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Comment on extracting α from $B \to \rho\rho$

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Abstract

Recent experimental results on $B \to \rho\rho$ decays [1–3] indicate that the CP asymmetry $S_{\rho^+\rho^-}$ will give an interesting determination of $\alpha = \arg\left[-(V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)\right]$. In the limit when the ρ width is neglected, the $B \to \pi\pi$ isospin analysis can also be applied to $B \to \rho\rho$, once an angular analysis is used to separate transversity modes. The present bound on the shift of $S_{\rho^+\rho^-}$ from the true $\sin 2\alpha$ is already stronger than it is for $S_{\pi^+\pi^-}$. We point out a subtle violation of the isospin relations when the two ρ mesons are observed with different invariant masses, and how to constrain this effect experimentally.

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I. INTRODUCTION

Rates and polarization fractions for various $B \to \rho\rho$ decays have been recently measured [1–3]. First measurements of CP asymmetries in these modes are expected in the near future. This note is a brief comment on the application of isospin analysis to these modes, similar to that for $\pi\pi$ channels [4] to extract Standard Model parameters, and in particular the CKM phase $\alpha \equiv \phi_2 \equiv \arg\left[-\left(V_{td}V_{tb}^*\right)/\left(V_{ud}V_{ub}^*\right)\right]$, from these measurements. It is important to constrain such CKM phases as precisely as possible in many independent ways. Inconsistent results from different approaches could be an indicator of new physics as various measurements that are related in the Standard Model can be affected differently by possible contributions from physics beyond the Standard Model. Here we comment on the need to parameterize the data to allow for the impact of possible $I = 1$ contributions that can occur if the two ρ mesons have different masses.[1]

In the standard parameterization for the CKM matrix, the phase dependence of the $B \to \rho^i\rho^j$ decay amplitudes can be written as

$$\begin{aligned} A_{ij} &= T_{ij}e^{+i\gamma} + P_{ij}e^{-i\beta}, \\ \bar{A}_{ij} &= T_{ij}e^{-i\gamma} + P_{ij}e^{+i\beta}, \end{aligned} \tag{1}$$

where A_{ij} describe B^+ and B^0 decays, $\bar{A}_{ij}$ describes B^- and $\overline{B}^0$ decays, and β, γ (and $\alpha = \pi - \beta - \gamma$) are the angles of the unitarity triangle (for their precise definitions, see *e.g.*, Ref. [5]). T_{ij} is dominated by the tree diagram, while P_{ij} comes primarily from so-called penguin diagrams. An important role in the CP asymmetries in neutral B decays is played by the $B^0 - \overline{B}^0$ mixing amplitude, which has the following CKM phase dependence:

$$M_{12} = |M_{12}|e^{2i\beta}. \tag{2}$$

The dominant CP violating effect in the $B \to \rho^+\rho^-$ decay comes from the interference between the $B^0 - \overline{B}^0$ mixing amplitude and T_{+-}. As can be deduced from Eqs. (1) and (2), this effect is sensitive to the phase $\alpha = \pi - \beta - \gamma$ (or $\phi_2 = \pi - \phi_1 - \phi_3$).

The time dependent CP asymmetry in $B \to \rho^+\rho^-$ can be parametrized as follows:

$$\frac{\Gamma(\overline{B}^0_{\rm phys}(t) \to \rho^+\rho^-) - \Gamma(B^0_{\rm phys}(t) \to \rho^+\rho^-)}{\Gamma(\overline{B}^0_{\rm phys}(t) \to \rho^+\rho^-) + \Gamma(B^0_{\rm phys}(t) \to \rho^+\rho^-)} = S_{+-}\sin(\Delta m\, t) - C_{+-}\cos(\Delta m\, t)\,. \tag{3}$$

[1] By ρ mass we mean throughout this paper the invariant mass of the pion pair from the decay of that ρ.

