

## **On Exclusive and Inclusive Rare B Decays: CKM Phenomenology and New Physics Reach**

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# On Exclusive and Inclusive Rare $B$ Decays: CKM Phenomenology and New Physics Reach \*

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We report recent results in rare  $B$  decays. Our focus will be on  $b \rightarrow d\gamma$  and  $b \rightarrow s\ell^+\ell^-$  transitions. We discuss their impact on the CKM phenomenology and their role within our search for new physics. In particular, we analyse the impact of a recent lattice QCD estimate of the  $B \rightarrow K^*$  form factor at zero recoil. We also briefly discuss the presently available optimal theoretical tools for the inclusive and exclusive modes.

## 1 Introduction

Rare  $B$  decays, as flavour changing neutral current processes (FCNC), are the most important tools within our (indirect) search for new physics in the present era of  $B$  factories (for a recent review see [1]). Among them, the inclusive  $b \rightarrow s\gamma$  mode is still the most prominent, because it has already been measured by several independent experiments [2–6]. The present world average is [7]:

$$\mathcal{B}(B \rightarrow X_s\gamma) = (3.34 \pm 0.38) \times 10^{-4}. \quad (1)$$

The present next-to-leading -logarithmic (NLL) QCD prediction, based on the original QCD calculations of several groups ([8–11]), has an additional charm mass renormalization scheme ambiguity as was first noticed in [12]. The  $\overline{MS}$  scheme is used in the most recent updates [12,13]. This choice is guided by the experience gained from many higher order calculations where the mass is dominantly off-shell and leads to the following theoretical prediction for the ‘total’ branching ratio:

$$\mathcal{B}(B \rightarrow X_s\gamma) = (3.70 \pm 0.30) \times 10^{-4}. \quad (2)$$

However, the renormalization scheme for  $m_c$  is an NNLL issue, and one should regard the preference for the  $\overline{MS}$  scheme in comparison with the pole mass scheme (which was used within all previous NLL predictions) just as an educated guess of the presently

unknown NNLL corrections. Therefore, one could also argue for a slightly larger theoretical error in (2). A complete resolution of this problem can be achieved by a NNLL calculation which is presently under study.

The stringent bounds obtained from the  $B \rightarrow X_s\gamma$  mode on various non-standard scenarios (see e.g. [14–18]) are a clear example of the importance of clean FCNC observables in discriminating new-physics models. The exclusive modes, however, often have large uncertainties due to the hadronic form factors and are not clean enough to disentangle possible new physics effects from hadronic uncertainties, but they can serve as important QCD tests. Exceptions are ratios of exclusive quantities like asymmetries in which a large part of the hadronic uncertainties cancel out.

Regarding the corresponding exclusive mode  $B \rightarrow K^*\gamma$ , quite recently a preliminary lattice determination of the  $B \rightarrow K^*$  form factor at zero recoil [19] was presented which is in perfect agreement with the previous indirect determinations. We analyse the impact of this new result on the CKM phenomenology in section 3.2.

Besides the  $b \rightarrow s\gamma$  mode, also the  $b \rightarrow s\ell^+\ell^-$  transitions are already accessible at  $B$  factories [20–22], inclusively and exclusively. The  $b \rightarrow s\ell^+\ell^-$  mode represents new sources of interesting observables, particularly kinematic observables such as the invariant dilepton mass spectrum and the forward–backward (FB) asymmetry. Rare  $B$  decays are also relevant to the CKM phenomenology; the  $b \rightarrow d\gamma$  is especially important in this respect. In the following, we will focus on the latter two rare modes and also briefly discuss the theoretical tools available for the analysis of exclusive and inclusive channels.

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## 2 Theoretical Tools

The effective field theory approach serves as a theoretical framework for both inclusive and exclusive modes. The standard method of the operator product expansion (OPE) allows for a separation of the  $B$  meson decay amplitude into two distinct parts, the long-distance contributions contained in the operator matrix elements and the short-distance physics described by the so-called Wilson coefficients. The  $W$  boson and the top quark with mass larger than the factorization scale are integrated out, i.e. removed from the theory as dynamical fields. The effective Hamiltonian for radiative and semileptonic  $b \rightarrow s/d$  transitions in the SM can be written as

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} \left[ \lambda_q^t \sum_{i=1}^{10} C_i \mathcal{O}_i + \lambda_q^u \sum_{i=1}^2 C_i (\mathcal{O}_i - \mathcal{O}_i^u) \right] \quad (3)$$

where  $\mathcal{O}_i(\mu)$  are dimension-six operators at the scale  $\mu \sim O(m_b)$ ;  $C_i(\mu)$  are the corresponding Wilson coefficients. Clearly, only in the sum of Wilson coefficients and operators, within the observable  $\mathcal{H}$ , does the scale dependence cancel out.  $G_F$  denotes the Fermi coupling constant and the explicit CKM factors are  $\lambda_q^t = V_{tb}V_{tq}^*$  and  $\lambda_q^u = V_{ub}V_{uq}^*$ . The unitarity relations  $\lambda_q^c = -\lambda_q^t - \lambda_q^u$  were already used by us in (3).

