

LIGHT-FRONT QUANTIZATION AND QCD PHENOMENA*

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Abstract

The light-front quantization of QCD provides an alternative to lattice gauge theory for computing the mass spectrum, scattering amplitudes, and other physical properties of hadrons directly in Minkowski space. Nonperturbative light-front methods for solving gauge theory and obtaining light-front wavefunctions, such as discretized light-front quantization, the transverse lattice, and light-front resolvents are reviewed. The resulting light-front wavefunctions give a frame-independent interpolation between hadrons and their quark and gluon degrees of freedom, including an exact representation of spacelike form factors, transition form factors such as $B \rightarrow \ell \bar{\nu} \pi$, and generalized parton distributions. In the case of hard inclusive reactions, the effects of final-state interactions must be included in order to interpret leading-twist diffractive contributions, nuclear shadowing, and single-spin asymmetries. I also discuss how the AdS/CFT correspondence between string theory and conformal gauge theory can be used to constrain the form and power-law fall-off of the light-front wavefunctions. In the case of electroweak theory, light-front quantization leads to a unitary and renormalizable theory of massive gauge particles, automatically incorporating the Lorentz and 't Hooft conditions as well as the Goldstone boson equivalence theorem. Spontaneous symmetry breaking is represented by the appearance of zero modes of the Higgs field, leaving the light-front vacuum equal to the perturbative vacuum.

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1 Introduction

In Dirac's "Front Form" [1], boundary conditions are specified at a given light-front time $x^+ \equiv t+z/c$; the value of x^+ is unchanged as a light front crosses a system. Thus, unlike ordinary time t , a moment of light-front time $x^+ = \tau$ "stands still forever" [2]. The generator of light-front time translations is $P^- = i\frac{\partial}{\partial\tau}$. Given the Lagrangian of a quantum field theory, P^- can be constructed as an operator on the Fock basis, the eigenstates of the free theory. In the case of QCD, light-front quantization provides an alternative to lattice gauge theory for computing the mass spectrum, scattering amplitudes, and other physical properties of hadrons directly in Minkowski space.

A remarkable advantage of light-front quantization is that the vacuum state $|0\rangle$ of the full QCD Hamiltonian coincides with the free vacuum. The light-front Fock space is a Hilbert space of non-interacting quarks and gluons, each of which satisfy $k^2 = m^2$ and $k^- = (m^2 + k_\perp^2)/k^+ \geq 0$. Note that all particles in the Hilbert space have positive energy $k^0 = \frac{1}{2}(k^+ + k^-)$, and thus positive k^\pm . Since the plus momenta $\sum k_i^+$ is conserved by the interactions, the perturbative vacuum can only couple to states with particles in which all $k_i^+ = 0$; *i.e.*, zero-mode states. Bassetto and collaborators [3] have shown that the computation of the spectrum of $QCD(1+1)$ in equal-time quantization requires the construction of the full spectrum of non perturbative contributions (instantons). In contrast, in the light-front quantization of gauge theory (where the $k^+ = 0$ singularity of the instantaneous interaction is defined by a simple infrared regularization), one obtains the correct spectrum of $QCD(1+1)$ without any need for vacuum-related contributions.

Light-front quantization can also be used to obtain a frame-independent formulation of thermodynamics systems, such as the light-front partition function [4, 5, 6, 7, 8, 9, 10]. This application is particularly useful for relativistic systems, such as the hadronic system produced in the central rapidity region of high energy heavy-ion collisions.

The light-front quantization of gauge theory [11, 12, 13] is usually carried out in the light-cone gauge $A^+ = A^0 + A^z = 0$. In this gauge the A^- field becomes a dependent degree of freedom, and it can be eliminated from the Hamiltonian in favor of a set of specific instantaneous light-front time interactions. In fact in $QCD(1+1)$ theory, the instantaneous interaction provides the confining linear x^- interaction between quarks. In $3+1$ dimensions, the transverse field A^\perp propagates massless spin-one gluon quanta with polarization vectors [14] which satisfy both the gauge condition $\epsilon_\lambda^+ = 0$ and the Lorentz condition $k \cdot \epsilon = 0$. The interaction Hamiltonian of QCD in light-cone gauge can be derived by systematically applying the Dirac bracket method to identify the independent fields [12, 15]. It contains the usual Dirac interactions between the quarks and gluons, the three-point and four-point gluon non-Abelian interactions, plus instantaneous gluon exchange and quark exchange contributions. The renormalization constants in the non-Abelian theory have been shown [15] to satisfy the identity $Z_1 = Z_3$ at one-loop order and are independent of the reference direction

