Measurements related to CKM angle α in BaBar.

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The BaBar collaboration measurements of the $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ decays are presented. New results, from a 113 fb^{-1} data sample, on the time-dependent CP asymmetries of the longitudinally polarized component of the $B^0 \to \rho^+\rho^-$ channel are $S_{\rho\rho,long} = -0.19 \pm 0.33 \pm 0.11$ and $C_{\rho\rho,long} = -0.23 \pm 0.24 \pm 0.14$. Constraints on the Unitarity Triangle angle α from the $\pi\pi$ and the $\rho\rho$ systems are derived.

The BaBar and Belle experiments have reported in 2001 the first observation of CP violation in the B meson system [1,2]. By measuring the value of the CP parameter $\sin 2\beta$, they have provided the first direct contraint on one of the Unitarity Triangle (UT) angles. In order to check the consistency of the CP violation description in the Standard Model, it is of main importance to measure the other angles of the UT. In this paper, we describe the measurements by the BaBar experiment of the time-dependent CP-violating asymmetries in three decay modes, $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$, related to the CKM angle $\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$.

1 Extraction of α from the decays $B \to hh'$ $(h, h' = \pi, \rho)$

1.1 Basic Formulae

The decay of a neutral B meson into two identical particles $B \to hh$ ($h = \pi$ or ρ[a] occurs via two topologies, illustrated in Fig. 1: a tree-level process (left) and one-loop penguin diagrams (right). The CP parameter λ, defined by $\lambda = \frac{q}{p}\frac{\bar{A}}{A}$, where q and p are the complex coefficients

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[a]Due to the finite width of the ρ, the two mesons in the $B \to \rho\rho$ in the final state are not necessarily identical. See the comment on this approximation in Section 3.

that link the mass and the flavour eigenstates in the B system, and A (resp. $\bar{A}$) is the B^0 (resp. $\bar{B}^0$) decay amplitude, can be expressed in terms of α as

$$\lambda = e^{2i\alpha} \frac{1 - \frac{|V_{td}^* V_{tb}|}{|V_{ud}^* V_{ub}|} P/T e^{-i\alpha}}{1 - \frac{|V_{td}^* V_{tb}|}{|V_{ud}^* V_{ub}|} P/T e^{i\alpha}} . \tag{1}$$

T and P are complex amplitudes dominated respectively by the tree and the penguin topologies.
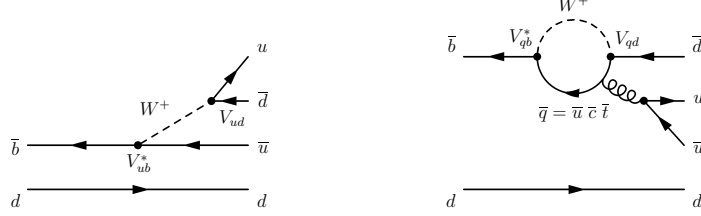

Figure 1: *Tree (left) and penguin (right) diagrams for the decays* $B^0 \to \pi^{\pm}\pi^{\mp}$, $B^0 \to \rho^{\pm}\pi^{\mp}$ *and* $B^0 \to \rho^{\pm}\rho^{\mp}$.

Experimentally, one measures the time-dependent decay rate

$$f_{Q_{tag}}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \left[1 + Q_{tag} S_{hh} \sin(\Delta m_d \Delta t) - Q_{tag} C_{hh} \cos(\Delta m_d \Delta t)\right], \tag{2}$$

where Δt is the decay time difference between the B decaying to the hh final state and the second B in the event, denoted B_{tag}. τ is the neutral B lifetime and Δm_d is the $B^0 \bar{B}^0$ oscillation frequency. Q_{tag} is set 1 (-1) if the B_{tag} is a B^0 ($\bar{B}^0$). The CP-violating asymmetries S_{hh} and C_{hh} are related to the parameter λ by

$$S_{hh} = \frac{2\Im m\lambda}{1 + |\lambda|^2}, \qquad C_{hh} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}. \tag{3}$$

S_{hh} reflects the CP violation induced by the interference between the mixing and the decay; C_{hh} is the direct CP-violating asymmetry and comes from the interference between decay processes. In absence of penguin contributions, C_{hh} vanishes and S_{hh} is simply related to the CKM angle α by $S_{hh} = \sin(2\alpha)$.

