

External Operators and Anomalous Dimensions in Soft-Collinear Effective Theory

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Abstract

It has recently been argued that soft-collinear effective theory for processes involving both soft and collinear partons contains a new soft-collinear mode, which can communicate between the soft and collinear sectors of the theory. The formalism incorporating the corresponding fields into the effective Lagrangian is extended to include external current and four-quark operators relevant to weak interactions. An explicit calculation of the anomalous dimensions of these operators reveals that soft-collinear modes are needed for correctly describing the ultraviolet behavior of the effective theory.

1 Introduction

Soft-collinear effective theory (SCET) [1, 2, 3, 4, 5] provides a systematic framework in which to discuss the factorization properties of exclusive B -decay amplitudes for processes in which the external hadronic states contain highly energetic, collinear partons inside light final-state mesons, and soft partons inside the initial B meson. Power counting in SCET is based on an expansion parameter $\lambda \sim \Lambda/E$, where $E \gg \Lambda_{\text{QCD}}$ is the large energy carried by collinear particles (typically $E \sim m_b$ in B decays) and $\Lambda \sim \Lambda_{\text{QCD}}$ is of order the QCD scale.

In a recent paper [6], three of us have argued that the intricate interplay between soft and collinear degrees of freedom makes it necessary to introduce modes with virtuality $E^2\lambda^3$, which have unsuppressed interactions with soft and collinear fields. In the strong-interaction sector of SCET, the leading-order couplings of these “soft-collinear” fields to soft or collinear particles can be removed using field redefinitions, leaving residual interactions that are suppressed by at least two powers of $\lambda^{1/2}$. A puzzling aspect of this analysis was the finding that soft-collinear modes have virtualities that are parametrically below the QCD scale. We argued that this is to some extent a consequence of dimensional regularization and analyticity. What matters is not the virtuality but the fact that the plus and minus components of soft-collinear momenta are commensurate with certain components of collinear or soft momenta. Yet, one might worry whether the scaling laws derived for interactions of the soft-collinear fields might be invalidated by some non-perturbative effects, thereby upsetting the power counting of the effective theory.

The goal of the present paper is to build up confidence in the new modes by showing explicitly that they are necessary to correctly reproduce the *ultraviolet* (UV) behavior of the effective theory, which has to match the scale dependence of short-distance coefficient functions derived in the matching of full-theory amplitudes onto matrix elements of SCET operators. Specifically, we compute the anomalous dimensions of the leading-order current operators containing a heavy and a collinear quark, a soft and a collinear quark, and four-quark operators obtained from combining these two currents. We find that without the inclusion of soft-collinear fields the results for the anomalous dimensions would be incorrect and violate fundamental principles of renormalization theory, such as the independence of renormalization-group (RG) functions of infrared (IR) regulators.

The explicit examples we investigate exhibit two other important features: first, in the presence of external operators such as flavor-changing currents, the soft-collinear fields can in general no longer be decoupled at leading order in λ using field redefinitions. Their effects must therefore be studied carefully in applications of SCET to exclusive B decays. This complicates proofs of QCD factorization theorems. Secondly, only the sum of soft, collinear and soft-collinear contributions to an amplitude is physically meaningful. Through the particular scaling $p_{sc}^2 \sim \Lambda^3/E$ of soft-collinear momenta the amplitude becomes sensitive to the large scale E . Part of this sensitivity has a short-distance interpretation, as reflected in the anomalous dimensions of SCET operators. However, in cases where the soft-collinear modes cannot be decoupled, amplitudes may contain additional dependence on the large scale that is of IR origin. In a strongly coupled theory such as QCD this dependence cannot be factorized using RG techniques.

2 SCET fields and interactions at leading order

The construction of an effective theory for collinear particles must account for the fact that different components of particle momenta and fields scale differently with the large scale E . To make this scaling explicit one introduces two light-like vectors n^μ and \bar{n}^μ satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$. Typically, $n^\mu = (1, 0, 0, 1)$ is chosen to be the direction of an outgoing fast hadron (or a jet of hadrons), and $\bar{n}^\mu = (1, 0, 0, -1)$ points in the opposite direction. Any 4-vector can be decomposed as

$$p^\mu = (n \cdot p) \frac{\bar{n}^\mu}{2} + (\bar{n} \cdot p) \frac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu, \quad (1)$$

where $p_\perp \cdot n = p_\perp \cdot \bar{n} = 0$. This relation defines the light-like vectors p_\pm^μ . The relevant SCET degrees of freedom describing the partons in the external hadronic states of exclusive B decays are soft and collinear, where $p_s^\mu \sim E(\lambda, \lambda, \lambda)$ for soft momenta and $p_c^\mu \sim E(\lambda^2, 1, \lambda)$ for collinear momenta. Here and below we indicate the scaling properties of the components $(n \cdot p, \bar{n} \cdot p, p_\perp)$. The corresponding effective-theory fields and their scaling relations are $h_v \sim \lambda^{3/2}$ (soft heavy quark), $q_s \sim \lambda^{3/2}$ (soft light quark), $A_s^\mu \sim (\lambda, \lambda, \lambda)$ (soft gluon), and $\xi \sim \lambda$ (collinear quark), $A_c^\mu \sim (\lambda^2, 1, \lambda)$ (collinear gluon). At leading order in power counting the effective strong-interaction Lagrangian splits up into separate Lagrangians for the soft and collinear fields. However, as mentioned above, the effective theory also contains soft-collinear quark and gluon fields, $\theta \sim \lambda^2$ and $A_{sc}^\mu \sim (\lambda^2, \lambda, \lambda^{3/2})$, which have leading-order couplings to both soft and collinear fields. The formalism incorporating these fields has been developed in [6], borrowing methods developed by Beneke and Feldmann [7]. It will be briefly reviewed here.

