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Conformal Aspects of QCD*

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Abstract

Theoretical and phenomenological evidence is now accumulating that the QCD coupling becomes constant at small virtuality; *i.e.*, $\alpha_s(Q^2)$ develops an infrared fixed point in contradiction to the usual assumption of singular growth in the infrared. For example, the hadronic decays of the τ lepton can be used to determine the effective charge $\alpha_\tau(m^2_{\tau'})$ for a hypothetical τ-lepton with mass in the range $0 < m_{\tau'} < m_\tau$. The τ decay data at low mass scales indicates that the effective charge freezes at a value of $s = m^2_{\tau'}$ of order $1\,\mathrm{GeV}^2$ with a magnitude $\alpha_\tau \sim 0.9 \pm 0.1$. The near-constant behavior of effective couplings suggests that QCD can be approximated as a conformal theory even at relatively small momentum transfer and why there are no significant running coupling corrections to quark counting rules for exclusive processes. The AdS/CFT correspondence of large N_C supergravity theory in higher-dimensional anti-de Sitter space with supersymmetric QCD in 4-dimensional space-time also has interesting implications for hadron phenomenology in the conformal limit, including an all-orders demonstration of counting rules for exclusive processes and light-front wavefunctions. The utility of light-front quantization and light-front Fock wavefunctions for analyzing nonperturbative QCD and representing the dynamics of QCD bound states is also discussed.

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1 The Infrared Behavior of the Effective QCD Couplings $\alpha_\tau(s)$

It is often assumed that color confinement in QCD can be traced to the singular behavior of the running coupling in the infrared, *i.e.* "infrared slavery." For example if $\alpha_s(q^2) \to \frac{1}{q^2}$ at $q^2 \to 0$, then one-gluon exchange leads to a linear potential at large distances. However, theoretical [1, 2, 3, 4, 5] and phenomenological [6, 7, 8] evidence is now accumulating that the QCD coupling becomes constant at small virtuality; *i.e.*, $\alpha_s(Q^2)$ develops an infrared fixed point in contradiction to the usual assumption of singular growth in the infrared. If QCD running couplings are bounded, the integration over the running coupling is finite and renormalon resummations are not required. If the QCD coupling becomes scale-invariant in the infrared, then elements of conformal theory [9] become relevant even at relatively small momentum transfers.

One can define the fundamental coupling of QCD from virtually any physical observable [10, 11]. Such couplings, called effective charges, are all-order resummations of perturbation theory, so they correspond to the complete theory of QCD; it is thus guaranteed that they are analytic and non-singular. For example, it has been shown that unlike the $\overline{\rm MS}$ coupling, a physical coupling is analytic across quark flavor thresholds [12, 13]. Furthermore, a physical coupling must stay finite in the infrared when the momentum scale goes to zero. In turn, this means that integrals over the running coupling are well defined for physical couplings. Once such a physical coupling $\alpha_{\rm phys}(k^2)$ is chosen, other physical quantities can be expressed as expansions in $\alpha_{\rm phys}$ by eliminating the $\overline{\rm MS}$ coupling which now becomes only an intermediary [14]. In such a procedure there are in principle no further renormalization scale (μ) or scheme ambiguities. The physical couplings satisfy the standard renormalization group equation for its logarithmic derivative, $d\alpha_{\rm phys}/d\ln k^2 = \widehat{\beta}_{\rm phys}[\alpha_{\rm phys}(k^2)]$, where the first two terms in the perturbative expansion of the Gell-Mann Low function $\widehat{\beta}_{\rm phys}$ are scheme-independent at leading twist, whereas the higher order terms have to be calculated for each observable separately using perturbation theory.

