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MIGHT DARK MATTER BE ACTUALLY BLACK?

Pisin Chen
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

There have been proposals that primordial black hole remnants (BHRs) are the dark matter, but the idea is somewhat vague. We argue here first that the generalized uncertainty principle (GUP) may prevent black holes from evaporating completely, in a similar way that the standard uncertainty principle prevents the hydrogen atom from collapsing. Secondly we note that the hybrid inflation model provides a plausible mechanism for production of large numbers of small black holes. Combining these we suggest that the dark matter might be composed of Planck-size BHRs and discuss the possible constraints and signatures associated with this notion.

It is by now widely accepted that dark matter (DM) constitutes a substantial fraction of the present critical energy density in the universe. However, the nature of DM remains an open problem. There exist many DM candidates, among which a contending category is weakly interacting massive particles, or WIMPs. It has been suggested that primordial black holes (PBHs) [1,2] are a natural candidate for WIMPs [3]. More recent studies [4] based on the PBH production from the "blue spectrum" of inflation demand that the spectral index $n \sim 1.3$, but this possibility may be ruled out by recent CMB observations [5].

In the standard view of black hole thermodynamics, based on the entropy expression of Bekenstein [6] and the temperature expression of Hawking [7], a small black hole should emit blackbody radiation, thereby becoming lighter and hotter, leading to an explosive end when the mass approaches zero. However Hawking's calculation assumes a classical background metric and ignores the radiation reaction, assumptions which must break down as the black hole becomes very small and light. Thus it does not provide an answer as to whether a small black hole should evaporate entirely, or leave something else behind, which we refer to as a black hole remnant (BHR).

Numerous calculations of black hole radiation properties have been made from different points of view [8], and some hint at the existence of remnants, but none appears to give a definitive answer. A cogent argument against the existence of BHRs can be made [9]: since there is no evident symmetry or quantum number preventing it, a black hole should radiate entirely away to photons and other ordinary stable particles and vacuum, just like any unstable quantum system.

In a recent paper [10], we invoked a generalized uncertainty principle (GUP) [11–13] and argued the contrary, that the total collapse of a black hole may be prevented by dynamics and not by symmetry, just like the prevention of hydrogen atom from collapse by the uncertainty principle [14]. Our arguments then lead to a modified black hole entropy and temperature, and as a consequence the existence of a BHR at around the Planck mass.

In this Letter we first briefly review these arguments. We then combine this idea with the hybrid inflation model [15–18] and show that primordial BHRs might in principle be the primary source for dark matter.

The uncertainty principle argument for the stability of hydrogen atom can be stated very briefly. The energy of the electron is $p^2/2m - e^2/r$, so the classical minimum energy is very large and negative, corresponding to the configuration $p = r = 0$, which is not compatible with the uncertainty principle. If we impose as a minimum condition that $p \approx \hbar/r$, we see that $E = \hbar^2/2mr^2 - e^2/r$, thus we find

$$r_{min} = \frac{\hbar^2}{me^2} \ , \quad \text{and} \quad E_{min} = -\frac{me^4}{2\hbar^2} \ . \tag{1}$$

That is the energy has a minimum, the correct Rydberg energy, when r is the Bohr radius, so the atom is stablized by the uncertainty principle.

As a result of string theory [11] or general considerations of quantum mechanics and gravity [12,13], the GUP gives the position uncertainty as

$$\Delta x \geq \frac{\hbar}{\Delta p} + l_p^2 \frac{\Delta p}{\hbar} \ , \tag{2}$$

where $l_p = (G\hbar/c^3)^{1/2} \approx 1.6 \times 10^{-33}$cm is the Planck length. A heuristic derivation may also be made on dimensional grounds. We think of a particle such as an electron being observed by means of a photon with momentum p. The usual Heisenberg argument leads to an electron position uncertainty given by the first term in Eq.(2). But we should add to this a term due to the gravitational interaction of the electron with the photon, and that term must be proportional to G times the photon energy, or Gpc. Since the electron momentum uncertainty Δp will be of order of p, we see that on dimensional grounds the extra term must be of order $G\Delta p/c^3$, as given in Eq.(2). Note that there is no $\hbar$ in the extra

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term when expressed in this way. The position uncertainty has a minimum value of $\Delta x = 2l_p$, so the Planck distance, l_p, plays the role of a fundamental length.

The Hawking temperature for a spherically symmetric black hole may be obtained in a heuristic way with the use of the standard uncertainty principle and general properties of black holes. We picture the quantum vacuum as a fluctuating sea of virtual particles; the virtual particles cannot normally be observed without violating energy conservation. However near the surface of a black hole there is an effective potential energy that is strong enough to negate the rest energy of a particle and give it zero total energy; of course the surface itself is a one-way membrane which can swallow particles so that they are henceforth not observable from outside. The net effect is that for a pair of photons one photon may be absorbed by the black hole with effective negative energy $-E$, and the other may be emitted to asymptotic distances with positive energy $+E$.

