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Light-Front Hadron Dynamics and AdS/CFT Correspondence *

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Abstract

A remarkable consequence of the AdS/CFT correspondence is the nonperturbative derivation of dimensional counting rules for hard scattering processes. Using string/gauge duality we derive the QCD power behavior of light-front Fock-state hadronic wavefunctions for hard scattering in the large-r region of the AdS space from the conformal isometries which determine the scaling of string states as we approach the boundary from the interior of AdS space. The nonperturbative scaling results are obtained for spin-zero and spin-$\frac{1}{2}$ hadrons and are extended to include the orbital angular momentum dependence of the constituents in the Fock-expansion in the light-cone. Quark interchange amplitudes emerge as the dominant large N_C scattering mechanisms for conformal QCD.

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Since the early identification of large N_C conformal QCD with the topological expansion of a string [1], the connection between string theories and the large-N_C limit of field theories has drawn major attention. However, string theory is not consistent in a four dimensional flat space, and at least one extra dimension has to be introduced [2]. It is only recently that a precise correspondence has been established between quantum field theories and string/M-theory on Anti-de Sitter spaces (AdS) [3], where strings live on the curved geometry of the AdS space and the observables of the conformal field theory are defined in the boundary of AdS. The partition function of the AdS theory is identified with the generating functional of the boundary conformal gauge field theory, and correlation functions are computed in the boundary of AdS where the quantum field theory operators are inserted [4, 5]. The AdS/CFT conjecture is indeed a concrete realization of the holographic principle [6, 7]. From the point of view of string theory/field theory duality, excitations near the boundary of the conformal AdS space correspond to states in the field theory. Hard-scattering interactions occur in the large-r region of the AdS space, so effectively we require the 't Hooft parameter, the product of the string coupling and the number of colors $g_s N_C$, to be sufficiently large so that the AdS radius $R = (4\pi g_s N_C)^{1/4} \alpha'^{1/2}$, with α' the string scale, is also large. We can map the string states degrees of freedom to the QCD degrees of freedom at the boundary of the AdS space [8].

A remarkable consequence of the AdS/CFT correspondence is the derivation [9] of dimensional counting rules for the leading power-law fall-off of hard exclusive processes [10, 11]. The derivation from supergravity/string theory in a $AdS_5 \times X$ background does not rely on perturbation theory, and thus is more general than perturbative QCD analysis [12]. The corrections from nonconformal effects in QCD can be incorporated into the effective scale of the running coupling, since the conformal symmetry is broken only by quantum corrections and should be moderate in the ultraviolet region. Indeed, QCD appears to be a nearly-conformal theory in the momentum regimes accessible to experiment. The amplitudes of confining gauge theories with superstring duals, in addition to the power-law hard dependence at large momentum transfer, have Regge behavior at small angles for different kinematic domains [9]. The String/Gauge duality for QCD in four dimensions can also be obtained from M-theory in a specific Black Hole deformation of $AdS^7 \times S^4$ [13] with Regge behavior in the near-forward limit [14].

In this paper counting rules for light-front wavefunctions, the probability amplitudes which relate the constituents degrees of freedom with the asymptotic hadronic states, are obtained from the AdS/CFT correspondence for large momentum transfer processes as we approach the boundary of the AdS space. The nonperturbative scaling results are obtained for spin-zero and spin-$\frac{1}{2}$ hadrons. The counting rules for light-front wavefunctions will also be extended to include the orbital angular momentum dependence of the constituents by computing the corresponding scaling dimension from the conformal algebra of the generators and their action on the field operators. As will be shown below, the quark interchange amplitudes for fixed-angle exclusive processes emerge as the dominant mechanism in the large 't Hooft limit [1, 15] $N_C \to \infty$, keeping the product $g^2_{QCD}(\mu) N_C$, with $g^2_{QCD} \sim g_s$, large but fixed at some given scale μ. The quark interchange amplitudes are shown to be the leading terms in a $1/N_C$ expansion with other contributions, such as gluon exchange, being suppressed by powers of $1/N_C$. This result is related to hadronic duality [16], since in the large N_C limit quark-interchange amplitudes are topologically equivalent to duality diagrams which keep track of the flow of quark quantum numbers; they represent string world-sheets with the quark lines defining the boundaries. The quark interchange amplitude scales as $1/N_C$ while other contributions scale faster as $N_C \to \infty$. The string/field theory duality predictions for hard scattering correspond precisely to the $N_C \to \infty$ limit, which is also the large $g_S N_C$ limit. For large transverse momentum the relevant scale is $\mu \sim Q$, and quark exchange interactions occur in the large-r conformal region of the AdS space.