If $|P_{+-}/T_{+-}|$ were zero, so that a single weak phase dominates the decay and if, in addition, the final state were purely CP even, then $S_{+-} = \sin 2\alpha$ (and $C_{+-} = 0$). A separation of final CP eigenstates is possible with angular analysis [6]; as we will see below the data show that the decays to charged ρ's are dominantly longitudinally polarized and thus CP even.

Other CP violating effects in $B \to \rho\rho$ decays arise from the interference between the T and P terms in Eq. (1) or from interference between mixing and P amplitudes. These effects not only have different weak phase dependences, but also depend on the amplitude ratio $|P/T|$ and the strong phase $\arg(P/T)$. These complicate the relationship between the measured CP violation and the phase α. For a given final transversity σ (see discussion below), this more complicated relation can be parametrized as follows:

$$S^{\sigma}_{+-} = \sqrt{1 - (C^{\sigma}_{+-})^2}\,\sin(2\alpha + 2\delta_{\sigma})\,. \tag{4}$$

In the case of two pions, Gronau and London [4] showed how to use the six flavor-tagged $B \to \pi\pi$ rates and isospin symmetry to precisely determine α even in the presence of the additional CP violating effects. Later work showed how one can use the isospin relations to bound the uncertainties in α, even when sufficient data to complete the full analysis is not available [7–9]. These methods can be applied also for the decays to two ρ mesons. The current experimental data implies that the $B \to \rho\rho$ case will give a better intermediate result.

II. BOSE STATISTICS AND BROAD RESONANCES

The vector-vector decays of a spin zero B meson can have orbital angular momentum $L = 0$, 1, or 2. Hence, for two vector particles, they include both even and odd CP modes. Since the decaying B meson is spin 0, the total spin of the two vector mesons must be equal to and oppositely aligned to the orbital angular momentum, L. Thus, in the case of two identical vector mesons, such as two equal mass ρ mesons, independent of the value of L, the combined space plus spin wave function of the the two identical vector mesons is symmetric under particle exchange. Bose statistics then tells us that, just as in the case of two pions, the isospin of the two ρ meson state must be symmetric under exchange of the particles, thereby eliminating any possible $I = 1$ contributions.[2]

While the above argument is made in terms of the amplitudes of a given L, it applies for all L. Thus it is equally valid when applied to the amplitudes expressed in any alternative angular decomposition. The set of basis functions for describing the decays used in the experimental analyses are labelled by the transversity $\sigma = 0, \|, \perp$ of the ρ mesons (which both must be the same since the initial state has spin zero). Thus, from this point on, our discussion will be in this basis. Note that once this basis is chosen there is no longer any sense in which one can separate the different orbital angular momentum contributions within a given transversity-labelled state. Since transversity-labelled amplitudes are a choice of three orthogonal angular basis functions for analyzing the decays, they contain the full angular momentum information. Thus we have a complete set of amplitudes, $A^{\sigma}_{ij} = A[B \to (\rho^i \rho^j)_\sigma]$, where σ is the transversity label and i and j are the charges of the two ρ mesons. The CP of a given transversity state is well-defined, in the case at hand the states $\sigma = 0$ and $\|$ are CP even, while the $\sigma = \perp$ states are CP odd [6].

The above arguments for the absence of $I = 1$ in each transversity state do not apply for general four-pion amplitudes. This contribution exists even when two pion pairs have the same invariant mass and angular momentum. Indeed the fact that ρ mesons have a significant width reintroduces the possibility of $I = 1$ contributions even for a pair of longitudinally polarized ρ particles. In each $B \to \rho\rho$ event the invariant mass of each ρ is measured, and the two values can differ by an amount of order of Γ_ρ, or rather by the width of the region allowed by experimental cuts on the data. The $B \to \rho\rho$ amplitude for two ρ mesons with charges $q_1,\ q_2$, masses $m_1,\ m_2$ and helicities $\lambda_1 = \lambda_2$ can have a part which is antisymmetric under the interchange of the values of m_1 and m_2, and thus, by Bose statistics, this amplitude is also antisymmetric in the combined (space, spin, isospin) wave function, thus allowing odd isospin, despite the fact that $L = S$. In contrast, the dependence of the even-isospin amplitudes on the ρ masses is symmetric under interchange of m_1 and m_2. The different isospin amplitudes do not interfere. Our main point in this note is that the fits to data should explicitly include the possibility of the odd-isospin contribution in $B \to \rho\rho$.