In a recent paper, Menke, Merino, and Rathsman [7] and I have presented a definition of a physical coupling for QCD which has a direct relation to high precision measurements of the hadronic decay channels of the $\tau^- \to \nu_\tau {\rm h}^-$. Let R_τ be the ratio of the hadronic decay rate to the leptonic one. Then $R_\tau \equiv R_\tau^0 \left[1 + \frac{\alpha_\tau}{\pi}\right]$, where R_τ^0 is the zeroth order QCD prediction, defines the effective charge α_τ. The data for τ decays is well-understood channel by channel, thus allowing the calculation of the hadronic decay rate and the effective charge as a function of the τ mass below the physical mass. The vector and axial-vector decay modes which can be studied separately.

Using an analysis of the τ data from the OPAL collaboration [15], we have found that the experimental value of the coupling $\alpha_\tau(s) = 0.621 \pm 0.008$ at $s = m_\tau^2$ corresponds to a value of $\alpha_{\overline{\rm MS}}(M_Z^2) = (0.117\text{-}0.122) \pm 0.002$, where the range corresponds to three different perturbative methods used in analyzing the data. This result is in good agreement with the world average $\alpha_{\overline{\rm MS}}(M_Z^2) = 0.117 \pm 0.002$. However, from the figure we also see that the effective charge only reaches $\alpha_\tau(s) \sim 0.9 \pm 0.1$ at $s = 1\,\mathrm{GeV}^2$, and it even stays within the same range down to $s \sim 0.5\,\mathrm{GeV}^2$. This result is in good agreement with the estimate of Mattingly and Stevenson [6] for the effective coupling $\alpha_R(s) \sim 0.85$ for $\sqrt{s} < 0.3\,\mathrm{GeV}$ determined from $\mathrm{e}^+\mathrm{e}^-$ annihilation, especially if one takes into account the perturbative commensurate scale relation, $\alpha_\tau(m_{\tau'}^2) = \alpha_R(s^*)$, where $s^* \simeq 0.10\, m_{\tau'}^2$. This behavior is not consistent with the coupling having a Landau pole, but rather shows that the physical coupling is close to constant at low scales, suggesting that physical QCD couplings are effectively constant or "frozen" at low scales.

Figure 1 shows a comparison of the experimentally determined effective charge $\alpha_\tau(s)$ with solutions to the evolution equation for α_τ at two-, three-, and four-loop order normalized at m_τ. At three loops the behavior of the perturbative solution drastically changes, and instead of diverging, it freezes to a value $\alpha_\tau \simeq 2$ in the infrared. The reason for this fundamental change is, the negative sign of $\beta_{\tau,2}$. This result is not perturbatively stable since the evolution of the coupling is governed by the highest order term. This is illustrated by the widely different results obtained for three different values of the unknown four loop term $\beta_{\tau,3}$ which are also shown† It is interesting to note that the central four-loop solution is in good agreement with the data all the way down to $s \simeq 1\,\mathrm{GeV}^2$.

It has also been argued that $\alpha_R(s)$ freezes perturbatively to all orders [3]. In fact since all observables are related by commensurate scale relations, they all should have an IR fixed point [4]. This result is also consistent with Dyson-Schwinger equation studies of the physical gluon propagator [1, 2].

The results for α_τ resemble the behavior of the one-loop "time-like" effective coupling [17, 18, 19]

$$\alpha_{\rm eff}(s) = \frac{4\pi}{\beta_0}\left\{\frac{1}{2} - \frac{1}{\pi}\arctan\left[\frac{1}{\pi}\ln\frac{s}{\Lambda^2}\right]\right\} \tag{1}$$

which is finite in the infrared and freezes to the value $\alpha_{\rm eff}(s) = 4\pi/\beta_0$ as $s \to 0$. It is instructive to expand the "time-like" effective coupling for large s,

$$\begin{aligned}\alpha_{\rm eff}(s) &= \frac{4\pi}{\beta_0 \ln(s/\Lambda^2)}\left\{1 - \frac{1}{3}\frac{\pi^2}{\ln^2(s/\Lambda^2)} + \frac{1}{5}\frac{\pi^4}{\ln^4(s/\Lambda^2)} + \ldots\right\} \\ &= \alpha_{\rm s}(s)\left\{1 - \frac{\pi^2\beta_0^2}{3}\left(\frac{\alpha_{\rm s}(s)}{4\pi}\right)^2 + \frac{\pi^4\beta_0^4}{5}\left(\frac{\alpha_{\rm s}(s)}{4\pi}\right)^4 + \ldots\right\}.\end{aligned}$$

