

Measurement of the Inclusive Charmless Semileptonic Branching Ratio of B Mesons and Determination of $|V_{ub}|$

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We report a measurement of the inclusive charmless semileptonic branching fraction of B mesons in a sample of 89 million $B\bar{B}$ events recorded with the BABAR detector at the $\Upsilon(4S)$ resonance. Events are selected by fully reconstructing the decay of one B meson and identifying a charged lepton from the decay of the other B meson. The number of signal events is extracted from the hadronic mass distribution and is used to determine the ratio of branching fractions $\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu})/\mathcal{B}(\bar{B} \rightarrow X \ell \bar{\nu}) = (2.06 \pm 0.25(stat) \pm 0.23(syst) \pm 0.36(theo)) \times 10^{-2}$.

Using the measured branching fraction for inclusive semileptonic B decays, we find $\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu}) = (2.24 \pm 0.27(stat) \pm 0.26(syst) \pm 0.39(theo)) \times 10^{-3}$ and derive the CKM matrix element $|V_{ub}| = (4.62 \pm 0.28(stat) \pm 0.27(syst) \pm 0.48(theo)) \times 10^{-3}$.

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The element $|V_{ub}|$ of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1] plays a critical role in testing the consistency of the Standard Model description of CP violation. In this paper, we present a determination of $|V_{ub}|$ from a measurement of inclusive charmless semileptonic decays $\bar{B} \rightarrow X_u \ell \bar{\nu}$ [2]. The analysis uses $\Upsilon(4S) \rightarrow B\bar{B}$ events in which one of the B meson decays hadronically and is fully reconstructed (B_{reco}) and the semileptonic decay of the recoiling \bar{B} meson is identified by the presence of an electron or muon. While this approach results in a low overall event selection efficiency, it allows for the determination of the momentum, charge, and flavor of the B mesons. We use the invariant mass m_X of the hadronic system to separate $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays from the dominant $\bar{B} \rightarrow X_c \ell \bar{\nu}$ background, which clusters above the D meson mass [3]. By ensuring a higher signal purity and acceptance than previously achieved [4], and by measuring the fraction of charmless semileptonic decays $R_u = \mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu})/\mathcal{B}(\bar{B} \rightarrow X \ell \bar{\nu})$, this analysis leads to substantially smaller systematic uncertainties [5].

The measurement presented here is based on a sample of about 89 million $B\bar{B}$ pairs collected near the $\Upsilon(4S)$ resonance by the BABAR detector [6] at the PEP-II asymmetric-energy e^+e^- storage ring operating at SLAC.

We use Monte Carlo (MC) simulations of the BABAR detector based on GEANT [7] to optimize selection criteria and to determine signal efficiencies and background distributions. Charmless semileptonic $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays are simulated as a combination (see Fig. 1a) of resonant three-body decays ($X_u = \pi, \eta, \rho, \omega, \dots$) [8] and decays to nonresonant hadronic final states X_u [9], for which the hadronization is performed by Jetset [10]. The motion of the b quark inside the B meson is implemented with the shape function parametrization given in Ref. [9]. The simulation of the $\bar{B} \rightarrow X_c \ell \bar{\nu}$ background uses an HQET parametrization of form factors for $\bar{B} \rightarrow D^* \ell \bar{\nu}$ [11], and models for $\bar{B} \rightarrow D\pi \ell \bar{\nu}$, $D^* \pi \ell \bar{\nu}$ [12], and $\bar{B} \rightarrow D \ell \bar{\nu}$, $D^* \ell \bar{\nu}$ [8].