In interactions with other fields, the soft-collinear fields (but not the soft and collinear fields) are multipole expanded as

$$\begin{aligned} \phi_{sc}(x) &= \phi_{sc}(x_-) + x_\perp \cdot \partial_\perp \phi_{sc}(x_-) + \dots && \text{in collinear interactions,} \\ \phi_{sc}(x) &= \phi_{sc}(x_+) + x_\perp \cdot \partial_\perp \phi_{sc}(x_+) + \dots && \text{in soft interactions.} \end{aligned} \quad (2)$$

The first correction terms are of $O(\lambda^{1/2})$, and the omitted terms are of $O(\lambda)$ and higher. Soft-collinear fields can couple to soft or collinear fields without altering their scaling properties. This motivates the treatment of the soft-collinear gluon field as a background field. However, in order to preserve the scaling properties of the fields under gauge transformations one must expand the transformation laws in λ . This leads to the following set of ‘‘homogeneous’’ gauge transformations for the quark fields:

$$\begin{aligned} \text{soft:} \quad q_s(x) &\rightarrow U_s(x) q_s(x), && \text{collinear and soft-collinear fields invariant} \\ \text{collinear:} \quad \xi(x) &\rightarrow U_c(x) \xi(x), && \text{soft and soft-collinear fields invariant} \\ \text{soft-collinear:} \quad q_s(x) &\rightarrow U_{sc}(x_+) q_s(x), && \xi(x) \rightarrow U_{sc}(x_-) \xi(x), \quad q_{sc}(x) \rightarrow U_{sc}(x) q_{sc}(x) \end{aligned} \quad (3)$$

The transformation laws for gluons [6] are more complicated and are not needed for the present work.

The effective Lagrangian of SCET can be split up as

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_s + \mathcal{L}_c + \mathcal{L}_{sc} + \mathcal{L}_{\text{int}}^{(0)} + \dots, \quad (4)$$

where the dots represent power-suppressed interaction terms. The integration measure d^4x in the action $S_{\text{SCET}} = \int d^4x \mathcal{L}_{\text{SCET}}$ scales like λ^{-4} for all terms except the soft-collinear Lagrangian \mathcal{L}_{sc} , for which it scales like λ^{-6} . The first three terms above correspond to the Lagrangians of soft particles (including heavy quarks), collinear particles, and soft-collinear particles. They are given by

$$\begin{aligned} \mathcal{L}_s &= \bar{q}_s i \not{D}_s q_s + \bar{h} i v \cdot D_s h + \mathcal{L}_s^{\text{glue}}, \\ \mathcal{L}_c &= \bar{\xi} \frac{\not{n}}{2} i n \cdot D_c \xi - \bar{\xi} i \not{D}_{c\perp} \frac{\not{n}}{2} \frac{1}{i \bar{n} \cdot D_c} i \not{D}_{c\perp} \xi + \mathcal{L}_c^{\text{glue}}, \\ \mathcal{L}_{sc} &= \bar{\theta} \frac{\not{n}}{2} i n \cdot D_{sc} \theta - \bar{\theta} i \not{D}_{sc\perp} \frac{\not{n}}{2} \frac{1}{i \bar{n} \cdot D_{sc}} i \not{D}_{sc\perp} \theta + \mathcal{L}_{sc}^{\text{glue}}, \end{aligned} \quad (5)$$

where $iD_s^\mu \equiv i\partial^\mu + gA_s^\mu$ etc., and v is the velocity of the hadron containing the heavy quark. Collinear, soft-collinear, and heavy-quark fields in the effective theory are described by 2-component spinors subject to the constraints $\not{n}\xi = 0$, $\not{n}\theta = 0$, and $\not{v}h = h$. The gluon Lagrangians in the three sectors retain the same form as in full QCD, but with the gluon fields restricted to the corresponding subspaces of their soft, collinear, or soft-collinear Fourier modes. The leading-order interactions between soft-collinear fields and soft or collinear fields are given by

$$\begin{aligned} \mathcal{L}_{\text{int}}^{(0)}(x) &= \bar{q}_s(x) \frac{\not{n}}{2} g \bar{n} \cdot A_{sc}(x_+) q_s(x) + \bar{h}(x) \frac{n \cdot v}{2} g \bar{n} \cdot A_{sc}(x_+) h(x) \\ &+ \bar{\xi}(x) \frac{\not{n}}{2} g n \cdot A_{sc}(x_-) \xi(x) + \text{pure glue terms}. \end{aligned} \quad (6)$$

Momentum conservation implies that soft-collinear fields can only couple to either soft or collinear modes, but not both. More than one soft or collinear particle must be involved in such interactions. The gluon self-couplings can be derived by substituting $A_s^\mu \rightarrow A_s^\mu + \frac{1}{2} n^\mu \bar{n} \cdot A_{sc}(x_+)$ for the gluon field in the soft Yang–Mills Lagrangian and $A_c^\mu \rightarrow A_c^\mu + \frac{1}{2} \bar{n}^\mu n \cdot A_{sc}(x_-)$ for the gluon field in the collinear Yang–Mills Lagrangian, and isolating terms containing the soft-collinear field. The precise form of these interactions will not be relevant to our discussion. Finally, let us note that none of the terms in the SCET Lagrangian (4) is renormalized beyond the usual renormalization of the strong coupling and the fields.

From (6) one can readily read off the Feynman rules for the couplings of soft-collinear gluons to soft or collinear quarks. The multipole expansion of the soft-collinear fields implies that momentum is *not* conserved at these vertices. When a soft (light or heavy) quark with momentum p_s absorbs a soft-collinear gluon with momentum k , the outgoing soft quark carries momentum $p_s + k_-$. Likewise, when a collinear quark with momentum p_c absorbs a soft-collinear gluon with momentum k , the outgoing collinear quark carries momentum $p_c + k_+$.

In order to match the quark and gluon fields of the full theory onto SCET fields obeying the homogeneous gauge transformations one first adopts specific gauges in the soft and collinear sectors, namely soft light-cone gauge $n \cdot A_s = 0$ (SLCG) and collinear light-cone gauge $\bar{n} \cdot A_c = 0$ (CLCG). At leading order in λ , one then introduces the corresponding SCET fields via the substitutions [7]

$$\psi_s|_{\text{SLCG}} \rightarrow R_s S_s^\dagger q_s, \quad b|_{\text{SLCG}} \rightarrow R_s S_s^\dagger h, \quad \psi_c|_{\text{CLCG}} \rightarrow R_c W_c^\dagger \xi. \quad (7)$$

The corresponding replacements for gluon fields can be found in [6]. The quantities

$$\begin{aligned} S_s(x) &= \text{P exp} \left(ig \int_{-\infty}^0 dt n \cdot A_s(x + tn) \right), \\ W_c(x) &= \text{P exp} \left(ig \int_{-\infty}^0 dt \bar{n} \cdot A_c(x + t\bar{n}) \right) \end{aligned} \quad (8)$$

are the familiar SCET Wilson lines in the soft and collinear sectors [2, 4], which effectively put the SCET fields into light-cone gauge. The objects R_s and R_c are short gauge strings of soft-collinear fields from x_+ to x (for R_s) and x_- to x (for R_c). They differ from 1 by terms of order $\lambda^{1/2}$ and so must be Taylor expanded. Note that S_s transforms as $S_s(x) \rightarrow U_s(x) S_s(x)$ and $S_s(x) \rightarrow U_{sc}(x_+) S_s(x) U_{sc}^\dagger(x_+)$ under soft and soft-collinear gauge transformations and is invariant under collinear gauge transformations. Likewise, W_c transforms as $W_c(x) \rightarrow U_c(x) W_c(x)$ and $W_c(x) \rightarrow U_{sc}(x_-) W_c(x) U_{sc}^\dagger(x_-)$ under collinear and soft-collinear gauge transformations and is invariant under soft gauge transformations. The short strings only transform under soft-collinear gauge transformations, in such a way that $R_s(x) \rightarrow U_{sc}(x) R_s(x) U_{sc}^\dagger(x_+)$ and $R_c(x) \rightarrow U_{sc}(x) R_c(x) U_{sc}^\dagger(x_-)$. It follows that the expressions on the right-hand side of (7) are invariant under soft and collinear gauge transformations and transform as ordinary QCD quark fields under soft-collinear gauge transformations.