This shows that the "time-like" effective coupling is a resummation of $(\pi^2\beta_0^2\alpha_{\rm s}^2)^n$-corrections to the usual running couplings. The finite coupling $\alpha_{\rm eff}$ given in Eq. (1)

†The values of $\beta_{\tau,3}$ used are obtained from the estimate of the four loop term in the perturbative series of R_τ, $K_4^{\overline{\rm MS}} = 25 \pm 50$ [16].

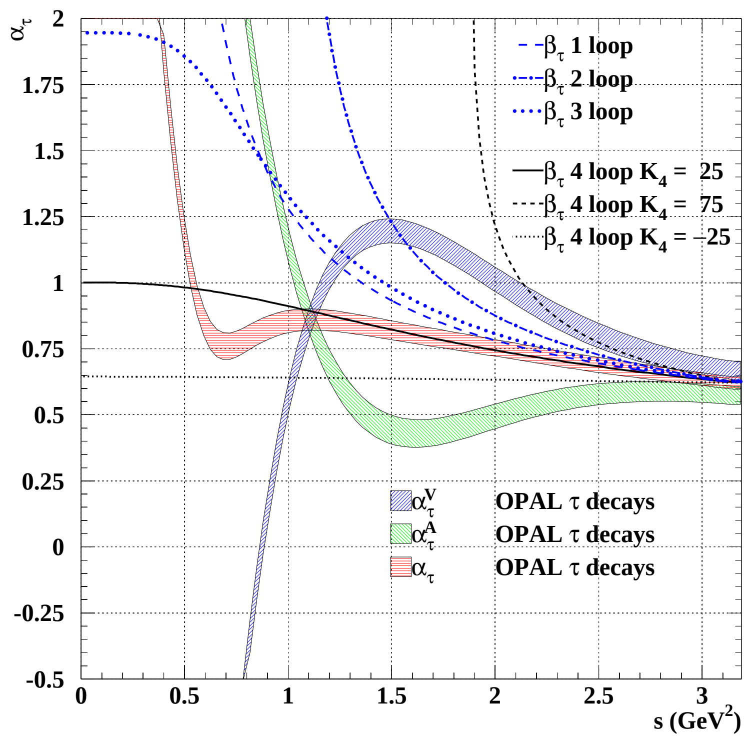

Figure 1: The effective charge α_τ for non-strange hadronic decays of a hypothetical τ lepton with $m_{\tau'}^2 = s$ compared to solutions of the fixed order evolution equation for α_τ at two-, three-, and four-loop order. The error bands include statistical and systematic errors.

obeys standard PQCD evolution at LO. Thus one can have a solution for the perturbative running of the QCD coupling which obeys asymptotic freedom but does not have a Landau singularity.

The near constancy of the effective QCD coupling at small scales helps explain the empirical success of dimensional counting rules for the power law fall-off of form factors and fixed angle scaling. As shown in the references [20, 21], one can calculate the hard scattering amplitude T_H for such processes [22] without scale ambiguity in terms of the effective charge α_τ or α_R using commensurate scale relations. The effective coupling is evaluated in the regime where the coupling is approximately constant, in contrast to the rapidly varying behavior from powers of $\alpha_{\rm s}$ predicted by perturbation theory (the universal two-loop coupling). For example, the nucleon form factors are proportional at leading order to two powers of $\alpha_{\rm s}$ evaluated at low scales in addition to two powers of $1/q^2$; The pion photoproduction amplitude at fixed angles is proportional at leading order to three powers of the QCD coupling. The essential variation from leading-twist counting-rule behavior then only arises from the anomalous dimensions of the hadron distribution amplitudes.

