# Constraints on $r_{B}$ and $\gamma$ in $B^{ \pm} \rightarrow D^{(*) 0} K^{ \pm}$decays by a Dalitz analysis of $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$ 

The BABAR Collaboration

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#### Abstract

We report on a study of direct $C P$ violation in the decay $B^{-} \rightarrow D^{(*) 0} K^{-}$with a Dalitz analysis of the $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$decay using a sample of 227 million $B \bar{B}$ pairs collected by the BABAR detector. Reference to the charge-conjugate state is implied here. We constrain the amplitude ratio $r_{B}=\frac{\left|A\left(B^{-} \rightarrow \bar{D}^{0} K^{-}\right)\right|}{\left|A\left(B^{-} \rightarrow D^{0} K^{-}\right)\right|}$to be $<0.19$ at the $90 \%$ confidence level and $r_{B}^{*}=$ $\frac{\left|A\left(B^{-} \rightarrow \bar{D}^{* 0} K^{-}\right)\right|}{A\left(B^{-} \rightarrow D^{* 0} K^{-}\right) \mid}=0.155_{-0.077}^{+0.070} \pm 0.040 \pm 0.020$ and we measure the relative strong phase $\delta_{B}=$ $(114 \pm 41 \pm 8 \pm 10)^{\circ}$ and $\delta_{B}^{*}=(303 \pm 34 \pm 14 \pm 10)^{\circ}$ between the amplitudes $A\left(B^{-} \rightarrow \bar{D}^{(*) 0} K^{-}\right)$ and $A\left(B^{-} \rightarrow D^{(*) 0} K^{-}\right)$. From these samples we measure $\gamma=(70 \pm 26 \pm 10 \pm 10)^{\circ}$. The first error is statistical, the second error accounts for experimental uncertainties and the third error reflects the Dalitz model uncertainty. For this preliminary result we have quoted confidence intervals obtained with a Bayesian technique assuming a uniform prior in $r_{B}, \gamma$ and $\delta_{B}$.


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## 1 INTRODUCTION

In the past years $C P$ violation in the $B$ meson system has been clearly established [1] and although there is good agreement with the expectations of the Standard Model, further measurements of $C P$ violation in $B$ decays are needed to over-constrain the unitarity triangle and look for New Physics effects. A crucial test will be represented by the measurement of $\gamma$, which is the complex phase of the Cabibbo-Kobayashi-Maskawa [2] quark mixing matrix element $V_{u b}$ in the Wolfenstein parameterization [3].
Various methods using $B^{-} \rightarrow D^{0} K^{-}$decays [4] have been proposed to measure the unitarity triangle angle $\gamma$, all exploiting the fact that a $B^{-}$can decay into a $D^{0} K^{-}$final state via a $b \rightarrow c$ transition or into a $\bar{D}^{0} K^{-}$final state via a $b \rightarrow u$ transition. $C P$ violation can be detected if the $D^{0}$ and $\bar{D}^{0}$ decay into the same final state. The measurement of direct $C P$ violation is sensitive to the phase difference between $V_{u b}$ and $V_{c b}$ and thus to the angle $\gamma$. Most of the experimental methods to extract $\gamma$ can be grouped in two categories: the $D^{0}$ and $\bar{D}^{0}$ decay into a $C P$ eigenstate [5]; or the $D^{0}$ decays to a common flavor state, either through a Cabibbo-allowed or a doubly Cabibbo-suppressed mode [6]. The measurement of $\gamma$ in both methods also requires the knowledge of $r_{B}$, the magnitude of the ratio of the amplitudes $\mathcal{A}\left(B^{-} \rightarrow \bar{D}^{0} K^{-}\right)$and $\mathcal{A}\left(B^{-} \rightarrow D^{0} K^{-}\right)$and of their relative strong phase $\delta_{B}$, which can be obtained from data.
In this paper we report on a measurement of direct $C P$ violation in $B^{-} \rightarrow D^{(*) 0} K^{-}$based on the analysis of the Dalitz distribution of the three-body decay $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}[7]$. The advantage of this method is that it involves the entire resonant substructure of the three-body decay, with Cabibboallowed and doubly Cabibbo-suppressed amplitudes interfering directly. It is therefore expected to have a higher statistical precision than the methods outlined above. Results of an analysis based on this procedure were reported by the Belle Collaboration in [8]. From the combination of the $B^{-} \rightarrow D^{0} K^{-}$and $B^{-} \rightarrow D^{* 0} K^{-}$mode they obtain the value $\gamma=77_{-19^{\circ}}^{\circ} \pm 13^{\circ} \pm 11^{\circ}$ where the first error is statistical, the second is experimental systematics and the third is model uncertainty. They also obtain a value of $r_{B}=0.26_{-0.14}^{+0.10} \pm 0.03 \pm 0.04$ for $B^{-} \rightarrow D^{0} K^{-}$and $r_{B}^{*}=0.20_{-0.17}^{+0.19} \pm 0.02 \pm 0.04$ for $B^{-} \rightarrow D^{* 0} K^{-}$.

### 1.1 Analysis outline

The $B^{-}$and $B^{+}$decay amplitudes for the $B^{-} \rightarrow D^{(*) 0} K^{-}$and $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$decays can be written assuming no CP asymmetry in $D$ decays as :

$$
\begin{align*}
& M_{-}\left(m_{-}^{2}, m_{+}^{2}\right)=\left|\mathcal{A}\left(B^{-} \rightarrow D^{0} K^{-}\right)\right|\left[f\left(m_{-}^{2}, m_{+}^{2}\right)+r_{B} e^{i\left(\delta_{B}-\gamma\right)} f\left(m_{+}^{2}, m_{-}^{2}\right)\right],  \tag{1}\\
& M_{+}\left(m_{-}^{2}, m_{+}^{2}\right)=\left|\mathcal{A}\left(B^{+} \rightarrow \bar{D}^{0} K^{+}\right)\right|\left[f\left(m_{+}^{2}, m_{-}^{2}\right)+r_{B} e^{i\left(\delta_{B}+\gamma\right)} f\left(m_{-}^{2}, m_{+}^{2}\right)\right], \tag{2}
\end{align*}
$$

where $m_{-}^{2}$ and $m_{+}^{2}$ are the squared invariant masses of the $K_{S} \pi^{-}$and $K_{S} \pi^{+}$combinations respectively and $f\left(m_{-}^{2}, m_{+}^{2}\right)$ is the amplitude of the $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$decay.