The light-front Fock-state wavefunction provides a frame-independent representation of relativistic composite systems in QCD at the amplitude level in terms of quark and gluon degrees of freedom which carry the symmetries within the hadrons. The basic constituents appear from the light-front quantization of the excitations of the dynamical fields expanded in terms of creation and annihilation operators on the transverse plane with coordinates $x^- = z - ct$ and $\vec{x}_\perp$ at $\tau = z + ct = 0$. The expansion of bound state hadronic systems in terms of Fock states provide an exact representation of the local matrix elements used for calculating form factors, distribution amplitudes, and generalized parton distributions [17]. In terms of the hadron four-momentum $P = (P^+, P^-, \vec{P}_\perp)$ with $P^\pm = P^0 \pm P^3$, the light-cone frame independent Hamiltonian for a hadronic composite system $H_{LC}^{QCD} = P^- P^+ - \vec{P}_\perp^2$, has eigenvalues given in terms of the eigenmass $\mathcal{M}$ squared corresponding to the mass spectrum of the color-singlet states in QCD, $H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$. The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle = \sum_{n,\lambda_i} \int \; [dx_i \; d^2\vec{k}_{\perp i}] \; \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i) \; |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle, \tag{1}$$

where the coefficients of the light-cone Fock expansion

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i) = \left\langle n : x_i, \vec{k}_{\perp i}, \lambda_i | \psi_h \right\rangle, \tag{2}$$

depend only on the relative partonic coordinates, the longitudinal momentum fraction $x_i = k_i^+/P^+$, $\sum_{i=1}^n x_i = 1$, the relative transverse momentum $\vec{k}_{\perp i}$, $\sum_{i=1}^n \vec{k}_{\perp i} = \vec{0}$ and λ_i, the projection of the constituents' spin along the z direction. The amplitudes $\psi_{n/h}$ represent the probability amplitudes to find on-mass-shell constituents i with momentum $x^i P^+$ and $x_i \vec{P}_\perp + \vec{k}_{\perp i}$ and spin projection λ_i in the hadron h. The measure of the constituents' phase-space momentum integration $[dx_i \; d^2\vec{k}_{\perp i}]$ depends on the normalization chosen. The complete basis of Fock-states $|n\rangle$ is constructed by applying free-field creation operators to the vacuum state $|0\rangle$ which has no particle content, $P^+|0\rangle = 0$, $\vec{P}_\perp|0\rangle = 0$. Since all the quanta have positive k^+, the vacuum state is unique and equal to the nonperturbative vacuum. A one-particle state is defined by $|q\rangle = \sqrt{2q^+} \; a^\dagger(q)|0\rangle$ so that its normalization has the Lorentz invariant form $\langle q|q'\rangle = 2q^+ \; (2\pi)^3 \; \delta(q^+ - q'^+) \; \delta^{(2)}(\vec{q}_\perp - \vec{q}'_\perp)$. The measure of the phase space integration is defined by

$$[dx_i \; d^2\vec{k}_{\perp i}] = (16\pi^3) \; \delta\left(1 - \sum_{j=1}^n x_j\right) \; \delta^{(2)}\left(\sum_{\ell=1}^n \vec{k}_{\perp \ell}\right) \prod_{i=1}^n \frac{dx_i}{x_i} \, \frac{d^2\vec{k}_{\perp i}}{16\pi^3}, \tag{3}$$

and a normalized hadronic state $\langle\psi|\psi\rangle = 1$, can be expressed as a sum of overlap integrals of light-front wavefunctions

$$\sum_n \int [dx_i \; d^2\vec{k}_{\perp i}] \; |\psi_{n/p}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1. \tag{4}$$

We will show that the scaling properties of light-front wave functions are obtained from the string/gauge theory duality, and their power-law dependence follows from the conformal dimension of string states dual to hadron states. According to the AdS/CFT correspondence, a quantum gravity theory in AdS space defines a conformal field theory on its boundary [3]. We thus expect that for large values of the 't Hooft parameter $g_s N_C$, near-conformal QCD has a dual string description.

