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The Geometrization of Matter Proposal in the Barrett-Crane Model and Resolution of Cosmological Problems

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We give an overview of the current issues in early universe cosmology and consider the potential resolution of these issues in an as yet nascent spin foam cosmology. The model is the Barrett-Crane model for quantum gravity along with a generalization of manifold complexes to complexes including conical singularities.

I. INTRODUCTION

The Barrett-Crane model [1] [2] is a constrained topological state sum model for quantum gravity. Recently [3] it was proposed that this model might incorporate matter and gauge interactions if the condition on triangulations to be manifolds were relaxed. That is, conical singularities would act as seeds of matter in the quantum geometry of the state sum. The purpose of this paper is to examine the consequences of this proposal, and of spin foam models in general, for early universe phenomenology. We feel that although the BC model, and the conical matter proposal in particular, are not yet sufficiently well understood, the connections are provocative. Conversely, one would like to use cosmology as a guiding principle for developing experimental techniques in spin foam models that could be used to explain and predict real observations.

The point of this paper is that natural approximations motivated by the conical matter proposal (CMP) seem to shed light on the *full* range of puzzles of the early universe. Although each argument is in need of further substantiation, the overall picture seems striking in itself.

To clarify this point, and since the communities of researchers in spin foam models and early universe phenomenology are not generally aware of each other's work, this paper begins with a self contained introduction to the range of early universe phenomenological issues. We then give a nontechnical introduction to the categorical state sum program and the BC model in particular, together with the conical matter proposal. We conclude with a discussion of a range of phenomenological problems which have plausible solutions in the state sum picture.

Although we have tried to make the paper accessible to cosmologists by giving physical explanations for the state sum models as far as possible, they do make use of branches of mathematics not generally familiar to physicists, so interested readers are strongly recommended to consult the references.

We believe that the current form of the model is likely to be less important than the fact that it leads to calculations that probe poorly understood phenomena.

II. CURRENT ISSUES IN EARLY UNIVERSE COSMOLOGY

A. The Flatness Problem

The Standard Big Bang predicts that in an expanding space-time $\Omega = 1$ is an unstable fixed point. However, present observations confirm that our universe is flat, i.e. $\Omega = 1$ to within 10 percent. This means that Ω had to be very close to one in the past. For example, at the era of nucleosynthesis, we are constrained to have $|\Omega - 1| < O(10^{-16})$. This is an extreme fine tuning of initial conditions. Unless initial conditions are chosen very accurately, the universe soon collapses, or expands quickly before structure can be formed. The suggestion discussed below of a topological phase may shed some light on the flatness problem.

B. The Horizon Problem

If we look back at the surface of last scattering we see homogeneity in the cosmic microwave radiation across distances subtending $10^{28} cm$. However the size of correlations described by causality at the surface of last scattering is predicted by SBB to be $d_{cor} \sim 10^{23} cm$, a large discrepancy between observations and theory. Therefore, the SBB provides no causal way to produce correlations to establish homogeneity on such large scales that are observed in the cosmic microwave background (CMB). Inflation was designed partially to solve this problem. We suggest a solution to the horizon problem where the initial conditions of the universe may already have established correlations that were homogeneous.

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C. Baryon Asymmetry

Current observations tell us that most of the universe is made up of non-baryonic degrees of freedom such as dark energy (2/3), dark matter (1/3) and only 5% baryonic matter. An even more striking observation of the smallness of baryon density is given by the ratio of baryons to photons in the CMB. Big Bang nucleosynthesis as well as anisotropies in the CMB yield the ratio

$$\frac{n_b}{n_\gamma} = 6.1 \times 10^{-10} \tag{1}$$

Why is this number so small, yet nonzero? If equal numbers of particles and antiparticles had been created in the early universe, they would have annihilated in pairs. It is well known that the Standard Model at the level of renormalizable terms possesses no interactions that can violate either baron number or lepton number violation. In 1967 Sakharov [4] studied how the baryon asymmetry could arise. He summarized the three conditions required of any process that would lead to baryon asymmetry:

- Baryon number violation must occur in the fundamental interactions
- CP violation
- Local violation of thermal equilibrium, including an Arrow of Time.