The operators can be chosen as (we only write the most relevant ones):

$$\mathcal{O}_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L), \quad (4)$$

$$\mathcal{O}_2^u = (\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L), \quad (5)$$

$$\mathcal{O}_7 = e/g_s^2 m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \quad (6)$$

$$\mathcal{O}_8 = 1/g_s m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a, \quad (7)$$

$$\mathcal{O}_9 = e^2/g_s^2 (\bar{s}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell} \gamma^\mu \ell), \quad (8)$$

$$\mathcal{O}_{10} = e^2/g_s^2 (\bar{s}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell} \gamma^\mu \gamma_5 \ell), \quad (9)$$

where the subscripts  $L$  and  $R$  refer to left- and right-handed components of the fermion fields. In  $b \rightarrow s$  transitions the contributions proportional to  $\lambda_s^u$  are rather small, while in  $b \rightarrow d$  decays  $\lambda_d^u$  is of the same order as  $\lambda_d^t$  and they play an important role in  $CP$  and isospin asymmetries. The operators  $\mathcal{O}_9$  and  $\mathcal{O}_{10}$  only occur in the semileptonic  $b \rightarrow s/d \ell^+ \ell^-$  modes.

While the Wilson coefficients  $C_i(\mu)$  enter both inclusive and exclusive processes and can be calculated with perturbative methods, the calculational approaches to the matrix elements of the operators differ in both cases. Within inclusive modes, one can use quark-hadron duality in order to derive a well-defined

heavy mass expansion of the decay rates in powers of  $\Lambda_{\text{QCD}}/m_b$  (HME). In particular, it turns out that the decay width of the  $B \rightarrow X_s \gamma$  is well approximated by the partonic decay rate, which can be calculated in renormalization-group-improved perturbation theory:

$$\Gamma(B \rightarrow X_s \gamma) = \Gamma(b \rightarrow X_s^{\text{parton}} \gamma) + \Delta^{\text{nonpert.}} \quad (10)$$

Non-perturbative effects,  $\Delta^{\text{nonpert.}}$ , are suppressed by inverse powers of  $m_b$  and are well under control thanks to the Heavy Mass Expansion (HME); they can be further estimated through the application of the Heavy Quark Effective Theory (HQET). In exclusive processes, however, one cannot rely on quark-hadron duality and has to face the difficult task of estimating matrix elements between meson states. A promising approach is the method of QCD-improved factorization that has recently been systemized for non-leptonic decays in the heavy quark limit. This method allows for a perturbative calculation of QCD corrections to naive factorization and is the basis for the up-to-date predictions for exclusive rare  $B$  decays. However, within this approach, a general, quantitative method to estimate the important  $1/m_b$  corrections to the heavy quark limit is missing. More recently, a more general quantum field theoretical framework for the QCD-improved factorization was proposed - known under the name of Soft Collinear Effective Theory (SCET).

### 2.1 Inclusive Modes

In contrast to the exclusive rare  $B$  decays, the inclusive ones are theoretically clean observables and dominated by the partonic contributions. Bound-state effects of the final states are eliminated by averaging over a specific sum of hadronic states. Moreover, also long-distance effects of the initial state are accounted for, through the heavy mass expansion in which the inclusive decay rate of a heavy  $B$  meson is calculated using an expansion in inverse powers of the  $b$  quark mass.

The optical theorem relates the inclusive decay rate of a hadron  $H_b$  to the imaginary part of certain forward scattering amplitudes

$$\Gamma(H_b \rightarrow X) = \frac{1}{2m_{H_b}} \text{Im} \langle H_b | \mathbf{T} | H_b \rangle, \quad (11)$$

where the transition operator  $\mathbf{T}$  is given by  $\mathbf{T} = i \int d^4x T[H_{eff}(x)H_{eff}(0)]$ . It is then possible to construct an OPE of the operator  $\mathbf{T}$ , which gets expressed as a series of *local* operators - suppressed by powers of the  $b$  quark mass and written in terms of the  $b$  quark field:

$$\mathbf{T} \stackrel{OPE}{=} \frac{1}{m_b} (\mathcal{O}_0 + \frac{1}{m_b} \mathcal{O}_1 + \frac{1}{m_b^2} \mathcal{O}_2 + \dots). \quad (12)$$

This construction is based on the parton–hadron duality, using the facts that the sum is done over all exclusive final states and that the energy release in the decay is large with respect to the QCD scale,  $\Lambda_{\text{QCD}} \ll m_b$ . With the help of the HQET, namely the new heavy-quark spin-flavour symmetries arising in the heavy quark limit  $m_b \rightarrow \infty$ , the hadronic matrix elements within the OPE,  $\langle H_b | \mathcal{O}_i | H_b \rangle$ , can be further simplified. The crucial observations within this well-defined procedure are the following: the free quark model turns out to be the first term in the constructed expansion in powers of  $1/m_b$  and therefore the dominant contribution. This contribution can be calculated in perturbative QCD. Second, in the applications to inclusive rare  $B$  decays one finds no correction of order  $1/m_b$  to the free quark model approximation, and the corrections to the partonic decay rate start with  $1/m_b^2$  only. The latter fact implies a rather small numerical impact of the non-perturbative corrections to the decay rate of inclusive modes.

The  $1/m_b^2$  corrections correspond to the OPE for  $T(\mathcal{O}_7^\dagger \mathcal{O}_7)$ . There are additional non-perturbative effects if one also takes into account the operator  $\mathcal{O}_2$ . They can be analysed in a model-independent way and scale with  $1/m_c^2$ . Due to small coefficients in the expansion also their impact is very small [23–25].

## 2.2 Exclusive Modes

The naive approach to the computation of exclusive amplitudes consists in writing the amplitude  $A \simeq C_i(\mu_b) \langle \mathcal{O}_i(\mu_b) \rangle$  and parametrizing  $\langle \mathcal{O}_i(\mu_b) \rangle$  in terms of form factors. A substantial improvement is obtained through the QCD-improved factorization [26,27] and SCET [28–33] approaches.