In the more general case of the $B^0(\bar{B}^0) \to \rho^{\pm}\pi^{\mp}$ decay, the time-dependent decay rate reads

$$f_{Q_{tag}}^{\rho^{\pm}\pi^{\mp}}(\Delta t) = (1 \pm A_{\rho\pi}) \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \;\; + \;\; Q_{tag}(S_{\rho\pi} \pm \Delta S_{\rho\pi}) \sin(\Delta m_d \Delta t)$$
$$- \;\; Q_{tag}(C_{\rho\pi} \pm \Delta C_{\rho\pi}) \cos(\Delta m_d \Delta t)], \tag{4}$$

where the $\pm$ sign depends on whether the ρ meson is emitted by the W boson or comes from the spectator quark. $A_{\rho\pi}$ is a direct CP violation parameter, measuring the asymmetry between the $\rho^+\pi^-$ and $\rho^-\pi^+$ final states, whereas $\Delta S_{\rho\pi}$ and $\Delta C_{\rho\pi}$, which arise from the fact that two production modes of the ρ are possible, are dilution terms and have no CP content.

1.2   The Isospin Analysis

Using strong isospin symmetry, one can extract α up to discrete ambiguities from the CP-violating asymmetries defined above[3]. The decay amplitudes of the isospin-related final states obey the pentagonal relations

$$\sqrt{2}\left(A_{\rho\pi}^{+0} + A_{\rho\pi}^{0+}\right) = 2A_{\rho\pi}^{00} + A_{\rho\pi}^{+-} + A_{\rho\pi}^{-+} , \tag{5}$$

$$\sqrt{2}\left(\overline{A}_{\rho\pi}^{+0} + \overline{A}_{\rho\pi}^{0+}\right) = 2\overline{A}_{\rho\pi}^{00} + \overline{A}_{\rho\pi}^{+-} + \overline{A}_{\rho\pi}^{-+} . \tag{6}$$

where $A_{\rho\pi}^{ij} = A(B^0 \text{ or } B^+ \to \rho^i\pi^j)$, $\overline{A}_{\rho\pi}^{ij} = A(\overline{B}^0 \text{ or } B^- \to \rho^i\pi^j)$, $i,j = +,-$ or 0. With the use of these relations, 12 unknowns (6 complex amplitudes with one arbitrary phase, and the CKM angle α) are to be determined while 13 observables are available: $S_{\rho\pi}$, $C_{\rho\pi}$, $\Delta S_{\rho\pi}$, $\Delta C_{\rho\pi}$, $A_{\rho\pi}$; the average branching fractions $\mathcal{B}(B^0/\overline{B}^0 \to \rho^\pm\pi^\mp)$, $\mathcal{B}(B^0/\overline{B}^0 \to \rho^0\pi^0)$, $\mathcal{B}(B^+ \to \rho^+\pi^0)$, $\mathcal{B}(B^+ \to \rho^0\pi^+)$; two time-dependent CP-violating asymmetries in the $B^0 \to \rho^0\pi^0$ decay ($S_{\rho\pi}^{00}$, $C_{\rho\pi}^{00}$) and two direct CP asymmetries in $B^+ \to \rho^+\pi^0$ and $B^+ \to \rho^0\pi^+$.

In the case of two identical mesons in the final state, Eqs. (5,6) simplify to two triangular relations

$$A_{hh}^{+0} = \frac{1}{\sqrt{2}}A_{hh}^{+-} + A_{hh}^{00} , \tag{7}$$

$$\overline{A}_{hh}^{+0} = \frac{1}{\sqrt{2}}\overline{A}_{hh}^{+-} + \overline{A}_{hh}^{00} . \tag{8}$$

The information counting leads then to 6 unknowns and 7 observables: three branching fractions $\mathcal{B}(B^0 \to h^+h^-)$, $\mathcal{B}(B^+ \to h^+h^0)$, $\mathcal{B}(B^0 \to h^0h^0)$; S_{hh}, C_{hh}, S_{hh}^{00}, C_{hh}^{00}. In the $\pi\pi$ system, S_{hh}^{00} is hard or impossible to measure and one is left with 6 observables: α can be extracted with an 8-fold ambiguity within $[0,\pi]$ [4].