Parisi [23] has shown that perturbative QCD becomes a conformal theory for $\beta \to 0$ and zero quark mass. There are a number of useful phenomenological consequences of near conformal behavior: the conformal approximation with zero β function can be used as template for QCD analyses [24, 25] such as the form of the expansion polynomials for distribution amplitudes [9, 26]. The near-conformal behavior of QCD is also the basis for commensurate scale relations [14] which relate observables to each other without renormalization scale or scheme ambiguities [27]. An important example is the generalized Crewther relation [28]. In this method the effective charges of observables are related to each other in conformal gauge theory; the effects of the nonzero QCD $\beta-$ function are then taken into account using the BLM method [29] to set the scales of the respective couplings. Since the QCD running couplings are bounded, integration over the running coupling is finite, and the arguments leading to renormalon resummations do not result. The magnitude of the corresponding effective charge [20] $\alpha_s^{\text{exclusive}}(Q^2) = F_\pi(Q^2)/4\pi Q^2 F^2_{\gamma\pi^0}(Q^2)$ for exclusive amplitudes is connected to α_τ by a commensurate scale relation. Its magnitude: $\alpha_s^{\text{exclusive}}(Q^2) \sim 0.8$ at small Q^2, is sufficiently large as to explain the observed magnitude of exclusive amplitudes such as the pion form factor using the asymptotic distribution amplitude.

2 AdS/CFT and Near-Conformal Field Theory

As shown by Maldacena [30], there is a remarkable correspondence between large N_C supergravity theory in a higher dimensional anti-de Sitter space and supersymmetric QCD in 4-dimensional space-time. String/gauge duality provides a framework for predicting QCD phenomena based on the conformal properties of the ADS/CFT correspondence. For example, Polchinski and Strassler [31] have shown that the power-law fall-off of hard exclusive hadron-hadron scattering amplitudes at large momentum transfer can be derived without the use of perturbation theory by using the scaling properties of the hadronic interpolating fields in the large-r region of AdS space. Thus one can use the Maldacena correspondence to compute the leading power-law falloff of exclusive processes such as high-energy fixed-angle scattering of gluonium-gluonium scattering in supersymmetric QCD. The resulting predictions for hadron physics effectively coincide [31, 32, 33] with QCD dimensional counting rules:[34, 35, 36]

$$\frac{d\sigma_{H_1 H_2 \to H_3 H_4}}{dt} = \frac{F(t/s)}{s^{n-2}} \tag{2}$$

where n is the sum of the minimal number of interpolating fields. (For a recent review of hard fixed θ_{CM} angle exclusive processes in QCD see the references [37].) As shown by Brower and Tan [32], the non-conformal dimensional scale which appears in the QCD analysis is set by the string constant, the slope of the primary Regge trajectory $\Lambda^2 = \alpha'_R(0)$ of the supergravity theory. Polchinski and Strassler [31] have also derived counting rules for deep inelastic structure functions at $x \to 1$ in agreement with perturbative QCD predictions [38] as well as Bloom-Gilman exclusive-inclusive duality.

The supergravity analysis is based on an extension of classical gravity theory in higher dimensions and is nonperturbative. Thus the usual analyses of exclusive processes, which were derived in perturbation theory can be extended by the Maldacena correspondence to all orders. An interesting point is that the hard scattering amplitudes which are normally or order α_s^p in PQCD appear as order $\alpha_s^{p/2}$ in the supergravity predictions. This can be understood as an all-orders resummation of the effective potential [30, 39].

The superstring theory results are derived in the limit of a large N_C [40]. For gluon-gluon scattering, the amplitude scales as $1/N_C^2$. Frampton has shown how to extend the analysis to the fundamental representation [41]. For color-singlet bound states of quarks, the amplitude scales as $1/N_C$. This large N_C-counting in fact corresponds to the quark interchange mechanism [42]. For example, for $K^+p \to K^+p$ scattering, the u-quark exchange amplitude scales approximately as $\frac{1}{u}\ \frac{1}{t^2}$, which agrees remarkably well with the measured large θ_{CM} dependence of the K^+p differential cross section [43]. This implies that the nonsinglet Reggeon trajectory asymptotes to a negative integer [44], in this case, $\lim_{-t\to\infty} \alpha_R(t) \to -1$.