3 Soft-collinear current operators

Flavor-changing currents and four-quark operators containing soft and collinear fields play an important role in many applications of SCET to exclusive B decays. The simplest example is that of a current $\bar{\psi}_c \Gamma b$ transforming a heavy quark into a collinear one, where Γ denotes an arbitrary Dirac structure. This current has been studied in detail in [1, 4] in another version of SCET, which contains only hard-collinear and soft fields.¹ Based on the discussion of the previous section, it follows that at tree level (and at leading power in λ) the QCD current is matched onto the following gauge-invariant object in SCET:

$$\begin{aligned} \bar{\psi}_c(x) \Gamma b(x) &\rightarrow e^{-im_b v \cdot x} [\bar{\xi} W_c R_c^\dagger](x) \Gamma [R_s S_s^\dagger h](x) \\ &= e^{-im_b v \cdot x} [\bar{\xi} W_c](x_+ + x_\perp) \Gamma [S_s^\dagger h](x_- + x_\perp) + O(\lambda), \end{aligned} \quad (9)$$

¹This theory is sometimes called SCET_I, and its degrees of freedom are often called collinear and ultra-soft. The effective theory considered in the present paper is also called SCET_{II}.

where the phase factor arises from the definition of the field h in HQET [8]. Note that the expression in the first line is not homogeneous in λ . In interactions of soft and collinear fields, the soft fields must be multipole expanded about $x_+ = 0$, while the collinear fields must be multipole expanded about $x_- = 0$. Also, as mentioned above, the quantities R_s and R_c must be expanded and equal 1 to first order. This leads to the result shown in the second line. The terms of $O(\lambda^{1/2})$ in the expansions of R_s and R_c cancel each other. The leading-order SCET current in the final expression is gauge invariant even without the R_s and R_c factors, since according to (3) soft fields at $x_+ = 0$ and collinear fields at $x_- = 0$ both transform with $U_{sc}(0)$ under soft-collinear gauge transformations. An analogous matching relation can be written for a soft-collinear current containing a light soft quark,

$$\bar{\psi}_c(x) \Gamma \psi_s(x) \rightarrow [\bar{\xi} W_c](x_+ + x_\perp) \Gamma [S_s^\dagger q_s](x_- + x_\perp) + O(\lambda). \quad (10)$$

Depending on the Dirac structure Γ , another operator containing an additional perpendicular collinear gluon field can appear in this case (even at tree level) [5]. We will not discuss such operators in the present paper.

When radiative corrections are taken into account, the currents in (9) and (10) mix with analogous operators at different positions on the light cone, and for the case of the heavy-collinear current different Dirac structures can be induced by hard gluon exchange. The correct matching relations are (setting $x = 0$ for simplicity) [4, 5]

$$\begin{aligned} \bar{\psi}_c(0) \Gamma b(0) &\rightarrow \sum_i \int ds \tilde{C}_i(s, \mu) [\bar{\xi} W_c](s\bar{n}) \Gamma_i [S_s^\dagger h](0) + O(\lambda), \\ \bar{\psi}_c(0) \Gamma \psi_s(0) &\rightarrow \int dsdt \tilde{D}(s, t, \mu) [\bar{\xi} W_c](s\bar{n}) \Gamma [S_s^\dagger q_s](tn) + O(\lambda). \end{aligned} \quad (11)$$

Translational invariance can be used to rewrite these relations in the local form

$$\begin{aligned} [\bar{\psi}_c \Gamma b](0) &\rightarrow \sum_i C_i(v \cdot P_-^c, \mu) [\bar{\xi} W_c \Gamma_i S_s^\dagger h](0) + O(\lambda), \\ [\bar{\psi}_c \Gamma \psi_s](0) &\rightarrow D(P_+^s \cdot P_-^c, \mu) [\bar{\xi} W_c \Gamma S_s^\dagger q_s](0) + O(\lambda), \end{aligned} \quad (12)$$

where

$$\begin{aligned} C_i(v \cdot P_-^c, \mu) &= \int ds \tilde{C}_i(s, \mu) e^{is\bar{n} \cdot P^c}, \\ D(P_+^s \cdot P_-^c, \mu) &= \int dsdt \tilde{D}(s, t, \mu) e^{i(s\bar{n} \cdot P^c - tn \cdot P^s)} \end{aligned} \quad (13)$$

are the Fourier transforms of the position-space Wilson coefficients. These are operator-valued coefficient functions, which depend on the momentum operators $P^c = P_{\text{out}}^c - P_{\text{in}}^c$ and $P^s = P_{\text{in}}^s - P_{\text{out}}^s$ acting on collinear and soft states. Invariance of the results under simultaneous rescalings $n \rightarrow n/\alpha$ and $\bar{n} \rightarrow \alpha\bar{n}$ of the light-cone basis vectors dictates that the momentum-space Wilson coefficients can only depend on the scalar products $2v \cdot P_-^c = (v \cdot n)(P^c \cdot \bar{n})$ and $2P_+^s \cdot P_-^c = (P^s \cdot n)(P^c \cdot \bar{n})$.

