

# Large scale power and running spectral index in New Old Inflation

Gia Dvali<sup>a</sup> and Shamit Kachru<sup>b</sup>

<sup>a</sup> *Center for Cosmology and Particle Physics  
Department of Physics, New York University, New York, NY 10003*

<sup>b</sup> *Department of Physics and SLAC  
Stanford University, Stanford, CA 94305*

## Abstract

We have proposed a new class of inflationary scenarios in which the first stage of expansion is driven by “old” false vacuum inflation. This ends by nucleation of a bubble, which then further inflates. Unlike the standard slow-roll scenarios the “clock” ending the second inflationary phase is not a local order parameter, but rather the average value of an oscillating scalar field, which locks the system at a saddle point of the potential in a temporary inflationary state. Inflation ends when the amplitude drops below a certain critical point and liberates the system from the false vacuum state. The second stage of inflation has only about 50 e-foldings, a number which is determined entirely by the ratio of the fundamental mass scales, such as the Planck/string scale and the supersymmetry breaking scale. The density perturbations are generated due to fluctuations of moduli-dependent Yukawa couplings. In this note we explore the observable imprints in the fluctuation spectrum of generic cross-couplings in the superpotential and in the Kähler potential. We show that in the presence of generic non-renormalizable interactions in the superpotential between the fluctuating modulus and the oscillating inflaton, the amplitude of the density perturbations is exponentially cut-off for sufficiently large wavelengths. With reasonable choices of scales and interactions, this long wavelength cutoff can occur at approximately the current horizon size. The perturbative corrections in the Kähler potential give non-trivial potentially observable tilt and a running of the spectral index which is different from the standard inflationary models.

# 1 Introduction

In a recent paper [1], we suggested an inflationary scenario which builds on Guth’s old inflation [2]. The basic idea is to start in a false vacuum as in old inflation, and construct our observed universe from a single bubble of false vacuum decay. This requires a brief period of inflation after bubble nucleation, and suitable mechanisms for reheating and production of density perturbations within the bubble. This cosmological sequence is similar in spirit to “New” inflation [3], but the way it is achieved in our scenario is dramatically different. The key difference is that the “clock” that controls the expansion of the bubble is not a local order parameter, i.e., the slowly rolling inflaton field. Instead, it is an averaged amplitude of a quickly oscillating scalar field, that locks the system in an inflationary state. Simple toy models which accomplish this aim were presented in [1]. They involve a rolling scalar field  $\Phi$  which stabilizes a second field  $\phi$  in its false vacuum.<sup>1</sup> The  $\Phi$  field oscillations are redshifted away during inflation, and eventually  $\phi$  is destabilized and rolls to its true vacuum.

The idea of separating the inflationary “clock” from the velocity of the slowly rolling scalar field goes back to Linde’s “Hybrid Inflation” [4], in which (in some regimes) the inflation can be terminated almost instantly by triggering a phase transition in the second field. However, the clock that triggers this transition is still a local slowly-rolling field. In this respect our attitude is more radical, since in our case what changes slowly is not a local scalar field, but rather its averaged amplitude.

The chief merit of these models is that the required inflationary potentials are natural from the viewpoint of softly broken supersymmetry, and do not have to satisfy any restrictive slow-roll conditions. However some restrictive assumptions are required to design full models with appropriate reheating and density perturbations.

In particular, for the choices of scales which are natural in the models of [1], it is necessary to find a nonstandard way of generating the observed density perturbations – given the low scale of inflation and the absence of slow-roll in the minimal scenario, the perturbations generated by the inflaton are negligible. The decay-rate fluctuation mechanism of [6, 7] is ideal for this purpose, and was suggested as an appropriate framework. In this note, we point out that in models of this sort, generic non-renormalizable couplings of the fluctuating modulus field  $\chi$  of [6, 7] to the rolling scalar field  $\Phi$  which produces the bubble inflation can have striking observational consequences. They give the  $\chi$  field an effective mass  $m_\chi > H_*$  during the early e-foldings of the bubble inflation, leading to an absence of perturbations on the largest scales! The perturbations turn on as the oscillation amplitude of  $\Phi$  decays away, and  $m_\chi$  relaxes to  $m_\chi \ll H_*$ . This can have two interesting consequences: 1) For reasonable choices of the  $\Phi - \chi$  coupling, one can design models where the

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<sup>1</sup>The potentials studied in [1] are essentially identical to those which occur in hybrid inflation [4], though we only use the potential in a small-field region where slow-roll inflation would not occur. With different choices of parameters than we make, these potentials have also been studied in connection with parametric resonance in [5].

perturbations become negligible at scales close to our present horizon size. CMB measurements at the largest scale are cosmic-variance limited, and so one would like to find other consequences/predictions of such a scenario.

