam-beam force by solving the two-dimensional Poisson equation [12]. Because of the crossing angle ϕ, the integrated beam-beam kick by a slice needs to be modified to

$$\begin{aligned}
\Delta p^{\pm}_{x} &= -\frac{e}{E^{\pm}_{0}}\cos\phi\int_{slice}E^{\mp}_{x}ds,\\
\Delta p^{\pm}_{y} &= -\frac{e}{E^{\pm}_{0}}\int_{slice}E^{\mp}_{y}ds,\\
\Delta\delta^{\pm} &= -\frac{e}{E^{\pm}_{0}}\sin\phi\int_{slice}E^{\mp}_{x}ds,
\end{aligned} \tag{4}$$

where E_x and E_y are the transverse electric fields and E_0 is the energy of the synchronous particle. Here we have assumed that the particles are ultra-relativistic and $E_0 = cp_0$.

4 Geometrical effect

The geometric degradation of luminosity due to the hourglass effect and the crossing angle is given by Hirata [2]

$$R_L=\frac{L}{L_0}=\sqrt{\frac{2}{\pi}}ae^{b}K_0(b), \tag{5}$$

$$a=\frac{\sigma^{*}_{y}}{\sqrt{2}\sigma^{*}_{z}\sigma^{*}_{p_y}}, b=a^2[1+(\frac{\sigma^{*}_{z}}{\sigma^{*}_{x}}\tan\phi)^2],$$

where L and L_0 is the luminosity with or without the hourglass effects and crossing angle and K_0 is a modified Bessel function.

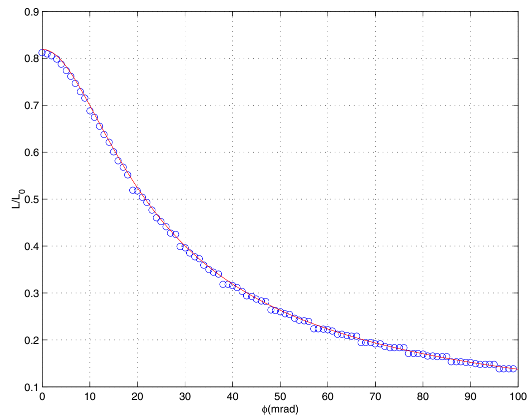

Figure 3: *The circles represent the simulation results using 50,000 macro particles on a mesh of* $128 \times 256 \times 31$ *and the solid is the plot of Eq. (6) with* $\sigma_x^* = 110.84$ *μm,* $\sigma_y^* = 1.16$ *μm,* $\sigma_{p_y}^* = 181.14$ *μrad, and* $\sigma_z^* = 7.9$ *mm.*

Since this is a purely geometrical and single-turn effect, we do not need to compute the electric and magnetic field during the collision. The simulated luminosity is calculated from the summation of overlapping beam distributions on the transverse head-on grids over all possible pairs of the colliding slice. The result of the simulation is shown in Fig. 3 for the symmetrized KEKB parameters. The excellent agreement between the simulation and the analytical analysis over a large range of the crossing angle provides an independent check of the accuracy of using these geometrical transformations.

5 Parallel computing

To achieve the required numerical convergence in the three-dimensional simulation forces the use of parallel supercomputers. One of the most important aspects of parallel computing is how to minimize the communication among processors. Each application may have a different optimal solution. For beam-beam simulations, we have developed an efficient strategy utilizing dual processors. Macro particles are evenly distributed across many processors. The processors are divided into two groups, one for the positron beam and the other for the electrons. Before the collision, the beam distribution on the grid is summed within each group, and the resulting distribution is distributed back to all processors in the group. Then the total distribution is exchanged between the groups. That allows us to solve the Poisson equation and compute the force on the macro particles in every local processor.

In this scheme, the macro particles always remain confined to the same computing processor. The division into two groups essentially allows us to double the speed without much penalty.

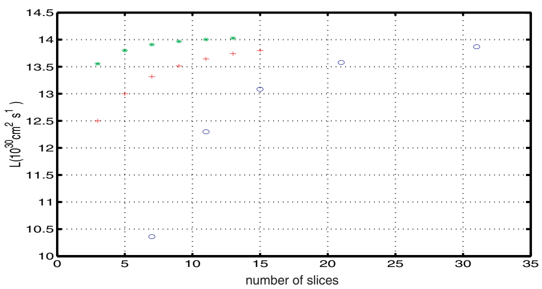

Figure 4: *Simulated luminosity as a function of number of longitudinal slices. The circles represent the results of using equal-spacing slices, the crosses for equal-area slices, and the starts for equal-area slices with linear interpolation between the slices.*

A linear and stochastic map [13] that includes the betatron and synchrotron oscillations, the radiation damping, and the quantum excitation is used in the arc to track the particles. The map also properly gives the effects of the dynamical beta and dynamical emittance [14] near the horizontal half integer.

Using 32 processors on a parallel computer at NERSC [15], we are able to achieve the required convergence with five linearly interpolated[10] and equal-strength slices[13] as shown in Fig 4. For a typical simulation, we use 160,000 macro particles for each beam with a mesh $128 \times 128 \times 5$. The area of the mesh has to be large enough to retain the particles in the tail of beam, especially in the vertical plane. In practice, we choose it so that the accumulated loss of the particles beyond the mesh during the whole run is less than a few percent even at the peak of the beam intensities. To reach an equilibrium of the beam distributions, each simulation takes about eight hours on the supercomputer.

6 Dynamical effect

The dynamical degradation of the luminosity from a finite crossing angle can be more severe than the geometrical reduction because of the synchro-betatron coupling introduced from the angle. To study this effect, a simulation is carried out for the present parameters, tabulated in Table 1, at KEKB to benchmark against a well-known code [16] based on Hirata's work. The results of the two simulation are shown in Fig. 5.