The momentum-space coefficient functions are renormalized multiplicatively and obey RG equations of the Sudakov type [1, 9],

$$\begin{aligned}\frac{d}{d \ln \mu} C_i(v \cdot p_{c-}, \mu) &= \gamma_{\xi h}(v \cdot p_{c-}, \mu) C_i(v \cdot p_{c-}, \mu), \\ \frac{d}{d \ln \mu} D(p_{s+} \cdot p_{c-}, \mu) &= \gamma_{\xi q}(p_{s+} \cdot p_{c-}, \mu) D(p_{s+} \cdot p_{c-}, \mu),\end{aligned}\tag{14}$$

where the anomalous dimensions take the form

$$\begin{aligned}\gamma_{\xi h}(v \cdot p_{c-}, \mu) &= -\frac{1}{2} \Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\mu^2}{(2v \cdot p_{c-})^2} + \Gamma_{\xi h}[\alpha_s(\mu)], \\ \gamma_{\xi q}(p_{s+} \cdot p_{c-}, \mu) &= -\Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\mu^2}{2p_{s+} \cdot p_{c-}} + \Gamma_{\xi q}[\alpha_s(\mu)].\end{aligned}\tag{15}$$

The coefficients of the logarithmic terms are determined in terms of the universal cusp anomalous dimension $\Gamma_{\text{cusp}} = C_F \alpha_s / \pi + O(\alpha_s^2)$, which plays a central role in the renormalization of Wilson lines with light-like segments [10]. In Section 5 we will show why the cusp anomalous dimension enters the above equation with a negative sign, and why an additional factor of 1/2 appears in the anomalous dimension of the heavy-collinear current.

The one-loop expressions for the non-logarithmic terms in the anomalous dimensions can be deduced from the explicit results for the Wilson coefficients derived in [1, 5].² They are

$$\Gamma_{\xi h}(\alpha_s) = -\frac{5}{4} \frac{C_F \alpha_s}{\pi} + O(\alpha_s^2), \quad \Gamma_{\xi q}(\alpha_s) = -\frac{3}{2} \frac{C_F \alpha_s}{\pi} + O(\alpha_s^2).\tag{16}$$

It may seem surprising that after hard and hard-collinear scales have been integrated out the operators of the low-energy theory still know about the large scales $v \cdot p_{c-} \sim E$ and $p_{s+} \cdot p_{c-} \sim E\Lambda$, as is evident from the appearance of the logarithms in (15). The reason is that in interactions involving both soft and collinear particles there is a large Lorentz boost $\gamma \sim p_s \cdot p_c / \sqrt{p_s^2 p_c^2} \sim E/\Lambda$ connecting the rest frames of soft and collinear hadrons, which is fixed by external kinematics and enters the effective theory as a parameter. This is similar to applications of heavy-quark effective theory to $b \rightarrow c$ transitions, where the fields depend on the external velocities of the hadrons containing the heavy quarks, and $\gamma = v_b \cdot v_c = O(1)$ is an external parameter that appears in matrix elements and anomalous dimensions of velocity-changing current operators [8, 11].

We will now explain how the results for the anomalous dimensions can be obtained from a calculation of UV poles of SCET loop diagrams. The relevant diagrams needed at one-loop order are shown in Figure 1. They must be supplemented by wave-function renormalization of the quark fields. The gluons connected to the current are part of the Wilson lines W_c and

²The Wilson coefficients $C_i(v \cdot p_{c-}, \mu)$ were computed in [1] using the effective theory SCET_I, in which the scaling of the collinear quark field relative to the heavy-quark field is different from that in the theory SCET_{II} considered here. The Wilson coefficients are the same in the two theories because they are independent of p_c^2 , and so the scaling of the collinear fields does not matter.

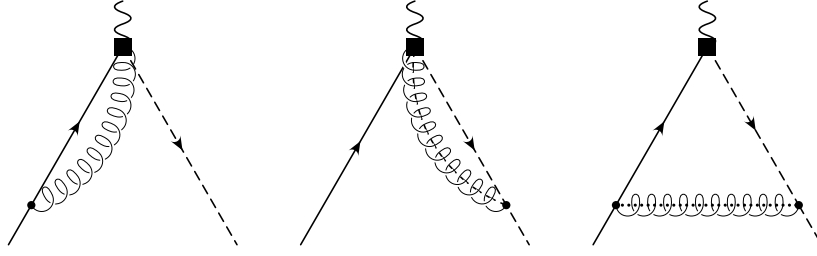


Figure 1: SCET graphs contributing to the anomalous dimension of a soft-collinear current. Full lines denote soft fields, dashed lines collinear fields, and dotted lines soft-collinear fields.

S_s . We regularize IR singularities by keeping the external lines off-shell. The results for the sum of all UV poles must be independent of the IR regulators. For the heavy-collinear current the pole terms obtained from the three diagrams are (here and below we omit the $-i0$ in the arguments of logarithms)

$$\begin{aligned} & \left(\frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{-2v \cdot p_s}{\mu} + \frac{1}{\epsilon} \right) + \left(\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{-p_c^2}{\mu^2} + \frac{3}{2\epsilon} \right) + \left(-\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{(-2v \cdot p_s)(-p_c^2)}{2v \cdot p_{c-} \mu^2} \right) \\ & = \frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu}{2v \cdot p_{c-}} + \frac{5}{2\epsilon}, \end{aligned} \quad (17)$$

while for the current containing a light soft quark we obtain

$$\begin{aligned} & \left(\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{-p_s^2}{\mu^2} + \frac{3}{2\epsilon} \right) + \left(\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{-p_c^2}{\mu^2} + \frac{3}{2\epsilon} \right) + \left(-\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{(-p_s^2)(-p_c^2)}{2p_{s+} \cdot p_{c-} \mu^2} \right) \\ & = \frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu^2}{2p_{s+} \cdot p_{c-}} + \frac{3}{\epsilon}. \end{aligned} \quad (18)$$

We quote the contributions to the operator renormalization constants Z^{-1} in units of $C_F \alpha_s / 4\pi$ (in the $\overline{\text{MS}}$ subtraction scheme with $D = 4 - 2\epsilon$). The three parentheses in the first line of the above equations correspond to the soft, collinear, and soft-collinear contributions, where the first two terms include the corresponding contributions from wave-function renormalization. Note that the $1/\epsilon$ poles of the soft and collinear graphs depend on the IR regulators, but that this dependence is precisely canceled by the soft-collinear contribution. By construction, the sum of the soft, collinear, and soft-collinear contributions is IR finite and only contains UV poles, whose coefficients depend on the ratios $v \cdot p_{c-}/\mu$ and $p_{s+} \cdot p_{c-}/\mu^2$. This follows since IR divergences in both the full and the effective theory (which are equivalent at low energy) are regularized by the off-shellness of the external quark lines. The one-loop contributions to the anomalous dimensions $\gamma_{\xi h}$ and $\gamma_{\xi q}$ are given by $-C_F \alpha_s / 2\pi$ times the coefficients of the $1/\epsilon$ poles in the above expressions. They are in agreement with the results (15) and (16) obtained from the scale dependence of Wilson coefficients.