The size of the $I = 1$ contribution is a dynamical question; we make no prediction. We

[2] In Section 6.1.2.2 of the Babar Physics Book [5] this argument is given correctly for the $L = 0, 2$ case. However, an incorrect conclusion that isospin analysis is not possible for the $L = 1$ component is stated. Mea culpa, HQ.

cannot rule out the presence of $I = 1$ contributions of order $(\Gamma_\rho/m_\rho)^2$ in the total rate. The fact that this amplitude must vanish for equal ρ meson masses gives it a distinct distribution as a function of m_1 and m_2 from the leading even-isospin terms. The leading contribution to the rate due to the amplitude antisymmetric in m_1 and m_2 can be parameterized by adding to the fits a term of the form

$$\left[c\,\frac{m_1 - m_2}{m_\rho}\right]^2 \left|B_\rho(m_1^2) B_\rho(m_2^2)\right|^2, \tag{5}$$

where $B_\rho(s)$ is the Breit-Wigner. This contribution vanishes where the even-isospin contribution peaks. The $I = 1$ contributions in the $\rho^+\rho^-$ and $\rho^\pm\rho^0$ channels are unrelated, while there is no such contribution to $\rho^0\rho^0$. Note that even-isospin contributions of the same form are also possible, *e.g.*, from the cross-term in

$$\left[a + b\,\frac{(m_1 - m_2)^2}{m_\rho^2}\right]^2 \left|B_\rho(m_1^2) B_\rho(m_2^2)\right|^2 . \tag{6}$$

We expect a, b and c to be of the same order, so the even-isospin contribution proportional to ab could be comparable to the $I = 1$ component.

The question is whether the extraction of the leading even-isospin amplitudes [the a^2 term in Eq. (6)] is sensitive to possible contributions of the form (5). Independent of whether the correction term is dominated by the c^2 term of Eq. (5) or the interference of a and b in Eq. (6), the stability of the fit for the a^2 term can be tested. If the addition of terms of the form (5) causes the value of the leading term to shift significantly then further tests must be made to ensure a stable value for the on-peak amplitudes. If adding such a term does not significantly change the result for the leading term, then we can be confident that the correct on-peak amplitudes have been measured.

While the $I = 1$ contribution must be positive, the subleading even-isospin contributions may have either sign. Thus, even if a fit to the data finds that contributions to the rate of the form in Eq. (5) are small, that could still be due to cancellations. Such a cancellation would be accidental in either the $\rho^+\rho^-$ or the $\rho^\pm\rho^0$ channels, and it is unlikely to occur in both. Thus, if the fits in both of these modes are insensitive to terms of the form (5), then it is probably safe to assume that the $I = 1$ contributions are likewise small. But, as we stress above, it is not the size of these terms that really matters here, but rather the stability of the fit to the on-peak, equal mass, $\rho\rho$ contribution, for which the isospin analysis is to be carried out. If the fits are sensitive to terms of the form in Eq. (5), then further analysis, and probably significantly more data is needed.