n^μ . The QCD β function has also been computed at one loop [15]. Dimensional regularization and the Mandelstam-Leibbrandt prescription [16, 17, 18] for LC gauge can be used to define the Feynman loop integrations [19]. The M-L prescription has the advantage of preserving causality and analyticity, as well as leading to proofs of the renormalizability and unitarity of Yang-Mills theories [20]. The ghosts which appear in association with the M-L prescription from the single poles have vanishing residue in absorptive parts, and thus do not disturb the unitarity of the theory. It is also possible to quantize QCD using light-front methods in covariant Feynman gauge [21].

The Heisenberg equation on the light-front is

$$H_{LC}|\Psi\rangle = M^2|\Psi\rangle . \quad (1)$$

The operator $H_{LC} = P^+P^- - P_\perp^2$, the “light-cone Hamiltonian”, is frame-independent. The Heisenberg equation can in principle be solved by diagonalizing the matrix $\langle n|H_{LC}|m\rangle$ on the free Fock basis: [13]

$$\sum_m \langle n|H_{LC}|m\rangle \langle m|\psi\rangle = M^2 \langle n|\Psi\rangle . \quad (2)$$

The eigenvalues $\{M^2\}$ of $H_{LC} = H_{LC}^0 + V_{LC}$ give the squared invariant masses of the bound and continuum spectrum of the theory. The projections $\{\langle n|\Psi\rangle\}$ of the eigensolution on the n -particle Fock states are the light-front wavefunctions. Thus finding the hadron eigenstates of QCD is equivalent to solving a coupled many-body quantum mechanical problem:

$$\left[M^2 - \sum_{i=1}^n \frac{m^2 + k_\perp^2}{x_i} \right] \psi_n = \sum_{n'} \int \langle n|V_{LC}|n'\rangle \psi_{n'} \quad (3)$$

where the convolution and sum is over the Fock number, transverse momenta, plus momenta, and spin projections of the intermediate states. The eigenvalues M are the invariant masses of the complete set of bound state and continuum solutions.

If one imposes periodic boundary conditions in $x^- = t - z/c$, then the plus momenta become discrete: $k_i^+ = \frac{2\pi}{L}n_i$, $P^+ = \frac{2\pi}{L}K$, where $\sum_i n_i = K$ [22, 23]. For a given “harmonic resolution” K , there are only a finite number of ways positive integers n_i can sum to a positive integer K . Thus at a given K , the dimension of the resulting light-front Fock state representation of the bound state is rendered finite without violating boost invariance. The eigensolutions of a quantum field theory, both the bound states and continuum solutions, can then be found by numerically diagonalizing a frame-independent light-front Hamiltonian H_{LC} on a finite and discrete momentum-space Fock basis. Solving a quantum field theory at fixed light-front time can thus be formulated as a relativistic extension of Heisenberg’s matrix mechanics. The continuum limit is reached for $K \rightarrow \infty$. This formulation of the non-perturbative light-front quantization problem is called “discretized light-cone quantization” (DLCQ) [23]. The method preserves the frame-independence of the Front form.

The DLCQ method has been used extensively for solving one-space and one-time theories [13], including applications to supersymmetric quantum field theories [24] and specific tests of the Maldacena conjecture [25]. There has been progress in systematically developing the computation and renormalization methods needed to make DLCQ viable for QCD in physical spacetime. For example, John Hiller, Gary McCartor, and I [26, 27, 28] have shown how DLCQ can be used to solve 3+1 theories despite the large numbers of degrees of freedom needed to enumerate the Fock basis. A key feature of our work is the introduction of Pauli Villars fields to regulate the UV divergences and perform renormalization while preserving the frame-independence of the theory. A recent application of DLCQ to a 3+1 quantum field theory with Yukawa interactions is given in Ref. [26] One can also define a truncated theory by eliminating the higher Fock states in favor of an effective potential [29, 30, 31]. As discussed below, spontaneous symmetry breaking and other nonperturbative effects associated with the instant-time vacuum are associated with zero mode degrees of freedom in the light-front formalism [32, 33].