The calculations presented above make it evident that there is an intricate interplay between the soft, collinear, and soft-collinear diagrams. In dimensional regularization the de-

pendence of the anomalous dimensions on the hard or hard-collinear scale enters through the loop integral involving the soft-collinear exchange and thus seems to be related to very small momentum scales. However, care must be taken when assigning physical significance to the scales associated with individual diagrams in SCET, because in the soft and collinear diagrams a cancellation of IR and UV divergences takes place. The logarithms appearing in their divergent parts should be interpreted as [cf. (18)]

$$\ln \frac{-p_s^2}{\mu^2} + \ln \frac{-p_c^2}{\mu^2} = \ln \frac{Q^2}{\mu^2} + \ln \frac{m_{sc}^2}{\mu^2}, \quad \text{with} \quad m_{sc}^2 = \frac{(-p_s^2)(-p_c^2)}{2p_{s+} \cdot p_{c-}}, \quad (19)$$

and thus arise from a cancellation of physics at the hard scale $Q^2 = 2p_{s+} \cdot p_{c-}$ and at the soft-collinear scale m_{sc}^2 . In the sum of all graphs, the soft-collinear contribution precisely cancels the IR piece of the soft and collinear parts, see (18). This interpretation is consistent with the fact that the anomalous dimensions measure the change of operator matrix elements under infinitesimal variations of the UV cutoff μ . They are therefore insensitive to the physics at low scales by construction, and the large logarithms in (15) are really of short-distance nature.

On the other hand, in the sum of the finite terms of the diagrams in Figure 1 (corresponding to SCET matrix elements) logarithms of the soft-collinear scale remain, which do not have an interpretation in terms of RG logarithms. In a weakly coupled theory such as QED, the large logarithms $\ln(\mu^2/m_{sc}^2) \sim \ln(\mu^2/(\Lambda^3/E))$ can be summed by matching SCET onto another effective theory, in which soft and collinear fields are integrated out and only soft-collinear fields remain as dynamical degrees of freedom, and by solving RG equations in this final theory. This is analogous to the evolution equations for the off-shell Sudakov form factor discussed in [12, 13], where large logarithms arise from two-stage evolution between the scales Q^2 to M^2 and M^2 to M^4/Q^2 . As argued in [6], the case of the current $\bar{\psi}_c \Gamma \psi_s$ can be mapped onto the Sudakov problem, such that Q^2 corresponds to the hard-collinear scale, M^2 corresponds to the QCD scale, and M^4/Q^2 corresponds to the soft-collinear scale. Since QCD is strongly coupled for scales of order Λ and below the second stage of running cannot be performed perturbatively, i.e., the low-energy hadronic matrix elements in SCET may contain a dependence on the small ratio Λ/E that cannot be factorized into a short-distance coefficient.

The observation that the soft-collinear contribution supplies a logarithm of a short-distance scale solves the following puzzle about soft-collinear current operators in SCET (which has confused some of the present authors for a considerable amount of time): We know from the explicit expressions for the Wilson coefficients of the currents that their anomalous dimensions must depend on a scalar product of the collinear momentum with a momentum characterizing the soft quark (i.e., $v \cdot p_{c-}$ or $p_{s+} \cdot p_{c-}$). However, the SCET Feynman rules imply that the first graph in Figure 1 can only be a function of the soft momentum (i.e., $v \cdot p_s$ or p_s^2), while the second one can only depend on the collinear momentum (i.e., p_c^2), as is in fact confirmed by our explicit calculation. The apparent “factorization” of soft and collinear degrees of freedom in SCET (in the absence of soft-collinear fields) would thus lead to the conclusion that the anomalous dimensions of the currents are independent of the products $v \cdot p_{c-}$ or $p_{s+} \cdot p_{c-}$, in contradiction with the results for the Wilson coefficients. It is possible to obtain the correct result for the anomalous dimensions by using a regularization scheme that breaks this factorization property; however, unavoidably this means that the regulator cannot preserve the

symmetries of the Lagrangian of the effective theory. (For instance, it is possible to suppress the soft-collinear contribution in one-loop graphs by putting the external lines on shell and using a different IR regulator such as a gluon mass, in which case the last diagram in Figure 1 vanishes in dimensional regularization, whereas the first two diagrams give expressions that cannot be regularized dimensionally. To give meaning to these expressions one may introduce additional analytic regulators, which break factorization and gauge invariance.) The formalism employing soft-collinear fields provides an elegant solution to this problem by making the non-factorization of the soft and collinear sectors of SCET explicit at the level of the Lagrangian, avoiding any subtleties related to regularization.

4 Four-quark operators

In many cases relevant to B physics, amplitudes calculated in SCET receive contributions from hadronic matrix elements of four-quark operators, which can be expressed in terms of the leading-order light-cone distribution amplitudes (LCDAs) of a light final-state meson and of the initial state B meson. An example is the hard spectator term in the QCD factorization formula for the exclusive decay $B \rightarrow K^* \gamma$ [14, 15]. The relevant SCET operators can be taken as [5]

$$\begin{aligned}
Q_{(C)}(s, t) &= [\bar{\xi} W_c](s\bar{n}) \frac{\not{n}}{2} \Gamma_1 T_1 [W_c^\dagger \xi](0) [\bar{q}_s S_s](tn) \frac{\not{n}}{2} \Gamma_2 T_2 [S_s^\dagger h](0) \\
&\equiv \int_0^\infty d\omega e^{-i\omega t} \int_0^{\bar{n}\cdot P} d\sigma e^{i\sigma s} Q_{(C)}(\omega, \sigma), \tag{20}
\end{aligned}$$

where the color label $C = S$ or O refers to the color singlet-singlet and color octet-octet structures $T_1 \otimes T_2 = \mathbf{1} \otimes \mathbf{1}$ or $T_A \otimes T_A$, respectively. The quantity $\bar{n}\cdot P$ is the total momentum carried by all collinear particles, which is fixed by kinematics. (Strictly speaking, this is a momentum operator.) The matrices Γ_i represent any of the Dirac basis matrices. Between two collinear fields only the three possibilities $\Gamma_1 = 1, \gamma_5, \gamma_\perp^\mu$ are allowed, whereas Γ_2 is not constrained. The factor $\not{n}/2$ between the fields \bar{q}_s and h ensures that the B -meson matrix element of the soft-quark current can be expressed in terms of the leading-order B -meson LCDA $\phi_+^B(\omega, \mu)$ [16]. In light-cone gauge, $\omega = n\cdot p_s$ corresponds to the plus component of the momentum of the spectator anti-quark in the B meson, while $\sigma = \bar{n}\cdot p_\xi$ denotes the minus component of the momentum of the quark inside a light final-state meson. It is conventional to introduce a dimensionless variable $u = \sigma/\bar{n}\cdot P \in [0, 1]$ corresponding to the longitudinal momentum fraction carried by the quark.