To reconstruct a large sample of B mesons, hadronic decays $B_{reco} \rightarrow \bar{D}Y^\pm, \bar{D}^*Y^\pm$ are selected. Here, the system Y^\pm consists of hadrons with a total charge of ± 1 , composed of $n_1\pi^\pm n_2K^\pm n_3K_S^0 n_4\pi^0$, where $n_1 + n_2 \leq 5$, $n_3 \leq 2$, and $n_4 \leq 2$. We reconstruct $D^{*-} \rightarrow \bar{D}^0\pi^-; \bar{D}^{*0} \rightarrow \bar{D}^0\pi^0, \bar{D}^0\gamma; D^- \rightarrow K^+\pi^-\pi^-, K^+\pi^-\pi^0, K_S^0\pi^-, K_S^0\pi^-\pi^0, K_S^0\pi^-\pi^-\pi^+$; and $\bar{D}^0 \rightarrow K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^-\pi^+, K_S^0\pi^+\pi^-$. The kinematic consistency of B_{reco} candidates is checked with

two variables, the beam energy-substituted mass $m_{ES} = \sqrt{s/4 - \vec{p}_B^2}$ and the energy difference $\Delta E = E_B - \sqrt{s}/2$. Here \sqrt{s} is the total energy in the $\Upsilon(4S)$ center of mass frame, and \vec{p}_B and E_B denote the momentum and energy of the B_{reco} candidate in the same frame. We require $\Delta E = 0$ within three standard deviations as measured for each mode.

For each of the 1097 reconstructed B decay modes, the purity \mathcal{P} is estimated as the fraction of signal events with $m_{ES} > 5.27 \text{ GeV}/c^2$. We only use events for which \mathcal{P} exceeds a decay mode dependent threshold in the range of 9% to 24%. In events with more than one reconstructed B decay, we select the decay mode with the highest purity. On average, we reconstruct one B candidate in 0.3% (0.5%) of the $B^0\bar{B}^0$ (B^+B^-) events. The purity for events with a high-momentum lepton is 67% (see Fig. 2a).

Semileptonic decays $\bar{B} \rightarrow X \ell \bar{\nu}$ of the \bar{B} recoiling against the B_{reco} candidate are identified by an electron or muon with a minimum momentum of $p^* > 1 \text{ GeV}/c$ in the \bar{B} rest frame. For charged B_{reco} candidates, we require the charge of the lepton to be consistent with a prompt semileptonic \bar{B} decay. For neutral B_{reco} candidates, both charge-flavor combinations are retained and the known average $B^0\text{-}\bar{B}^0$ mixing rate is used to extract the prompt lepton yield. Electrons are identified [13] with 92% average efficiency and a hadron misidentification rate ranging between 0.05% and 0.1%. Muons are identified [6] with an efficiency ranging between 60% ($p^* > 1 \text{ GeV}/c$) and 75% ($p^* > 2 \text{ GeV}/c$) and hadron misidentification rate between 1% and 3%. Efficiencies and misidentification rates are estimated from data control samples.

The hadronic system X in the decay $\bar{B} \rightarrow X \ell \bar{\nu}$ is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the B_{reco} candidate or the identified lepton. Care is taken to eliminate fake charged tracks, as well as low-energy beam-generated photons and energy depositions in the calorimeter from charged and neutral hadrons. The neutrino four-momentum p_ν is estimated from the missing momentum four-vector $p_{miss} = p_{Y(4S)} - p_{B_{reco}} - p_X - p_\ell$, where all momenta are measured in the laboratory frame and $p_{Y(4S)}$ refers to the $\Upsilon(4S)$ meson. The mass of the hadronic system is determined by a kinematic fit that imposes four-momentum conservation, the equality of the masses of the two B mesons, and forces $p_\nu^2 = 0$. The resulting m_X resolution is $350 \text{ MeV}/c^2$ on average.

To select $\bar{B} \rightarrow X_u \ell \bar{\nu}$ candidates we require exactly

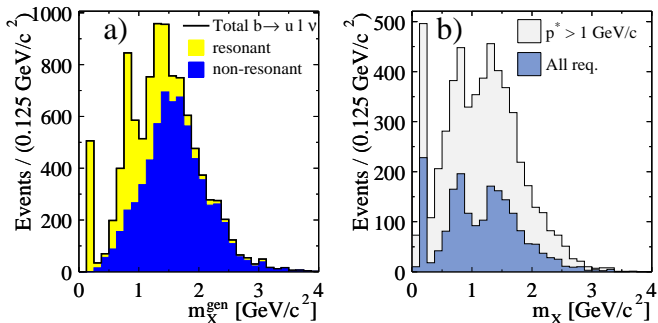


FIG. 1: Signal MC m_X distributions with the requirement of a lepton with $p^* > 1 \text{ GeV}/c$: a) generated m_X distributions for the two components of the signal model, and b) measured m_X distribution before and after all other requirements.