2) In the same class of scenarios, one finds that the spectral index  $n(k)$  has striking behavior at the wavelengths where the  $\chi$  field mass is passing through  $H_*$ . The form of  $n(k)$  that one finds is unlike that which occurs in other inflationary models, and is in principle a testable prediction.

The organization of this note is as follows. In §2, we describe the scenario of [1]. In §3, we show how the imprinting of density perturbations can be influenced by generic non-renormalizable couplings between the inflaton  $\Phi$  and the fluctuating modulus field  $\chi$ . In three brief appendices, we address several questions about the approximations that yield our inflationary scenario. We explain how the small numbers in our potentials come naturally from soft susy breaking scenarios, we show that tunneling to nonzero values of  $\phi$  or  $\dot{\phi}$  is highly unlikely (such paths are quickly drawn back to the one we considered in [1]), and we show that perturbative annihilation of  $\Phi$  quanta has a negligible influence on our scenario.

Several other ideas about producing features in the CMB at low  $l$  have been proposed, see e.g. [8, 9, 10, 11, 12, 13, 14] and references therein. In particular in section IIIB of [13], similar potentials to ours are discussed in a different regime, in exploring a different idea about the same problem.

## 2 Scenario

The basic building block of our “locked” inflationary scenario is a system of two scalar fields  $\Phi$  and  $\phi$  with an unsuppressed cross-coupling in the potential,

$$\Phi^2\phi^2, \tag{1}$$

such that when either of the fields is large, the other one acquires a big mass and gets “nailed” to a zero value. This is a characteristic feature of the “hybrid” model [4], where this coupling traps one field in a false vacuum, while the other is slowly-rolling. The regime that we are exploring, however, does not rely on the slow-roll.

We assume that the self-interaction potentials  $V(\Phi)$  and  $V(\phi)$  are generated as a result of supersymmetry-breaking effects in the Kähler potential, and are such that the system has: 1) a false minimum at some  $\phi = 0, \Phi = \Phi_{false} \sim M_P$ , 2) a true minimum at  $\phi = \phi_{true}, \Phi = 0$ , and 3) a saddle point at  $\phi = \Phi = 0$ . Inflation starts in the false vacuum, where  $\phi$  has a mass  $\sim M_P$ , and cannot affect the tunneling dynamics. So  $\Phi$  tunnels towards the saddle point, and materializes at some initial value  $\sim M_P$  or so. Throughout this process, the  $\phi$  field is superheavy and is firmly fixed at zero value (since this has caused some confusion, see the appendix for elaboration on this point). After bubble nucleation,  $\Phi$  rolls towards the saddle point and oscillates about it. These oscillations induce an effective positive mass<sup>2</sup> term for the  $\phi$  field, and prevent an instability in the  $\phi$ -direction from developing.

Hence the system is locked in a temporary false vacuum state, with energy density consisting of the constant false vacuum potential energy at the saddle point, plus the energy density of the oscillator, which redshifts as matter.

Eventually, the oscillator energy density becomes sub-dominant and the system inflates. Since  $\Phi$  is a very weakly self-coupled field, we shall ignore the non-linear part of its self-interactions after tunneling and will only keep the harmonic term in the potential. The potential then has the form

$$m_{\Phi}^2 \Phi^2 + \Phi^2 \phi^2 + V(\phi) \quad (2)$$

where  $V(\phi)$  is a self-coupling potential of  $\phi$ , which has a maximum at  $\phi = 0$  and a minimum at  $\phi = \phi_{true}$ .