Another important nonperturbative light-front method is the transverse lattice [34, 35, 36, 37] which utilizes DLCQ for the x^- and x^+ light-front coordinates together with a spatial lattice in the two transverse dimensions. A finite lattice spacing a can be implemented by choosing the parameters of the effective theory in a region of renormalization group stability to respect the required gauge, Poincaré, chiral, and continuum symmetries. For example, Dalley has recently computed the impact parameter dependent quark distribution of the pion [38].

The Dyson-Schwinger method [39] can also be used to predict light-front wavefunctions and hadron distribution amplitudes by integrating over the relative k^- momentum of the Bethe-Salpeter wavefunctions to project dynamics at $x^+ = 0$. Explicit nonperturbative light-front wavefunctions have been found in this way for the Wick-Cutkosky model, including states with non-zero angular momentum [40]. One can also implement variational methods, using the structure of perturbative solutions as a template for the numerator of the light-front wavefunctions. I will discuss the use of another light-front nonperturbative method, the light-front resolvent, below.

Light-front wavefunctions are the interpolating functions between hadrons and their quark and gluon degrees of freedom in QCD [41]. For example, the eigensolution of a meson, projected on the eigenstates $\{|n\rangle\}$ of the free Hamiltonian $H_{LC}^{QCD}(g=0)$ at fixed light-front time $\tau = t + z/c$ with the same global quantum numbers, has the expansion:

$$\begin{aligned}
|\Psi_M; P^+, \vec{P}_\perp, \lambda\rangle &= \sum_{n \geq 2, \lambda_i} \int \prod_{i=1}^n \frac{d^2 k_{\perp i} dx_i}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_j^n x_j\right) \delta^{(2)}\left(\sum_\ell^n \vec{k}_{\perp \ell}\right) \\
&\times |n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle \psi_{n/M}(x_i, \vec{k}_{\perp i}, \lambda_i).
\end{aligned} \tag{4}$$

The set of light-front Fock state wavefunctions $\{\psi_{n/M}\}$ represents the ensemble of quark and gluon states possible when the meson is intercepted at the light-front. The

light-front momentum fractions $x_i = k_i^+ / P_\pi^+ = (k^0 + k_i^z) / (P^0 + P^z)$ with $\sum_{i=1}^n x_i = 1$ and $\vec{k}_{\perp i}$ with $\sum_{i=1}^n \vec{k}_{\perp i} = \vec{0}_\perp$ represent the relative momentum coordinates of the QCD constituents; the scalar light-front wavefunctions $\psi_{n/M}(x_i, \vec{k}_{\perp i}, \lambda_i)$ are independent of the hadron's momentum $P^+ = P^0 + P^z$, and P_\perp , reflecting the kinematical boost invariance of the front form. The physical transverse momenta are $\vec{p}_{\perp i} = x_i \vec{P}_\perp + \vec{k}_{\perp i}$. The λ_i label the light-front spin S^z projections of the quarks and gluons along the quantization z direction. The physical gluon polarization vectors $\epsilon^\mu(k, \lambda = \pm 1)$ are specified in light-cone gauge by the conditions $k \cdot \epsilon = 0$, $\eta \cdot \epsilon = \epsilon^+ = 0$. Each light-front Fock state component then satisfies the angular momentum sum rule: $J^z = \sum_{i=1}^n S_i^z + \sum_{j=1}^{n-1} l_j^z$; the summation over orbital angular momenta $l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$ derives from the $n - 1$ relative momenta. The numerator structure of the light-front wavefunctions is in large part determined by the angular momentum constraints. Thus wavefunctions generated by perturbation theory [42] provides a guide to the numerator structure of nonperturbative light-front wavefunctions. Karmanov and Smirnov [40] have formulated a covariant version of light-front quantization by introducing a general null vector $n^\mu, n^2 = 0$ to specify the light-front direction $x^+ = x \cdot n$. All observables must be invariant under variation of n^μ ; this generalized rotational invariance provides an elegant generalization of angular momentum on the light-front [43].