We will observe that the conical matter proposal plausibly satisfies the Sakharov conditions.

D. Dark Matter

Our current understanding of structure formation necessitates the existence of pressureless non-baryonic dark matter. Observations on galactic and cosmological scales reveal that 22 percent of the matter in our universe consists of non-baryonic dark matter (NBDM). NBDM does not interact with radiation so it can not be detected by standard astronomical means. More importantly, there are a few candidates for NBDM, such as SUSY neutralinos, axions, shadow universes and Planck mass black hole remnants. These are plausible candidates because they are fields which are weakly interacting. However, they are all based on extensions of the Standard Model and will likewise suffer the same type of initial condition fine tuning as scalar fields coupled to gravity in a cosmological setting. Below, we suggest a plausible new candidate for DM in our model, namely higher genus conical matter, and also a relationship between these and Primordial Black Holes (PBH).

A very concrete challenge to theoretical physics is posed by the recent detailed observations of dwarf galaxies [5]. The density of dark matter in dwarf galaxies as a function of radius can be computed very accurately because they are dominated by dark matter even near their cores. The observed distribution does not agree with the theoretical prediction for non-interacting dark matter in that it lacks a central cusp.

E. Inflation

The idea that the early universe underwent a phase of exponential growth has many attractions, such as its ability to reproduce the acoustic peaks in deviations to the CMB power spectrum. Unfortunately, current theoretical models require an unmotivated fine tuning of the potential of an as yet hypothetical scalar field. We will propose that the unusual thermodynamics of conical matter provides a plausible mechanism for inflation, and the decoupling of the higher genus particles could provide an exit scenario.

III. THE STANDARD BIG BANG

The Standard Big Bang Scenario is based on three observational pillars: primordial nucleosynthesis, isotropy and homogeneity seen in the cosmic microwave background and the Hubble redshift relation. SBB is the simplest general relativistic scenario which predicts these observations. Despite its success the SBB suffers from other observational and theoretical problems. By now these problems are well known [6]. A key issue for model builders is to simultaneously resolve these problems while keeping the SBB in the regime where it predicts observations consistently. The main theoretical underpinning of SBB is the use of Einstein's general theory of relativity minimally coupled to a gas of particles (a hot perfect fluid in thermal equilibrium). However, at high curvatures and hence early times, this approximation is no longer valid and the SBB needs to be modified. Some of the main problems of the SBB were partially solved by the inflationary paradigm. Without changing general relativity but by relaxing the assumption of the equation of state of matter to include a contribution from a quantum field, the inflaton, the inflationary paradigm was able to simultaneously resolve the problems of the SBB and even provide a causal mechanism of generating a scale invariant power spectrum. The conventional wisdom stemming from QFT is to minimally couple gravity to quantum fields, which realizes inflation, although it has been shown that this assumption breaks down in the early universe. Unfortunately, inflation does not shed much light on the dark energy and dark matter problems. In fact, inflation seems to be pointing to the roots of its own demise, namely the transplanckian problem. Briefly, the transplanckian problem of inflation is that structures on scales of cosmological interest today were generically generated deep within the Planckian regime where the assumption of a scalar field minimally coupled to gravity breaks down.

With these issues in mind, we consider a different approach. There are features, to be discussed below, of the categorical state sum (CSS) approach to quantum general relativity which are useful for proposing new solutions to some of the cosmological problems, which interestingly are not soluble by inflation or other modifications to the SBB.

The problems we will discuss are inflation, dark matter and the flatness, horizon and chirality problems, as these appear to have distinctive realizations in the BC model of quantum gravity and yield a new perspective as to how quantum gravity can resolve these problems in cosmology without resorting to the logic of fine tuning an effective field theory.

IV. STATE SUM MODELS, QUANTUM GRAVITY AND CONICAL MATTER

The basic idea of the state sum approach to quantum gravity [7] is that quantum geometry is a superposition of discrete quantum processes of the same form as Feynman diagrams. Geometries are quantized by using representation theory to obtain Hilbert spaces on which geometric quantities act as operators. Thus, a categorical state sum is a discretized version of a Feynman vacuum, where the fields and vertices correspond to Lorentzian geometry.