To conserve Poincaré invariance in four dimensional Minkowsky space the metric should have the form

$$ds^2 = w(z)^2[dx^2 + dz^2], \tag{5}$$

where $dx^2 = \eta_{\mu\nu}dx^\mu dx^\nu$. Reparametrization invariance allows us to factor out the warp factor $w(z)^2$. The coordinates x^μ are Minkowsky spacetime variables and z labels the new spatial dimension. If the four-dimensional gauge theory is conformal, it is invariant under the transformation $x \to \lambda x$. If $z \to \lambda z$ is also an isometry of (5), then $w(z) = R/z$ and the space is a five-dimensional Anti-de Sitter space, AdS_5, and the conformal group $SO(2,4)$ in Minkowski space is isomorphic to the isometry group of AdS_5. The value $z = 0$ corresponds to the four-dimensional boundary where the gauge theory is defined. Introducing a radial coordinate $r = R^2/z$ we can express the AdS_5 metric (5) as

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2. \tag{6}$$

If the gauge theory is dual to a critical string, a superstring theory in 10 dimensions, spacetime is the product space AdS_5 with a five-dimensional manifold X. For example, in the original Maldacena conjecture [3] a correspondence was established between Type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ large N_C superconformal Yang-Mills theory in the boundary, with the solution $R = (4\pi g_s N_C)^{1/4}\alpha'^{1/2}$. Coordinates in the compact space X will be labeled by Ω.

The observables of the gauge theory are defined at the boundary of the AdS space. The AdS metric implies that a distance $L_{\rm local}$, measured in the local inertial coordinates in the full space shrinks by a warp factor as one approaches the AdS boundary. Thus the length in the Minkowski space is

$$L = \left(\frac{R}{r}\right) L_{\rm local}. \tag{7}$$

Consequently a local scattering state in the bulk with local momentum $Q_{\rm local}$ as determined in the inertial frame is shifted to the ultraviolet as measured by an observer in Minkowsky space

$$Q = \left(\frac{r}{R}\right) Q_{\rm local}. \tag{8}$$

Since the characteristic energy scale in the ten-dimensional space is $Q_{\rm local} \sim R^{-1}$, it follows that $r \sim QR^2$. Thus large Q pointlike hard scattering processes occur in the large-r conformal region of the AdS space.

The geometry at small values of r and the form of the compact space X depend on the nature of the gauge theory and the mechanisms of conformal symmetry breaking. QCD is a nearly conformal theory in the ultraviolet region and a confining theory in the infrared with a mass gap corresponding to the scale Λ_o of the lightest glueball scale. We thus expect that a non-conformal metric will be manifest at $r \sim r_o = \Lambda_o\, R^2$. However, we are interested in probing the wavefunction at large transverse momentum of order $Q \sim r/R^2 \gg \Lambda_o$. The interaction occurs in the $r \sim QR^2 \gg r_o$ region where the conformal geometry (6) is valid and where the specific form of the compact space X is not relevant [9].

According to the AdS/CFT correspondence we expect hadronic states at the boundary to have dual scattering states in the bulk. Consider first a spin-zero hadron state dual to a string state represented by a 10-dimensional wavefunction corresponding to the massless mode of the spinless dilaton field Φ. For large values of $r \gg r_o$ spacetime geometry is the product space $AdS \times X$ and $\Phi = \Psi(r, x)\, Y(\Omega)$. For a given eigenvalue of the Laplace equation in the compact manifold X, the field Ψ satisfies a d+1 dimensional Laplace equation in AdS_{d+1} space with the linearly independent solutions [18]

$$\Psi(r, x) = C r^{-\frac{d}{2}} e^{-ip\cdot x} \left\{ \begin{array}{l} I_\alpha\left(\frac{pR^2}{r}\right) \\ K_\alpha\left(\frac{pR^2}{r}\right) \end{array} \right. , \tag{9}$$

with $\alpha^2 = (d/2)^2 + (mR)^2$. The dominant contribution at large-r scales as