As an alternative to fitting the data including terms of the form (5), one can eliminate effects of any contributions of this form by decreasing the width of the ρ bands, Δ, used in the fit (or imposing a cut on $|m_1 - m_2|$). Once the accepted ρ band is small enough, the result will be stable against further reduction in its width, and also against changes to the leading fit parameters when a term of the form (5) is added. At present, BaBar uses a band $0.52\,\mathrm{GeV} < m_{\pi\pi} < 1.02\,\mathrm{GeV}$ [2, 3] whereas BELLE accepts a narrower range, $0.65\,\mathrm{GeV} < m_{\pi\pi} < 0.89\,\mathrm{GeV}$ [1]. The possible $I = 1$ contamination in the $B \to \rho\rho$ signal diminishes for $\Delta < \Gamma_\rho$ at least as $(\Delta/m_\rho)^2$. If the extracted values of the rates are stable for different values of Δ that would indicate that the $I = 1$ contamination is small and we need not worry further about these types of terms, whereas results that are sensitive to Δ would indicate that there is a contribution of this type that must be more carefully investigated, or excluded by taking a smaller acceptance.

Clearly both the approach of adding parameters to the fit and the approach of narrowing the acceptance have a statistical cost. We are hopeful that, even with the present data set, one will be able to see that the impact of possible $I = 1$ terms is not large. If their effect turns out to be important, then more data will be needed to eliminate their impact.

III. ISOSPIN RELATIONS

For each transversity, σ, the even-isospin amplitudes have relationships similar to that for the two-pion amplitudes [4],

$$
\begin{aligned}
\frac{1}{\sqrt{2}}\, A^\sigma_{+-} + A^\sigma_{00} &= A^\sigma_{+0}\,, \\
\frac{1}{\sqrt{2}}\, \bar{A}^\sigma_{+-} + \bar{A}^\sigma_{00} &= \bar{A}^\sigma_{-0}\,.
\end{aligned} \tag{7}
$$

Each of these equations can be represented as a triangle in the complex plane. Note that the triangles corresponding to the different transversity states can be different.

Tree diagrams contribute to both $\Delta I = 1/2$ and $3/2$ transitions to $I = 0$ and $I = 2$ final states, respectively. Since the gluon is isospin singlet, penguin diagrams contribute only to $\Delta I = 1/2$ transitions to $I = 0$ final states. Since the final $\rho^\pm\rho^0$ states have no $I = 0$ component, A^σ_{+0} and $\bar{A}^\sigma_{-0}$ are pure tree amplitudes. Therefore, $|A^\sigma_{+0}| = |\bar{A}^\sigma_{-0}|$ and the relative phase of these amplitudes is 2γ [see Eq. (1)]. The two triangles originating from

Eqs. (7) for any given σ can thus be superimposed with a common base, A^{σ}_{+0}, if all the $\bar{A}^{\sigma}_{ij}$ amplitudes are multiplied by a factor $e^{2i\gamma}$.

Electroweak penguin amplitudes, unlike gluonic penguins, contribute to both $\Delta I = 1/2$ and $3/2$ and hence cannot be distinguished from the tree amplitudes by their isospin structure. Since electroweak penguins contribute to both T_{ij} and P_{ij} in Eq. (1), one impact of such terms would be a possible difference between A_{+0} and $e^{2i\gamma}\bar{A}_{-0}$. The size of corrections that contribute to $|A_{+0}| \neq |\bar{A}_{-0}|$ can be constrained by measuring these two rates. The average of the BABAR [2] and BELLE [1] results is

$$\begin{aligned} \mathcal{A}_{\mp 0} &= \frac{|\bar{A}_{-0}|^2 - |A_{+0}|^2}{|\bar{A}_{-0}|^2 + |A_{+0}|^2} = -0.09 \pm 0.16\,, \\ f_0 &= \frac{|\bar{A}^0_{-0}|^2 + |A^0_{+0}|^2}{|\bar{A}_{-0}|^2 + |A_{+0}|^2} = 0.96^{+0.04}_{-0.06}\,. \end{aligned} \tag{8}$$

These results are consistent with the isospin relationship $\mathcal{A}_{\mp 0} = 0$, though with current precision the test is not particularly stringent. Given this, there is residual uncertainty in the extracted value of α that is not constrained by the isospin analysis. There is no calculation of electroweak penguin amplitudes from first principles; estimates of their impact on the determination of α in the $B \to \pi\pi$ isospin analysis ranges from negligible to less than $5^\circ - 10^\circ$ [10]. This effect is expected to be similar in $B \to \rho\rho$, although a dedicated analysis is warranted since the matrix elements are different. At the present level of accuracy it is reasonable to assume that this contribution is small compared to the uncertainties that are bounded by the isospin analysis; we will neglect it in what follows.