At present, S_{hh}^{00} and C_{hh}^{00} have not been measured, neither in the $\pi\pi$ nor in the $\rho\rho$ system. Therefore, one cannot measure α but rather set a bound [5]

$$\cos\left(2\alpha - 2\alpha_{\text{eff}}\right) \geq \frac{1}{D}\left(1 - 2\frac{\mathcal{B}^{00}}{\mathcal{B}^{+0}}\right) + \frac{1}{D}\frac{\left(\mathcal{B}^{+-} - 2\mathcal{B}^{+0} + 2\mathcal{B}^{00}\right)^2}{4\mathcal{B}^{+-}\mathcal{B}^{+0}} , \tag{9}$$

where the effective angle α_{eff} is defined by $\alpha_{\text{eff}} \equiv \arg(\lambda)$ and $D \equiv \sqrt{1 - C_{hh}^2}$. Note that Eq. (9) fully exploits the isospin relations while the well-known Grossman-Quinn bound [6] is recovered by neglecting the second term on the right-hand side of Eq. (9) and setting D to 1.

2 Data Analysis

2.1 Data Selection

Results on $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ decays are presented. Signal events are selected by combining the relevant number of charged tracks and/or neutral clusters to form a B candidate. Other particles in the event form the B_{tag}. A vertexing algorithm [7] is used to determine the decay time difference Δt between the two $B's$ from their distance along the z axis (Δz). The typical resolution on Δz is 180 μm. The tagging procedure, based on a multivariate technique [8], is applied on the B_{tag} to determine the flavour of the B at $\Delta t = 0$. The total effective tagging efficiency is $(28.4 \pm 0.7)\%$. The data selection relies on several common aspects, which are summarized below.

Useful variables to discriminate signal from background are primarily: the beam-energy-substituted mass $m_{ES} = \sqrt{(s/2 + \vec{p}_i \cdot \vec{p}_B)^2/E_i^2 - \vec{p}_B^2}$, where $\sqrt{s}$ is the total energy in the e^+e^- center of mass (CM), $(E_i, \vec{p}_i)$ is the four-momentum of the initial state and $\vec{p}_B$ the momentum of the B, both measured in the laboratory frame; the energy difference, ΔE, between the CM energy of the B and $\sqrt{s}/2$.

The topogical properties of $B\bar{B}$ decays in the $\Upsilon(4S)$ rest frame are used to discriminate the signal from the $B \to q\bar{q}$ ($q = u, d, s, c$) background. All analyses use the L_0 and L_2 moments, defined in the $\Upsilon(4S)$ rest frame as

$$L_0 \equiv \sum_{i \notin B} p_i^* \ , \ L_2 \equiv \sum_{i \notin B} p_i^* \cos^2 \theta_i \tag{10}$$

where p_i^* is the momentum of particle i not included in the B candidate, and θ_i is the angle between p_i^* and the thrust of the B candidate. L_0 and L_2 are combined with a Fisher algorithm. In order to increase the discrimination, they can be further combined with variables such as the cosine of the angle between the beam axis and the B candidate momentum or the B thrust axis.

All channels but $B^0 \to \pi^+\pi^-$ decays suffer from B background. In addition to ΔE, other discriminating variables are the ρ candidate mass ($0.4 < m(\pi^+\pi^0) < 1.3$ GeV/c^2 or $0.53 < m(\pi^+\pi^0) < 0.9$ GeV/c^2) and the helicity angle ($|cos\theta_\rho| > 0.25$, where θ_ρ is the angle of one daughter pion mementum and the B momentum in the ρ rest frame).

Particle identification mainly relies on the DIRC[9], the Cherenkov detector, which provides a kaon-pion separation greater than 2.1σ over a $[1.7 - 4.2]$ GeV/c momentum range.

An unbinned likelihood fit is finally performed on selected events: for each event, a probability density function is built from discriminating variables, including the Δt-dependence, either in its simple exponential form for charged $B's$ or following Eqs. (2) and (4).

2.2 Results

Branching fraction and time-dependent CP asymmetries of the $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ decays are summarized in Table 1.

The branching fractions of the three isospin partner decays in the $\pi\pi$ system are measured[10,11,12]. The statistical significance of the recently observed $B^0 \to \pi^0\pi^0$ mode is 4.2σ [12]. No evidence for CP violation is observed in the $B^0 \to \pi^+\pi^-$ channel[13].