Given a known $f$, the bi-dimensional Dalitz ( $m_{-}^{2}, m_{+}^{2}$ ) distributions for $B^{-}$and $B^{+}$can be simultaneously fitted to $\left|M_{-}\left(m_{-}^{2}, m_{+}^{2}\right)\right|^{2}$ and $\left|M_{+}\left(m_{-}^{2}, m_{+}^{2}\right)\right|^{2}$ respectively. A maximum likelihood technique may be used to estimate $r_{B}, \delta_{B}$, and $\gamma$. Since the measurement of $\gamma$ arises from the interference in Eq. 1 and Eq. 2, the uncertainty in the knowledge of the complex form of $f\left(m_{-}^{2}, m_{+}^{2}\right)$ can lead to a systematic uncertainty. A model describing the $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$decay in terms of two-body amplitudes has been assumed in this analysis. This model has been characterized using a high statistics flavor tagged $D^{0}$ sample $\left(D^{*+} \rightarrow D^{0} \pi_{s}^{+}\right)$, obtained from $e^{+} e^{-} \rightarrow c \bar{c}$ events as described in Section 4.

A similar analysis is also performed using $B^{-} \rightarrow D^{* 0} K^{-}$decays, and $\gamma$ is extracted along with the amplitude ratio $r_{B}^{*}$ and strong phase difference $\delta_{B}^{*}$ taking into account the effective strong phase shift of $\pi$ radians between the $D^{* 0} \rightarrow D^{0} \pi^{0}$ and $D^{* 0} \rightarrow D^{0} \gamma$ channels [9]. By convention $\delta_{B}^{*}$ is the strong phase of $D^{* 0} \rightarrow D^{0} \pi^{0}$ decay mode.

## 2 THE BABAR DETECTOR AND DATASET

The analysis is based on a sample of 227 million $B \bar{B}$ pairs collected by the BABAR detector at the SLAC PEP-II $e^{+} e^{-}$asymmetric-energy storage ring. BABAR is a solenoidal detector optimized for the asymmetric-energy beams at PEP-II and is described in [10]. We summarize briefly the components that are crucial to this analysis. Charged-particle tracking is provided by a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH). In addition to providing precise spatial hits for tracking, the SVT and DCH also measure the ionization energy loss $(d E / d x)$, which is used for particle identification of low-momentum charged particles. At higher momenta ( $\mathrm{p}>0.7$ $\mathrm{GeV} / c$ ) pions and kaons are identified by Cherenkov radiation detected in a ring-imaging device (DIRC). The typical separation between pions and kaons varies from $8 \sigma$ at $2 \mathrm{GeV} / c$ to $2.5 \sigma$ at $4 \mathrm{GeV} / c$. Neutral cluster (photon) positions and energies are measured with an electromagnetic calorimeter (EMC) consisting of 6580 thallium-doped CsI crystals. Candidate $\pi^{0}$ mesons are reconstructed as pairs of photons, spatially separated in the EMC, with an invariant mass within $3 \sigma$ of the $\pi^{0}$ mass. These systems are mounted inside a $1.5-\mathrm{T}$ solenoidal super-conducting magnet.

## 3 EVENT SELECTION

We reconstruct the decays $B^{-} \rightarrow D^{0} K^{-}$and $B^{-} \rightarrow D^{* 0} K^{-}$with $D^{* 0} \rightarrow D^{0} \pi^{0}, D^{0} \gamma$. A larger sample of $B^{-} \rightarrow D^{(*) 0} \pi^{-}$is also reconstructed and is used as a control sample to determine the Probability Density Function (PDF) of the discriminating variables used in the likelihood fit for $\gamma$. $D^{0}$ candidates are reconstructed in the $K_{S} \pi^{+} \pi^{-}$final state with the $K_{S}$ reconstructed from pairs of oppositely charged pions with an invariant mass within $9 \mathrm{MeV} / c^{2}$ of the nominal $K_{S}$ mass [11]. The two pions are constrained to originate from the same point. The angle $\alpha_{K_{S}}$ between the $K_{S}$ line of flight and its momentum is required to satisfy the condition $\cos \alpha_{K_{S}}>0.99 . D^{0}$ candidates are selected by making all possible combinations of the $K_{S}$ candidate and two oppositely charged pions with an invariant mass within $12 \mathrm{MeV} / c^{2}$ of the nominal $D^{0}$ mass.

The photon candidates for $D^{* 0} \rightarrow D^{0} \gamma$ are reconstructed from clusters in the electromagnetic calorimeter with energy greater than 30 MeV and consistent with a photon shower profile. We select $\pi^{0}$ candidates from pairs of photon candidates and require $115<m(\gamma \gamma)<150 \mathrm{MeV} / c^{2}$ and with total energy greater than 70 MeV . To improve the momentum resolution, the $\pi^{0}$ candidates are kinematically fitted with their mass constrained to the nominal $\pi^{0}$ mass. The $D^{0}$ candidates are combined with a low energy $\pi^{0}$ or $\gamma$. The $D^{* 0}-D^{0}$ mass difference $\Delta m$ is required to be within $2.5(10) \mathrm{MeV} / c^{2}$ of the nominal $\Delta m$ for $D^{* 0} \rightarrow D^{0} \pi^{0}(\gamma)$.