Once the branching ratios $\mathcal{B}[B \to (\rho^i\rho^j)_\sigma] = |A^{\sigma}_{ij}|^2$ are measured, one can construct the two triangles and use this construction to measure the relative phase between A^{σ}_{+-} and $e^{2i\gamma}\bar{A}^{\sigma}_{+-}$ [4]. This phase is $2\delta_\sigma$ defined in Eq. (4). It arises from a combination of relative weak and strong phases and the relative magnitudes of the T_{+-} and P_{+-} contributions, none of which can be reliably calculated. Using the two-triangle construction to determine $2\delta_\sigma$, there is a fourfold ambiguity in the value of this phase, coming from the four possible orientations of the two triangles relative to their common base.

Until the flavor-tagged branching fractions, $\mathcal{B}[B^0 \to (\rho^0\rho^0)_\sigma]$ and $\mathcal{B}[\overline{B}^0 \to (\rho^0\rho^0)_\sigma]$, are separately measured, one cannot determine δ_σ. However, one can bound it. Among the three averaged branching ratios (summed over transversities),

$$\mathcal{B}_{+-} = \frac{1}{2}\left(|A_{+-}|^2 + |\bar{A}_{+-}|^2\right),$$

$$\mathcal{B}_{+0} = \frac{1}{2}\left(|A_{+0}|^2 + |\bar{A}_{-0}|^2\right),$$
$$\mathcal{B}_{00} = \frac{1}{2}\left(|A_{00}|^2 + |\bar{A}_{00}|^2\right), \tag{9}$$

the first two have been measured and there is an upper bound on the third. This provides an upper bound on $\mathcal{B}_{00}^{\sigma}$ for any σ. It is significantly smaller than the rate for the dominant longitudinal mode in the other channels. This allows us to place a significant bound on δ_0, using the construction described above. Explicitly, the bound reads [7, 8][3]

$$\cos 2\delta_0 \geq 1 - \frac{2\mathcal{B}_{00}^0}{\mathcal{B}_{+0}^0} + \frac{(\mathcal{B}_{+-}^0 - 2\mathcal{B}_{+0}^0 + 2\mathcal{B}_{00})^2}{4\mathcal{B}_{+-}^0\mathcal{B}_{+0}^0}. \tag{10}$$

This bound can be further strengthened if experiments constrain C_{+-}^0 [9] and C_{00}^0.

IV. CORRECTIONS PROPORTIONAL TO $1 - f_0$

For both $B \to \rho^+\rho^-$ and $B \to \rho^\pm\rho^0$, experiments have determined that the longitudinal fraction f_0 is close to 100% [see Eq. (14)]. Thus, even if the experiments do not distinguish the asymmetry in the longitudinal mode alone, one can use the total asymmetry to constrain the longitudinal asymmetry. Since we already know from the data that the decay is almost purely longitudinal, the correction is small, of $\mathcal{O}(1-f_0)$. Using $S_{+-} = \sum_\sigma f_\sigma S_{+-}^\sigma$ and $C_{+-} = \sum_\sigma f_\sigma C_{+-}^\sigma$, the differences between the transversity-summed CP violating asymmetries and those in the longitudinal mode are given by

$$S_{+-}^0 - S_{+-} = (1-f_0)\left(S_{+-}^0 - \frac{S_{+-}^{\parallel} + S_{+-}^{\perp}}{2}\right) - \left(f_{\parallel} - f_{\perp}\right)\frac{S_{+-}^{\parallel} - S_{+-}^{\perp}}{2},$$
$$C_{+-}^0 - C_{+-} = (1-f_0)\left(C_{+-}^0 - \frac{C_{+-}^{\parallel} + C_{+-}^{\perp}}{2}\right) - \left(f_{\parallel} - f_{\perp}\right)\frac{C_{+-}^{\parallel} - C_{+-}^{\perp}}{2}. \tag{11}$$