A novel way to measure the light-front wavefunction of a hadron is to diffractively or Coulomb dissociate it into jets [44]. Measurements by Ashery *et al.* [45] of the diffractive dissociation of pions into dijets on heavy nuclei $\pi A \rightarrow q\bar{q}A$ at FermiLab show that the pion's light-front $q\bar{q}$ wavefunction resembles the asymptotic solution to the evolution equation for the pion's distribution amplitude. The results also demonstrate QCD color transparency – the color dipole moment of the pion wavefunction producing high k_\perp jets interact coherently throughout the nucleus without absorption [46]. It would be very interesting to extend these measurements to the diffractive dissociation of high energy protons into trijets.

Matrix elements of spacelike currents such as spacelike electromagnetic form factors at $q^+ = 0$ have an exact representation in terms of simple overlaps of the light-front wavefunctions in momentum space with the same x_i and unchanged parton number n [47, 48, 49]. The Pauli form factor and anomalous moment are spin-flip matrix elements of j^+ and thus connect states with $\Delta L_z = 1$ [49]. Thus, these quantities are nonzero only if there is nonzero orbital angular momentum of the quarks in the proton. The formulas for electroweak current matrix elements of j^+ can be easily extended to the T^{++} coupling of gravitons. In, fact, one can show that the anomalous gravitomagnetic moment $B(0)$, analogous to $F_2(0)$ in electromagnetic current interactions, vanishes identically for any system, composite or elementary [42]. This important feature, which follows in general from the equivalence principle [50, 51, 52, 53, 54], is obeyed explicitly in the light-front formalism [42].

The light-front Fock representation also has direct application for the study of exclusive B decays. For example, one can write an exact frame-independent repre-

resentation of decay matrix elements such as $B \rightarrow D\ell\bar{\nu}$ from the overlap of $n' = n$ parton conserving wavefunctions and the overlap of $n' = n - 2$ from the annihilation of a quark-antiquark pair in the initial wavefunction [55]. The off-diagonal $n + 1 \rightarrow n - 1$ contributions give a new perspective for the physics of B -decays. A semileptonic decay involves not only matrix elements where a quark changes flavor, but also a contribution where the leptonic pair is created from the annihilation of a $q\bar{q}'$ pair within the Fock states of the initial B wavefunction. The semileptonic decay thus can occur from the annihilation of a nonvalence quark-antiquark pair in the initial hadron. Intrinsic charm $|b\bar{u}c\bar{c}\rangle$ states of the B meson, although small in probability, can play an important role in its weak decays because they facilitate CKM-favored weak decays [56]. The “handbag” contribution to the leading-twist off-forward parton distributions measured in deeply virtual Compton scattering has a similar light-front wavefunction representation as overlap integrals of light-front wavefunctions [57, 58].

The distribution amplitudes $\phi(x_i, Q)$ which appear in factorization formulae for hard exclusive processes are the valence LF Fock wavefunctions integrated over the relative transverse momenta up to the resolution scale Q [14]. These quantities specify how a hadron shares its longitudinal momentum among its valence quarks; they control virtually all exclusive processes involving a hard scale Q , including form factors, Compton scattering and photoproduction at large momentum transfer, as well as the decay of a heavy hadron into specific final states [59, 60].