The characteristic energy E of the emitted photons may be estimated from the uncertainty principle. In the vicinity of the black hole surface there is an intrinsic uncertainty in the position of any particle of about twice the Schwarzschild radius, $\Delta x = 2r_s$, due to the behavior of its field lines [19] - as well as on dimensional grounds. This leads to a momentum uncertainty

$$\Delta p \approx \frac{\hbar}{\Delta x} = \frac{\hbar}{2r_s} = \frac{\hbar c^2}{4GM_{\rm BH}} \ , \tag{3}$$

and hence to an energy uncertainty of $\Delta pc = \hbar c^3/4GM_{\rm BH}$. We identify this as the characteristic energy of the emitted photon, and thus as a characteristic temperature; it agrees with the Hawking temperature up to a factor 2π, which we will henceforth include as a "calibration factor" and write (with $k_B = 1$),

$$T_{\rm H} \approx \frac{\hbar c^3}{8\pi GM_{\rm BH}} = \frac{M_p^2 c^2}{8\pi M_{\rm BH}} \ , \tag{4}$$

where $M_p = (\hbar c/G)^{1/2} \approx 1.2 \times 10^{19}$GeV is the Planck mass. We know of no way to show heuristically that the emitted photons should have a thermal blackbody spectrum except on the basis of thermodynamic consistency.

If the energy loss is dominated by photons we may use the Stefan-Boltzmann law to estimate the mass and energy output as functions of time. With use of the Hawking temperature and a mass in units of the Planck mass, $x = M_{\rm BH}/M_p$, the rate of energy loss is

$$\frac{dx}{dt} = \dot{x} = -\frac{1}{60(16)^2\pi t_p}\frac{1}{x^2} = -\frac{1}{t_{ch}}\frac{1}{x^2} \ , \tag{5}$$

where $t_p = (\hbar G/c^5)^{1/2} \approx 0.54 \times 10^{-43}$sec is the Planck time and $t_{ch} = 60(16)^2\pi t_p \approx 4.8 \times 10^4 t_p$ is a characteristic time for BH evaporation. It follows that the mass and the energy output rates are given by

$$x(t) = \left[x_i^3 - \frac{3t}{t_{ch}}\right]^{1/3} \ , \tag{6}$$

$$\dot{x} = \frac{1}{t_{ch}(x_i^3 - 3t/t_{ch})^{2/3}} \ , \tag{7}$$

where x_i refers to the initial mass of the hole. The black hole thus evaporates to zero mass in a time given by $t/t_{ch} = x_i^3/3$, and the rate of radiation has an infinite spike at the end of the process.

We may use the GUP to derive a modified black hole temperature. The momentum uncertainty according to the GUP is

$$\frac{\Delta p}{\hbar} \approx \frac{\Delta x}{2l_p^2}\left[1 \mp \sqrt{1 - 4l_p^2/(\Delta x)^2}\right] . \tag{8}$$

Therefore

$$T_{\rm GUP} = \frac{M_{\rm BH}c^2}{4\pi}\left[1 \mp \sqrt{1 - 1/x^2}\right] . \tag{9}$$

This agrees with the Hawking result for large mass if the negative sign is chosen, whereas the positive sign has no evident physical meaning. Note that the temperature becomes complex and unphysical for mass less than the Planck mass and Schwarzschild radius less than $2l_p$, the minimum size allowed by the GUP. At the Planck mass the slope is infinite, which corresponds to zero heat capacity of the black hole, at which the evaporation comes to a stop.

If there are g species of relativistic particles, then the BH evaporation rate is

$$\dot{x} = -\frac{16g}{t_{ch}}x^6\left[1 - \sqrt{1 - 1/x^2}\right]^4 . \tag{10}$$

Thus the hole with an initial mass x_i evaporates to a Planck mass remnant in a time given by

$$\begin{aligned}\tau &= \frac{t_{ch}}{16g}\Big[\frac{8}{3}x_i^3 - 8x_i - \frac{1}{x_i} + \frac{8}{3}(x_i^2-1)^{3/2} \\ &\qquad -4\sqrt{x_i^2 - 1} + 4\cos^{-1}\frac{1}{x_i} + \frac{19}{3}\Big] \\ &\approx \frac{x_i^3}{3g}t_{ch}, \qquad x_i \gg 1 \ .\end{aligned} \tag{11}$$

The energy output given by Eq.(10) is finite at the end point where $x = 1$, i.e., $dx/dt|_{x=1} = -16g/t_{ch}$, whereas for the Hawking case it is infinite at the endpoint where $x = 0$. The present result thus appears to be more physically reasonable. The evaporation time in the $x_i \gg 1$ limit agrees with the standard Hawking picture.