A $B^{-}$candidate is obtained by combining a $D^{(*) 0}$ candidate with a track ("bachelor" track) identified as a kaon as described in [10]. We improve the momentum resolution for the $D^{0}$ daughters by applying a kinematic mass constraint. For every $B$ candidate two standard variables are defined,
the beam-energy-substituted mass $m_{\mathrm{ES}} \equiv \sqrt{\left(\frac{1}{2} s+\vec{p}_{0} \cdot \vec{p}_{B}\right)^{2} / E_{0}^{2}-p_{B}^{2}}$ and the energy difference $\Delta E \equiv E_{B}^{*}-\frac{1}{2} \sqrt{s}$, where the asterisk denotes the CM frame, $s$ is the square of the total energy in the CM frame, $p$ and $E$ are, respectively, momentum and energy, and the subscripts 0 and $B$ refer to $\Upsilon(4 S)$ and $B^{ \pm}$, respectively. The resolutions, evaluated on simulated signal events, are $2.6 \mathrm{MeV} / c^{2}$ and 17 MeV for $m_{E S}$ and $\Delta E$, respectively.


Figure 1: $B^{-} \rightarrow D^{0} K^{-}$(top left), $B^{-} \rightarrow D^{* 0}\left(D^{0} \pi^{0}\right) K^{-}$(top right) and $B^{-} \rightarrow D^{* 0}\left(D^{0} \gamma\right) K^{-}$ (bottom) $m_{E S}$ distribution in the $\Delta E$ region $[-30,30] \mathrm{MeV}$ in the sample of 211 million $B \bar{B}$ pairs. The signal contribution is shown in red, $B^{-} \rightarrow D^{* 0} \pi^{-}$in blue, generic $B \bar{B}$ in green, and continuum in magenta.

To distinguish between $B \bar{B}$ and continuum events the following topological variables are used: $\cos \theta_{B}^{*}$, where $\theta_{B}^{*}$ is the polar angle of the $B$ candidate with respect to the beam axis in the CM frame; $L_{0}=\sum_{i} p_{i}$ and $L_{2}=\sum_{i} p_{i} \cos ^{2} \theta_{i}$ calculated in the CM frame, where $p_{i}$ and $\theta_{i}$ are the momenta and the angles of tracks and neutral clusters not used to reconstruct the $B$ candidate with
respect to its thrust axis; $\cos \theta_{T}$, where $\theta_{T}$ is the angle in the CM frame between the thrust axes of the $B$ candidate and of the rest of the event. $B$ candidates are selected by requiring $\left|\cos \theta_{T}\right|<0.8$. Under this condition a Fisher discriminant $\mathcal{F}$ is constructed from the variables discussed above. This Fisher discriminant is used in the likelihood fit to help distinguish between signal and continuum $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ events. The PDF of $\mathcal{F}$ is obtained for continuum events using $m_{E S}$ sideband data. The $B^{-} \rightarrow D^{(*) 0} \pi^{-}$events are used to obtain the $\mathcal{F}$ PDF for $B \bar{B}$ and signal events.
Finally, the $B^{-} \rightarrow D^{(*) 0} K^{-}$sample events are required to satisfy $m_{E S}>5.2 \mathrm{GeV} / c^{2},|\Delta E|<30$ MeV . The $B^{-} \rightarrow D^{(*) 0} \pi^{-}$candidates are selected using criteria similar to those applied for $B^{-} \rightarrow D^{(*) 0} K^{-}$but requiring that the bachelor pion not be consistent with the kaon hypothesis. The overall reconstruction efficiencies are $18 \%, 4.3 \%, 8.1 \%$ for the $D^{0} K^{-}, D^{* 0}\left(D^{0} \pi^{0}\right) K^{-}$, and $D^{* 0}\left(D^{0} \gamma\right) K^{-}$decay modes, respectively. Fig. 1 shows the $m_{E S}$ distributions after all the selection criteria are applied.

## 4 DETERMINATION OF $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$DECAY MODEL

The amplitude $f\left(m_{ \pm}^{2}, m_{\mp}^{2}\right)$ has been constructed from a Dalitz analysis of a $97 \%$ pure flavor tagged $D^{0}$ sample obtained from $81496 D^{*+} \rightarrow D^{0} \pi^{+}$events corresponding to a luminosity of $91.5 \mathrm{fb}^{-1}$ (Fig. 2). The Dalitz ( $m_{ \pm}^{2}, m_{\mp}^{2}$ ) distribution (Fig. 3) is fitted in the context of the isobar formalism described in [12]. In this formalism the amplitude $f$ can be written as a sum of two-body decay matrix elements and a non-resonant term according to the expression

$$
\begin{equation*}
f=a_{n r} e^{i \phi_{n r}}+\Sigma_{r} a_{r} e^{i \phi_{r}} \mathcal{A}_{s}\left(K_{S} \pi^{-} \pi^{+} \mid r\right) . \tag{3}
\end{equation*}
$$

Each term of the sum is parameterized with an amplitude and a phase. The factor $\mathcal{A}_{s}\left(K_{S} \pi^{-} \pi^{+} \mid r\right)$ gives the Lorentz invariant expression for the matrix element of a $D^{0}$ meson decaying into $K_{S} \pi^{-} \pi^{+}$ through an intermediate resonance $r$ as a function of the position in the Dalitz plot. It is, in general, parameterized by a relativistic Breit-Wigner with a functional form dependent on the spin of the resonance. For the $\rho$ a more complex parametrization is used as suggested in [13]. We fit the Dalitz distribution with a model consisting of 13 resonances leading to 17 two-body decay amplitudes and phases (Table 1). Of the 13 resonances eight involve a $K_{S}$ plus a $\pi \pi$ resonance and the remaining five are made of a $\left(K_{S} \pi^{-}\right)$resonance plus a $\pi^{+}$. We also include the corresponding doubly Cabibbo-suppressed amplitudes for most of the ( $K_{S} \pi^{-}$) $\pi^{+}$decays. All the resonances considered in this model are well established except for the two scalar $\pi \pi$ resonances, $\sigma_{1}$ and $\sigma_{2}$, whose masses and widths are obtained from our sample. Those are introduced in order to obtain an acceptable fit to the data, but their existence as true, scalar particles is a matter outside the scope of the paper.

An unbinned maximum likelihood fit is performed to measure the amplitudes $a_{n r}, a_{r}$ and the phases $\phi_{n r}, \phi_{r}$. The fit fraction for each decay channel is defined as the integral of a single component divided by the coherent sum of all components. The results of the fit are shown in Fig. 3. Amplitudes, phases and fit fractions as obtained by the likelihood fit are reported in Table 1.