The quark and gluon probability distributions $q_i(x, Q)$ and $g(x, Q)$ of a hadron can be computed from the absolute squares of the light-front wavefunctions, integrated over the transverse momentum. All helicity distributions are thus encoded in terms of the light-front wavefunctions [14]. The DGLAP evolution of the structure functions can be derived from the high k_\perp properties of the light-front wavefunctions. Similarly, the transversity distributions and off-diagonal helicity convolutions are defined as a density matrix of the light-front wavefunctions. However, it is not true that the leading-twist structure functions $F_i(x, Q^2)$ measured in deep inelastic lepton scattering are identical to the quark and gluon distributions. It is usually assumed, following the parton model, that the F_2 structure function measured in neutral current deep inelastic lepton scattering is at leading order in $1/Q^2$ simply $F_2(x, Q^2) = \sum_q e_q^2 x q(x, Q^2)$, where $x = x_{bj} = Q^2/2p \cdot q$ and $q(x, Q)$ can be computed from the absolute square of the proton’s light-front wavefunction. Hoyer, Marchal, Peigne, Sannino, and I have shown that this standard identification is incomplete [61]; one cannot neglect the interactions which occur between the times of the currents in the current correlator even in light-cone gauge. For example, the final-state interactions lead to the Bjorken-scaling diffractive component $\gamma^*p \rightarrow pX$ of deep inelastic scattering. Since the gluons exchanged in the final state carry negligible k^+ , the Pomeron structure function closely resembles that of the primary gluon. The structure function of the Pomeron distribution of a hadron is not derived from the hadron’s light-front wavefunction and thus is not a universal quantity. The diffractive scattering of the fast outgoing quarks on spectators in the target in turn causes shadowing

in the DIS cross section. Thus the depletion of the nuclear structure functions is not intrinsic to the wave function of the nucleus, but is a coherent effect arising from the destructive interference of diffractive channels induced by final-state interactions.

Measurements from HERMES, SMC, and Jlab show a significant single-spin asymmetry in semi-inclusive pion leptonproduction $\gamma^*(q)p \rightarrow \pi X$ when the proton is polarized normal to the photon-to-pion production plane. Hwang, Schmidt, and I [62] have shown that final-state interactions from gluon exchange between the outgoing quark and the target spectator system lead to single-spin asymmetries in deep inelastic lepton-proton scattering at leading twist in perturbative QCD; *i.e.*, the rescattering corrections are not power-law suppressed at large photon virtuality Q^2 at fixed x_{bj} . The existence of such single-spin asymmetries (the Sivers effect) requires a phase difference between two amplitudes coupling the proton target with $J_p^z = \pm\frac{1}{2}$ to the same final-state, the same amplitudes which are necessary to produce a nonzero proton anomalous magnetic moment. The single-spin asymmetry which arises from such final-state interactions is in addition to the Collins effect which measures the transversity distribution $\delta q(x, Q)$. These effects highlight the unexpected importance of final- and initial-state interactions in QCD observables—they lead to leading-twist single-spin asymmetries, diffraction, and nuclear shadowing, phenomena not included in the light-front wavefunctions of the target. Alternatively, as discussed by Belitsky, Ji, and Yuan [63], one can augment the light-front wavefunctions by including the phases induced by initial and final state interactions. Such wavefunctions correspond to solving the light-front bound state equation in an external field.

2 Light-Front Hadron Dynamics and the AdS/CFT Correspondence

A precise correspondence has been established between quantum field theories and string/M-theory on Anti-de Sitter spaces (AdS) [64], where strings live on the curved geometry of the AdS space and the observables of the corresponding conformal field theory are defined on the boundary of the AdS space. A remarkable consequence of the AdS/CFT correspondence is the derivation [65] of dimensional counting rules for the leading power-law fall-off of hard exclusive processes [66, 67]. The derivation from supergravity/string theory does not rely on perturbation theory and thus is more general than perturbative QCD analyses [14].

The corrections from nonconformal effects in QCD are caused by quantum corrections and quark masses and should be moderate in the ultraviolet region. Theoretical [68, 69, 70, 71] and phenomenological [72, 73] evidence is now accumulating that the QCD coupling becomes constant at small virtuality; *i.e.*, $\alpha_s(Q^2)$ develops an infrared fixed point. Indeed, QCD appears to be a nearly-conformal theory even at moderate momentum transfers [74].