The S_{+-}^σ and C_{+-}^σ asymmetries in each of the transversity channels can in principle be anywhere from -1 to $+1$ subject to the constraints $(S_{+-}^\sigma)^2 + (C_{+-}^\sigma)^2 \leq 1$. Thus, the maximal deviations of the measured asymmetries from those for the longitudinal modes are

$$|S_{+-}^0 - S_{+-}| \leq (1-f_0)\left(1 + |S_{+-}^0|\right),$$
$$|C_{+-}^0 - C_{+-}| \leq (1-f_0)\left(1 + |C_{+-}^0|\right). \tag{12}$$

[3] The bound in [8], quoted in Eq. (10), is the same as the one in Eq. (2.15) of [7] up to terms of $\mathcal{O}[(\mathcal{B}_{00}/\mathcal{B}_{+0})^2, (\mathcal{B}_{+-}/\mathcal{B}_{+0} - 2)^2]$, where [8] is more restrictive. In [8], the weaker bound in Eq. (2.12) of [7], $\cos 2\delta_0 \geq 1 - 2\mathcal{B}_{00}^0/\mathcal{B}_{+0}^0$, is referred to as the Grossman-Quinn bound.

In reality, we expect the error in estimating S^0_{+-} to be smaller than this upper bound. To zeroth order in $|P^\sigma_{+-}/T^\sigma_{+-}|$ we have $S^{\parallel}_{+-} = -S^{\perp}_{+-} = S^0_{+-}$. Consequently, we obtain

$$S^0_{+-} - S_{+-} = (1 - f_0 - f_\parallel + f_\perp)\, S^0_{+-} + \mathcal{O}[(1-f_0)\,|P_{+-}/T_{+-}|]\,. \tag{13}$$

One further issue that must be considered is the impact of non-resonant contributions to B meson decays to four pions, and that of other resonances that yield the same final state in this analysis. These could contribute with opposite CP to that of the dominant longitudinal mode. Since the angular distribution given by the decay of a spin-1 longitudinally-polarized meson is quite restrictive, the contamination due to all such contributions is effectively included in the error of $1 - f_0$, the fraction of non-longitudinal contributions. Thus the uncertainty due to these contributions is taken into account by allowing for the uncertainties in Eq. (12) when determining the CP asymmetries in the longitudinal mode.

V. NUMERICAL RESULTS

The experimental values given by BABAR for the three averaged branching ratios defined in Eq. (9) are [2, 3]

$$\begin{aligned}
\mathcal{B}_{+-} &= (27^{+7+5}_{-6-7}) \times 10^{-6}\,, \qquad & (f_0)_{+-} = 0.99^{+0.01}_{-0.07} \pm 0.03\,,\\
\mathcal{B}_{+0} &= (22.5^{+5.7}_{-5.4} \pm 5.8) \times 10^{-6}\,, \qquad & (f_0)_{+0} = 0.97^{+0.03}_{-0.07} \pm 0.04\,,\\
\mathcal{B}_{00} &< 2.1 \times 10^{-6} \quad (90\%\ \mathrm{CL})\,, &
\end{aligned} \tag{14}$$

while BELLE obtained [1]

$$\mathcal{B}_{+0} = (31.7 \pm 7.1^{+3.8}_{-6.7}) \times 10^{-6}, \qquad (f_0)_{+0} = 0.948 \pm 0.106 \pm 0.021\,. \tag{15}$$