Let us consider processes involving the decay of a heavy meson into fast moving light particles ( $B \rightarrow \gamma e \nu$ ,  $B \rightarrow (\rho, K^*) \gamma$ ,  $B \rightarrow K \pi$ , ...) and indicate with  $Q \sim O(m_b)$  their typical large energy. The idea is to isolate *all* the relevant degrees of freedom necessary to correctly describe the infrared structure of QCD below the scale  $Q$  and associate independent fields to each of them. It is possible to identify two distinct *perturbative* modes, called hard ( $p^2 \sim Q^2$ ) and semi-hard ( $p^2 \sim \Lambda_{\text{QCD}} Q$ ). These modes are produced, for instance, in interactions of energetic light particles with the heavy quark and the  $B$ -meson spectator, respectively. These two modes do not appear in the initial and final states and, therefore, have to be integrated out. We do not wish to entertain here a comprehensive discussion of the technicalities involved in this step. It will suffice to say that the resulting theory (also called SCET<sub>II</sub> in the literature) contains only *non-perturbative* degrees of freedom with virtualities  $O(\Lambda_{\text{QCD}}^2)$  and that hard and semi-hard modes are re-

flected in the coefficient functions in front of the operators of that (SCET<sub>II</sub>) theory. We note that these coefficients depend, in general, on energies of order  $Q$  and  $\Lambda_{\text{QCD}}$ . Moreover, the hierarchy  $\Lambda_{\text{QCD}} \ll Q$  allows for an expansion in the small parameter  $\lambda = \Lambda_{\text{QCD}}/Q$ .

Given a process, one has to construct the most general set of (SCET<sub>II</sub>) operators at a given order in  $\lambda$ , and show that all the possible gluon exchanges can be reabsorbed, at all orders in perturbation theory, into form factors and meson light-cone wave functions. The resulting amplitude is a convolution of these non-perturbative universal objects with the coefficient functions encoding the contribution of hard and semi-hard modes. Questions regarding the convergence of these convolution integrals can be addressed using symmetries, power counting and dimensional analysis (a discussion on this point is presented in Ref. [33]).

The few form factors that describe the transition  $B \rightarrow M$  (where  $M$  denotes a pseudo-scalar or vector meson) can be written as [34]:

$$F_i^{B \rightarrow M} = C_i \xi^{B \rightarrow M} + \phi_B \otimes T_i \otimes \phi_M + O\left(\frac{\Lambda}{m_b}\right) \quad (13)$$

where  $\xi^{B \rightarrow M}$  is the so-called non-factorizable (or soft) contribution to the form factors (actually there is one soft form factor for the decay into pseudoscalar meson and two for the decay into vector mesons);  $\phi_{B,M}$  are the  $B$  and  $M$  meson light-cone wave functions;  $C_i$  are Wilson coefficients that depend on hard scales; and  $T_i$  are perturbative hard scattering kernels generated by integrating out hard and semi-hard modes. In Ref. [35] the factorization formula Eq. (13) has been proved at all orders in perturbation theory and at leading order in  $\Lambda_{\text{QCD}}/m_b$ , using SCET techniques.<sup>3</sup> The strength of Eq. (13) is that it allows us to express several independent QCD form factors in terms of only one soft form factor (two in the case of vector mesons) and moments of the light-cone wave functions of the light pseudo-scalar (vector) and  $B$  mesons.

Let us now briefly discuss the form of factorization for the decays  $B \rightarrow V \gamma$  (with  $V = K^*, \rho$ ). At leading order, only the operator  $\mathcal{O}_7$  contributes and its matrix element between meson states is given by an expression similar to (13). The choice of using either the full QCD form factor  $T^{B \rightarrow V}$  or the soft one  $\xi_\perp$  clearly is a matter of taste (note that non-perturbative methods, such as lattice-QCD and light-cone QCD sum rules, give only informations on the full QCD form factors and not on the soft contributions alone). The advantage of the QCD-improved factorization approach is

<sup>3</sup>We note that a discussion of the convergence of the convolution integrals is still missing.

evident in the computation of the next-to-leading order (in  $\alpha_s$ ) corrections. In fact, one can show that the matrix elements of the operators  $\mathcal{O}_2$  and  $\mathcal{O}_8$ , which are expected to contribute at this order, are given by the matrix element of  $\mathcal{O}_7$  times a computable hard scattering kernel. Moreover, spectator interactions can be computed and are given by convolutions involving the light-cone wave functions of the  $B$  and  $V$  mesons. It must be mentioned that light-cone wave functions of pseudo-scalar and vector mesons have been deeply studied using light-cone QCD sum rules methods [36–39]. On the other hand, not much is known about the  $B$  meson light-cone distribution amplitude, whose first negative moment enters the factorized amplitude at NLO. Since this moment enters the factorized expression for the  $B \rightarrow \gamma$  form factor as well, it might be possible to extract its value from measurements of decays like  $B \rightarrow \gamma e \nu$ , if it can be shown that power corrections are under control [40].

Finally, let us stress that a breakdown of factorization is expected at order  $\Lambda_{\text{QCD}}/m_b$  [27,41,42]. In Ref. [41], in particular, the authors have shown that in the analysis of  $B \rightarrow K^* \gamma$  decays at subleading order an infrared divergence is encountered in the matrix element of  $\mathcal{O}_8$ . Nevertheless, some very specific power corrections might still be computable. Indeed, this is the case for the annihilation and weak exchange amplitudes in  $B \rightarrow \rho \gamma$  at the one-loop level.