Recently Guy de Téramond and I have shown how counting rules for the nominal

power-law fall-off of light-front wavefunctions at large relative transverse momentum can be obtained from the AdS/CFT correspondence [75]. The goal is to use the AdS/CFT correspondence in the conformal domain to constrain the form of the light-front wavefunctions of hadrons in QCD. To do this, we consider the dual string theory at finite 't Hooft coupling $g_s N_C$, the product of the string constant $g_s \sim g_{YM}^2$ and the number of colors. The power-law fall-off of light-front Fock-state hadronic wavefunctions then follows from the scaling properties of string states in the large- r region of the AdS space as one approaches the boundary from the interior of AdS space.

Consider an operator $\Psi_h^{(n)}$ which creates an n -partonic Fock state by applying n -times $a^\dagger(k^+, \vec{k}_\perp)$ to the vacuum state, creating n -constituent individual states with plus momentum k^+ and transverse momentum \vec{k}_\perp . Integrating over the relative coordinates x_i and $\vec{k}_{\perp i}$ for each constituent, we find the ultraviolet behavior of $\Psi_h^{(n)}$

$$\Psi_h^{(n)}(Q) \sim \int^{Q^2} [d^2 \vec{k}_\perp]^{n-1} [a^\dagger(\vec{k}_\perp)]^n \psi_{n/h}(\vec{k}_\perp) \sim Q^{-\Delta}, \quad (5)$$

where the operator $a^\dagger(\vec{k}_\perp)$ scales as $1/k_\perp$ at large \vec{k}_\perp^2 . The string state scales as $Q^{-\Delta}$ near the AdS boundary. The dimension of the state Δ tracks with the number of constituents since each interpolating fermion and gauge field operator has a minimum twist (dimension minus spin) of one. With the identification $\Delta = n$ (modulo anomalous dimensions), the power-law behavior of the light-front wavefunctions for large \vec{k}_\perp^2 then follows: $\psi_{n/h}(\vec{k}_\perp) \rightarrow (\vec{k}_\perp^2)^{1-n}$.

The angular momentum dependence of the light-front wavefunctions also follow from the near-conformal properties of the AdS/CFT correspondence [75]. The orbital angular momentum component of the hadron wavefunction is constructed in terms of powers $|\ell_i^z|$ of the $n - 1$ transverse momenta $k_{i\perp}^\pm = k_i^1 \pm i k_i^2$. We thus can obtain a model the hard component of the light-front wavefunction

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{zi}) \sim \frac{(g_s N_C)^{\frac{1}{2}(n-1)}}{\sqrt{N_C}} \prod_{i=1}^{n-1} (k_{i\perp}^\pm)^{|l_{zi}|} \left[\frac{\Lambda_o}{\mathcal{M}^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} + \Lambda_o^2} \right]^{n+|l_z|-1}, \quad (6)$$

The scaling properties of the hadronic interpolating operator in the extended AdS/CFT space-time theory thus determines the scaling of light-front hadronic wavefunctions at high relative transverse momentum. The scaling predictions agree with the perturbative QCD analysis given in Ref. [76], but the AdS/CFT analysis is performed at strong coupling without the use of perturbation theory. Remarkably, the usual perturbative normalization factor $(g_{YM}^2 N_C)^{n-1}$ is replaced by $(g_{YM}^2 N_C)^{\frac{n-1}{2}}$. The normalization factor The near-conformal scaling properties of light-front wavefunctions lead to a number of other predictions for QCD which are normally discussed in the context of perturbation theory, such as constituent counting scaling laws for the leading power fall-off of form factors and hard exclusive scattering amplitudes for QCD

processes. The ratio of Pauli to Dirac baryon form factor have the nominal asymptotic form $F_2(Q^2)/F_1(Q^2) \sim 1/Q^2$, modulo logarithmic corrections, in agreement with the perturbative results of Ref. [77]. This analysis can also be extended to study the spin structure of scattering amplitudes at large transverse momentum and other processes which are dependent on the scaling and orbital angular momentum structure of light-front wavefunctions.