one charged lepton with $p^* > 1 \text{ GeV}/c$, charge conservation ($Q_X + Q_\ell + Q_{B_{reco}} = 0$), and a missing mass consistent with zero ($m_{miss}^2 < 0.5 \text{ GeV}^2/c^4$). These criteria suppress the dominant $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decays, many of which contain additional neutrinos or an undetected K_L^0 meson. We suppress the $\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}$ background by reconstructing the π_s^+ from the $D^{*+} \rightarrow D^0 \pi_s^+$ decay and the lepton: since the momentum of the π_s^+ is almost collinear with the D^{*+} momentum in the laboratory frame, we can approximate the energy of the D^{*+} as $E_{D^{*+}} \simeq m_{D^{*+}} \cdot E_{\pi_s^+} / 145 \text{ MeV}/c^2$ and require for the neutrino $m_\nu^2 = (p_B - p_{D^{*+}} - p_\ell)^2 < -3 \text{ GeV}^2/c^4$. We veto events with charged or neutral kaons in the recoil \bar{B} to reduce the background from $\bar{B} \rightarrow X_c \ell \bar{\nu}$ decays. Charged kaons are identified [6] with an efficiency varying between 60% at the highest and almost 100% at the lowest momenta. The pion misidentification rate is about 2%. The $K_S^0 \rightarrow \pi^+ \pi^-$ decays are reconstructed with an efficiency of 80% from pairs of oppositely charged tracks with an invariant mass between 486 and 510 MeV/c^2 . The impact of the event selection on the m_X distribution is illustrated in Fig. 1b.

We determine R_u from N_u , the observed number of $\bar{B} \rightarrow X_u \ell \bar{\nu}$ candidates with $m_X < 1.55 \text{ GeV}/c^2$, and N_{sl} , the number of events with at least one charged lepton:

$$R_u = \frac{\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu})}{\mathcal{B}(\bar{B} \rightarrow X \ell \bar{\nu})} = \frac{N_u / (\varepsilon_{sel}^u \varepsilon_{m_X}^u)}{N_{sl}} \times \frac{\varepsilon_l^{sl} \varepsilon_{reco}^{sl}}{\varepsilon_l^u \varepsilon_{reco}^u}.$$

Here $\varepsilon_{sel}^u = (34.2 \pm 0.6)\%$ is the efficiency for selecting $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays once a $\bar{B} \rightarrow X \ell \bar{\nu}$ candidate has been identified, $\varepsilon_{m_X}^u = (73.3 \pm 0.9)\%$ is the fraction of signal events with $m_X < 1.55 \text{ GeV}/c^2$, $\varepsilon_l^{sl} / \varepsilon_l^u = 0.887 \pm 0.008$ corrects for the difference in the efficiency of the lepton momentum cut for $\bar{B} \rightarrow X \ell \bar{\nu}$ and $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays, and $\varepsilon_{reco}^{sl} / \varepsilon_{reco}^u = 1.00 \pm 0.03$ accounts for a possible efficiency difference in the B_{reco} reconstruction in events with $\bar{B} \rightarrow X \ell \bar{\nu}$ and $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays.

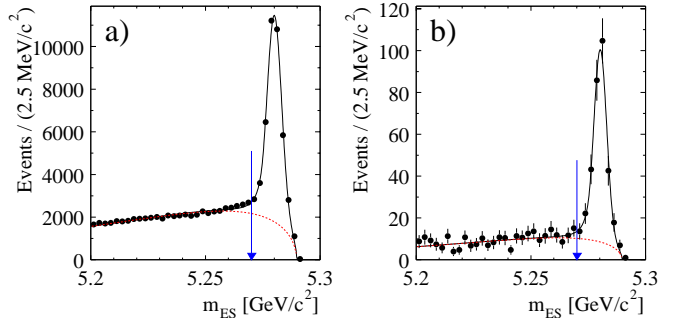


FIG. 2: Fit to the m_{ES} distributions for a) the sample with a $p^* > 1 \text{ GeV}/c$ lepton and b) the sample after all requirements and with $m_X < 1.55 \text{ GeV}/c^2$. The arrow indicates the lower limit of the signal region.