Before discussing  $V(\phi)$ , let us briefly discuss corrections to the potential of  $\Phi$ . Since inflation breaks supersymmetry (spontaneously), the perturbative corrections to the Kähler potential arising from  $\phi$ -loops will correct the potential of  $\Phi$ , even if all other corrections are absent [15]. The resulting one-loop Coleman-Weinberg potential for  $\Phi \gg m_{soft}^2$  behaves as

$$V_{one-loop} \simeq \frac{m_{soft}^2}{32\pi^2} |\Phi|^2 \ln(|\Phi|), \quad (3)$$

where  $m^2$  is a soft mass of  $\phi$ . The cancellation of the  $\Phi^4$  term among bosons and fermions is a general consequence of supersymmetry, which holds even though we evaluate corrections along the inflationary trajectory where SUSY is spontaneously broken [15]. It is clear that in our case, these corrections are so small that they are unimportant for the inflationary dynamics. However, as we shall see, similar corrections may play an important role in creating a non-trivial tilt in the spectrum of perturbations.

We shall assume that  $V(\phi)$  is a typical potential for a field which is a supersymmetric flat direction, whose potential comes entirely from the Kähler potential after supersymmetry-breaking. The corrections to the Kähler potential that induce the  $\phi$  VEV may come either from tree-level gravity-mediated susy-breaking or from perturbative renormalization due to matter loops. In the former case the VEV at the true minimum is typically  $\phi_{true} \sim M_P$ , whereas in the latter case we may have  $\phi_{true} \ll M_P$ . Both possibilities are considered in the appendix. Irrespective of the precise form of these corrections, for the flat direction fields whose potentials and preferred VEVs are generated as a result of soft supersymmetry breaking, the following is true in general. The curvature at  $\phi = 0$  is  $\sim m_{soft}^2$ , and the value of the false vacuum energy is

$$V(0) = m_{soft}^2 \left( \frac{\phi_{true}^p}{M_P^{p-2}} \right) \quad (4)$$

for some  $p$ . For instance for the model discussed in §2 of [1], one has  $p = 4$ .

The inflationary Hubble parameter in such a model is given by  $H_* \sim m_{soft} (\frac{\phi_{true}}{M_P})^{p/2}$ . The bubble size today is:

$$R_{today} \sim c^{\frac{5p}{6}} \left( \frac{M_P}{H_*} \right)^{\frac{7}{6}} \frac{1}{T_{today}} \quad (5)$$

where  $c = \phi_{true}/M_P$ . This expression was derived as follows. The initial value of the  $\Phi$  field inside the bubble is  $\Phi_{in} \sim M_P$ . Hence the initial energy density is  $m_{soft}^2 M_P^2$ , and the initial Hubble  $H \sim m_{soft}$ , which also sets the size of most generic bubble to be  $\sim 1/m_{soft}$ . If  $\phi_{true} \ll M_P$ , initially the energy of  $\Phi$ -oscillations dominates and there is an interval of matter-dominated expansion until the energy of oscillations becomes sub-dominant to  $V(0)$  (that is until the amplitude of  $\Phi$  becomes less than  $M_P c^{p/2}$ ). During the matter-dominated interval the bubble interior grows by a factor  $c^{-\frac{p}{3}}$ , after which inflation begins and the universe begins to exponentially expand. The expansion factor during the subsequent phase of locked inflation is given by

$$e^N = \left( c^{p/2-1} \frac{\phi_{true}}{m_{soft}} \right)^{\frac{2}{3}}. \quad (6)$$

Finally there is an additional expansion factor after reheating

$$\frac{T_R}{T_{today}} \sim \frac{\sqrt{H_* M_P}}{T_{today}} \quad (7)$$

Combining all the factors, we find equation (5).

### 3 Density perturbations

The density perturbations are generated through the ‘‘decay-rate-fluctuation’’ mechanism of references [6, 7]. The idea is that the decay rate  $\Gamma$  of the field  $\phi$  (or some other field responsible for the reheating, as in §4.2 of [1]) is controlled by a fluctuating modulus field. This is reasonably motivated by string theory, where couplings of the low energy fields are set by the expectation values of moduli. Let  $\chi$  be a modulus that controls the decay rate of  $\phi$ . If the mass of  $\chi$  is an order of magnitude smaller than  $H_*$ , fluctuations will be imprinted in  $\chi$  during inflation. These  $\chi$  fluctuations will translate into density perturbations because they will lead to fluctuations in the  $\phi$  decay rate during reheating. The resulting density perturbations are given by

$$\frac{\delta\rho}{\rho} = -\frac{2}{3} \frac{\delta\Gamma}{\Gamma} \quad (8)$$

Because  $\delta\Gamma \propto \delta\chi$ , the spectrum of perturbations will be set by the spectrum of  $\chi$  fluctuations.