The diagrams are not considered to be embedded in a background classical spacetime. Rather, the combinatorial structure of the diagrams themselves yields spacetime, and the quantum fields on it represent a sum over metrics. The structure of spacetime in this picture is given by a simplicial complex, in which the individual Feynman diagrams are quantum geometries of four dimensional simplices in Minkowski space.

In other words, quantum geometry in this approach is represented by families of Hilbert spaces on which the sort of quantities typically measured in classical geometries act as operators. The most basic geometric quantities are bivectors, i.e. skew symmetric rank two tensors, which describe oriented area elements. Utilizing their expedient quantization, we define the other geometric quantities in terms of these bivectors, which are attached to the faces of a triangulation.

In the quantization procedure of the BC model [1], bivectors are represented by unitary representations of the Lorentz algebra. The bivectors on faces and tetrahedra are constrained to be simple, i.e. to correspond to oriented area elements. This has a natural quantization in the restriction to the balanced unitary representations. We construct the model by using harmonic analysis to describe these representations.

Specifically, the balanced representations are given by families of functions on the hyperboloid H^3. The projection onto the balanced representation with real parameter ρ is given by

$$\frac{1}{2\pi^2}\int_{H^3} K_\rho(x,y)h(y)dy \tag{2}$$

where

$$K_\rho(x,y) = \frac{\sin \rho r(x,y)}{\rho \sinh r(x,y)} \tag{3}$$

for $r(x,y)$ the hyperbolic distance. The $10j$ symbol which corresponds to the quantum geometry of a Lorentzian 4-simplex s is represented by the integral

$$\begin{aligned}(10j)_s = \int dx_1 dx_2 dx_3 dx_4 dx_5 K_{\rho_1}(x_1,x_5) \\ K_{\rho_2}(x_1,x_4)K_{\rho_3}(x_1,x_3)K_{\rho_4}(x_1,x_2) \\ K_{\rho_5}(x_2,x_5)K_{\rho_6}(x_2,x_4)K_{\rho_7}(x_2,x_3) \\ K_{\rho_8}(x_3,x_5)K_{\rho_9}(x_3,x_4)K_{\rho_{10}}(x_4,x_5)\end{aligned} \tag{4}$$

depending on the 10 face labels. The complete state sum as a sum (and integral) over labellings c of the triangulation takes the form

$$Z_{BC} = N \sum_c \prod_t A_t \prod_f {\rho_f}^2 \prod_s (10j)_s \tag{5}$$

where for the tetrahedron t

$$A_t = \int dx K_{\rho_1}(x)K_{\rho_2}(x)K_{\rho_3}(x)K_{\rho_4}(x) \tag{6}$$

See [1] and [8] for details. Since we think of the individual triangulations themselves as Feynman diagrams for a more fundamental theory called Group Field Theory [9], we want to make a summation over triangulated complexes to produce the full theory.

The actual construction of the theory depends on the idea that general relativity can be described as a constrained version of BF theory [7] [2], which has the action

$$S = \int B \wedge F \tag{7}$$

where B is a 2-form and F the curvature of a connection A. In fact, the state sum of the BC model is a constrained version of the topological CKY model of [10], which is a quantization of BF theory. The quantization in [10] also has bivectors as variables. The constraint which transforms the CKY model to the BC one is the restriction to bivectors that are simple, i.e. that correspond to oriented area elements rather than superpositions of them.

The steepest descent condition (the analog of the classical equation of motion) for the CKY model corresponds to the condition of flat geometry, while the constraint introduced in the BC model converts this to an analog of