The momentum-space operators $Q_{(C)}(\omega, \sigma)$ obey the integro-differential RG equation

$$\frac{d}{d \ln \mu} Q_{(C)}(\omega, u \bar{n}\cdot P) = - \int_0^\infty d\omega' \int_0^1 du' \gamma_{(C)}(\omega, \omega', u, u', \bar{n}\cdot P, \mu) Q_{(C)}(\omega', u' \bar{n}\cdot P). \tag{21}$$

To obtain the anomalous dimensions at leading order we compute the $1/\epsilon$ poles of the diagrams shown in Figure 2 in dimensional regularization and add the contributions from wave-function

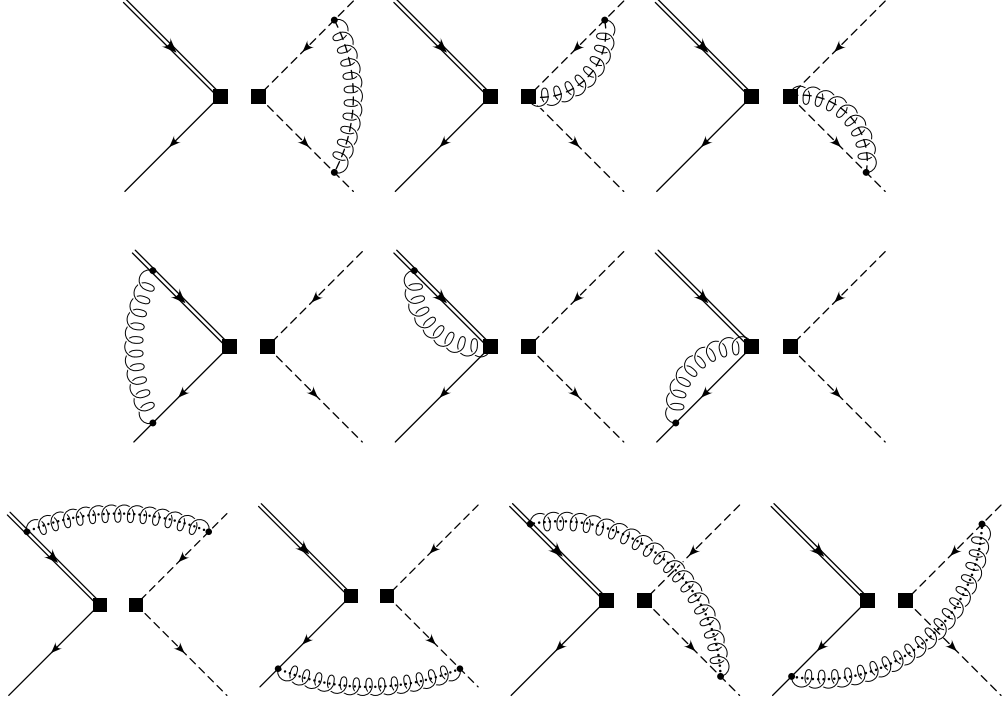


Figure 2: SCET graphs contributing to the anomalous dimension of the four-quark operators $Q_{(C)}(\omega, \sigma)$. Full lines denote soft fields, dashed lines collinear fields, and dotted lines soft-collinear fields.

renormalization. Note that only soft-collinear gluons can be exchanged between the soft and collinear currents. For the color-singlet case $T_1 \otimes T_2 = \mathbf{1} \otimes \mathbf{1}$ we find that the sum of the four diagrams with soft-collinear exchanges (but not each diagram separately) is UV finite. The anomalous dimension is then a combination of the anomalous dimensions for the two non-local currents in (20). At one-loop order we obtain

$$\gamma_{(S)}(\omega, \omega', u, u', \bar{n} \cdot P, \mu) = \frac{C_F \alpha_s}{\pi} \left[\delta(\omega - \omega') V(u, u') + \delta(u - u') H(\omega, \omega', \mu) \right], \quad (22)$$

where (with $\bar{u} \equiv 1 - u$)

$$\begin{aligned} V(u, u') = & - \left[\frac{u}{u'} \left(\frac{1}{u' - u} + c(\Gamma_1) \right) \theta(u' - u) + \frac{\bar{u}}{\bar{u}'} \left(\frac{1}{u - u'} + c(\Gamma_1) \right) \theta(u - u') \right]_+ \\ & + \frac{1 - c(\Gamma_1)}{2} \delta(u - u') \end{aligned} \quad (23)$$

with $c(1) = c(\gamma_5) = 1$, $c(\gamma_\perp^\mu) = 0$ is the Brodsky–Lepage kernel [17] for the evolution of the leading-twist LCDA of a light meson, which we have reproduced here using the Feynman rules of SCET. The plus distribution is defined as

$$[f(u, u')]_+ = f(u, u') - \delta(u - u') \int_0^1 dw f(w, u'), \quad (24)$$

which coincides with the conventional definition if the distribution acts on functions $g(u)$ but not if it acts on functions $g(u')$. The function

$$H(\omega, \omega', \mu) = \left(\ln \frac{\mu v \cdot n}{\omega} - \frac{5}{4} \right) \delta(\omega - \omega') - \omega \left[\frac{\theta(\omega - \omega')}{\omega(\omega - \omega')} + \frac{\theta(\omega' - \omega)}{\omega'(\omega' - \omega)} \right]_+ \quad (25)$$

is the analogous kernel governing the evolution of the leading-order B -meson LCDA [18]. Here the plus distribution is symmetric in the two arguments and defined as

$$\int_0^\infty d\omega' [f(\omega, \omega')]_+ g(\omega') = \int_0^\infty d\omega' f(\omega, \omega') [g(\omega') - g(\omega)]. \quad (26)$$