We derive N_{sl} from a fit to the m_{ES} distribution shown in Fig. 2a. The fit uses an empirical description [14] of the combinatorial background from continuum and $B\bar{B}$ events, together with a narrow signal [15] peaked at the B meson mass. The small tail accounts for energy losses in the reconstruction of π^0 mesons. The residual background in N_{sl} from misidentified leptons and semileptonic charm decays amounts to 6.8% and is subtracted.

We extract N_u from the m_X distribution by a minimum χ^2 fit to the sum of three contributions: the signal, the background N_c from $\bar{B} \rightarrow X_c \ell \bar{\nu}$, and a background of $< 1\%$ from other sources (misidentified leptons, secondary τ and charm decays). In each bin of the m_X distribution, the combinatorial B_{reco} background for $m_{ES} > 5.27$ is subtracted on the basis of a fit to the m_{ES} distribution (Fig. 2b). Fig. 3a shows the fitted m_X distribution. To minimize the model dependence, the first bin is extended to $m_X < 1.55 \text{ GeV}/c^2$. The fit reproduces the data well with $\chi^2/dof = 7.6/6$. Fig. 3b shows the m_X distribution after background subtraction with finer binning. Table I summarizes the results of fits with different requirements on m_X , for electrons and muons, for neutral and charged B_{reco} candidates, and for different ranges of the B_{reco} purity \mathcal{P} . The results are all consistent within the uncorrelated statistical errors.

We have performed extensive studies to determine the systematic uncertainties on R_u . To establish that the background from $\bar{B} \rightarrow X_c \ell \bar{\nu}$ events is adequately simulated we use previously excluded events with charged or neutral kaons as a control sample. The relative systematic error due to uncertainties in the detection of photons is estimated to be 4.7% by varying the corrections applied to the MC simulation to match the data control samples. An additional error of 1.0% is ascribed to the uncertainty in the simulation of showers generated by K_L^0 interactions; it is equal to the shift caused by the removal of the K_L^0 energy depositions in the MC simulation.

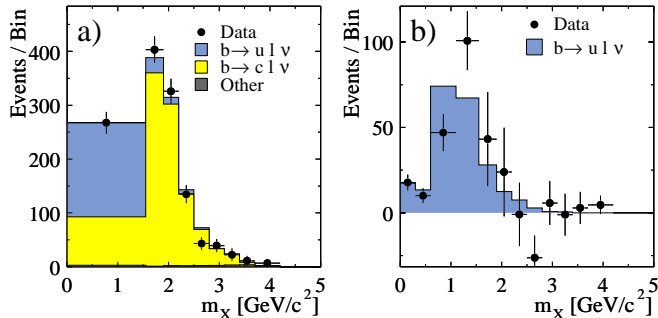


FIG. 3: The m_X distribution for $\bar{B} \rightarrow X\ell\bar{\nu}$ candidates: a) data (points) and fit components, and b) data and signal MC after subtraction of the $b \rightarrow c\ell\nu$ and the “other” backgrounds.

TABLE I: Fit results for data subsamples.