We take $\mathcal{B}_{+-} = \mathcal{B}^0_{+-}$ and $\mathcal{B}_{+0} = \mathcal{B}^0_{+0}$, thus introducing errors of order $(1 - f_0)$. These are much smaller than the present experimental errors on $\mathcal{B}_{+-}$ and $\mathcal{B}_{+0}$ and therefore can be neglected. We use the following averages, based on (14) and (15):

$$\begin{aligned}
\mathcal{B}^0_{+-} &= (27 \pm 9) \times 10^{-6},\\
\mathcal{B}^0_{+0} &= (26 \pm 6) \times 10^{-6},\\
\mathcal{B}^0_{00} &= (0.6^{+0.8}_{-0.6}) \times 10^{-6}.
\end{aligned} \tag{16}$$

The value of $\mathcal{B}_{00}$ is based on scaling the number of signal events given in Ref. [2] and conservatively assuming the efficiency for $(f_0)_{00} = 1$, which yields the largest rate [11].

The first question to be asked is whether the rates in Eq. (16) are consistent with isospin symmetry. Note that in the $\mathcal{B}^0_{00} \to 0$ limit, we must have $\mathcal{B}^0_{+-} = 2\mathcal{B}^0_{+0}$. The central values in (16) imply a small $\mathcal{B}^0_{00}$ but $\mathcal{B}^0_{+-} \sim \mathcal{B}^0_{+0}$, thus the consistency with the isospin constraints is limited. Indeed, a statistical analysis [12] of the rates in Eq. (16) finds the goodness of the fit is only 24%. Since this confidence level is not extremely small, in the following we derive limits on δ_0 assuming that isospin symmetry holds.[4] Using the isospin constraints as coded in [12] and the branching ratios in Eq. (16), we obtain the 90% CL bound:

$$\cos 2\delta_0 > 0.83\,, \tag{17}$$

or, equivalently,

$$|\delta_0| < 17^\circ. \tag{18}$$

Note that even though the statistical significance of $\mathcal{B}^0_{+-} - 2\mathcal{B}^0_{+0} + 2\mathcal{B}_{00} \neq 0$ is small, the last term in Eq. (10) does play a role. Had we ignored it, we would have obtained $\cos 2\delta_0 > 0.80$.

It is interesting that the small value of $\mathcal{B}_{00}/\mathcal{B}_{+0}$ already puts an upper bound on C_{+-}, the measure of direct CP violation. For each transversity component, the isospin relations imply, for $\mathcal{B}^\sigma_{00}/\mathcal{B}^\sigma_{+0} < 1/2$,

$$|C^\sigma_{+-}| < 2\sqrt{\frac{\mathcal{B}^\sigma_{00}}{\mathcal{B}^\sigma_{+0}} - \left(\frac{\mathcal{B}^\sigma_{00}}{\mathcal{B}^\sigma_{+0}}\right)^2}\,. \tag{19}$$

The 90% CL bound on $\mathcal{B}^0_{00}/\mathcal{B}^0_{+-}$ that can be extracted from Eq. (16) yields, to leading order in the small quantity $1 - f_0$,

$$|C^0_{+-}| < 0.53\,. \tag{20}$$

VI. CONCLUSIONS

The present measurements of the rates of the various $B \to \rho\rho$ decays already yield significant limits on the uncertainty in the extraction of α from the CP violating asymmetry in B^0 and $\overline{B}^0$ decays to $\rho^+\rho^-$. Given the large branching fractions of these channels, we look forward to an asymmetry measurement in the near future which will determine α with

[4] It was pointed out in Ref. [3] that the small upper bound on $\mathcal{B}_{00}/\mathcal{B}_{+0}$ constrains the penguin pollution.

interesting precision. To ensure the accuracy of the results it is important to include an isospin-1 contribution in the fits to data, as in Eq. (5), constrained to vanish when the two ρ mesons have equal masses. We do not expect the impact of this contribution to be large, but it could introduce changes of order $(\Gamma_\rho/m_\rho)^2$ to the best fit parameters. Once this effect is constrained experimentally and the CP-violating quantity S_{+-} is measured, $B \to \rho\rho$ decays promise to provide the best model independent determination of the parameter α for some time to come.

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