For the color-octet case $T_1 \otimes T_2 = T_A \otimes T_A$ things are more complicated. In this case the diagrams in the first two lines of Figure 2 reproduce the singlet anomalous dimension except for a different overall color factor, but in addition these graphs contain $1/\epsilon$ poles that depend on the IR regulators. In units of the tree-level matrix element $\langle Q_{(O)}(\omega, \sigma) \rangle$, the extra terms are

$$\frac{N}{2} \frac{\alpha_s}{2\pi} \left\{ \left(\frac{3}{2\epsilon^2} - \frac{1}{\epsilon} \ln \frac{-2v \cdot l_h}{\mu} - \frac{1}{\epsilon} \ln \frac{-l_q^2}{\mu} + \frac{5}{4\epsilon} \right) + \left(\frac{2}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{-p_\xi^2}{\mu^2} - \frac{1}{\epsilon} \ln \frac{-p_{\bar{\xi}}^2}{\mu^2} + \frac{3}{2\epsilon} \right) \right\}, \quad (27)$$

where l_i are the incoming soft momenta, and p_i denote the outgoing collinear momenta. The first parenthesis shows the soft contribution, while the second one gives the collinear contribution. In addition, in the color-octet case the sum of the soft-collinear exchange graphs shown in the last line in Figure 2 does not vanish, but adds up to

$$-\frac{N}{2} \frac{\alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{(-2v \cdot l_h)(-l_q^2)(-p_\xi^2)(-p_{\bar{\xi}}^2)}{(n \cdot v)(n \cdot l_q)(\bar{n} \cdot p_\xi)(\bar{n} \cdot p_{\bar{\xi}}) \mu^4} \right). \quad (28)$$

In the sum of the two terms (27) and (28) the dependence on the IR regulators drops out. Our final result for the anomalous dimension in the octet case is

$$\begin{aligned} \gamma_{(O)}(\omega, \omega', u, u', \bar{n} \cdot P, \mu) &= -\frac{1}{2N} \frac{\alpha_s}{\pi} \left[\delta(\omega - \omega') V(u, u') + \delta(u - u') H(\omega, \omega', \mu) \right] \\ &\quad - \frac{N}{2} \frac{\alpha_s}{\pi} \delta(\omega - \omega') \delta(u - u') \left(\ln \frac{\mu^3}{n \cdot v \omega (\bar{n} \cdot P)^2} - \ln u \bar{u} + \frac{11}{4} \right). \end{aligned} \quad (29)$$

5 Decoupling transformation

The leading-order interactions between soft-collinear fields and soft or collinear fields in the SCET Lagrangian (4) can be removed by a redefinition of the soft and collinear fields [6]. In analogy with the decoupling of ultra-soft gluons in SCET_I [2], we define new fields

$$\begin{aligned} q_s(x) &= W_{sc}(x_+) q_s^{(0)}(x), & h(x) &= W_{sc}(x_+) h^{(0)}(x), & \xi(x) &= S_{sc}(x_-) \xi^{(0)}(x), \\ A_s^\mu(x) &= W_{sc}(x_+) A_s^{(0)\mu}(x) W_{sc}^\dagger(x_+), & A_c^\mu(x) &= S_{sc}(x_-) A_c^{(0)\mu}(x) S_{sc}^\dagger(x_-). \end{aligned} \quad (30)$$

The quantities W_{sc} and S_{sc} are yet another set of Wilson lines. They are defined in analogy with W_c and S_s in (8), however with the gluon fields replaced by soft-collinear gluon fields in both cases. These objects are invariant under soft and collinear gauge transformations, while under a soft-collinear gauge transformation they transform as

$$W_{sc}(x_+) \rightarrow U_{sc}(x_+) W_{sc}(x_+), \quad S_{sc}(x_-) \rightarrow U_{sc}(x_-) S_{sc}(x_-). \quad (31)$$

Consequently, the new fields with “(0)” superscripts are invariant under soft-collinear gauge transformations. When they are introduced in the SCET Lagrangian the terms \mathcal{L}_s , \mathcal{L}_c , and \mathcal{L}_{sc} retain their original form, while the leading-order interaction Lagrangian $\mathcal{L}_{\text{int}}^{(0)}$ vanishes. Residual interactions between soft-collinear and soft or collinear fields start at $O(\lambda)$ [6]. After the field redefinition it is convenient to introduce the gauge-invariant building blocks [5]

$$\begin{aligned} \mathcal{Q}_s(x) &= S_s^{(0)\dagger}(x) q_s^{(0)}(x) = W_{sc}^\dagger(x_+) S_s^\dagger(x) q_s(x), \\ \mathcal{H}(x) &= S_s^{(0)\dagger}(x) h^{(0)}(x) = W_{sc}^\dagger(x_+) S_s^\dagger(x) h(x), \\ \mathcal{X}(x) &= W_c^{(0)\dagger}(x) \xi^{(0)}(x) = S_{sc}^\dagger(x_-) W_c^\dagger(x) \xi(x), \end{aligned} \quad (32)$$

which are invariant under all three types of gauge transformations.

The fact that interactions of soft-collinear fields with other fields can be decoupled from the strong-interaction Lagrangian does not necessarily imply that these fields can be ignored at leading order in power counting. The question is whether the decoupling transformation (30) leaves external operators such as weak-interaction currents invariant. The analysis of the previous sections indicates that in some cases the soft-collinear exchange graphs contribute to the calculation of the anomalous dimensions. Let us then study what happens when the decoupling transformation is applied to the various types of operators.

Under the transformation (30), the soft-collinear currents in (9) and (10) transform into (setting $x_\perp = 0$ for simplicity)

$$\begin{aligned} [\bar{\xi} W_c](x_+) \Gamma [S_s^\dagger h](x_-) &\rightarrow \bar{\mathcal{X}}(x_+) S_{sc}^\dagger(0) \Gamma W_{sc}(0) \mathcal{H}(x_-), \\ [\bar{\xi} W_c](x_+) \Gamma [S_s^\dagger q_s](x_-) &\rightarrow \bar{\mathcal{X}}(x_+) S_{sc}^\dagger(0) \Gamma W_{sc}(0) \mathcal{Q}_s(x_-). \end{aligned} \quad (33)$$

We observe that the soft-collinear fields do not decouple from these currents but rather form a light-like Wilson loop with a cusp at $x = 0$. The anomalous dimension of the combination $S_{sc}^\dagger W_{sc}$ is the universal cusp anomalous dimension times a logarithm of the soft-collinear scale, see the last terms in the first lines in (17) and (18). After adding the contributions from the soft and collinear sectors, the dependence on the IR regulators drops out. However, the coefficient of the logarithm of $v \cdot p_{c-}$ in the heavy-collinear current and $p_{s+} \cdot p_{c-}$ in the soft-collinear current is unchanged, since both the soft and the collinear part are independent of these large scales. This cancellation also explains why the anomalous dimensions of the soft-collinear and heavy-collinear currents involve $-\Gamma_{\text{cusp}}$ and $-\frac{1}{2}\Gamma_{\text{cusp}}$, respectively:

$$\begin{aligned} \Gamma_{\text{cusp}} \left[\ln \frac{2p_{s+} \cdot p_{c-} \mu^2}{(-p_s^2)(-p_c^2)} + \ln \frac{-p_s^2}{\mu^2} + \ln \frac{-p_c^2}{\mu^2} \right] &= -\Gamma_{\text{cusp}} \ln \frac{\mu^2}{2p_{s+} \cdot p_{c-}}, \\ \Gamma_{\text{cusp}} \left[\ln \frac{2v \cdot p_{c-} \mu^2}{(-2v \cdot p_s)(-p_c^2)} + \ln \frac{-2v \cdot p_s}{\mu} + \ln \frac{-p_c^2}{\mu^2} \right] &= -\frac{1}{2} \Gamma_{\text{cusp}} \ln \frac{\mu^2}{(2v \cdot p_{c-})^2}. \end{aligned} \quad (34)$$

Similar arguments were used by Korchemsky in his analysis of the off-shell Sudakov form factor [12].