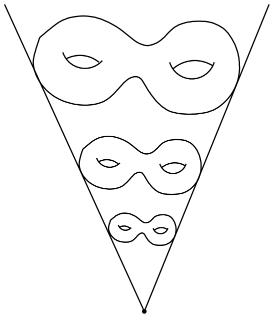

FIG. 1. cone over surface

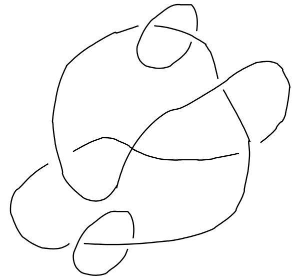

FIG. 2. three manifold with boundary components

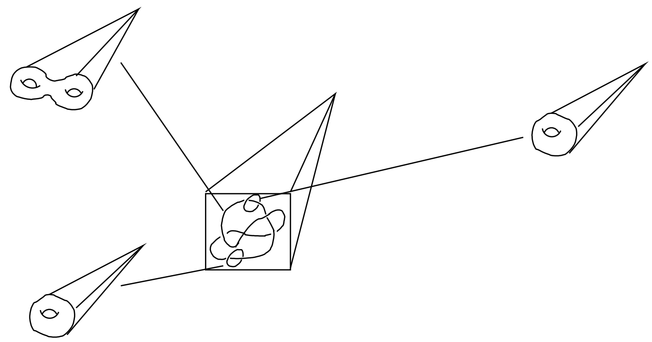

FIG. 3. conical vertex as Feynman vertex

Ricci flatness, i.e. to a discretization of Einstein's equation. In order to make the sum over triangulations finite, it is very tempting to consider the hypothesis of a 'phase transition' in the early universe, in which the constraint of the BC model emerges as a result of a kind of collapse of the wave packet of the unmeasured universe. The form of the constraint mentioned above is suggestive of that, in that it suppresses superpositions of simple bivectors. We warn the reader that there is as yet no model of how this could come about, and that the work in quantum information theory [11] which suggests it is also lacking a mechanism. A plausible model for the transition is outlined below, and can be thought of as an attempt to make precise the 'topological phase' suggestion of Witten [12].

At this point, we discover that the class of triangulated spaces which occur as Feynman graphs for GFT is broader than the class of triangulated four dimensional manifolds. As explained in [3], it is possible to have edges and vertices whose points are conical singularities. The edge points can appear as cones over closed surfaces, while the vertices can appear as cones over general three manifolds, which contain the conical singularities over all surfaces corresponding to edges incident on the vertex.

Let us restate this. Every Feynman diagram for the GFT approach to the BC model has the topology of a four manifold containing a web of conical singularities taking the form of a graph, the points of whose edges have neighborhoods which are cones over surfaces, while the vertices of the graph have neighborhoods which are cones over 3-manifolds with boundaries, the boundary components fitting to the boundary surfaces on the incident edges. This is a theorem of combinatorial topology; it just summarizes the ways that the 4-simplices in the model can be glued together.

Differently put, in passing from physical theories described by differential equations to discrete models, we find that the natural class of spacetimes to consider has broadened, from smooth manifolds to what mathematicians would call PL pseudomanifolds.

A. How to Get Matter

The conical matter proposal (CMP) is to consider a) the conical singularities on edges as generating particles which propagate through space, and b) the conical singularities on vertices as interaction vertices.

Perhaps it is useful to try to picture this. Feynman taught us to think of the vacuum as full of processes described by Feynman diagrams. Instead, we are proposing a picture where spacetime is full of edges, each point of which is a cone over a surface (fig 1), joined at vertices which are cones over three manifolds with boundary; in the simplest case, cones over link complements (fig 2). These would fit together into complexes with a combinatorial structure analogous to Feynman diagrams (fig 3).

It is interesting to note that this is a direct extension of the most popular technique for adding matter in 2+1 dimensional gravity to the 3+1 setting. In 2+1 dimensions, matter is added in the form of conical curvature singularities. In the quantum theory, this is expressed by adding punctures to the Riemann surfaces of the spatial slices. In 2+1 dimensions, a conical singularity is not a topological defect; that is a new feature in 3+1 dimensions.

In comparison to other fundamental physical theories involving matter, the CMP has one advantage: matter is naturally included in the theory of quantum gravity, rather than added by hand. There is *no* new element; neither a gauge group, nor extra dimensions, nor a topology on a compact manifold. The surfaces are not insertions into the space-time; they are only descriptions of part of its topological structure. It is therefore highly remarkable, as we will explain, that the most natural approximation scheme available suggests that the Standard Model may emerge from it.