The quasi-two-body analysis parameters of the $B^0 \to \rho^\pm\pi^\mp$ decay are also reported[14]. Direct CP violation information carried by the $C_{\rho\pi}$ and $A_{\rho\pi}$ parameters has a 2.5σ significance, which is likely to be a statistical fluctuation since the well measured branching fraction of the $B^0 \to \rho^+K^-$ decay is rather small ($\mathcal{B}(B^0 \to \rho^+K^-) = (9.0 \pm 1.6) \ 10^{-6}$)[14].

The $B^0 \to \rho^+\rho^-$ and $B^+ \to \rho^+\rho^0$ modes are observed[15,16], while only an upper limit is set on the branching fraction of the $B^0 \to \rho^0\rho^0$ channel[16]. Dominance of the longitudinally polarized component in the first two decays is observed. Recently, the BABAR collaboration has reported on the measurement of the CP violating asymmetries in the $B^0 \to \rho^+\rho^-$ longitudinal component decay on 81 fb^{-1} [15,17]. The measurement has been updated on a 113 fb^{-1} sample and found in agreement with the previous result. A detailed analysis of the background due to other B decays is performed. The main systematic uncertainty on the asymmetries $S_{\rho\rho,long}$ and $C_{\rho\rho,long}$ is found to be the unknown CP violation in B background events.

3 Constraints on α

At present, SU(2)-based analysis of the $B \to \rho\pi$ system does not lead to useful constraint on α, since the construction of the pentagons described by Eqs. (5) requires more precise measurements than currently available. Data sample with a luminosity of the order of 10 fb^{-1} is needed. More promising is a Dalitz plot analysis that would bring informations on the strong phases involved in the $B^0 \to \rho^\pm\pi^\mp$ decay. If the validity of QCD Factorization was established, such a model could also help to constraint α, with an accuracy of $9°$ with current data[18].

The confidence level as a function of α obtained from the isospin analysis of the $B \to \pi\pi$ decays is shown on Fig. 2 (light shaded histogram) [19]. The CP asymmetries $C_{\pi\pi}$ and $S_{\pi\pi}$

$\mathcal{B}(B^0 \to \pi^+\pi^-)(^*)$	$\mathcal{B}(B^+ \to \pi^+\pi^0)(^*)$	$\mathcal{B}(B^0 \to \pi^0\pi^0)$
$4.7 \pm 0.6 \pm 0.2$	$5.5^{+1.0}_{-0.9}\pm0.6$	$2.1 \pm 0.6 \pm 0.3$
$C_{\pi\pi}$	$S_{\pi\pi}$	Correlation coeff.
$-0.19 \pm 0.19 \pm 0.05$	$-0.40 \pm 0.22 \pm 0.03$	-0.02
$C_{\rho\pi}$	$S_{\rho\pi}$	$A_{\rho\pi}$
$0.35 \pm 0.13 \pm 0.05$	$-0.13 \pm 0.18 \pm 0.04$	$-0.114 \pm 0.062 \pm 0.027$

$\Delta C_{\rho\pi}$	$\Delta S_{\rho\pi}$
$0.20 \pm 0.13 \pm 0.05$	$0.33 \pm 0.18 \pm 0.03$

$C_{\rho\rho,long}$	$S_{\rho\rho,long}$	$\mathcal{B}(B^0 \to \rho^+\rho^-)\,(^*)$	$f_L(B^0 \to \rho^+\rho^-)\,(^*)$
$-0.23 \pm 0.24 \pm 0.14$	$-0.19 \pm 0.33 \pm 0.11$	$30 \pm 4 \pm 5$	$0.99 \pm 0.03^{+0.04}_{-0.03}$

$\mathcal{B}(B^+ \to \rho^+\rho^0)\,(^*)$	$f_L(B^+ \to \rho^+\rho^0)\,(^*)$	$\mathcal{B}(B^0 \to \rho^0\rho^0)\,(^*)$
$22.5^{+5.74}_{-5.4} \pm 5.8$	$0.97^{+0.03}_{-0.07} \pm 0.04$	< 2.1 (90% CL)

Table 1: *Branching fractions and time-dependent CP asymmetries in $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ decays. Measurements are performed on samples of 81 fb^{-1} (marked with $(^*)$) or 113 fb^{-1}. The first error is statistical and the second is systematic. Branching fractions are given in 10^{-6} units.*

quoted in Table 1 are used, together with the world average values of the branching fractions of the $B^0 \to \pi^+\pi^-$, $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$ channels [21]. The plateau reflects the unfruitful bound on $\alpha - \alpha_{\text{eff}}$: $-54° < \alpha - \alpha_{\text{eff}} < 52°$ (90% CL), largely dominated by the uncertainty on the penguin contribution.[b]