The origin of BHRs is of importance and their relevance to dark matter is of interest. Our attention is on scenarios in cosmology that can naturally provide copious PBHs. We note that the hybrid inflation, first proposed by A. Linde [15], can in principle offer that [20].

In the hybrid inflation model two inflaton fields, (ϕ, ψ), are invoked. Governed by the inflation potential,

$$V(\phi,\psi) = \Big(M^2 - \frac{\sqrt{\lambda}}{2}\psi^2\Big)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\gamma\phi^2\psi^2 \ , \quad (12)$$

ϕ first executes a "slow-roll" down the potential, and is responsible for the more than 60 e-folds expansion while ψ remains zero. When ϕ eventually reduces to a critical value, $\phi_c = (2\sqrt{\lambda}M^2/\gamma)^{1/2}$, it triggers a phase transition that results in a "rapid-fall" of the energy density of the ψ field, which lasts only for a few e-folds, that ends the inflation.

The equations of motion for the fields are

$$\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} = -(m^2 + \gamma\psi^2)\phi \ , \\ &\ddot{\psi} + 3H\dot{\psi} = (2\sqrt{\lambda}M^2 - \gamma\phi^2 - \lambda\psi^2)\psi \ , \end{aligned} \quad (13)$$

subject to the Friedmann constraint,

$$H^2 = \frac{8\pi}{3M_p^2}\Big[V(\phi,\psi) + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2\Big] \ . \quad (14)$$

The solution for the ψ field in the small ϕ regime, measured backward from the end of inflation, is

$$\psi(N(t)) = \psi_e \exp(-sN(t)) \ , \quad (15)$$

where $N(t) = H_*(t_e - t)$ is the number of e-folds from t to t_e and $s = -3/2 + (9/4 + 2\sqrt{\lambda}M^2/H_*^2)^{1/2}$ and $H_* \simeq \sqrt{8\pi/3}M^2/M_p$.

We now show how large number of small black holes can result from the second stage of inflation. Quantum fluctuations of ψ induce variations of the starting time of the second stage inflation, i.e., $\delta t = \delta\psi/\dot{\psi}$. This translates into perturbations on the number of e-folds, $\delta N = H_*\delta\psi/\dot{\psi}$, and therefore the curvature contrasts.

It can be shown that [21] the density contrast at the time when the curvature perturbations re-enter the horizon is related to δN by

$$\delta \equiv \frac{\delta\rho}{\rho} = \frac{2+2w}{5+3w}\delta N \ , \quad (16)$$

where $p = w\rho$ is the equation of state of the universe at reentry. From Eq.(15), it is easy to see that $\dot{\psi} = sH_*\psi$. At horizon crossing, $\delta\psi \sim H_*/2\pi$. So with the initial condition (at $\phi = \phi_c$) $\psi \sim H_*/2\pi$, we find that $\delta N \sim 1/s$. Thus

$$\delta \sim \frac{2+2w}{5+3w}\frac{1}{s} \ . \quad (17)$$

As w is always of order unity, we see that the density perturbation can be sizable if s is also of order unity. With an initial density contrast $\delta(m) \equiv \delta\rho/\rho|_m$, the probability that a region of mass m becomes a PBH is [22]

$$P(m) \sim \delta(m)e^{-w^2/2\delta^2} \ . \quad (18)$$

Let us assume that the universe had inflated e^{N_c} times during the second stage of inflation. From the above discussion we find [20]

$$e^{N_c} \sim \Big(\frac{2M_p}{sH_*}\Big)^{1/s} \ . \quad (19)$$

At the end of inflation the physical scale that left the horizon during the phase transition is $H_*^{-1}e^{N_c}$. If the second stage of inflation is short, i.e., $N_c \sim \mathcal{O}(1)$, then the energy soon after inflation may still be dominated by the oscillations of ψ with $p = 0$, and the scale factor of the universe after inflation would grow as $(tH_*)^{2/3}$. The scale $(tH_*)^{2/3}H_*^{-1}e^{N_c}$ became comparable to the particle horizon ($\sim t$), or $t \sim (tH_*)^{2/3}H_*^{-1}e^{N_c}$, when

$$t \sim t_h = H_*^{-1}e^{3N_c} \ . \quad (20)$$

At this time if the density contrast was $\delta \sim 1$, then BHs with size $r_s \sim H_*^{-1}e^{3N_c}$ would form with an initial mass