Parameter	Description	e^+	e^-
E (Gev)	beam energy	3.5	8.0
N (10^{10})	bunch population	7.36	5.28
β_x^* (cm)	beta x at the IP	59.0	58.0
β_y^* (mm)	beta y at the IP	5.8	7.0
ϵ_x (nm-rad)	emittance x	18.0	24.0
ϵ_y (nm-rad)	emittance y	0.18	0.24
ν_x	x tune	0.506	0.513
ν_y	y tune	0.545	0.586
ν_s	z tune	0.0249	0.0207
σ_z (mm)	bunch length	8.7	7.1
σ_p (10^{-4})	energy spread	7.26	6.67
τ_t (turn)	x, y damping time	4000	4000
τ_s (turn)	z damping time	2000	2000

Table 1: The present parameters of KEKB. The crossing angle is $\pm 11 mrad$.

The equilibrium luminosities obtained from the simulations and the measurement agree within 5%. The measured luminosity is nearly at the middle of the two simulations. The equilibrium beam sizes agree within a few percentages between the two codes. At the peak beam intensities, the total luminosity reduction due to the crossing angle of $\phi = \pm 11$ mrad is 58%, which is significantly higher than its geometric degradation 17%.

The success of reaching its design luminosity at KEKB has clearly demonstrates many advantages of the design with the crossing angle. Still, the simulation shows that its luminosity could be doubled if one simply compensates the crossing angle with crab cavities [17]. However, this result also implies that the head-on collision has a potential to produce twice the luminosity at extremely high intensities of beam compared to the collision with an angle.

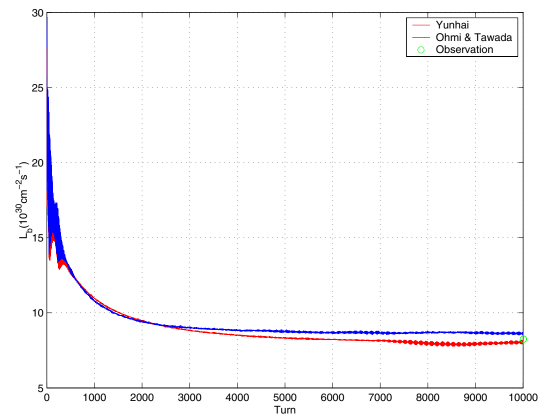

Figure 5: *The simulated bunch luminosity is compared with the result obtained by Ohmi and Tawada for the present KEKB parameters at the peak beam currents and the measurement on May 3, 2003.*

In additional to these simulations, we also benchmark the two codes at the current PEP-II working point and the super KEKB parameters. The results in agreement or disagreement are similar. Since they are all head-on collisions, the results are not shown in this letter.

7 Discussions

We have demonstrated that this geometrical method works just as well as the traditional method using the Lorentz boost in e^+e^- storage rings. Since the method is based entirely on the Lie operators during the collision, it is manifestly sympletic. Moreover, it has a geometric interpretation at each step of the operation.

Acknowledgments

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References

[1] A. Piwinski, “Satellite resonances due to beam-beam interaction”, 1977 Particle Accel. Conf., IEEE Trans. V. NS-24, No.3 p.1408 (1977).

[2] K. Hirata, “Analysis of beam-beam interactions with a large crossing angle”, Phys. Rev. Lett. **74** 2228 (1995).

[3] “PEP-II conceptual design report”, SLAC-418, LBL-PUB-3579, June (1993).

[4] “KEKB B-factory design report”, KEK-Report-95-7, (1995).

[5] G. Vignola *el al*, 1996 Erupean Particle Accel. Conf., p.22 (1996).

[6] “BEPCII design report”, IHEP-AC-Report/2002-01, BEPCII-Report/2002-01, May (2002).

[7] F. Ruggiero and F. Zimmermann, “Luminosity optimization near the beam-beam limit by increasing bunch length or crossing angle”, Phys. Rev. ST Accel. Beams **5**, 061001 (2002).

[8] “Run II handbook”, Fermilab Technical Design Report, November (1996).

[9] D. Douglas, E. Forest, and R. V. Servranckx, “A method to render second order beam optics programs symplectic,” IEEE Trans. Nucl. Sci. **32** 2279 (1985).

[10] K. Ohmi, *et al*, “Study of Beam-Beam Interactions with or without Crossing Angle,” KEK-PREPRINT-2004-17, May 2004, SLAC-PUB-10532.

[11] E. Forest, M. F. Reusch, D. L. Bruhwiler, and A. Amiry, “The Correct Local Description for Tracking in Rings,” Part. Accel. **45** 66 (1994).

[12] Y. Cai, A. W. Chao, S. I. Tzenov, and T. Tajima, “Simulation of the beam-beam effects in e^+e^- storage rings with a method of reducing the region of mesh,” Phys. Rev. ST Accel. Beams **4**, 011001 (2001).

[13] K. Hirata, H. Moshammer, and F. Ruggiero, “A symplectic beam-beam interaction with energy change,” Particle Accelerator, **40**, 205 (1993).

[14] K. Hirata and F. Ruggiero, "Treatment of Radiation in electron storage rings," LEP notes 661, August 8, (1988).

[15] The National Energy Research Scientific Computing Center supported by the Office of Science of the U.S. Department of Energy.

[16] K. Ohmi, "Simulation of beam-beam effects in a circular e^+e^- collider," Phys. Rev. E **62**, 7287 (2000).

[17] K. Oide and K. Yokoya, "Beam-beam collision scheme for storage-ring collider," Phys. Rev. A **40**, 315 (1989).