### 3 $b \rightarrow d\gamma$ Transitions

#### 3.1 $B \rightarrow X_d \gamma$

Most of the theoretical improvements on the perturbative contributions and the power corrections in  $1/m_b^2$  and  $1/m_c^2$ , carried out in the context of the decay  $B \rightarrow X_s \gamma$ , can straightforwardly be adapted to the decay  $B \rightarrow X_d \gamma$ ; thus, the NLL-improved decay rate for  $B \rightarrow X_d \gamma$  decay has much reduced theoretical uncertainty [43]. But as  $\lambda_d^u = V_{ub} V_{ud}^*$  for  $b \rightarrow d\gamma$  is not small with respect to  $\lambda_d^t = V_{tb} V_{td}^*$  and  $\lambda_d^c = V_{cb} V_{cd}^*$ , one also has to take into account the operators proportional to  $\lambda_d^u$  and, moreover, the long-distance contributions from the intermediate  $u$ -quark in the penguin loops might be important. However, there are three *soft* arguments that indicate a small impact of these non-perturbative contributions: first, one can derive a model-independent suppression factor  $\Lambda_{\text{QCD}}/m_b$  within these long-distance contributions [44]. Second, model calculations, based on vector meson dominance, also suggest this conclusion [45]. Furthermore, estimates of the long-distance contributions in exclusive decays  $B \rightarrow \rho \gamma$  and  $B \rightarrow \omega \gamma$  in the light-cone sum rule approach do not exceed 15% [46]. Finally, it must be stressed that there

is no spurious enhancement of the form  $\log(m_u/\mu_b)$  in the perturbative contribution, as was shown in [9,47]. All these observations exclude very large long-distance intermediate  $u$ -quark contributions in the decay  $B \rightarrow X_d \gamma$ . Nevertheless, the theoretical status of the decay  $B \rightarrow X_d \gamma$  is not as clean as that of  $B \rightarrow X_s \gamma$ .

While the  $b \rightarrow s$  transitions like  $B \rightarrow X_s \gamma$  have no relevant impact on the CKM phenomenology because of the flatness of the corresponding unitarity triangle (for example:  $V_{ts}$  cannot be further constrained by the  $B \rightarrow X_s \gamma$  measurement if the unitarity constraint is not used in the theoretical prediction, because the charm contribution is twice as large as the top contribution),  $b \rightarrow d$  transitions give important complementary information on the unitarity triangle, which is also tested by the measurements of  $V_{ub}/V_{cb}$ ,  $\Delta M_{B_d}$ , and  $\Delta M_{B_d}/\Delta M_{B_s}$ . Thus, a future measurement of the  $B \rightarrow X_d \gamma$  decay rate will help to reduce the currently allowed region of the CKM Wolfenstein parameters  $\rho$  and  $\eta$  significantly.

Regarding new physics, the branching ratio of  $B \rightarrow X_d \gamma$  might be of interest, because its CKM suppression by the factor  $|V_{td}|^2/|V_{ts}|^2$  in the SM may not be true in extended models. We also emphasize that in the ratio

$$R(d\gamma/s\gamma) \equiv \frac{\mathcal{B}(B \rightarrow X_d \gamma)}{\mathcal{B}(B \rightarrow X_s \gamma)}, \quad (14)$$

a good part of the theoretical uncertainties cancel out. It is therefore of particular interest for CKM phenomenology and for the new physics search.

A measurement of the  $B \rightarrow X_d \gamma$  is rather difficult but perhaps within the reach of the high-luminosity  $B$  factories. Such a measurement will rely on high statistics and on powerful methods for the kaon-pion discrimination. At present only upper bounds on corresponding exclusive modes are available (see next section).

The direct *normalized* CP asymmetry of the inclusive decay modes represent another interesting observable:

$$\alpha_{CP} = \frac{\Gamma(\bar{B} \rightarrow X_{s/d} \gamma) - \Gamma(B \rightarrow X_{\bar{s}/\bar{d}} \gamma)}{\Gamma(\bar{B} \rightarrow X_{s/d} \gamma) + \Gamma(B \rightarrow X_{\bar{s}/\bar{d}} \gamma)}. \quad (15)$$

CLEO has already presented a measurement of the CP asymmetry in the inclusive decay  $B \rightarrow X_s \gamma$ , actually a measurement of a weighted sum,  $\alpha_{CP} = 0.965\alpha_{CP}(B \rightarrow X_s \gamma) + 0.02\alpha_{CP}(B \rightarrow X_d \gamma)$  [48], which already excludes very large effects. The same conclusion can be deduced from the measurements of the CP asymmetry in the exclusive modes (see next section).

Theoretical NLL QCD predictions of the *normalized* CP asymmetries of the inclusive channels ([43,49]) within the SM can be expressed by the approximate formulae (see [50]):

$$\begin{aligned}\alpha_{CP}(B \rightarrow X_s \gamma) &\approx 0.334 \times \text{Im}[\epsilon_s] \approx +0.6\%, \\ \alpha_{CP}(B \rightarrow X_d \gamma) &\approx 0.334 \times \text{Im}[\epsilon_d] \approx -16\%,\end{aligned}\quad (16)$$

where

$$\epsilon_s = \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \simeq -\lambda^2(\rho - i\eta), \quad \epsilon_d = \frac{V_{ud}^* V_{ub}}{V_{td}^* V_{tb}} \simeq \frac{\rho - i\eta}{1 - \rho + i\eta}$$

Numerically, the best-fit values of the CKM parameters are used. The two CP asymmetries are connected by the relative factor  $\lambda^2((1-\rho)^2 + \eta^2)$ . Moreover, the small SM prediction for the CP asymmetry in the decay  $B \rightarrow X_s \gamma$  is a result of three suppression factors. There is an  $\alpha_s$  factor needed in order to have a strong phase; moreover, there is a CKM suppression of order  $\lambda^2$  and there is a GIM suppression of order  $(m_c/m_b)^2$ , reflecting the fact that in the limit  $m_c = m_u$  any CP asymmetry in the SM would vanish.