$$\Psi(r, x) \sim e^{-ip\cdot x} \left(\frac{r}{r_o}\right)^{-\Delta}, \tag{10}$$

where

$$\Delta = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + (mR)^2} \tag{11}$$

is the conformal dimension of the state. The limiting values of the operators in the bulk are the Heisenberg field operators of the gauge field theory [3] specified by the boundary conditions [4, 5]

$$\lim_{r\to\infty} \Psi(r, x) = r^{-\Delta}\Psi_o(x), \tag{12}$$

where $\Psi_o(x)$ is the corresponding renormalized operator of the quantum field theory [19]. At large transverse momentum transfer, $Q \sim r/R^2$, Ψ has the scaling behavior

$$\Psi(Q) \sim Q^{-\Delta} \tag{13}$$

in the large-r region of AdS space.

In QCD the hadron eigenfunctions of the light-cone Hamiltonian H_{LC}^{QCD} are expanded in terms of light-front amplitudes $\psi_{n/h}$ according to (1). If the light-front wavefunctions do not fall quickly enough, infinities appear in the unitarity sum given by (4). To avoid this problem we truncate the Fock-states for light-cone transverse momenta $\vec{k}_\perp^2$ above an ultraviolet scale Λ^2 [12]. Certainly the introduction of a cutoff explicitly breaks the conformal invariance of the theory, but we know that the net effect of renormalization of the quantum theory is to introduce logarithmic quantum corrections. Our results are thus valid up to logarithms. The cutoff is usually taken in the limit $\Lambda \to \infty$ when performing calculations. In practice, the cutoff has no effect on the results provided that Λ is much greater than all mass scales, so calculations are carried out with a finite cutoff by defining the wavefunctions, masses and couplings at the scale Λ. In a field theory where only a single scale Q is relevant, it is natural to take the cutoff $\Lambda \sim Q$ and redefine the basic parameters and wavefunctions at the scale Q. Discarding Fock states with light-cone momenta above Λ and taking the effective cutoff Λ as the scale Q for large values of the momentum transfer, we can determine the large $\vec{k}_\perp^2$ momentum dependence of the light front amplitudes since we know the conformal behavior of the bulk state as we approach the boundary from the interior of AdS space. The correspondence follows indeed from the conformal isometry which determines the scaling of the string state as expressed by (13).

To determine the precise counting rule, consider an operator $\Psi_h^{(n)}$ which creates an n-partonic state by applying n-times $a^\dagger(k^+, \vec{k}_\perp)$ to the vacuum state, creating n-constituent individual states with plus momentum k^+ and transverse momentum $\vec{k}_\perp$. Integrating over the relative coordinates x_i and $\vec{k}_{\perp i}$ for each constituent using the expression for the phase space (3), we find the ultraviolet behavior of $\Psi_h^{(n)}$

$$\Psi_h^{(n)}(Q) \sim \int^{Q^2} [d^2\vec{k}_\perp]^{n-1} [a^\dagger(\vec{k}_\perp)]^n \; \psi_{n/h}(\vec{k}_\perp) \sim Q^{-\Delta}, \tag{14}$$

where the operator $a^\dagger(\vec{k}_\perp)$ scales as $1/k_\perp$ at large $\vec{k}_\perp^2$. The conformal dimension Δ corresponds to the number of constituents since each interpolating fermion and gauge field operator has a minimum twist (dimension minus spin) of one. With the identification $\Delta = n$ the power-law behavior of the light-front wavefunctions for large $\vec{k}_\perp^2$ follows immediately

$$\psi_{n/h}(\vec{k}_\perp) \to \left(\frac{1}{\vec{k}_\perp^2}\right)^{n-1}. \tag{15}$$

The field operator $\Psi_h^{(n)}$ behaves as $Q^{-\Delta}$, and the light-front wavefunctions $\psi_{n/h}$ as $(\vec{k}_\perp^2)^{-\Delta+1}$. Notice that the light-front amplitude $\psi_{n/h}$ does not scale as its mass dimension M^{-n+1}.