We estimate the goodness of the fit for our model with a $\chi^{2}$ fit using adaptive binning of the Dalitz plot. We obtain $\chi^{2} /$ d.o.f. $=3824 /(3054-32)=1.27$.

To illustrate the region of the Dalitz plot most sensitive to $\gamma$ measurement, we show in Fig. 4 the distribution of simulated $B^{-} \rightarrow D^{0} K^{-}$events based on our Dalitz model, where each event is given a weight of $\frac{d^{2} \ln \mathcal{L}}{d^{2} \gamma}$ where $\mathcal{L}$ is the likelihood function described in the following section. The

| Resonance | Amplitude | Phase <br> (degrees) | Fraction <br> $(\%)$ | Mass <br> $\mathrm{MeV} / c^{2}$ | Width <br> $\mathrm{MeV} / c^{2}$ | Functional <br> form |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $K^{*}(892)$ | $1.777 \pm 0.018$ | $131.0 \pm 0.81$ | 58.51 | 891.66 | 50.8 | BW |
| $\rho^{0}(770)$ | 1 (fixed) | 0 (fixed) | 22.33 | 775.8 | 146.4 | GS |
| $K^{*}(892)$ DCS | $0.1789 \pm 0.0080$ | $-44.0 \pm 2.4$ | 0.59 | 891.66 | 50.8 | BW |
| $\omega(782)$ | $0.0391 \pm 0.0016$ | $114.8 \pm 2.5$ | 0.56 | 782.6 | 8.5 | BW |
| $f_{0}(980)$ | $0.469 \pm 0.011$ | $213.4 \pm 2.2$ | 5.81 | 975 | 44 | BW |
| $f_{0}(1370)$ | $2.32 \pm 0.31$ | $114.1 \pm 4.4$ | 3.39 | 1434 | 173 | BW |
| $f_{2}(1270)$ | $0.915 \pm 0.041$ | $-22.0 \pm 2.9$ | 2.95 | 1275.4 | 185.1 | BW |
| $K_{0}^{*}(1430)$ | $2.454 \pm 0.074$ | $-7.9 \pm 2.0$ | 8.37 | 1412 | 294 | BW |
| $K_{0}^{*}(1430)$ DCS | $0.350 \pm 0.069$ | $-344 . \pm 10$. | 0.60 | 1412 | 294 | BW |
| $K_{2}^{*}(1430)$ | $1.045 \pm 0.045$ | $-53.1 \pm 2.6$ | 2.70 | 1425.6 | 98.5 | BW |
| $K_{2}^{*}(1430) \mathrm{DCS}$ | $0.074 \pm 0.038$ | $-98 \pm 30$ | 0.01 | 1425.6 | 98.5 | BW |
| $K^{*}(1410)$ | $0.524 \pm 0.073$ | $-157 \pm 10$ | 0.39 | 1414 | 232 | BW |
| $K^{*}(1680)$ | $0.99 \pm 0.31$ | $-144 \pm 18$ | 0.35 | 1717 | 322 | BW |
| $\rho(1450)$ | $0.554 \pm 0.097$ | $35 \pm 12$. | 0.28 | 1406 | 455 | GS |
| $\sigma_{1}$ | $1.346 \pm 0.044$ | $-177.5 \pm 2.5$ | 9.11 | $484 \pm 9$ | $383 \pm 14$ | BW |
| $\sigma_{2}$ | $0.292 \pm 0.025$ | $-206.8 \pm 4.3$ | 0.98 | $1014 \pm 7$ | $88 \pm 13$ | BW |
| Non resonant | $3.41 \pm 0.48$ | $-233.9 \pm 5.0$ | 6.82 | - | - | - |

Table 1: Amplitudes, phases and fit fractions of the different components obtained from the likelihood fit of the $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$Dalitz distribution in $D^{* \pm} \rightarrow D^{0} \pi_{s}^{ \pm}$data. Masses and widths of all resonances except $\sigma_{1}$ and $\sigma_{2}$ are taken from [11]. The abbreviations BW and GS stand for relativistic Breit-Wigner and Gounaris-Sakurai [13] respectively. The total fit fraction is 1.24 .


Figure 2: The $K_{S} \pi^{-} \pi^{+}$invariant mass distribution in the flavor tagged $D^{*}$ sample.
regions of interference between doubly Cabibbo suppressed and Cabibbo allowed decays and $C P$ eigenstate decays exhibit the highest sensitivity to $\gamma$.

## 5 CP FIT TO $B \rightarrow D^{(*) 0} K$ SAMPLES

A maximum likelihood fit ( $C P$ fit) is performed on the $B^{-} \rightarrow D^{(*) 0} K^{-}$samples to extract simultaneously the $C P$ violation parameters $\gamma, \delta_{B}^{(*)}$, and $r_{B}^{(*)}$ and the signal and background yields. The likelihood for each candidate $j$ is obtained by summing the product of the event yield $N_{i}$ and the probability $\mathcal{P}_{i}$ over the signal and the three background hypotheses. The extended likelihood function is

$$
\begin{equation*}
\mathcal{L}=\exp \left(-\sum_{i} N_{i}\right) \prod_{j}\left[\sum_{i} N_{i} \mathcal{P}_{i}\left(\vec{x}_{j}\right) \mathcal{P}_{i}^{\text {Dal }}\left(m_{+}^{2}, m_{-}^{2}\right)\right] . \tag{4}
\end{equation*}
$$

The probabilities $\mathcal{P}_{i}$ are evaluated as the product of the PDFs for each of the independent variables $\vec{x}_{j}=\left\{m_{\mathrm{ES}}, \Delta E, \mathcal{F}\right\} . \mathcal{P}_{i}^{\text {Dal }}\left(m_{+}^{2}, m_{-}^{2}\right)$ is the PDF for the Dalitz distribution for the $i^{\text {th }}$ category. The categories in the fit are signal $B^{-} \rightarrow D^{(*) 0} K^{-}$, the continuum background, $B \bar{B}$ background, and $B^{-} \rightarrow D^{* 0} \pi^{-}$and are shown in Fig. 1. The $m_{E S}$ and $\Delta E$ distributions for signal events are described by a Gaussian. The Fisher PDF is parametrized with a double Gaussian function. The signal PDF parameters are determined from the $B^{-} \rightarrow D^{(*) 0} \pi^{-}$control sample.