It will be rather difficult to make an inclusive measurement of the CP asymmetry in the  $b \rightarrow d$  channel. However, based on CKM unitarity, one can derive the following U-spin relation between the *un-normalized* CP asymmetries [51]:

$$\Delta\Gamma(B \rightarrow X_s \gamma) + \Delta\Gamma(B \rightarrow X_d \gamma) = 0. \quad (17)$$

Within the inclusive channels, one can rely on parton-hadron duality and can actually compute the U-spin breaking by keeping a non-vanishing strange quark mass [52]. Going beyond the leading partonic contribution, one can further check if the large suppression factor from the U-spin breaking is still effective, in addition to the natural suppression factors already present in the corresponding branching ratios [53]; this finally leads to the SM prediction

$$\Delta\Gamma(B \rightarrow X_s \gamma) + \Delta\Gamma(B \rightarrow X_d \gamma) = 1 \times 10^{-9}. \quad (18)$$

This prediction provides a very clean SM test, whether generic new CP phases are active or not. Any significant deviation from the estimate would be a direct hint of non-CKM contributions to CP violation.

### 3.2 $B \rightarrow \rho\gamma$

In the analysis of exclusive  $B \rightarrow V\gamma$  decays (with  $V = K^*, \rho, \omega$ ) we will construct the various observables in terms of the CP-averaged quantities - which are much easier to measure than the individual channels - unless otherwise stated. In the NLL approximation,

this procedure is equivalent to defining two distinct observables for the charge-conjugate modes and *then* perform the average. The ratios  $R(\rho\gamma/K^*\gamma)$  are given by

$$\begin{aligned}R^\pm(\rho\gamma/K^*\gamma) &= \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3} \zeta^2 (1 + \Delta R^\pm), \\ R^0(\rho\gamma/K^*\gamma) &= \frac{1}{2} \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3} \zeta^2 (1 + \Delta R^0),\end{aligned}$$

where  $\zeta = \xi_\perp^\rho(0)/\xi_\perp^{K^*}(0)$ , and  $\xi_\perp^\rho(0)$  and  $\xi_\perp^{K^*}(0)$  are the form factors at  $q^2 = 0$  in the effective heavy quark theory for the decays  $B \rightarrow \rho(K^*)\gamma$  [54]. There are several estimates of the quantity  $\zeta$  in the present literature coming from light-cone QCD sum rules (LCSR) [55], hybrid LCSR [56], improved LCSR [57] and quark models [58]. In the numerical analysis we adopt the value  $\zeta = 0.76 \pm 0.10$ ; the central value is taken from the LCSR approach while the error is increased in order to accommodate all the other determinations. The quantities  $(1 + \Delta R^{\pm,0})$  entail the explicit  $O(\alpha_s)$  corrections as well as the power-suppressed annihilation contributions proportional to  $\lambda_q^4$ . The latter contribution, in particular, is effective only for charged  $B$  decays (weak annihilation); in fact, W-exchange amplitudes are smaller because of the ratio  $Q_d/Q_u = -1/2$  and of colour suppression. The ratio  $R(\rho\gamma/K^*\gamma)$  acquires, therefore, a tiny dependence on the CKM angle  $\alpha$ . Within the SM, the numerical value of these NLO corrections are  $\Delta R^\pm = 0.055 \pm 0.13$  and  $\Delta R^0 = 0.015 \pm 0.11$ . Explicit expressions for these quantities, which are valid in the presence of beyond-the-SM physics, can be found in Refs. [54,59].

Further important observables are the isospin breaking ratio given by

$$\Delta(\rho\gamma) = \frac{\Gamma(B^+ \rightarrow \rho^+\gamma) - \Gamma(B^- \rightarrow \rho^-\gamma)}{2(\Gamma(B^0 \rightarrow \rho^0\gamma) + \Gamma(\bar{B}^0 \rightarrow \bar{\rho}^0\gamma))} - 1 \quad (19)$$

and the CP asymmetries in the charged and neutral modes,

$$A_{CP}^\pm(\rho\gamma) = \frac{\Gamma(B^- \rightarrow \rho^-\gamma) - \Gamma(B^+ \rightarrow \rho^+\gamma)}{\Gamma(B^- \rightarrow \rho^-\gamma) + \Gamma(B^+ \rightarrow \rho^+\gamma)}, \quad (20)$$

$$A_{CP}^0(\rho\gamma) = \frac{\Gamma(\bar{B}^0 \rightarrow \rho^0\gamma) - \Gamma(B^0 \rightarrow \rho^0\gamma)}{\Gamma(\bar{B}^0 \rightarrow \rho^0\gamma) + \Gamma(B^0 \rightarrow \rho^0\gamma)}. \quad (21)$$