In order to investigate the implications of the CMP for low energy physics, two approximation techniques had to be adopted. It was assumed that the low energy behavior, both at conical singularities along edges and at vertices, would be dominated by the flat metrics on the regions around the singularities. It was further assumed that the state propagating along an edge would correspond to the states of the topological quantum field theory of the Lorentz algebra on the boundary surface of the conical singularity.

We should justify these assumptions by saying that any configuration around the singularity which required curvature would therefore be very massive, and that since the CSW TQFT [12] has flatness for an equation of motion, it is a quantization of the flat geometries. We regard this approach as preliminary, and hope that further progress in understanding the CSS models will provide a better underpinning for it.

Another way of thinking of this approximation is as a novel form of the holographic principle [13] [14]. The conical regions are analogous to black holes with more complex topology. The spaces of states on them is described by the states of CSW on their boundaries.

The implications of these assumptions are interesting. The classification of flat Lorentzian metrics over a cone is equivalent to the classification of hyperbolic metrics on its boundary. This means that the space of flat geometries on the cone over a surface is described by its Teichmuller space. If we take this as confirmation that the effective low energy states of the theory are given by the CSW theory for the q-deformed Lorentz algebra on the once punctured torus, then this has the structure of the quantum group $U_q(su(2))$, which as an algebra has the structure

$$U_q(su(2)) = \bigoplus_{n=1}^{\infty} Mat_n(\mathbf{C}) \tag{8}$$

of a direct sum of one matrix algebra of each dimension. The unitary part of this reproduces the gauge group of the Standard Model in a manner analogous to the Connes-Lott model [15], if truncated after the first three terms. Since noncommutative geometries arise naturally in the description of singular spaces, in particular of spaces which admit descriptions as quotients of more regular spaces, as conical singularities do, it would be reasonable to try to connect our picture more directly with the noncommutative geometry approach.

The truncation can either be done by picking a third root of unity for q as in [16], or one can simply consider the possibility that particles with $su(4)$ or higher quantum numbers are extremely massive, a possibility which is at least open to study.

Consider the easiest nontrivial cobordism (3-manifold with boundary) linking three tori. The most obvious interaction between them corresponds to multiplication in the algebra $U_q(su(2))$, so that the states on the torus get a natural interpretation as gauge bosons for the Standard Model. The question of the orientation of the surface needs careful study. Thus, we have candidates for photons, gluons etc. in our model, with the right sort of interactions. In order to determine the masses, propagation and spins of these, we will need better approximations.

The problem of incorporating fermions is more difficult to understand. The space of states on the cone over a Klein bottle seems a natural candidate, but more developed approximations will be necessary to study this question. The one thing we can say is that one might look for chiral asymmetry in the interaction vertices represented by 3-manifolds with boundary; in particular, link complements. Given the discovery of neutrino masses, this at least seems a plausible approach.

The case of flat metrics over a vertex is more complex. Hyperbolic three manifolds with complete metrics correspond to Kleinian groups and play an important role in the topology of three dimensional manifolds.

A number of important facts are known about hyperbolic 3-manifolds with boundary. For instance, if any of the boundary components have genus greater than 1, then the complete hyperbolic metric has infinite volume, which in fact grows exponentially as we approach the end, while the genus 1 (tori and Klein bottle) have finite total volume. Another rather obvious fact is that 3-manifolds with boundary give natural examples of topological objects which are not equivalent to their mirror images, such as the class of 3-manifolds arising from taking the complement of an ordinary link in S^3.

A simple physical interpretation of these facts is that at low energy the states corresponding to singularities over higher genus surfaces decouple both from the genus 1 states and from one another. The states on tori and Klein bottles remain interacting, giving rise to a world of gauge bosons and fermions, as conjectured above. The higher genus states, therefore, were interacting with the genus 1 states in the early universe, but decoupled at some later stage.

The assumptions that we can take the TQFT Hilbert space on the boundary of a conic singularity as the physical Hilbert space of its states, and the CSW amplitude for the three dimensional boundary of a conical vertex as its amplitude, have the further interesting implication that the dimension of the Hilbert space goes up exponentially with the area of the boundary, while the amplitude of the vertex goes up exponentially with its volume.