### 5.1 Background Composition

The numbers of events for the various background components in the $B^{-} \rightarrow D^{(*) 0} K^{-}$samples are summarized in Table 2. The dominant background contribution is from the random combination of a real or fake $D^{(*) 0}$ meson with a charged track in continuum events or other $B \bar{B}$ decays. The combinatorial background in the $m_{E S}$ distribution is described by a threshold function [14]


Figure 3: (a) The $\bar{D}^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$Dalitz distribution from the $D^{*+} \rightarrow D^{0} \pi^{+}$events. Projections on (b) $m_{+}^{2}$, (c) $m_{-}^{2}$, and (d) $m^{2}\left(\pi^{+} \pi^{-}\right)$are shown. The result of the fit is superimposed as a solid line.
whose free parameter $\xi$ is determined from the $B^{-} \rightarrow D^{(*) 0} \pi^{-}$data sample. The shape of the combinatorial background $m_{E S}$ distribution in generic $B \bar{B}$ decays is taken from simulated events. The $\Delta E$ distribution is described by a straight line whose slope is extracted from a fit to the $B^{-} \rightarrow D^{(*) 0} \pi^{-}$sample. The PDF of the Fisher distribution for continuum events is determined from the $m_{E S}$ sideband in the same data sample. The Fisher PDF for $B \bar{B}$ events is assumed to be the same as that for the signal.

An important class of background events arises from continuum where a real $D^{0}$ is produced back-to-back with a kaon. Depending on the flavor-charge correlation this background can mimic either the $b \rightarrow c$ or the $b \rightarrow u$ signal component. In the likelihood function we take this effect


Figure 4: $\bar{D}^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$Dalitz distribution of simulated ( $B^{+} \rightarrow \bar{D}^{0} K^{+}$) events. Each event is given a weight $\frac{d^{2} \ln \mathcal{L}}{d^{2} \gamma}$. The black points represent the same events with weight equal to unity.
into account with two parameters, the fraction $f_{D^{0}}$ of background events with a real $D^{0}$ and the parameter $R$, the fraction of background events with a real $D^{0}$ associated with an oppositely flavored kaon (same charge correlation as the $b \rightarrow u$ signal component).

The fraction of real $D^{0}$ from continuum events has been evaluated in events satisfying $\mathrm{m}_{E S}$ $<5.272 \mathrm{GeV} / c^{2}$ after removing the requirement on the $D^{0}$ mass. The fraction $R$ of background events with a genuine $D^{0}$ associated with a negatively charged kaon is obtained from simulated events. The values of $f_{D^{0}}$ and $R$ for continuum $q \bar{q}$ are summarized in Table 3. The fraction of events with a real $D^{0}$ in generic $B \bar{B}$ events is found to be few percent.

A small background originates from $B^{-} \rightarrow D^{(*) 0} \pi^{-}$where the bachelor pion is misidentified as a kaon. These events have the same $m_{E S}$ distribution as signal but can be distinguished using their $\Delta E$ information.

### 5.2 Likelihood fit on control samples

We test our $C P$ fit procedure on two high statistics control samples: $D^{*+} \rightarrow D^{0} \pi^{+}$from $c \bar{c}$ continuum events and $B^{-} \rightarrow D^{(*) 0} \pi^{-}$. The $D^{*+} \rightarrow D^{0} \pi^{+}$sample mimics a $B^{-} \rightarrow D^{0} K^{-}$sample with $r_{B}=0$. The $B^{-} \rightarrow D^{0} \pi^{-}$sample is similar to $B^{-} \rightarrow D^{0} K^{-}$, but its $r_{B}$ is expected to be approximately 0.007 [15]. In the $C P$ fit to $D^{*+} \rightarrow D^{0} \pi^{+}$we obtain $r_{B}=(-5.2 \pm 5.2) \times 10^{-3}$. In the $B^{-} \rightarrow D^{0} \pi^{-}$we obtain $r_{B}=(1.8 \pm 1.5) \times 10^{-2}, \gamma=(18 \pm 45)^{\circ}, \delta_{B}=(246 \pm 43)^{\circ}$ and in $B^{-} \rightarrow D^{* 0} \pi^{-}$we find $r_{B}^{*}=(4.6 \pm 2.1) \times 10^{-2}, \gamma=(90 \pm 35)^{\circ}, \delta_{B}^{*}=(117 \pm 35)^{\circ}$. The results

| Background components | $D^{0} K^{-}$ | $\left(D^{0} \pi^{0}\right) K^{-}$ | $\left(D^{0} \gamma\right) K^{-}$ |
| :---: | :---: | :---: | :---: |
| Continuum $q \bar{q}$ | $125 \pm 6$ | $9 \pm 2$ | $38 \pm 3$ |
| $B \bar{B}$ | $28 \pm 7$ | $4 \pm 2$ | $14 \pm 5$ |
| $D \pi$ | $9 \pm 8$ | $0 \pm 5$ | $6 \pm 4$ |

Table 2: Estimates of the numbers of background events in the $m_{E S}>5.272 \mathrm{GeV} / c^{2}$ region from the fit to data in the full $m_{E S}$ region for the $B^{-} \rightarrow D^{0} K^{-}, B^{-} \rightarrow D^{* 0}\left(D^{0} \pi^{0}\right) K^{-}$, and $B^{-} \rightarrow D^{* 0}\left(D^{0} \gamma\right) K^{-}$samples.

| $q \bar{q}$ background parameters | $D^{0} K^{-}$ | $\left(D^{0} \pi^{0}\right) K^{-}$ | $\left(D^{0} \gamma\right) K^{-}$ |
| :---: | :---: | :---: | :---: |
| $f_{D^{0}}$ | $0.27 \pm 0.06$ | $0.27 \pm 0.13$ | $0.13 \pm 0.02$ |
| $R$ | $0.21 \pm 0.03$ | $0.23 \pm 0.11$ | $0.16 \pm 0.06$ |

Table 3: $q \bar{q}$ background parameters for the $B^{-} \rightarrow D^{0} K^{-}, B^{-} \rightarrow D^{* 0}\left(D^{0} \pi^{0}\right) K^{-}$, and $B^{-} \rightarrow$ $D^{* 0}\left(D^{0} \gamma\right) K^{-}$samples.
obtained are consistent with the expectations of Monte Carlo experiments.