Our main point is the following. In the presence of generic Planck scale suppressed interactions between  $\chi$  and the oscillating inflaton field, the resulting spectrum is very peculiar and exhibits a sharp cut-off at large wavelengths, which is

potentially observable. Imagine that  $\chi$  and  $\Phi$  have the following generic coupling in the potential, arising from a coupling in the superpotential:

$$\frac{\Phi^n}{M_P^{n-2}} \chi^2 \quad (9)$$

Then, the fluctuations in  $\chi$  at the beginning of inflation will be strongly suppressed, due to its high effective mass

$$\mu_{eff}^2 = \frac{\langle \Phi_{in}^n \rangle}{M_P^{n-2}} e^{-\frac{3}{2}nN} + \mu^2, \quad (10)$$

where  $\langle \Phi \rangle$  is the amplitude of oscillations and  $N$  is the number of e-foldings since the start of inflation.  $\mu$  is the “bare” mass of  $\chi$ , which for us means the full contribution to the mass arising from the the Kähler potential (generally field-dependent). We assume it to be somewhat below  $H_*$ . Fluctuations of  $\chi$  then are governed by the following equation

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + (\mu_{eff}^2 + k^2/a^2)\delta\chi_k = 0 \quad (11)$$

So  $\chi$  can only start to fluctuate after its effective mass drops below  $\mu_c \simeq H_*$ , which happens only some critical number  $N_c$  of e-foldings after the onset of inflation. Hence, density perturbations should be cut-off at large scales.

To estimate  $N_c$  recall that inflation begins when the amplitude of  $\Phi \sim M_P c^{p/2}$ , and after this point it decays as  $\langle \Phi \rangle = M_P c^{p/2} e^{-\frac{3}{2}N}$ . Thus, we find

$$e^{-N_c} \sim \left( \frac{H_*}{M_P} \right)^{\frac{4}{3n}} c^{-\frac{p}{3}} \quad (12)$$

Correspondingly the maximal wavelength beyond which perturbations will be suppressed is given by

$$k_c^{-1} \sim c^{\frac{p}{3}} \left( \frac{M_P}{H_*} \right)^{\frac{7}{6} - \frac{4}{3n}} \frac{1}{T_{today}} \quad (13)$$

Taking  $c = 1$  and  $p = 4$  (as in [1]), with  $H_* \sim 10^{-12}$  GeV (to allow TeV reheating), this is roughly the size of the present horizon ( $\sim 10^{28}$  cm) for  $n = 6$ . For the more generically expected  $n = 4$ , one would have to choose smaller values for  $H_*$  and/or  $c$  to accommodate a sufficiently small wavelength to be relevant to observations in our horizon.

It is well known that cosmic variance bounds make it difficult to improve our certainty about the significance of the observed low quadrupole and octupole in the CMB. However, the measurements of the spectral index  $n(k)$  will improve in the future. With this in mind, let us now discuss the  $k$ -dependence of the cut-off. The perturbations generated at  $N < N_c$  (that today have  $k < k_c$ ) are suppressed as

$$\delta_{N < N_c} \sim H_* e^{\frac{-2\pi\mu(t)}{H_*}} = H_* e^{-2\pi e^{-(N-N_c)\frac{3n}{4}}} \quad (14)$$

This can be simply understood: de Sitter space has a Gibbons-Hawking temperature  $\frac{H_*}{2\pi}$  [18], and (14) is just a reflection of the Boltzmann-suppressed excitations of a massive field at this temperature. Hence today we should see a sharp cut-off in the perturbation amplitude at large scales with the following  $k$ -dependence

$$\sim e^{-2\pi\left(\frac{k_c}{k}\right)^{\frac{3n}{4}}}. \quad (15)$$

This is a very sharp cut-off even for the lowest possible choice of  $n = 4$ , and implies an abrupt decline in the spectrum for  $k < k_c$ .

The fit of the observed data to a perturbation spectrum with a sharp cut-off was done in reference [13], according to which the best fit occurs for  $k_c^{-1} \sim 10^{28}\text{cm}$ . Although our cutoff is smoother than the one postulated by the authors of [13], from the existing analysis it seems difficult to distinguish between the two possibilities, and more precise studies are needed.