Sample	N_{sl}	N_u	N_c	$R_u(\%)$
$m_X < 1.55 \text{ GeV}/c^2$	29982 ± 233	175 ± 21	90 ± 5	2.06 ± 0.25
$m_X < 1.40 \text{ GeV}/c^2$	29982 ± 233	143 ± 18	54 ± 3	1.89 ± 0.24
$m_X < 1.70 \text{ GeV}/c^2$	29982 ± 233	214 ± 26	145 ± 9	2.35 ± 0.28
neutral B_{reco}	10862 ± 133	76 ± 15	22 ± 3	2.53 ± 0.50
charged B_{reco}	19080 ± 191	100 ± 16	67 ± 4	1.82 ± 0.30
Electrons	17320 ± 173	101 ± 15	46 ± 3	2.27 ± 0.34
Muons	12622 ± 157	73 ± 15	41 ± 4	1.83 ± 0.37
$\mathcal{P} > 80\%$	4187 ± 68	20 ± 7	12 ± 1	1.68 ± 0.57
$50\% < \mathcal{P} < 80\%$	12373 ± 141	68 ± 13	41 ± 3	1.94 ± 0.37
$\mathcal{P} < 50\%$	13144 ± 170	86 ± 15	34 ± 3	2.31 ± 0.41

An error of 1.0% is attributed to the uncertainty in the track-finding efficiency. The error due to identification of electrons, muons, and kaons is estimated to be 1.0%, 1.0%, and 2.3%, respectively, by varying identification efficiency by $\pm 2\%$, $\pm 3\%$, and $\pm 2\%$ for e^\pm , μ^\pm , and K^\pm , and the misidentification rates by $\pm 15\%$ for all particle types.

The uncertainty in the B_{reco} combinatorial background subtraction contributes 3.8%. It is estimated by changing the empirical m_{ES} signal function to a Gaussian distribution and by varying the parameters within one standard deviation of the default values. The limited statistics of the simulated event samples adds an uncertainty of 4.5%. The choice of bins for $m_X > 1.55 \text{ GeV}/c^2$ impacts the fit result at a level of 1.2%. All the above mentioned experimental errors add up to 8.7%.

The uncertainties in the background modeling due to branching fraction measurements for $B \rightarrow D\ell\nu, D^*\ell\nu, \dots$ and for inclusive and exclusive D meson decays [16] contribute 4.4%.

The error due to the hadronization in the $\bar{B} \rightarrow X_u\ell\bar{\nu}$ final state is estimated to be 3.0% by measuring R_u as a function of the charged and neutral particle multiplic-

ities and performing the fit with only the nonresonant part of the signal model. We assign an additional 2.8% error to account for the uncertainties in the inclusive and exclusive branching fractions for charmless semileptonic B decays [16], plus 3.7% for the veto on strange particles. Here, we assume a 100% uncertainty in the $s\bar{s}$ contents for the resonant and 30% for the nonresonant component [17]. These three uncertainties contribute a combined error of 5.5%.

The efficiencies ε_{sel}^u and $\varepsilon_{m_X}^u$ are sensitive to the detailed modeling of the $\bar{B} \rightarrow X_u\ell\bar{\nu}$ decays. We assess these uncertainties by varying the nonperturbative parameters in the model [9] within their errors, $\bar{\Lambda} = 0.48 \pm 0.12 \text{ GeV}$ and $\lambda_1 = -0.30 \pm 0.11 \text{ GeV}^2$, obtained from the results in Ref. [18] by removing terms proportional to $1/m_b^3$ and α_s^2 from the relation between the measured observables and $\bar{\Lambda}$ and λ_1 . Taking into account the correlation of -0.8 between $\bar{\Lambda}$ and λ_1 , we arrive at a theoretical error of 17.5%.

In summary, we obtain

$$R_u = (2.06 \pm 0.25 \pm 0.23 \pm 0.36) \times 10^{-2},$$

where the errors are statistical, systematic (experimental plus signal and background modeling), and theoretical, respectively. Taking into account common errors we compute the double ratio $\frac{\mathcal{B}(\bar{B}^0 \rightarrow X_u\ell\bar{\nu})}{\mathcal{B}(\bar{B}^0 \rightarrow X\ell\bar{\nu})} \frac{\mathcal{B}(B^- \rightarrow X\ell\bar{\nu})}{\mathcal{B}(B^- \rightarrow X_u\ell\bar{\nu})} = 0.72 \pm 0.18(stat) \pm 0.19(syst)$. Combining the ratio R_u with the measured inclusive semileptonic branching fraction $\mathcal{B}(\bar{B} \rightarrow X\ell\bar{\nu}) = (10.87 \pm 0.18(stat) \pm 0.30(syst))\%$ [13], we have

$$\mathcal{B}(\bar{B} \rightarrow X_u\ell\bar{\nu}) = (2.24 \pm 0.27 \pm 0.26 \pm 0.39) \times 10^{-3}.$$