Recently, the BABAR collaboration has reported a significant improvement on the upper limits of the branching ratios for the decays  $B^0(\bar{B}^0) \rightarrow \rho^0\gamma$  and  $B^\pm \rightarrow \rho^\pm\gamma$ . Averaged over the charge-conjugated modes, the current 90% C.L. upper limits are [60]:

$$\mathcal{B}(B^0 \rightarrow \rho^0\gamma) < 1.4 \times 10^{-6}, \quad (22)$$

$$\mathcal{B}(B^\pm \rightarrow \rho^\pm\gamma) < 2.3 \times 10^{-6}, \quad (23)$$

$$\mathcal{B}(B^0 \rightarrow \omega\gamma) < 1.2 \times 10^{-6}. \quad (24)$$

They have been combined, using isospin weights for  $B \rightarrow \rho\gamma$  decays and assuming  $\mathcal{B}(B^0 \rightarrow \omega\gamma) = \mathcal{B}(B^0 \rightarrow \rho^0\gamma)$ , to yield the improved upper limit

$$\mathcal{B}(B \rightarrow \rho\gamma) < 1.9 \times 10^{-6}. \quad (25)$$

Note that the equality between the  $\rho$  and  $\omega$  branching ratios receives  $SU(3)$ -breaking corrections that can be as large as 20%. The current measurements of the branching ratios for  $B \rightarrow K^*\gamma$  decays by BABAR [61],

$$\mathcal{B}(B^0 \rightarrow K^{*0}\gamma) = (4.23 \pm 0.40 \pm 0.22) \times 10^{-5} \quad (26)$$

$$\mathcal{B}(B^+ \rightarrow K^{*+}\gamma) = (3.83 \pm 0.62 \pm 0.22) \times 10^{-5} \quad (27)$$

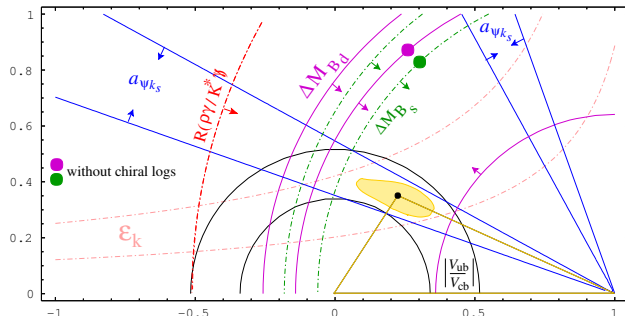
are then used to set a 90% C.L. upper limit on the ratio of the branching ratios [60]:

$$R(\rho\gamma/K^*\gamma) \equiv \frac{\mathcal{B}(B \rightarrow \rho\gamma)}{\mathcal{B}(B \rightarrow K^*\gamma)} < 0.047. \quad (28)$$

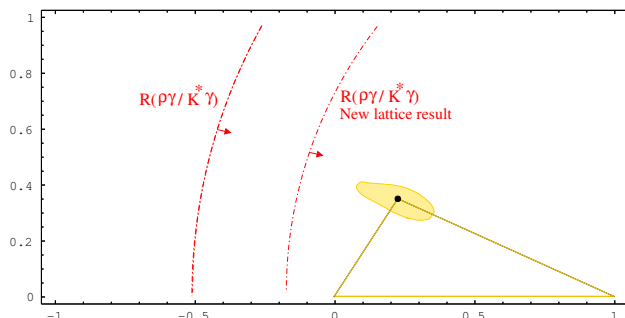
This bound is typically a factor 2 away from the SM estimates [54], which we quantify more precisely in this letter. In beyond-the-SM scenarios, this bound provides a highly significant constraint on the relative strengths of the  $b \rightarrow d\gamma$  and  $b \rightarrow s\gamma$  transitions.

Let us present an updated analysis of the constraints in the  $(\bar{\rho}, \bar{\eta})$  plane from the unitarity of the CKM matrix, including the measurements of the CP asymmetry  $a_{\psi K_s}$  in the decays  $B^0/\bar{B}^0 \rightarrow J/\psi K_s$  (and related modes), and show the impact of the upper limit  $R(\rho\gamma/K^*\gamma) \leq 0.047$  [60]. The SM expressions for  $\epsilon_K$  (CP-violating parameter in  $K$  decays),  $\Delta M_{B_d}$  ( $B_d^0 - \bar{B}_d^0$  mass difference),  $\Delta M_{B_s}$  ( $B_s^0 - \bar{B}_s^0$  mass difference) and  $a_{\psi K_s}$  are fairly standard and can be found, for instance, in Ref. [62]; the values of the theoretical parameters and experimental measurements that we use are taken from Ref. [59]. The SM fit of the unitarity triangle is presented in Fig. 1. Note that where the hadronic parameters  $f_{B_d} \sqrt{\hat{B}_{B_d}}$  and  $\zeta_s$  are concerned, we adopt very recent lattice estimates that take into account uncertainties induced by the so-called chiral logarithms [63]. These errors are extremely asymmetric and, once taken into account, reduce sizeably the impact of the  $\Delta M_{B_s}/\Delta M_{B_d}$  lower bound on the UT analysis. In Fig. 1 we explicitly show what happens to the allowed regions once these errors are taken into account. The 95% C.L. contour is drawn, taking into account chiral logarithms uncertainties.

As the bound from the current upper limit on  $R(\rho\gamma/K^*\gamma)$  is not yet competitive to the ones from either the measurement of  $\Delta M_{B_d}$  or the current bound on  $\Delta M_{B_s}$ , we use the allowed  $\bar{\rho} - \bar{\eta}$  region to work out the SM predictions for the observables in the radiative  $B$ -decays described above. Taking into account these



**Figure 1.** Unitarity triangle fit in the SM and the resulting 95% C.L. contour in the  $\bar{\rho} - \bar{\eta}$  plane. The solid lines show the upper bounds (with and without chiral logs) due to  $\Delta M_{B_d}$ , the dot-dashed lines the ones due to  $\Delta M_{B_s}$ . The impact of the  $R(\rho\gamma/K^*\gamma) < 0.047$  constraint is also shown.