3 Spontaneous Symmetry Breaking and Light-Front Quantization

An important question is how one implements spontaneous symmetry breaking in the light-front, such as chiral symmetry in QCD or the Higgs mechanism in the Standard Model. In the case of the Schwinger model QED(1+1), degenerate vacua arise when one allows for a nonzero contribution x^- -independent contribution to the constrained $\psi^- = \psi^-(x^+)$ field. Thus zero modes of auxiliary fields distinguish the θ -vacua of massless QED(1+1) [33, 78, 79] corresponding to large gauge transformations. Zero-modes are also known to provide the light-front representation of spontaneous symmetry breaking in scalar theories [80]. It is expected that chiral symmetry breaking in QCD arises from a τ - independent contribution to the constrained ψ^- fields when the u and d quark masses are ignored. The existence of such vacua also leads to new effective interactions in the light-front Hamiltonian.

One can use light-front quantization of the $SU(2)_W \times U(1)_Y$ [81] standard model to obtain a new perspective on the Higgs mechanism and spontaneous symmetry breaking [82, 83, 84]. One first separates the quantum fluctuations from the corresponding zero-longitudinal-momentum-mode variables and then applies the Dirac procedure in order to construct the Hamiltonian. The interaction Hamiltonian of the Standard Model can be written in a compact form by retaining the dependent components A^- and ψ_- in the formulation. Its form closely resembles the interaction Hamiltonian of covariant theory, except for the presence of additional instantaneous four-point interactions. The resulting Dyson-Wick perturbation theory expansion based on equal-LF-time ordering allows one to perform higher-order computations in a straightforward fashion. The singularities in the noncovariant pieces of the field propagators can be defined using the causal ML prescription for $1/k^+$. The power-counting rules in LC gauge then become similar to those found in covariant gauge theory. The only ghosts which appear in the formalism are the $n \cdot k = 0$ modes of the gauge field associated with regulating the light-cone gauge prescription. For example, consider the Abelian Higgs model. The interaction Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi^\dagger\phi) \quad (7)$$

where

$$D_\mu = \partial_\mu + ieA_\mu, \quad (8)$$

and

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (9)$$

with $\mu^2 < 0, \lambda > 0$. The complex scalar field ϕ is decomposed as

$$\phi(x) = \frac{1}{\sqrt{2}}v + \varphi(x) = \frac{1}{\sqrt{2}}[v + h(x) + i\eta(x)] \quad (10)$$

where v is the $k^+ = 0$ zero mode determined by the minimum of the potential: $v^2 = -\frac{\mu^2}{\lambda}$, $h(x)$ is the dynamical Higgs field, and $\eta(x)$ is the Nambu-Goldstone field. The quantization procedure determines $\partial \cdot A = M\eta$, the 't Hooft condition. One can now eliminate the zero mode component of the Higgs field v which gives masses for the fundamental quantized fields. The A_\perp field then has mass $M = ev$ and the Higgs field acquires mass $m_h^2 = 2\lambda v^2 = -2\mu^2$. Similarly, in the case of the Standard model, the zero mode of the Higgs field couples to the gauge boson and Fermi fields through its Yukawa interaction. The zero mode can then be eliminated from the theory in favor of mass terms for the fundamental matter fields in the effective theory. The resulting masses are identical to those of the usual Higgs implementation of spontaneous symmetry breaking in the Standard Model. A new aspect of LF quantization is that the third polarization of the quantized massive vector field A^μ with four momentum k^μ has the form $E_\mu^{(3)} = n_\mu M / n \cdot k$. Since $n^2 = 0$, this non-transverse polarization vector has zero norm. However, when one includes the constrained interactions of the Goldstone particle, the effective longitudinal polarization vector of a produced vector particle is $E_{\text{eff}\mu}^{(3)} = E_\mu^{(3)} - k_\mu k \cdot E^{(3)} / k^2$ which is identical to the usual polarization vector of a massive vector with norm $E_{\text{eff}}^{(3)} \cdot E_{\text{eff}}^{(3)} = -1$. Thus, unlike the conventional quantization of the Standard Model, the Goldstone particle only provides part of the physical longitudinal mode of the electroweak particles. The massive gauge field propagator has well-behaved asymptotic behavior in accordance with a renormalizable theory, and the massive would-be Goldstone fields can be taken as physical degrees of freedom. Spontaneous symmetry breaking is thus implemented in a novel way when one quantizes the Standard Model at fixed light-front time $\tau = x^+$. The LF vacuum remains equal to the perturbative vacuum; it is unaffected by the occurrence of spontaneous symmetry breaking. In effect, one can interpret the $k^+ = 0$ zero mode Higgs field as an x^- -independent external field, analogous to an applied constant electric or magnetic field in atomic physics. In this interpretation, the zero mode is a remnant of a Higgs field which persists from early cosmology; the LF vacuum however remains unchanged and unbroken.