Pinch contributions corresponding to the independent scattering mechanism of Landshoff [45] are absent in the superstring derivation. This can be understood by the fact that amplitudes based on gluon exchange between color-singlet hadrons is suppressed at large N_C [46]. Furthermore, the independent scattering amplitudes are suppressed by Sudakov form factors which fall faster than any power in a theory with a fixed-point coupling such as conformal QCD [36, 47].

The leading-twist results for hard exclusive processes correspond to the suppression of hadron wave functions with non-zero orbital angular momentum, which is the principle underlying the selection rules corresponding to hadron helicity conservation [48]. The suppression can be understood as follows: the LF wave function with nonzero angular momentum in the constituent rest frame $\sum \vec{k}_i = 0$ can be determined by iterating the one gluon exchange kernel. They then have the structure [49, 50]

$$\psi_{L_z=1} = \frac{\vec{S}\cdot\widehat{n}\times\vec{k}_\perp}{D(k_\perp^2, x)}\psi_{L_z=0} \tag{3}$$

or

$$\psi_{L_z=1} = \frac{\widehat{\epsilon}\cdot\widehat{n}\times\vec{k}_\perp}{D(k_\perp^2, x)}\psi_{L_z=0} \tag{4}$$

where the light-front energy denominator $D(k_\perp^2, x) \sim k_\perp^2$ at high transverse momentum, $\widehat{n}$ is the light-front quantization direction, and $\widehat{\epsilon}$ is a spin-one polarization vector. This leads to the Λ/Q suppression of spin-flip amplitudes in QCD. For example, such wave functions lead to the large momentum transfer prediction $A_{LL} \sim 1/3$ for $pp \to pp$

elastic scattering [46] at large angles and momentum transfer and the asymptotic prediction $F_2(t)/F_1(t) \propto t^{-2}$ modulo powers of $\log t$ [51].

3 Light-Front Quantization

The concept of a wave function of a hadron as a composite of relativistic quarks and gluons is naturally formulated in terms of the light-front Fock expansion at fixed light-front time, $\tau = x\cdot\omega$. The four-vector ω, with $\omega^2 = 0$, determines the orientation of the light-front plane; the freedom to choose ω provides an explicitly covariant formulation of light-front quantization [52]. Although LFWFs depend on the choice of the light-front quantization direction, all observables such as matrix elements of local current operators, form factors, and cross sections are light-front invariants – they must be independent of ω_μ.

The light-front wave functions (LFWFs) $\psi_n(x_i, k_{\perp_i}, \lambda_i)$, with $x_i = \frac{k_i\cdot\omega}{P\cdot\omega}$, $\sum_{i=1}^n x_i = 1$, $\sum_{i=1}^n k_{\perp_i} = 0_\perp$, are the coefficient functions for n partons in the Fock expansion, providing a general frame-independent representation of the hadron state. Matrix elements of local operators such as spacelike proton form factors can be computed simply from the overlap integrals of light front wave functions in analogy to nonrelativistic Schrödinger theory. In principle, one can solve for the LFWFs directly from the fundamental theory using methods such as discretized light-front quantization, the transverse lattice, lattice gauge theory moments, or Bethe–Salpeter techniques. The determination of the hadron LFWFs from phenomenological constraints and from QCD itself is a central goal of hadron and nuclear physics. Reviews of nonperturbative light-front methods may be found in the references [53, 52, 54, 55]. One can also project the known solutions of the Bethe–Salpeter equation to equal light-front time, thus producing hadronic light-front Fock wave functions. A potentially important method is to construct the $q\bar{q}$ Green's function using light-front Hamiltonian theory, with DLCQ boundary conditions and Lippmann-Schwinger resummation. The zeros of the resulting resolvent projected on states of specific angular momentum J_z can then generate the meson spectrum and their light-front Fock wavefunctions. The DLCQ properties and boundary conditions allow a truncation of the Fock space while retaining the kinematic boost and Lorentz invariance of light-front quantization.