We combine this result with the average B lifetime of $\tau_B = 1.608 \pm 0.012 \text{ ps}$ [16, 19] and obtain [20]

$$|V_{ub}| = (4.62 \pm 0.28 \pm 0.27 \pm 0.40 \pm 0.26) \times 10^{-3}.$$

The first error is statistical, the second systematic, the third gives the theoretical uncertainty in the signal efficiency and the extrapolation of R_u to the full m_X range, and the fourth combines the perturbative and nonperturbative uncertainties in the extraction of $|V_{ub}|$ from the total decay rate.

This result is consistent with previous inclusive measurements [4], but has a smaller systematic error, primarily due to larger phase-space acceptance and much higher sample purity. In the future, improved understanding of the signal composition and charm background will significantly reduce the experimental errors, and this, together with independent measurements of $b \rightarrow s$ transitions and semileptonic B decays, is expected to constrain the theoretical uncertainties.

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- [1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Th. Phys. **49**, 652 (1973).
- [2] Charge-conjugation is implied throughout the Letter.
- [3] V. D. Barger, C. S. Kim, and R. J. Phillips, Phys. Lett. B **251**, 629 (1990); R. D. Dikeman and N. G. Uraltsev, Nucl. Phys. B **509**, 378 (1998); I. I. Bigi, R. D. Dikeman, and N. Uraltsev, Eur. Phys. J. C **4**, 453 (1998); A. F. Falk, Z. Ligeti, and M. B. Wise, Phys. Lett. B **406**, 225 (1997).
- [4] R. Barate *et al.* [ALEPH Collaboration], Eur. Phys. J. C **6**, 555 (1999); M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **436**, 174 (1998); P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. B **478**, 14 (2000); G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **21**, 399 (2001); A. Bornheim *et al.* [CLEO Collaboration], Phys. Rev. Lett. **88**, 231803 (2002).
- [5] The results of this analysis depend on the assumption of parton-hadron duality to which no error is assigned.
- [6] B. Aubert *et al.* [BABAR Collaboration], Nucl. Instrum. Meth. A **479**, 1 (2002).
- [7] S. Agostinelli *et al.* [GEANT4 Collaboration], SLAC-PUB-9350, CERN-IT-2002-003 (2002).
- [8] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).
- [9] F. De Fazio and M. Neubert, JHEP **9906**, 017 (1999).
- [10] T. Sjöstrand, Comput. Phys. Commun. **82**, 74 (1994).
- [11] J. E. Duboscq *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76**, 3898 (1996).
- [12] J. L. Goity and W. Roberts, Phys. Rev. D **51**, 3459 (1995).
- [13] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **67**, 031101 (2003).
- [14] H. Albrecht *et al.* [ARGUS Collaboration], Z. Phys. C **48**, 543 (1990).
- [15] T. Skwarnicki [Crystal Ball Collaboration], DESY F31-86-02.
- [16] K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002).
- [17] M. Althoff *et al.* [TASSO Collaboration], Z. Phys. C **27**, 27 (1985).
- [18] D. Cronin-Hennessy *et al.* [CLEO Collaboration], Phys. Rev. Lett. **87**, 251808 (2001). We assume that the shape function parameters in Ref. [9] can be related to the HQET parameters $\bar{\Lambda}$ and λ_1 .
- [19] The impact of the uncertainty of the fraction of neutral and charged *B* mesons is negligible.
- [20] We follow the prescription on page 708 of [16], based on N. Uraltsev, Int. J. Mod. Phys. A **14**, 4641 (1999) and A. H. Hoang, Z. Ligeti, and A. V. Manohar, Phys. Rev. D **59**, 074017 (1999).