Because our cut-off is so sharp for  $k < k_c$ , it would be difficult to distinguish this mechanism from others which produce a sharp cutoff based on that feature alone. However, we also have a very different prediction for the behavior of the spectral index  $n(k)$  on scales  $k > k_c$ . In particular, the way the tilt is imprinted in the spectral index is very different from the standard inflationary case. In slow roll inflation, the primary source for the tilt is the fact that the Hubble parameter inevitably changes during the last 60 or so e-foldings, resulting in a subsequent change of the perturbation amplitude (this is the reason that in standard inflationary models, having  $n = 1$  is unnatural). For a thorough discussion of the perturbations in standard inflation, see e.g. [16]. In our scenario, the Hubble parameter is essentially constant throughout the inflationary period and its evolution cannot result in a tilt. A tilt is nevertheless generated during the evolution of  $\chi$ -fluctuations on superhorizon scales, due to the simple fact that different wavelengths spent different amounts of time before finally being translated into density perturbations by reheating.

After a given wavelength of  $\chi$  crosses outside the inflationary horizon (or more precisely, after  $k/a < 1/\mu$ ), the  $k^2$ -term in equation (11) becomes subdominant and the mode is almost frozen, apart from the fact that the small “bare” mass term  $\mu^2$  very slowly pushes it down. As stated above, this mass term comes from the Kähler potential, and is assumed to be somewhat below  $H_*$ . Because  $\chi$  and  $\Phi$  are most likely coupled through the Kähler potential,  $\mu$  is not in general a constant throughout the inflation, but rather undergoes some change itself.

For  $\mu^2(t) \ll H_*^2$ , the  $\delta\ddot{\chi}_k$  term in equation (11) is subdominant, and the evolution of the fluctuations on superhorizon scales is given by the following equation:

$$\delta\chi_k \propto e^{-\int_0^t \frac{\mu^2(\tau)}{3H_*^2} d\tau} \quad (16)$$

Remembering that  $k \propto e^{-H_*t}$ , this can be recast as

$$\delta\chi_k \propto k^{\frac{1}{t}} \int_0^t \frac{\mu^2(\tau)}{3H_*^2} d\tau \quad (17)$$

This implies that the spectral index is given by

$$n - 1 = \frac{1}{t} \int_0^t \frac{\mu^2(\tau)}{3H_*^2} d\tau \quad (18)$$

and for the running we have

$$\frac{dn}{d \ln k} = \frac{1}{(\ln k)^2} \int_0^t \frac{\mu^2(\tau)}{3H_*^2} d\tau + \frac{1}{\ln k} \frac{\mu^2(t)}{3H_*^2} \quad (19)$$

For the particular case of a constant  $\mu^2$  we would immediately get

$$n - 1 = \frac{\mu^2}{3H_*^2} \quad (20)$$

However, in general  $\mu(t)$  won't be constant, even after the contribution from the cross couplings in the superpotential become negligible. This is because of the cross couplings between  $\chi$  and  $\Phi$  in the Kähler potential. It follows from the standard rules for supersymmetric Lagrangians that these couplings are universally suppressed by the soft supersymmetry breaking scale (the Kähler potential couplings can only contribute in the scalar potential if the auxiliary  $F$ -component of at least one of the chiral superfields is non-zero, see the appendix for a detailed discussion). In our case the susy breaking scale is  $\sim H_*^2$ , and so the corrections to  $\mu^2$  coming from the Kähler potential can generically be parameterized as

$$\mu^2 = H_*^2 \left( \alpha_0 + \alpha_1 \ln(|\Phi|) + \alpha_2 (\ln(|\Phi|))^2 + \dots + (|\Phi|/M_P)^n + \dots \right) \quad (21)$$

where we have explicitly separated the  $\Phi$ -independent ( $\alpha_0$ -term) and the different types of  $\Phi$ -dependent contributions. The  $\alpha$ -coefficients parameterize their strength relative to the tree-level soft supersymmetry breaking measured by  $H_*$ .