**Figure 2.** The impact of the new lattice QCD estimate of the ratio  $\zeta$  on the  $R(\rho\gamma/K^*\gamma)$  constraint.

errors and the uncertainties on the theoretical parameters, we find the following SM expectations for the radiative decays:

$$R^\pm(\rho\gamma/K^*\gamma) = 0.023 \pm 0.012, \quad (29)$$

$$R^0(\rho\gamma/K^*\gamma) = 0.011 \pm 0.006, \quad (30)$$

$$\Delta(\rho\gamma) = 0.04_{-0.07}^{+0.14}, \quad (31)$$

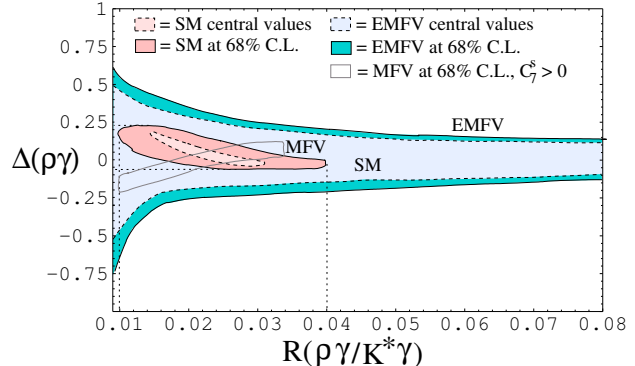
$$A_{CP}^\pm(\rho\gamma) = -0.10_{-0.03}^{+0.02}, \quad (32)$$

$$A_{CP}^0(\rho\gamma) = -0.06 \pm 0.02. \quad (33)$$

In the CP asymmetries the uncertainties due to form-factors cancel out to a large extent, however, the scale dependence is rather large because the CP asymmetries arise at the  $O(\alpha_s)$ . It is interesting to work out the extremal values of  $R(\rho\gamma/K^*\gamma)$  compatible with the SM UT analysis. Any measurement of  $R(\rho\gamma/K^*\gamma)$  whose central value lies in the range (0.013, 0.037) would be compatible with the SM, irrespective of the size of the experimental error. The error induced by the imprecise determination of the isospin breaking parameter  $\zeta$  currently limits the possibility of having a very sharp impact from  $R(\rho\gamma/K^*\gamma)$  on the UT analysis.

Let us comment on the impact of a recent preliminary lattice determination of the  $B \rightarrow (K^*, \rho)$  form factor at zero recoil [19]:  $F^{K^*}(0) = 0.25(5)(2)$  and  $\zeta = 0.91(8)$ . In the first place, note that the central value of the  $K^*$  form factor is in perfect agreement with the indirect determinations obtained in Refs. [64,65,54]. In these papers, the authors compute  $BR(B \rightarrow K^*\gamma)$  at NLL, using the inclusive channel  $B \rightarrow X_s\gamma$  to extract the value of the Wilson coefficient  $C_7$ . Their conclusion is that in order to accommodate the experimental data, the form factor at zero-recoil has to be substantially smaller than the typical light-cone QCD estimate ( $T_{s,r}^{K^*}(0) = 0.38 \pm 0.06$ ). For instance, in Ref. [54], the fitted value of the form factor is  $T^{K^*}(0) = 0.27 \pm 0.04$ . In second place, the higher central value for the ratio  $\zeta$  given in Ref. [19] strengthens the impact of the current  $R(\rho\gamma/K^*\gamma)$  upper limit. In Fig. 2, the additional line has been obtained using this new determination of the form factors ratio and gives a bound comparable to the  $\Delta m_{B_s}$  one. In order to fully trust this new lattice estimate of the ratio  $\zeta$ , an independent cross-check of this result is mandatory; moreover it is necessary to analyse the old QCD sum rules estimate deeper so as to understand the reasons of this discrepancy.

Let us finally discuss the analysis of the exclusive modes in supersymmetric models and entertain two variants of the MSSM called in the literature MFV [66] and Extended-MFV [67] models. In MFV models, all the flavour changing sources other than the CKM matrix are neglected. In this class of models there are essentially no additional contributions (on top of the SM ones) to  $a_{\psi K_S}$  and  $\Delta M_{B_s}/\Delta M_{B_d}$ , while the impact on  $\epsilon_K$ ,  $\Delta M_{B_d}$  and  $\Delta M_{B_s}$  is described by a single parameter,  $f$ , whose value depends on the parameters of the supersymmetric models [62]. EMFV models are based on the assumption that all the superpartners are heavier than 1 TeV with the exception of the lightest stop; no constraints are imposed on the off diagonal structure of the soft breaking terms. It can be shown [67] that under these assumptions there are only two new parameters in addition to the MFV ones, namely:  $\delta_{\tilde{u}_L \tilde{t}} = M_{\tilde{u}_L \tilde{t}}^2 / (M_{\tilde{t}} M_{\tilde{q}}) \times V_{td} / |V_{td}|$  and  $\delta_{\tilde{c}_L \tilde{t}} = M_{\tilde{c}_L \tilde{t}}^2 / (M_{\tilde{t}} M_{\tilde{q}}) \times V_{ts} / |V_{ts}|$ . Where  $\tilde{t}$  is the lightest stop mass eigenstate and  $M^2$  is the up-squark mass matrix given in a basis obtained from the SCKM one after the diagonalization of the  $2 \times 2$  stop submatrix. Since we are interested in the phenomenology of  $b \rightarrow d$  transitions, we will consider here only  $\delta_{\tilde{u}_L \tilde{t}}$ . With the inclusion of this new parameter, the description of the UT-related observables needs one more complex parameter,  $g = g_R + i g_I$  [67]. A signature of these models is the presence of a new phase in the  $B_d^0 - \bar{B}_d^0$  mixing amplitude. Using the parametrization  $M_{12}^d = r_d^2 e^{2i\theta_d} M_{12}^{\text{SM}}$ , we get  $r_d^2 = |1 + f + g|$  and



**Figure 3.** Correlation between  $R(\rho\gamma/K^*\gamma)$  and  $\Delta(\rho\gamma)$  in the SM and in MFV and EMFV models. The light-shaded regions are obtained varying  $\bar{\rho}$ ,  $\bar{\eta}$ , the supersymmetric parameters (for the MFV and EMFV models) and using the central values of all the hadronic quantities. The darker regions show the effect of  $\pm 1\sigma$  variation of the hadronic parameters.