### 5.3 Likelihood fit on $B^{-} \rightarrow D^{(*) 0} K^{-}$sample

In the sample of 227 million $B \bar{B}$ events we obtain the following signal yields

$$
\begin{align*}
N\left(B^{-} \rightarrow D^{0} K^{-}\right) & =282 \pm 20,  \tag{5}\\
N\left(B^{-} \rightarrow D^{* 0}\left(D^{0} \pi^{0}\right) K^{-}\right) & =89 \pm 11, \\
N\left(B^{-} \rightarrow D^{* 0}\left(D^{0} \gamma\right) K^{-}\right) & =44 \pm 8,
\end{align*}
$$

in agreement with our expectation from simulation and measured branching ratios. We obtain the following $C P$ parameters from the fit for $B^{-} \rightarrow D^{0} K^{-}, r_{B}=0.117 \pm 0.053, \delta_{B}=(109 \pm 28)^{\circ}$, and $\gamma=(66 \pm 28)^{\circ}$ and for $B^{-} \rightarrow D^{* 0} K^{-}, r_{B}^{*}=0.167 \pm 0.065, \delta_{B}^{*}=(294 \pm 28)^{\circ}$, and $\gamma=(68 \pm 29)^{\circ}$. These errors are estimated with a Gaussian assumption for the likelihood. However, for this small sample, these low $r_{B}$ and $r_{B}^{*}$ fitted values lead to a non-Gaussian behavior of the likelihood function as shown in Fig. 5, necessitating a different approach, described next, in the computation of the confidence intervals for $r_{B}, \gamma$ and $\delta_{B}$. Fig. 6 shows for $r_{B}$ values generated in the $[0.0,0.3]$ range the $r_{B}$ values obtained in fits to Monte Carlo experiments of the same size as data. While the $C P$ fit is linear for large values of $r_{B}$, it is not sensitive to $r_{B}$ values below 0.1 . This problem did not exist for the larger $D^{*+} \rightarrow D^{0} \pi^{+}$and $B^{-} \rightarrow D^{(*) 0} \pi^{-}$samples as we have verified using Monte Carlo simulation.

In Fig. 7 and Fig. 8 we show the Dalitz distribution and the $m_{+}^{2}$ and $m_{-}^{2}$ projections for events with $m_{E S}>5.272 \mathrm{GeV} / c^{2}$ for $B^{-} \rightarrow D^{0} K^{-}$and $B^{-} \rightarrow D^{* 0} K^{-}$respectively. $B^{+}$and $B^{-}$ candidates distributions are separately shown with the total PDF superimposed.

### 5.4 Confidence intervals of the $C P$ parameters

We evaluate the likelihood function $\mathcal{L}\left(r_{B}, \gamma, \delta_{B}\right)$ after fixing all parameters except ( $r_{B}, \gamma, \delta_{B}$ ) which are varied in their range of definition, $[0,1],[-\pi, \pi]$, and $[0,2 \pi]$, respectively. We then estimate


Figure 5: $\ln \mathcal{L}$ contour plots in $\gamma$ versus $r_{B}$ in $B^{-} \rightarrow D^{0} K^{-}$(top) and $B^{-} \rightarrow D^{* 0} K^{-}$(bottom). $\delta_{B}$ is fixed to the value obtained from the fit. Each contour represent a $\ln \mathcal{L}$ variation of 0.5 .


Figure 6: $r_{B}$ values obtained in the likelihood fit versus generated $r_{B}$ values in Monte Carlo $B^{-} \rightarrow D^{0} K^{-}$experiments. The error bars represent the RMS of the $r_{B}$ values returned by the fit. The dashed horizontal line indicates the $r_{B}$ value (0.106) found in data.
the confidence region for the $C P$ parameters using a Bayesian technique. This implies a choice of a priori distribution. For this preliminary result we arbitrarily assume a uniform a priori distribution for each of the $C P$ parameters $r_{B}, \gamma$ and $\delta_{B}$.

In $\gamma-r_{B}$ space we define a two-dimensional confidence region $\mathcal{D}(\mathcal{C})$ corresponding to a given confidence level $\mathcal{C}$

$$
\frac{\int_{D(\mathcal{C})} d r_{B} d \gamma \int_{0}^{2 \pi} d \delta_{B} \mathcal{L}\left(r_{B}, \gamma, \delta_{B}\right)}{\int_{0}^{1} d r_{B} \int_{-\pi}^{\pi} d \gamma \int_{0}^{2 \pi} d \delta_{B} \mathcal{L}\left(r_{B}, \gamma, \delta_{B}\right)}=\mathcal{C}
$$

We uniquely define $D(\mathcal{C})$ by requiring that the likelihood value at any point on the boundary of $\mathcal{D}$ be the same and integrating over all likelihood values larger than the value at the boundary.