The first observation is closely analogous to the observation that the dimension of the Hilbert space on a punctured sphere goes up exponentially with the number of punctures, which led Smolin [2] to conjecture a connection between TQFT and quantum gravity. Higher genus surfaces are very similar to surfaces with many punctures in TQFT. The second observation has interesting implications for a universe with higher genus conical dark matter.

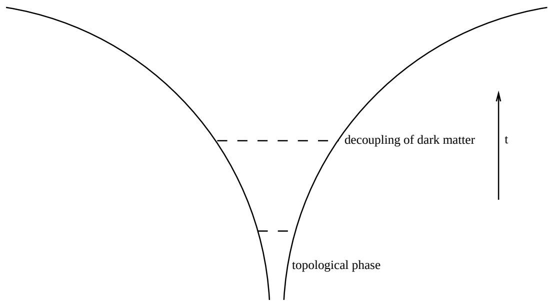

FIG. 4. history of early universe

V. CONNECTIONS

The picture which thus arises from considering CSS models, including the suggestion of a TQFT-BC type phase transition and the CMP, makes contact with a range of puzzles in early universe phenomenology.

Let us summarize the picture of the history of the universe which our model seems to suggest (fig 4). There would be an early (or rather sub-Planckian) phase, in which the universe would be modelled by a topological quantum field theory, but with conic singularities included in the manifold. This phase would be a substitute for the initial singularity of the SBB model. It would be followed by a phase of quantum gravity, in which genus 1 and higher genus conic singularities would be interacting, while the universe expanded and cooled. Next would come a decoupling, in which further interactions involving higher genus singularities would be suppressed by topological obstructions, leaving an interacting world composed of genus 1 singularities.

We now discuss how features of this CSS program relate to the specific phenomenological issues outlined above.

A. The Cosmological Constant and Quantum Groups

Since it is very natural to pass from classical to quantum groups in constructing categorical state sums, and in particular since the q-BC model is well behaved [17], our discussion above easily accommodates a cosmological constant, and indeed may even require it. The representation theory of the quantum Lorentz algebra seems to give a quantum geometry of space with constant curvature very similar to the quantum geometry of flat space provided by the unitary representations of the usual Lorentz algebra. It also provides additional regularization, which may be useful for suitable normalizations; in particular, for the contribution of higher genus singularities.

B. The Flatness and Horizon Problems and the Topological Phase

The idea of a 'phase transition' from the BF TQFT to a BC type model suggests an approach to the flatness problem, since the equations of motion for the BF theory are exactly flatness. Also, since a TQFT has no light cones, the distribution of conical matter would be random, transforming into a thermal distribution when the metric materialized. This also suggests an approach to the horizon problem: we regard the substitution of a topological phase for a point-like singularity as a positive feature of our model.

The phase transition might arise as a result of coarse graining of the topological universe, which at the origin of time fluctuated into a combination of quantum variables corresponding to a sufficiently large 'size'. Small black holes would cause a loss of phase information, which would mediate a transformation to a regime described by a sum over simple bivectors only, which, as we explained above, has GR as a classical limit. The GR equations of motion would then prevent the escape of information from the black holes. We believe this process could be modeled using techniques similar to those of [11].

Although this proposal is self-consistent, it has a disturbing chicken versus egg quality. The question of the phase transition is the point which most strongly suggests to us that still deeper theoretical constructions will ultimately be needed in this approach.

There is at least one toy candidate for investigating the phase transition within the timelike q-BC model. The constraint condition in terms of representations of the (quantum) Lorentz group is that a real spin label $k \in \frac{1}{2}\mathbf{Z}$ should equal zero. One could consider a state sum cut off at any maximum k, the topological theory being recovered when $k \to \infty$. If a spacelike model is considered instead, this 'thermal' parameter becomes continuous. This idea is strengthened by [18] and [19] in which a generalized action of the form

$$S(B, A, \phi, \mu) = \int B^{IJ} \wedge F_{IJ} - \frac{1}{2}\phi_{IJKL} B^{IJ} \wedge B^{KL} + \mu H \tag{9}$$

is discussed, where A is an $SO(3,1)$ connection, μ and ϕ Lagrange multipliers such that ϕ satisfies $\phi_{IJKL} = -\phi_{JIKL} = -\phi_{IJLK} = \phi_{KILJ}$ and

$$H = a_1 \phi^{IJ}_{IJ} + a_2 \phi_{IJKL} \epsilon^{IJKL}$$

The Immirzi parameter γ is introduced by

$$\frac{a_2}{a_1} = \frac{1}{4}(\gamma - \frac{1}{\gamma}) \tag{10}$$

In the Lorentzian case the generalized action always corresponds to the q-deformed Barrett-Crane model. When $\gamma = 0$ the topological theory is recovered.