Similarly, one can apply the isospin analysis to the longitudinal components of the $B \to \rho\rho$ decays. BABAR measurements of the time-dependent asymmetries, branching fraction and polarization fraction in $B^0 \to \rho^+\rho^-$ mode (Tab. 1) are used, as well as the BABAR and Belle average branching fraction [21] and polarization fraction [16,22] for the $B^+ \to \rho^+\rho^0$ channel. The analysis includes the value leading to the upper limit on $\mathcal{B}(B^0 \to \rho^0\rho^0)$ quoted in Tab. 1, $\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.6^{+0.7}_{-0.6} \pm 0.1)\ 10^{-6}$, and it is assumed conservatively that the decay is 100% longitudinally polarized. Interference with higher radial excitations of the ρ meson, non-resonant contributions or possible isospin violating effects due to the finite width of the ρ are neglected [23]. Powerful constraint on α is obtained (Fig. 2, dark shaded histogram), in agreement with and with comparable accuracy to the standard CKM fit, which includes the constraints from CP violation measurements in neutral kaon mixing, $|V_{ub}|$, $|V_{cb}|$, $B^0\bar{B}^0$ and $B^0_s\bar{B}^0_s$ oscillations, and $\sin 2\beta$ (hatched area) [19]. Choosing the solution closest to the standard fit constraint, one estimates $\alpha = (96 \pm 10_{stat} \pm 4_{syst} \pm 13_{peng})°$. Note that the peak-like shape of the CL function, in contrast with the plateau expected from Eq. (9), is due central values violating the isospin relations (7). However, this "incompatibility" is well covered by the present experimental uncertainties.

4   Conclusion

The BABAR collaboration has published evidence or observation of the three decay modes of the $B \to \pi\pi$ system. The measurements of the time-dependent CP asymmetries in $B^0 \to \pi^+\pi^-$ channel do not lead to a useful constraint on α, due to the present uncertainty on the

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[b]A strict application of Eq. (9) gives a symmetric bound on $\alpha - \alpha_{\text{eff}}$. However, in the study of Ref. [19], electroweak penguins are taken into account, following the recipe proposed by Neubert and Rosner [20] (so that no additional degrees of freedom is introduced), leading to the asymmetric bound given above.
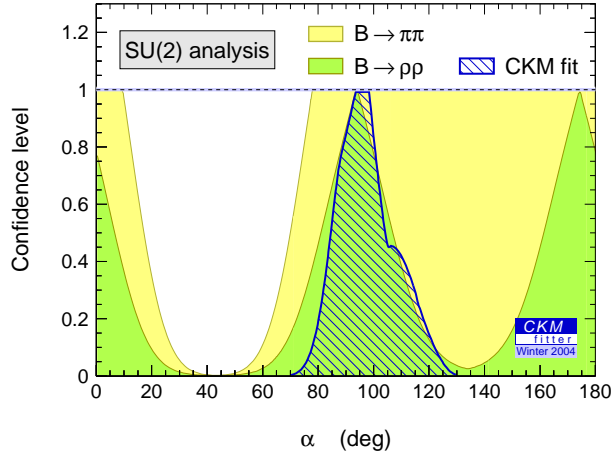
Figure 2: *Confidence level from the SU(2) analysis of the $B \rightarrow \pi\pi$ (light shaded) and $B \rightarrow \rho\rho$ (dark shaded) decays as a function of α. Also shown is the result from the standard CKM fit (hatched area, see text).*

penguin contribution. The quasi-two-body CP asymmetries in the $B \rightarrow \rho\pi$ decay have been measured. Next step is to perform the Dalitz plot analysis in order to constrain the strong phases involved in the decay and that are needed to extract α. In contrast with the $\pi\pi$ and $\rho\pi$ systems, a powerful constraint on α in obtained from the measurements of the time-dependent asymmetries of the longitudinally polarized component of the $B^0 \rightarrow \rho^+\rho^-$ channel. Performing the isospin analysis and choosing the solution closest to the standard CKM fit, BABAR quotes $\alpha = (96 \pm 10_{stat} \pm 4_{syst} \pm 13_{peng})°$.

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