Similarly we define one dimensional confidence intervals $I(\mathcal{C})$ corresponding to a confidence level $\mathcal{C}$. For example, the interval for $r_{B}$ is defined as

$$
\frac{\int_{I(\mathcal{C})} d r_{B} \int_{-\pi}^{+\pi} d \gamma \int_{0}^{2 \pi} d \delta_{B} \mathcal{L}\left(r_{B}, \gamma, \delta_{B}\right)}{\int_{0}^{1} d r_{B} \int_{-\pi}^{\pi} d \gamma \int_{0}^{2 \pi} d \delta_{B} \mathcal{L}\left(r_{B}, \gamma, \delta_{B}\right) d r_{B}}=\mathcal{C}
$$

The two dimensional confidence regions $\mathcal{D}(\mathcal{C})$ in $\gamma$ versus $r_{B}$ and $\gamma$ versus $r_{B}^{*}$ are shown in Fig. 9. The red (dark) and yellow (light) regions correspond to the $68 \%$ and $95 \%$ confidence levels respectively. The likelihood distributions for $r_{B}, \gamma$, and $\delta_{B}$ obtained by integrating $\mathcal{L}\left(r_{B}, \gamma, \delta_{B}\right)$ over the other two variables are shown in Figs. 10, 11 and 12 respectively. The $68 \%$ and $95 \%$ confidence intervals are shown in red (dark) and yellow (light) respectively. The intervals for $\gamma$ and $\delta_{B}$ are disjoint as a consequence of the $\gamma \rightarrow \gamma \pm \pi$ and $\delta_{B} \rightarrow \delta_{B} \pm \pi$ ambiguities.

### 5.5 Constraints on $r_{B}, \delta_{B}$ and $\gamma$

From the procedure above we obtain Bayesian $68 \%$ confidence intervals. We quote as the central values for $\gamma$ and $\delta_{B}$ the average values weighted by the likelihood distribution. The errors associated


Figure 7: Dalitz distribution (first row) from the $B^{-} \rightarrow D^{0} K^{-}$events with $m_{E S}>5.272 \mathrm{GeV} / c^{2}$. Projections on $m_{+}^{2}$ (second row) and $m_{-}^{2}$ (third row) are shown with the result of the fit superimposed. In the left column $B^{+}$candidates are shown, in the right $B^{-}$candidates.


Figure 8: Dalitz distribution (first row) from the $B^{-} \rightarrow D^{* 0} K^{-}$events with $m_{E S}>5.272 \mathrm{GeV} / c^{2}$. Projections on $m_{+}^{2}$ (second row) and $m_{-}^{2}$ (third row) are shown with the result of the fit superimposed. In the left column $B^{+}$candidates are shown, in the right $B^{-}$candidates.


Figure 9: Bayesian confidence regions for $\gamma$ versus $r_{B}$ (top) and $r_{B}^{*}$ (bottom) for the $B^{-} \rightarrow D^{0} K^{-}$ and $B^{-} \rightarrow D^{* 0} K^{-}$samples respectively. The red (dark) region corresponds to the $68 \%$ confidence level region, the yellow (light) region to the $95 \%$ C.L. region


Figure 10: Probability density function for $r_{B}$ (left) and $r_{B}^{*}$ (right). The red (dark) region corresponds to the Bayesian $68 \%$ confidence level region, the yellow (light) region to the $95 \%$ C.L. region.


Figure 11: Probability density function for $\gamma$ in $B^{-} \rightarrow D^{0} K^{-}$(left) and $B^{-} \rightarrow D^{* 0} K^{-}$(right) sample . The red (dark) region corresponds to the Bayesian $68 \%$ confidence interval region, the yellow (light) region to the $95 \%$ C.L. region. The intervals for $\gamma$ are disjoint as a consequence of the $\gamma \rightarrow \gamma \pm \pi$ and $\delta_{B} \rightarrow \delta_{B} \pm \pi$ ambiguities.


Figure 12: Probability density function for $\delta_{B}$ in $B^{-} \rightarrow D^{0} K^{-}$(left) and $B^{-} \rightarrow D^{* 0} K^{-}$(right) sample. The red (dark) region corresponds to the Bayesian $68 \%$ confidence interval region, the yellow (light) region to the $95 \%$ C.L. region. The intervals for $\delta_{B}$ are disjoint as a consequence of the $\gamma \rightarrow \gamma \pm \pi$ and $\delta_{B} \rightarrow \delta_{B} \pm \pi$ ambiguities.
with the central values are defined by the boundaries of the $68 \%$ confidence interval. We obtain $r_{B}=0.087 \pm{ }_{-0.074}^{+0.041}, \delta_{B}=114^{\circ} \pm 41^{\circ}\left(294^{\circ} \pm 41^{\circ}\right), \gamma=70^{\circ} \pm 44^{\circ}\left(-110^{\circ} \pm 44^{\circ}\right)$ for $B^{-} \rightarrow D^{0} K^{-}$and $r_{B}^{*}=0.155_{-0.077}^{+0.070}, \delta_{B}^{*}=303^{\circ} \pm 34^{\circ}\left(123^{\circ} \pm 34^{\circ}\right), \gamma=73^{\circ} \pm 35^{\circ}\left(-107^{\circ} \pm 35^{\circ}\right)$ for $B^{-} \rightarrow D^{* 0} K^{-}$. We constrain $r_{B}$ to be $<0.16$ at $90 \%$ confidence level.

As illustrated in Figs. 5 and 13, the data have no sensitivity to $\gamma$ and $\delta_{B}$ for small values of $r_{B}$. Those values are not excluded by our data.

We construct a combined likelihood function from the product of the individual likelihoods for $B^{-} \rightarrow D^{0} K^{-}$and $B^{-} \rightarrow D^{* 0} K^{-}$and we repeat the procedure outlined above. From this we obtain $\gamma=70^{\circ} \pm 26^{\circ}\left(-110^{\circ} \pm 26^{\circ}\right)$. Fig. 14 shows the $\gamma$ likelihood distribution.


Figure 13: Statistical uncertainty in Gaussian hypothesis for $\gamma$ as a function of the value of $r_{B}$ obtained by the likelihood fit in Monte Carlo experiments.