4 The Non-Perturbative Light-Front T-Matrix

The light-front formalism can be used to construct the T -matrix of QCD or other quantum field theories using light-front time-ordered perturbation theory. The application of the light-front time evolution operator P^- to an initial state systematically

generates the tree and virtual loop graphs of the T -matrix in light-front time-ordered perturbation theory. Given the interactions of the light-front interaction Hamiltonian, any amplitude in QCD and the electroweak theory can be computed. At higher orders, loop integrals only involve integrations over the momenta of physical quanta and physical phase space $\prod d^2k_{\perp i} dk_i^+$. Renormalized amplitudes can be explicitly constructed by subtracting from the divergent loops amplitudes with nearly identical integrands corresponding to the contribution of the relevant mass and coupling counter terms (the “alternating denominator method”) [85]. The natural renormalization scheme to use for defining the coupling in the event amplitude generator is a physical effective charge such as the pinch scheme [86]. The argument of the coupling is then unambiguous [87]. The DLCQ boundary conditions can be used to discretize the phase space and limit the number of contributing intermediate states without violating Lorentz invariance. This provides an “event amplitude generator” for high energy physics reactions where each particle’s final state is completely labelled in momentum, helicity, and phase. Since one avoids dimensional regularization and nonphysical ghost degrees of freedom, this method of generating events at the amplitude level could provide a simple but powerful tool for simulating events both in QCD and the Standard Model.

One can use a similar method to construct the T matrix to any given order of perturbation theory or maximal Fock number. The Lippmann-Schwinger method $T = H_I + H_I G T$ provides nonperturbative resummation at any stage. One can also use an elementary field to project out states with specific hadronic quantum numbers. The zeroes of the resolvent of the projected Green’s function should determine the mass and light-front wavefunctions of the bound states of the theory with the same hadronic numbers as that of the elementary field. A related method for calculating scattering amplitudes using the Lanczos algorithm has been proposed by Hiller [88].

5 Nonperturbative Anomalous Moment Calculations

One of the most challenging problems in quantum electrodynamics is to compute the anomalous magnetic moment of the leptons without recourse to perturbation theory [89, 90]. In recent work, John Hiller and Gary McCartor and I [91, 92, 93] have shown how such a program can be implemented using light-front methods to construct the Fock components of the physical electron nonperturbatively. The generalized Pauli-Villars method with ghost metric fermion fields ψ_{PV} can be used to regulate the ultraviolet divergences. If one rewrites the theory in terms of the zero mode fermion fields, $\psi \pm \psi_{PV}$, then instantaneous fermion exchange interactions do not appear in the light-front Hamiltonian. In addition, the constraint equation for the zero-norm fermion field does not require inverting a covariant derivative. Thus one can implement light-front quantization of gauge theory in a covariant gauge such as

the Feynman gauge [21]. If one truncates the Fock space with a maximal number of constituents N , the method includes perturbative contributions to order $n = N - 1$. The N -particle truncated result for the lepton anomalous moment thus has the form

$$a_N = \sum_{i=1}^n c_i \alpha^i + \Delta \alpha^N$$

where Δ remains dependent on the PV masses since the mass renormalization counter term is not itself cancelled in the N -particle Fock state. This residual dependence is similar to the factorization scale dependence which occurs when one separates hard and soft effects in factorization analyses. It is possible to make nonperturbative predictions by relating the anomalous moment to other observables, or to optimize the values of the cutoffs using theoretical criteria, such as minimization of estimated errors.

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