Because the amplitude of  $\Phi$  decreases exponentially quickly, the power-law terms very quickly diminish after the onset of inflation, and cannot produce any observable effect. The log-terms in contrast change very slowly, and have a potentially observable late time effect. So we shall now focus on these terms. Their origin is the *perturbative* renormalization of the Kähler potential. Because, unlike the superpotential, the Kähler metric is not protected by any non-renormalization theorem, it is unnatural to assume any strong suppression of the  $\alpha$ -coefficients. We shall take their typical value to be roughly a one or two-loop factor times a number of order one. Taking into account the explicit time-dependence of the  $\Phi$ -amplitude, and using equations (18) and (19), we arrive at the following expressions for the tilt

$$n - 1 = \frac{\alpha_0}{3} + \frac{\alpha_1}{4} \ln \frac{k}{k_0} + \frac{\alpha_2}{4} \ln^2 \frac{k}{k_0} \quad (22)$$

and the running

$$\frac{dn}{d \ln k} = \frac{\alpha_1}{4} + \frac{\alpha_2}{2} \ln \frac{k}{k_0} \quad (23)$$



The  $k$ -dependence of the above cosmological parameters is rather striking and different from the known inflationary models. The precise values of the  $\alpha$ -coefficients are model dependent, but their natural values are consistent with the present WMAP data and are potentially detectable in the future. According to this data [19], for instance, for  $k_0 = 0.002 \text{ Mpc}^{-1}$ , the central value of our  $\frac{\alpha_1}{2}$  is around 0.077. This is certainly consistent with the natural assumption that  $\alpha_1$  is a one-loop factor.

## A Appendix

### A.1 Moduli potentials

Let us briefly discuss the origin of  $V(\phi)$ . According to our assumption,  $\phi$  is some supersymmetric flat direction field, and thus  $V(\phi)$  should vanish in an exact susy limit. Supersymmetry breaking effects transmitted through the Kähler potential induce a non-zero  $V(\phi)$  with a minimum at  $\phi = \phi_{true}$ . We shall distinguish two possibilities. The first is when the relevant Kähler terms come from tree-level gravity mediated supersymmetry breaking. In such a case the Kähler metric can be taken to be

$$K = \Theta^+\Theta + \Theta^+\Theta f\left(\frac{\phi}{M_P}\right) + \dots \quad (24)$$

Here  $\Theta$  is the ‘‘spurion’’ superfield whose  $F$ -term breaks supersymmetry  $\Theta_F = M^2$ , and  $f$  is some generic polynomial function

$$f = \lambda_1 \frac{\phi^2}{M_P^2} + \lambda_2 \frac{\phi^4}{M_P^4} + \dots \quad (25)$$

with  $\lambda_i \sim 1$ . After substituting the value of this  $F$ -term into (24), we generate the following effective potential for  $\phi$ , at the leading order in the expansion in powers of  $f$

$$V(\phi) \simeq -M^4 f\left(\frac{\phi}{M_P}\right) \quad (26)$$

Note the over-all minus sign in the potential relative to the Kähler potential. If  $\lambda_1 > 0$ , the potential has a maximum at  $\phi = 0$ , as we need in our inflationary scenario. Since the higher order terms only become significant for  $\phi \sim M_P$ , the minimum of the potential will be established only at some large value  $\phi_{true} \sim M_P$ .

If, however,  $\phi$  has some unsuppressed interactions with other gauge or chiral superfields, the minimum can be established at values much smaller than  $M_P$ . The reason is that the perturbative renormalization of the Kähler potential can dominate over the gravity-mediated higher corrections. The classic example of spontaneous symmetry breaking due to perturbative Kähler renormalization is Witten’s ‘‘inverse hierarchy’’ mechanism [20]. Another simple example is the flip of the sign of the gravity-mediated soft mass for  $\phi$  due to perturbative running. The effective potential

for  $\phi$  then can be approximated as

$$V(\phi) = m_{soft}^2 \phi^2 (1 + \alpha \ln(\phi/M_P)) + M_P - \text{suppressed corrections} \quad (27)$$

where  $\alpha$  is the renormalization loop factor, and  $m_{soft}^2 = -\lambda_1 M^4/M_P^2$  is the leading term in the gravity mediated contribution in (26). Choosing  $\lambda_1 < 0$ , the minimum at  $\phi = 0$  will only get destabilized after renormalization effects are included. For  $\alpha \ll 1$  the minimum then will be developed at

$$\phi_{true} \sim e^{-\frac{2}{\alpha}} M_P \quad (28)$$

This gives us a natural mechanism for designing models that have  $c < 1$ .