## 6 SYSTEMATIC UNCERTAINTIES

The principal systematic uncertainty on the measurement of $\gamma$ comes from the choice of the model used to describe $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$decay. We evaluate this uncertainty by considering alternative models. For each model we generate events and fit both the alternative model and the nominal model (defined in Section 4) to these events. We quantify this uncertainty using the differences in the fitted values for $r_{B}, \delta_{B}$ and $\gamma$. For models where the $\rho(1450)$, the $K^{*}(1680)$ and/or the doubly Cabibbo suppressed $K_{0}^{*}(1430)$ and $K_{2}^{*}(1430)$ are removed or a different description of resonances is used, the $\chi^{2}$ of the fit is not significantly different from that of the nominal model. For these models the biases on $\gamma$ and $r_{B}$ are negligible and the RMS of the distribution of the differences is at most $1^{\circ}$ and 0.002 for $\gamma$ and $r_{B}$ respectively. As an extreme we consider a model without the $\sigma_{1}$ and/or $\sigma_{2}$ scalar, or the CLEO Model [12]. Fits to these models result in a significantly larger $\chi^{2}$ than that of the nominal model. To illustrate the magnitude of this variation Fig. 15 shows the result of a fit with the CLEO model. For these extreme models the biases for $C P$ parameters are still small. The RMS of the differences for $\gamma$ and $r_{B}$ are approximately $10^{\circ}$ and 0.02 respectively. We conservatively assign $\sigma_{\gamma}, \sigma_{\delta_{B}}=10^{\circ}$ and $\sigma_{r_{B}}=0.02$ as systematic uncertainties associated with


Figure 14: Probability density function for $\gamma$ from the combined samples.The red (dark) region corresponds to the Bayesian $68 \%$ confidence interval region, the yellow (light) region to the $95 \%$ C.L. region.
the Dalitz model.
The summary of the estimates of other systematic uncertainties is given in Table 4. The most important effect is due to the uncertainties on the knowledge of the Dalitz distribution of background events and of the $m_{E S}, \Delta E$, and $\mathcal{F}$ PDF parameters for both background and signal. Uncertainties in the efficiency variation across the Dalitz distribution are estimated. The statistical uncertainty on the amplitude and phases of the nominal Dalitz model also contributes significantly to the systematic uncertainties on the $C P$ parameters.

|  | $B^{-} \rightarrow D^{0} K^{-}$ |  |  | $B^{-} \rightarrow D^{* 0} K^{-}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | $r_{B}$ | $\gamma$ | $\delta_{B}$ | $r_{B}^{*}$ | $\gamma$ | $\delta_{B}^{*}$ |
| Combinatorial background Dalitz shape | 0.008 | $6.7^{\circ}$ | $3.3^{\circ}$ | 0.010 | $2.9^{\circ}$ | $5.1^{\circ}$ |
| $m_{E S}, \Delta E, \mathcal{F}$ PDF shapes | 0.007 | $5.4^{\circ}$ | $4.2^{\circ}$ | 0.025 | $1.8^{\circ}$ | $8.2^{\circ}$ |
| $R$ | 0.018 | $3.1^{\circ}$ | $3.0^{\circ}$ | 0.018 | $3.1^{\circ}$ | $3.0^{\circ}$ |
| Efficiency | 0.004 | $3.0^{\circ}$ | $2.7^{\circ}$ | 0.005 | $3.0^{\circ}$ | $2.8^{\circ}$ |
| Dalitz amplitude and phase uncertainties | 0.004 | $1.6^{\circ}$ | $4.7^{\circ}$ | 0.014 | $6.1^{\circ}$ | $8.8^{\circ}$ |
| Total | 0.022 | $9.8^{\circ}$ | $8.3^{\circ}$ | 0.036 | $8.2^{\circ}$ | $13.8^{\circ}$ |

Table 4: Summary of the contributions to the systematic errors on $r_{B}, \gamma$ and $\delta_{B}$.

## 7 RESULTS AND SUMMARY

We report preliminary results of the measurement of $r_{B}$ and of the angle $\gamma$ using the $B^{-}$meson decays into $D^{0} K^{-}$and $D^{* 0} K^{-}$with a technique based on the Dalitz analysis of the $D^{0} \rightarrow K_{S} \pi^{-} \pi^{+}$ three-body decay. From 227 million $B \bar{B}$ pairs collected by the $B A B A R$ detector, we reconstruct


Figure 15: CLEO model fit: a) the $\bar{D}^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$Dalitz distribution from the $D^{*-} \rightarrow \bar{D}^{0} \pi^{-}$. Projection on (b) $m_{+}^{2}$, (c) $m_{-}^{2}$ and (d) $m^{2}\left(\pi^{+} \pi^{-}\right)$are shown. The result of the fit is superimposed and it is used to estimate the uncertainty due to the Dalitz model.
$282 \pm 20 B^{-} \rightarrow D^{0} K^{-}, 89 \pm 11 B^{-} \rightarrow D^{* 0} K^{-}, D^{* 0} \rightarrow D^{0} \pi^{0}$ and $44 \pm 8 B^{-} \rightarrow D^{* 0} K^{-}, D^{* 0} \rightarrow D^{0} \gamma$ signal events.

Values of the ratio of $b \rightarrow u$ and $b \rightarrow c$ amplitudes for the processes $B^{-} \rightarrow D^{0} K^{-}$and $B^{-} \rightarrow$ $D^{* 0} K^{-}$at the small end of our measurements allow no determination of $\gamma$ at this statistical level. Accounting for systematic uncertainties, we constrain these ratios to be $r_{B}<0.19$ at $90 \%$ confidence level and $r_{B}^{*}=0.155_{-0.077}^{+0.070} \pm 0.040 \pm 0.020$. The relative phases between these two amplitudes are $\delta_{B}=114^{\circ} \pm 41^{\circ} \pm 8^{\circ} \pm 10^{\circ}\left(294^{\circ} \pm 41^{\circ} \pm 8^{\circ} \pm 10^{\circ}\right)$ and $\delta_{B}^{*}=303^{\circ} \pm 34^{\circ} \pm 14^{\circ} \pm 10^{\circ}\left(123^{\circ} \pm 34^{\circ} \pm 14^{\circ} \pm 10^{\circ}\right)$. The first error is statistical, the second error accounts for experimental uncertainties and the third error reflects the Dalitz model uncertainty. By combining the information from the two samples we obtain $\gamma=70^{\circ} \pm 26^{\circ} \pm 10^{\circ} \pm 10^{\circ}\left(-110^{\circ} \pm 26^{\circ} \pm 10^{\circ} \pm 10^{\circ}\right)$. For this preliminary result we have quoted confidence intervals obtained with a Bayesian technique assuming a uniform prior in $r_{B}, \gamma$ and $\delta_{B}$.

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