Although we have derived the scaling behavior of light-front amplitudes for a spinless hadron, the results are identical for a spin-$\frac{1}{2}$ hadron corresponding to the supergravity modes of a dilatino state λ. In the large-r conformal region the dilatino field is written as a product $\lambda = \Psi(r,x)\;\eta(\Omega)$, where Ψ is a spinor field defined in a conformal d+1 space and η is an m-dimensional compact manifold spinor. For an eigenvalue of the Dirac equation in the compact manifold X, the field ψ satisfies the d + 1 dimensional Dirac equation with the solution [18]

$$\psi(r,x)_\sigma = C r^{-\frac{d}{2}} e^{-ip\cdot x} \left[I_\alpha(\frac{pR^2}{r})\;(\mu_+)_\sigma + K_\alpha(\frac{pR^2}{r})\;(\mu_-)_\sigma \right]. \tag{16}$$

The dominant contribution at large-r also scales as $r^{-\Delta}$ since the spinors are defined in the tangent space. We thus obtain the same nominal power-law behavior of the light-front wavefunctions for baryons and mesons.

The scaling dimension for a state with orbital angular momentum ℓ can be obtained from the algebra of the generators of the conformal group in d-dimensional Minkowsky space SO(2,d), which is isomorphic to the isometry group of AdS_{d+1}. In particular, the generator of scaling transformations D and the generator of translations P_μ obey the commutation relations $[D, P_\mu] = -iP_\mu$. Consider a state with orbital angular momentum ℓ, $\Psi_\ell \sim p^\ell \; \Psi$, where Ψ is an eigenfunction of the scaling operator D with eigenvalue $-i\Delta$ [20]

$$[D, \Psi] = i(-\Delta + x^\mu \partial_\mu)\Psi. \tag{17}$$

Using the commutation relation of the operator D with P_μ as written above, we obtain the scaling dimension Δ_ℓ of the field Ψ^ℓ representing a state with orbital angular momentum ℓ

$$[D, \Psi_\ell] = i(-\Delta_\ell + x^\mu \partial_\mu)\Psi_\ell, \tag{18}$$

with $\Delta_\ell = \Delta + \ell$. The dimensional scaling law for the light-front wavefunctions at large Q including constituent orbital angular momentum ℓ follows directly from the identification $\Delta \to \Delta_\ell = n + \ell$ in (14)

$$\psi_{n/h}(\vec{k}_\perp) \to k_\perp^\ell \left[\frac{1}{\vec{k}_\perp^2}\right]^{n+\ell-1}. \tag{19}$$

Note that the scaling properties of hadronic states with orbital angular momentum follow as well from the AdS/CFT duality with the corresponding exited orbital string states moving in a d+1 AdS space, since a string mode on AdS with orbital angular momentum ℓ on the compact space X has the product of its mass and angular momentum quantized according to $(mR)^2 = \ell(\ell + d)$. From (11) it follows that $\Delta_\ell = \Delta + \ell$, where Δ is the conformal dimension for the massless string mode with zero orbital angular momentum.

In QCD each Fock state is an eigenfunction of the total angular momentum projection $J^z = \sum_{i=1}^n S_i^z + \sum_{i=1}^{n-1} L_i^z$, where the sum over the spin S_i^z corresponds to the intrinsic spins of the n-constituent Fock states and the sum over the $n-1$ orbital angular momentum excludes the contribution from the motion of the center-of-mass [21]. To specify the angular momentum properties of hadronic states [22], the orbital angular momentum component of the hadron wavefunction is constructed in terms of the $n-1$ transverse momenta of the components and has the general structure [23]: $(k_{1\perp}^\pm)^{|l_{z1}|}(k_{2\perp}^\pm)^{|l_{z2}|}...(k_{(n-1)\perp}^\pm)^{|l_{z(n-1)}|}$, with $k_{i\perp}^\pm = k_i^1 \pm i k_i^2$. We thus write for the hard component of the light-front wavefunction

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{zi}) \sim \frac{(g_s\ N_C)^{\frac{1}{2}(n-1)}}{\sqrt{N_C}} \prod_{i=1}^{n-1} (k_{i\perp}^\pm)^{|l_{zi}|} \left[\frac{\Lambda_o}{\mathcal{M}^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} + \Lambda_o^2}\right]^{n+|l_z|-1}, \tag{20}$$

where $\Lambda_o^2 = 1/\alpha'_{QCD}$ gives the basic QCD mass scale. The form (20) is compatible with the scaling properties predicted by the AdS/CFT correspondence (19) including orbital angular momentum [24]. It agrees with the normalization of Ref. [9]. Equation (20) has mass dimensions $-(n + |l_z| - 1)$ consistent with [23].