## A.2 Off-trail tunneling

We are assuming that for the generic bubble the initial condition of the system after tunneling is  $\Phi = \Phi_{in}, \phi = 0$ . This is well justified for the following reason. When the system sits in the false vacuum state  $\Phi = \Phi_{false} \sim M_P, \phi = 0$ , the curvature of the potential in the  $\phi$  direction is  $\sim M_P^2$ , and in the  $\Phi$  direction it is only  $\sim H^2$ . The energetically most favorable trajectory goes through the barrier, which is the maximum of the self-interaction potential of  $\Phi$ . In the generic bubble,  $\Phi$  cannot tunnel all the way to  $\Phi = 0$ , but instead will be materialized at some  $\Phi_{in} \sim M_P$  on the other side of the barrier. In the language of tunneling, this is because the relevant instanton is the Hawking-Moss instanton [17] which tunnels to the top of the barrier, not the true vacuum. In the language of stochastic inflation [21], this is because one fluctuates to the top of the hill and then rolls down, instead of fluctuating directly to the true vacuum.

Hence everywhere around the tunneling trajectory of interest  $\phi$  has a Planck scale mass. Since the energy scales involved in the tunneling dynamics are much smaller, it is obvious that  $\phi$  can be integrated out and cannot influence the tunneling dynamics. However, let us try to keep the track of  $\phi$  explicitly, and see what happens if the system instead tunnels to some point  $\Phi_{in} \sim M_P, \phi_{in} \neq 0$ . Since  $\phi$  has a Planckian mass, tunneling to such a point would be equivalent to exciting a condensate of  $\phi$  particles with an occupation number  $N_\phi \sim M_P \phi_{in}^2$ , and an energy density

$$\rho_\phi \sim M_P^2 \phi_{in}^2. \quad (29)$$

Unless  $\phi_{in} < H_*$ , the resulting energy density is bigger than the inflationary energy density, and nucleation of such a bubble will be suppressed by an additional exponential factor  $\sim \frac{\phi_{in}}{H_*}$ . On the other hand if  $\phi_{in} < H_*$ , the system will very quickly be pulled back to the point  $\phi = 0$  due to an almost instant decay of the  $\phi$ -condensate. Indeed, the decay rate of such a condensate into  $\Phi$ -particles is  $\Gamma \sim m_\phi \sim M_P$  (given the Planckian expectation value of  $\Phi$ ). This is much faster than one inverse oscillation time in the  $\Phi$ -direction (and also much faster than the expansion rate of the Universe  $H_*$ ). So  $\phi_{in}$  will decay to  $\phi = 0$  well before the system has any chance to reach the saddle point. Similar remarks obviously apply to tunneling to a configuration with  $\dot{\phi} \neq 0$ .

### A.3 Decay rate of $\Phi$

One might worry that perturbative annihilation of  $\Phi$  particles to  $\phi$  particles could lead to an earlier end of the locked inflation than the naive estimate suggests. Here we argue that in fact, these decays are negligible.

The naive perturbative decay rate  $\Phi\Phi \rightarrow \phi\phi$  is

$$\Gamma_{2\Phi \rightarrow 2\phi} \sim m_\Phi \quad (30)$$

(the dimensionless coupling is set to one). This is only applicable, however, for small oscillation amplitudes. It does not take into account the non-perturbative effect of the  $\Phi$  condensate on the  $\phi$  mass, which blocks the process. This can be understood as follows. During most of the oscillation time  $\phi$  is much heavier than  $\Phi$ , and annihilation is blocked. The process is allowed in a very narrow time interval (per each oscillation), during which  $\phi$  is lighter than  $\Phi$ . The corresponding fraction of time per oscillation when the annihilation is allowed is  $\frac{m_\Phi}{\langle\Phi\rangle}$ . Therefore to first approximation, we can model the system by saying that the annihilation rate will be suppressed to roughly

$$\Gamma_{2\Phi \rightarrow 2\phi} \sim \frac{m_\Phi^2}{\langle\Phi\rangle} \quad (31)$$

This is negligible compared to the expansion rate of the Universe  $H_*$ , and is highly inefficient.

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