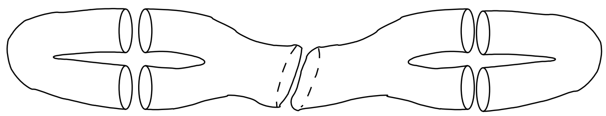

FIG. 5. surface decomposition in $2D$ TFT

C. Inflation

Let us now briefly recall our current understanding of the Bekenstein bound for the entropy of a black hole in terms of TQFTs. The hyperbolic area of a uniform Riemann surface, normalized to a constant curvature of -1, depends only its genus. Such a surface may be cut up into cylindrical and trinion pieces, as shown in figure 5. These pieces are clearly homeomorphic to the punctured spheres that represent black hole horizons. In a TQFT that assigns a Hilbert space to each such puncture, the invariant Z thus depends on the hyperbolic area and increases exponentially with the number of punctures. A simple corollary of this is that the entropy of the black hole scales with horizon area. For an overview of entropy bounds see [14] and references therein.

In an analogous simple analysis of the thermodynamics near vertices in the conical matter proposal, one considers the 3-dimensional topological CSW invariant on the hyperbolic link complement. Now it so happens that there is a strongly supported mathematical conjecture due to Kashaev [20] [21] which states that the closely related colored Jones invariant J_N is asymptotically related to the normalized hyperbolic volume $V(L)$ of a link complement in S^3 by

$$2\pi \log |J_N(L)| \sim NV(L) \qquad N \to \infty \qquad (11)$$

where $q = e^{2\pi i/N}$ and $N \in \mathbf{Z}$.

Assuming that it is reasonable to define thermodynamic quantities using our partition function, this conjecture seems to suggest that the contribution of the conical singularities to the pressure (normalized with respect to energy density) is given by

$$\frac{P}{\rho} = -\frac{2\pi}{N}\frac{\partial log J}{\partial V} \sim -1 \qquad (12)$$

A $P/\rho \leq -1/3$ drives inflation, so this result is in accord with an inflation scenario. In this new scenario *no* inflaton field is introduced. Inflation arises naturally as a result of the negative pressure. Decoupling of higher genus modes at low temperature could end inflation. Observe that temperature scales as N.

Restating this argument more physically: the free energy of the vertex, as given by the partition function for the TQFT, is equal to the Jones polynomial. Since this equals the exponential of the volume, the system has a negative pressure equal to the energy density. This is analogous to the situation for conventional inflation, where the stress-energy of the inflaton field must be proportional to the Lorentz metric $g_{\mu\nu} = diag(1, -1, -1, -1)$ by Lorentz invariance.

We are thus led to conjecture that the appropriate configuration for a vertex inserted into a nearly flat region (yet to be analysed) will not depend on the choice of a preferred frame. That is, it will be manifestly Lorentz invariant. If true, this would give an interesting physical interpretation to Kashaev's deep mathematical conjecture.

The main assumption which goes into the above argument is that the behavior of the state sum in a region around a singular vertex can be well approximated by GR coupled to an effective stress-energy with pressure as above. Since the state sum reproduces Einstein's equation as a classical limit, this is plausible, but needs to be more carefully studied. We have also assumed that the volume of a spatial slice is proportional to the boundary volume of a neighborhood of a conical vertex.

A definite weakness of this stage of development of the model is that we cannot really justify the above assignation of an energy to conical matter configurations. To investigate this question further, one might find the 'best' metrics on the union of a conical singularity and a patch of surrounding nearly flat smooth spacetime, and compute the ADM energy. At this point we can only claim that we are making the most natural guesses in arriving at phenomenologically interesting conclusions.