$$M_{\mathrm{BH}i} \simeq \frac{M_p^2}{H_*}e^{3N_c} \ . \quad (21)$$

Following the numerical example given in Ref.20, and for reasons that will become clear later, we let $H_* \sim 5 \times 10^{13}$ GeV and $s \sim 3$. Assume that the universe was radiation-dominated (so $w = 1/3$) when the curvature perturbation reentered the horizon. Then the density contrast is $\delta \sim 1/7$, and the fraction of matter in the BH is $P(m) \sim 10^{-2}$. From Eq.(19), $e^{N_c} \sim 54$. So the total number of e-folds is $N_c \sim 4$. The black holes were produced at the moment $t_h \sim 2 \times 10^{-33}$ sec, and had a typical mass $M_{\mathrm{BH}i} \sim 4 \times 10^{10} M_p$. Let $g \sim 100$. Then the time it took for the BHs to reduce to remnants, according to Eq.(11), is

$$\tau \sim \frac{x_i^3}{3g}t_{ch} \sim 5 \times 10^{-10}\mathrm{sec} \ . \quad (22)$$

The "black hole epoch" thus ends in time for baryogenesis and other subsequent epochs in the standard cosmology. As suggested in Ref.20, such a post-inflation PBH evaporation provides an interesting mechanism for reheating. Note that due to the continuous evaporation process and the redshift, the BH reheating should result in an effective temperature which is sufficiently lower than the Planck scale.

This process also provides a natural way to create cold dark matter. Although in our example $P(m) \sim 10^{-2}$, PBHs would soon dominate the energy density by the time $t \sim P(m)^{-2}t_h \sim 2 \times 10^{-29}$s, because the original relativistic particles would be diluted much faster than non-relativistic PBHs. By the time $t \sim \tau$, all the initial BH mass (x_i) had turned into radiation except one unit of M_p preserved by each BHR. As BH evaporation rate rises sharply towards the end, the universe at $t \sim \tau$ was dominated by the BH evaporated radiation.

Roughly, $\Omega_{\mathrm{BHR},\tau} \sim 1/x_i$ and $\Omega_{\gamma,\tau} \sim 1$ at $t \sim \tau$, and since the universe resumed its standard evolution after the black hole epoch ($t > \tau$), we find the density parameter for the BHR at present to be

$$\Omega_{\mathrm{BHR},0} \sim \left(\frac{t_{eq}}{\tau}\right)^{1/2} \left(\frac{t_0}{t_{eq}}\right)^{2/3} \frac{1}{x_i} \Omega_{\gamma,0} \ , \tag{23}$$

where $t_0 \sim 4 \times 10^{17}$s is the present time, and t_{eq} is the time when the density contributions from radiation and matter were equal. It is clear from our construction that $(t_{eq}/\tau)^{1/2} \sim x_i$. So $t_{eq} \sim 10^{12}$ sec, which is close to what the standard cosmology assumes, and Eq.(23) is reduced to a simple and interesting relationship:

$$\Omega_{\mathrm{BHR},0} \sim \left(\frac{t_0}{t_{eq}}\right)^{2/3} \Omega_{\gamma,0} \sim 10^4 \Omega_{\gamma,0} \ . \tag{24}$$

In the present epoch, $\Omega_{\gamma,0} \sim 10^{-4}$. So we find $\Omega_{\mathrm{BHR},0} \sim \mathcal{O}(1)$, about the right amount for dark matter!

PBH evaporation can in principle emit late decaying massive supersymmetric (SUSY) particles, such as gravitinos and moduli. The abundance of these particles, and in turn that of PBHs, is limited by, e.g., big bang nucleosynthesis (BBN), and stringent constraints have been derived [23]. We note that these are typically based on the gravity-mediated SUSY breaking scenario, where gravitino and moduli masses are often at the electroweak scale and they decay at very late times. To the contrary, in the alternative low energy gauge-mediated SUSY breaking mechanism [24] both gravitino and moduli are light, with masses below $\mathcal{O}(1)$eV. In addition, in this mechanism the gravitino is naturally the lightest SUSY particle, while the decay of the "next to lightest SUSY particle" (NLSP) to it is very efficient. As a result of these, in this scenario the above-mentioned constraints can be evaded.

As interactions with BHR are gravitational, the cross section is extremely small, and direct observation of BHR seems unlikely. One possible indirect signature may be associated with the cosmic gravitational wave background. Unlike photons, the gravitons radiated during evaporation would be instantly frozen. Since, according to our notion, the BH evaporation would terminate when it reduces to a BHR, the graviton spectrum should have a cutoff at Planck mass. Such a cutoff would have by now been redshifted to $\sim \mathcal{O}(10^4)$ GeV. It would be interesting to investigate whether such a spectrum is in principle observable. Another possible signature may be some imprints on the CMB fluctuations due to the thermodynamics of BH-radiation interactions. This will be further investigated.

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