One of the central issues in the analysis of fundamental hadron structure is the presence of non-zero orbital angular momentum in the bound-state wave functions. The evidence for a "spin crisis" in the Ellis-Jaffe sum rule signals a significant orbital contribution in the proton wave function [56, 57]. The Pauli form factor of nucleons is computed from the overlap of LFWFs differing by one unit of orbital angular momentum $\Delta L_z = \pm 1$. Thus the fact that the anomalous moment of the proton is non-zero requires nonzero orbital angular momentum in the proton wavefunction [58]. In the light-front method, orbital angular momentum is treated explicitly; it includes the orbital contributions induced by relativistic effects, such as the spin-orbit effects normally associated with the conventional Dirac spinors.

In recent work, Dae Sung Hwang, John Hiller, Volodya Karmonov [50], and I have studied the analytic structure of LFWFs using the explicitly Lorentz-invariant formulation of the front form. Eigensolutions of the Bethe-Salpeter equation have specific angular momentum as specified by the Pauli-Lubanski vector. The corresponding LFWF for an n-particle Fock state evaluated at equal light-front time $\tau = \omega \cdot x$ can be obtained by integrating the Bethe-Salpeter solutions over the corresponding relative light-front energies. The resulting LFWFs $\psi^I_n(x_i, k_{\perp i})$ are functions of the light-cone momentum fractions $x_i = k_i \cdot \omega / p \cdot \omega$ and the invariant mass squared of the constituents $M_0^2 = (\sum_{i=1}^n k_i^\mu)^2 = \sum_{i=1}^n \left[\frac{k_\perp^2 + m^2}{x}\right]_i$ and the light-cone momentum fractions $x_i = k \cdot \omega / p \cdot \omega$ each multiplying spin-vector and polarization tensor invariants which can involve ω^μ. The resulting LFWFs for bound states are eigenstates of the Karmanov–Smirnov kinematic angular momentum operator [59]. Thus LFWFs satisfy all Lorentz symmetries of the front form, including boost invariance, and they are proper eigenstates of angular momentum.

4 AFS/CFT Correspondence and Light-Front Wavefunctions

One can also use the scaling properties of the hadronic interpolating operator in the extended AdS/CFT space-time theory to determine the scaling of light-front hadronic wavefunctions at high relative transverse momentum. De Teramond and I [46] have also shown how the angular momentum dependence of the light-front wavefunctions also follow from the conformal properties of the AdS/CFT correspondence. The scaling and conformal properties of the AdS/CFT correspondence leads to a hard component of the light-front Fock state wavefunctions of the form:

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{zi}) \sim \frac{(g_s \; N_C)^{\frac{1}{2}(n-1)}}{\sqrt{N_C}} \prod_{i=1}^{n-1} (k_{i\perp}^{\pm})^{|l_{zi}|} \left[\frac{\Lambda_o}{\mathcal{M}^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} + \Lambda_o^2} \right]^{n + |l_z| - 1}, \tag{5}$$

where g_s is the string scale and Λ_o represents the basic QCD mass scale. The scaling predictions agree with the perturbative QCD analysis given in the references [49], but the AdS/CFT analysis is performed at strong coupling without the use of perturbation theory. The near-conformal scaling properties of light-front wavefunctions lead to a number of other predictions for QCD which are normally discussed in the context of perturbation theory, such as constituent counting scaling laws for the leading power fall-off of form factors and hard exclusive scattering amplitudes for QCD processes. The ratio of Pauli to Dirac baryon form factor have the nominal asymptotic form $F_2(Q^2)/F_1(Q^2) \sim 1/Q^2$, modulo logarithmic corrections, in agreement with the perturbative results [51]. Our analysis can also be extended to study the spin structure of scattering amplitudes at large transverse momentum and other processes which are dependent on the scaling and orbital angular momentum structure of light-front wavefunctions.

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