D. Dark Matter and Higher Genus Singularities

One potentially exciting use of the model is in describing dark matter. Currently there are a few dark matter candidates, most of which are modelled by weakly interacting scalar fields, like the axion or the neutralino. But these CDM candidates are problematic for explaining the lack of cuspiness of dwarf galaxies [5]. Naively, one could imagine that the weak residual interactions of higher genus conical matter would cause the halo near the center of a galaxy to thermalize, thus eliminating the (empirically nonexistent) cusps. This should be susceptible to computational study, and the detail with which the halos can be observed provides a demanding test.

E. Baryon Asymmetry

The most difficult of Sakharov's conditions is that concerning baryon and lepton number violating processes. In the early universe our model would allow baryons or leptons to couple into higher genus singularities, which would later freeze out. The necessary asymmetries could arise from the 3-manifold topology of fermionic interactions.

F. Higher Genus Matter and PBH Remnants

Another interesting idea is to realize dark matter as black hole remnants [22] which survive after black holes evaporate.

Higher genus conical matter could easily emerge as PBH remnants. Their topology would be an obstruction to complete evaporation. Currently there is no first principle realization of PBH from quantum gravity, so it will be a major development to make this connection more concrete.

G. CP Violation and 3-Manifolds

We observed above that couplings which arise from 3-manifolds with boundary, and can therefore easily have chiral asymmetry, are also interesting as a possible mechanism for CP violation.

H. The Phase Transition and Variable Speed of Light Theories

It would be interesting to see whether or not the TQFT can be considered a mathematically more elegant version of the variable speed of light idea, which solves the horizon problem but still lacks a precise quantum gravitational description [23]. Perhaps a more detailed model of the phase transition would interpolate between acausal and causal propagation.

VI. CONCLUSIONS

In summary, spin foam cosmology appears to satisfy the Sakharov conditions and has the potential to explain a whole host of major cosmological problems. The computations which go into this picture are very rudimentary. At the semiclassical level, the problem of connecting together flat metrics around vertices and edges into composites for whole spacetimes needs to be studied much more carefully. The claim that the considerations of phenomenology discussed here suggest interesting natural questions for research in the CSS picture, at least, is well grounded.

It seems very plausible to us that the picture we are outlining here could play a role analogous to the old quantum mechanics. It has the flavor of a model constructed out of well understood mathematical physical tools, which bears a reasonable resemblance to otherwise puzzling phenomenology. It may seem crazy to substitute discrete categorical diagrams for a continuum lagrangian, but it may turn out to be not crazy enough! Judging by historical experience, only a prolonged dialog with phenomenology will guide us to a theory which is sufficiently crazy.

It is therefore an attractive feature of this model that it suggests natural procedures for generating refined approximations. We could use classical GR techniques to study flat or low curvature metrics on combinations of conical singularities, on edges and on vertices, and patches of nearly flat smooth spacetime about these. Calculations based on such approximations could be combined into models for dark matter or inflationary cosmologies, among other possibilities, and compared to empirical data.

We have some speculative ideas about the emergence of a deeper theory. One esthetic drawback to the model we are proposing is that we begin with spacetime, and produce matter as a sort of pinch within it. One could wish, rather, for a theory in which spacetime and matter played dual roles. A hint that such a model might be possible is that the genus 1 states form the Hopf algebra object in the category [24] which is a geometric realization of the braided group of Majid [25]. The representations of the braided group reproduce the category of representations of the quantum group; this is suggestive of a deeper model with matter-spacetime duality.

There is enormous scope for investigating early universe phenomenology in other categorical state sum models. For example, it appears worthwhile investigating the variation of the deformation parameter q. The relation of q to both a Planck scale (as a UV cutoff) and a cosmological scale is very suggestive of a duality such as the aforementioned. One might also consider a 4-dimensional model constructed from CSW S^3 slices of varying q. The discrete parameter $N \in \mathbf{Z}$ labels a dimensionless 'time' for the topological phase, and might heuristically enforce a real arrow of time.

Of course, it is much too soon to say anything stronger about the suggestion that this line of development could lead to a unified field theory than that it leads to a computational program which appears tractable and deserves further study.

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