Fixed-angle large transverse momentum exclusive collision processes in QCD take place in the large conformal region of AdS space. Consider first hard meson-meson scattering in a theory with gauge symmetry $SU(N_C)$ for large N_C [25]. We use the 't Hooft double-line representation [1] of Feynman diagrams where a quark propagator is represented by a single-index line and a gluon propagator by two-index lines. To obtain the $1/N_C$ expansion for meson-meson scattering, we note that there is a factor of N_C from a closed color quark loop from quark interchange and a normalization factor of $1/\sqrt{N_C}$ for each meson wave function; thus $M_{QIM} \propto 1/N_C$. The counting rule is not changed at fixed $g^2_{QCD}\ N_C$ by any number of ladder gluon exchanges between quarks within the same meson, as would result from the iteration of the equation of motion of the meson wavefunction. In the case of two-gluon exchange in meson-meson scattering, the index color-counting of the gluon exchange in terms of $q\overline{q}$ pairs gives an additional factor N_C relative to the quark interchange diagram from the additional quark color loop, but each vertex has a factor $1/\sqrt{N_C}$ and thus $M_{2g} \propto 1/N_C^2$. For three gluon exchange, there are three quark color-loop diagrams and six vertices which cancel the N_C factors, and the scaling is given by the wavefunction normalization; thus $M_{3g} \propto 1/N_C^2$.

Baryons constitute a difficult problem in the large N_C limit since they are represented by a totally antisymmetric color state with N_C different quarks, and the number of quark lines in a given diagram grows with N_C. It is expected, however, that baryons will follow simple scaling laws in a $1/N_C$ expansion. In his theory of baryons at large N_C, Witten uses graphical methods to describe the n-body force and introduces a bound state which consists of a large number of weakly interacting particles described in the Hartree approximation [15]. In the case under discussion, large transverse momentum hadron-hadron scattering, the problem is simplified, since all the N_C constituents (N_C large but finite) change their collinear direction in the collision process and acquire high transverse momentum. Thus one requires the high-Q components of the hadronic wavefunction corresponding to large-r values of AdS space. We introduce a normalization factor of $1/\sqrt{N_C}$ for each baryon wave function as in Eq. (20), and compute the factors giving the non-trivial $1/N_C$ expansion with the usual index counting in the diagrams. Repeating the analysis for the $1/N_C$ power counting for meson-baryon processes, or for baryon-baryon or baryon-antibaryon scattering, we obtain identical results: the quark interchange amplitudes scale as $1/N_C$, whereas two, three or multiple gluon exchange amplitudes scale as $1/N_C^2$. The suppression of the gluon exchange amplitudes as compared to quark interchange could explain the suppression of Landshoff pinch contributions [26] in the large transverse momentum fixed t/s proton-proton scattering data. Quark interchange thus becomes the dominant mechanism at large momentum transfer in the $N_C \to \infty$ limit of QCD.

In this paper we have shown how the scaling properties of the hadronic interpolating operator in the extended AdS/CFT space-time theory determines the scaling of light-front hadronic wavefunctions at high relative transverse momentum. The angular momentum dependence of the light-front wavefunctions also follow from the conformal properties of the AdS/CFT correspondence. The scaling predictions agree with the perturbative QCD analysis given in Ref. [23], but here the analysis is performed at strong coupling without the use of perturbation theory. The near-conformal scaling properties of light-front wavefunctions lead to a number of other predictions for QCD which are normally discussed in the context of perturbation theory, such as constituent counting scaling laws for the leading power fall-off of form factors and hard exclusive scattering amplitudes for QCD processes. The ratio of Pauli to Dirac baryon form factor have the nominal asymptotic form $F_2(Q^2)/F_1(Q^2) \sim 1/Q^2$, modulo logarithmic corrections, in agreement with the perturbative results of Ref. [27]. Our analysis can also be extended to study the spin structure of scattering amplitudes at large transverse momentum and other processes which are dependent on the scaling and orbital angular momentum structure of light-front wavefunctions.

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