The effect of the field redefinition (30) on the four-quark operators is different. The color singlet-singlet operator is invariant, namely (setting $x = 0$ for simplicity)

$$Q_{(S)}(s, t) \rightarrow \tilde{\mathcal{X}}(s\bar{n}) \Gamma_1 \mathcal{X}(0) \bar{\mathcal{Q}}_s(tn) \Gamma_2 \mathcal{H}(0), \quad (35)$$

since the additional soft-collinear Wilson lines come in pairs $W_{sc}^\dagger W_{sc} = 1$ and $S_{sc}^\dagger S_{sc} = 1$. The color octet-octet operator is however not invariant. It transforms into

$$Q_{(O)}(s, t) \rightarrow \tilde{\mathcal{X}}(s\bar{n}) \Gamma_1 [S_{sc}^\dagger T_A S_{sc}](0) \mathcal{X}(0) \bar{\mathcal{Q}}_s(tn) \Gamma_2 [W_{sc}^\dagger T_A W_{sc}](0) \mathcal{H}(0). \quad (36)$$

Because the objects $W_{sc}^\dagger T_A W_{sc}$ and $S_{sc}^\dagger T_A S_{sc}$ are pure color octets, the result can be rewritten as

$$Q_{(O)}(s, t) \rightarrow c_{AB}[A_{sc}] \tilde{\mathcal{X}}(s\bar{n}) \Gamma_1 T_A \mathcal{X}(0) \bar{\mathcal{Q}}_s(tn) \Gamma_2 T_B \mathcal{H}(0), \quad (37)$$

where

$$c_{AB}[A_{sc}] = 2 \text{Tr}[S_{sc} T_A S_{sc}^\dagger W_{sc} T_B W_{sc}^\dagger](0) \quad (38)$$

is a functional of the soft-collinear gluon field. Since after the decoupling transformation the SCET Lagrangian no longer contains leading-order interactions between soft-collinear and soft or collinear fields, it follows that at leading power the soft, collinear, and soft-collinear parts of the current operators in (35) and (37) only interact among themselves. The color structure of the color-singlet and color-octet currents built up of soft or collinear fields are preserved in these interactions. Hence, both types of four-quark operators are multiplicatively (in the convolution sense) renormalized in the effective theory – unlike in full QCD, where they mix under renormalization.

The presence of the functional $c_{AB}[A_{sc}]$ in the octet case explains why soft-collinear modes give a non-zero contribution to the anomalous dimension of the operator $Q_{(O)}$. However, since this operator does not mix into the singlet-singlet operator $Q_{(S)}$, this effect does not propagate into physical decay amplitudes (as hadronic matrix elements of color-octet currents vanish). The decoupling of soft-collinear fields from the color singlet-singlet operator implies that, to all orders in perturbation theory, the anomalous dimension of the four-quark operator $Q_{(S)}$ is the sum of the anomalous dimensions of the two currents $\tilde{\mathcal{X}}(s\bar{n}) \Gamma_1 \mathcal{X}(0)$ and $\bar{\mathcal{Q}}_s(tn) \Gamma_2 \mathcal{H}(0)$, in accordance with the one-loop result obtained in the previous section. This observation has important implications for applications of SCET to proofs of QCD factorization theorems. For instance, the potential effects of soft-collinear modes have been ignored in studies of factorization for the exclusive decay $B \rightarrow D\pi$ [19, 20]. Our results justify this treatment *a posteriori*, thereby completing the proof of factorization for this decay.

6 Conclusions

We have argued that soft-collinear effective theory (SCET) for processes involving both soft and collinear partons is a more complicated (yet more interesting) theory than previously assumed. In addition to soft and collinear particles, which make up the external hadron

states, there exist soft-collinear messenger modes, which can communicate between the soft and collinear sectors of the theory. The presence of these modes, and the fact that they have leading-order interactions with both soft and collinear particles, destroys the trivial factorization of soft and collinear physics that was thought to be a property of the effective theory. As a consequence, a careful analysis of soft-collinear exchange contributions must be part of any proof of QCD factorization theorems.

We have extended the construction of the SCET Lagrangian to include external sources such as current and four-quark operators containing both soft and collinear fields. To build up confidence in the soft-collinear modes, we have explicitly shown that they are needed for obtaining the correct ultraviolet behavior of effective theory amplitudes. The explicit examples we have investigated show that only the sum of soft, collinear and soft-collinear contributions to an amplitude is physically meaningful. In cases where the soft-collinear modes cannot be decoupled by field redefinitions, SCET amplitudes become sensitive to the large scale E through the particular scaling $m_{sc}^2 \sim \Lambda^3/E$ of soft-collinear momenta. Only part of this sensitivity is of a short-distance nature, as described by the anomalous dimensions of SCET operators. In addition, amplitudes may contain a long-distance dependence on the large scale that cannot be factorized using renormalization-group techniques.

In the strong-interaction sector of SCET the leading-order interactions of soft-collinear fields with soft or collinear fields can be removed using field redefinitions, leaving residual interactions that are power suppressed. In the presence of external operators this decoupling property is no longer guaranteed, but depends on whether external operators remain invariant under the decoupling transformation. We have shown, for instance, that currents containing a soft and a collinear quark are not invariant, implying that the effects of soft-collinear contributions to current matrix elements must be studied carefully.

In summary, we have completed the discussion of the SCET Lagrangian at leading power and including external operators relevant to weak interactions. The framework developed here forms the basis for systematic, complete proofs of QCD factorization theorems for exclusive B -meson decay amplitudes. In particular, our finding of the decoupling of soft-collinear contributions for color singlet-singlet four-quark operators completes the proof of factorization for the decay $B \rightarrow D\pi$.

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