$\theta_d = 1/2 \arg(1 + f + g)$ . This implies new supersymmetric contributions to the CP asymmetry  $a_{\psi K_S}$ .

The phenomenology of the MFV and EMFV models, analysed by scatter plots over the supersymmetric parameter space, shows the discrimination power of exclusive modes, if one focus on ratios of exclusive observables and their correlation. If one also scan over  $\bar{\rho}$  and  $\bar{\eta}$ , and require that each point satisfy the bounds that come from direct searches, from the  $B \rightarrow X_s\gamma$  branching ratio, and from the UT-related observables, one finally finds the surviving regions presented in Fig. 3. It shows the correlation of the isospin breaking ratio  $\Delta(\rho\gamma)$  and the ratio of the branching ratios  $R(\rho\gamma/K^*\gamma)$ . The light-shaded regions are obtained using the central values of the input parameters while the dark-shaded ones result from the inclusion of their  $1\sigma$  errors. In the MFV case, there are two distinct regions that correspond to the negative (SM-like) and positive  $C_7^s$  case. For  $C_7^s < 0$ , the allowed regions in MFV almost coincide with the SM ones and we do not draw them. For  $C_7^s > 0$ , the allowed regions are different and, in general, a change of sign of both the CP-asymmetries (compared to the SM) is expected. We note that the latter scenario needs very large SUSY contributions to  $C_7^s$ , arising from the chargino-stop diagrams, and for fixed values of  $\tan\beta_S$  it is possible to set an upper limit on the mass of the lightest stop squark.

## 4 $b \rightarrow s\ell^+\ell^-$ Transitions

### 4.1 $B \rightarrow X_s\ell^+\ell^-$

In comparison to the  $B \rightarrow X_s\gamma$ , the inclusive  $B \rightarrow X_s\ell^+\ell^-$  decay presents a complementary and also



more complex test of the SM. This decay is dominated by perturbative contributions if the  $c\bar{c}$  resonances that show up as large peaks in the dilepton invariant mass spectrum are removed by appropriate kinematic cuts. In the 'perturbative windows', namely in the low- $\hat{s}$  region  $0.05 < \hat{s} = q^2/m_b^2 < 0.25$  and also in the high- $\hat{s}$  region with  $0.65 < \hat{s}$ , theoretical predictions for the invariant mass spectrum are dominated by the purely perturbative contributions, and a theoretical precision comparable with the one reached in the decay  $B \rightarrow X_s \gamma$  is in principle possible. Regarding the choice of precise cuts in the dilepton mass spectrum, it is important that one directly compares theory and experiment using the same energy cuts and avoids any kind of extrapolation.

In the high- $\hat{s}$  region, one should encounter the breakdown of the heavy mass expansion at the endpoint. Integrated quantities are still defined; nevertheless one finds sizeable  $\Lambda_{QCD}^2/m_b^2$  non-perturbative corrections within this region.

The decay  $B \rightarrow X_s \ell^+ \ell^-$  is particularly attractive because of kinematic observables such as the invariant dilepton mass spectrum and the forward-backward (FB) asymmetry. They are usually normalized by the semi-leptonic decay rate in order to reduce the uncertainties due to bottom quark mass and CKM angles and are defined as follows:

$$R_{quark}^{\ell^+ \ell^-}(\hat{s}) = \frac{d}{d\hat{s}} \Gamma(b \rightarrow X_s \ell^+ \ell^-) / \Gamma(b \rightarrow X_c e \bar{\nu}), \quad (34)$$

$$A_{FB}(\hat{s}) = \frac{1}{\Gamma(b \rightarrow X_c e \bar{\nu})} \times \int_{-1}^1 d \cos \theta_\ell \frac{d^2 \Gamma(b \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} d \cos \theta_\ell} \text{sgn}(\cos \theta_\ell), \quad (35)$$

here  $\theta_\ell$  is the angle between  $\ell^+$  and  $B$  momenta in the dilepton centre-of-mass frame. The so-called 'normalized' FB asymmetry, which is also often used, is given by

$$\overline{A}_{FB}(\hat{s}) = \frac{\int_{-1}^1 d \cos \theta_\ell \frac{d^2 \Gamma(B \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} d \cos \theta_\ell} \text{sgn}(\cos \theta_\ell)}{\int_{-1}^1 d \cos \theta_\ell \frac{d^2 \Gamma(B \rightarrow X_s \ell^+ \ell^-)}{d\hat{s} d \cos \theta_\ell}}. \quad (36)$$

For the low- $\hat{s}$  region the present partonic NNLL prediction is given by (see [68,69,72]):

$$\int_{0.05}^{0.25} d\hat{s} R_{quark}^{\ell^+ \ell^-}(\hat{s}) = (1.27 \pm 0.08_{scale}) \times 10^{-5} \quad (37)$$

The error quoted in (37) reflects only the renormalization scale uncertainty and is purely perturbative. There is no additional problem due to the charm mass renormalization scheme ambiguity within the decay  $B \rightarrow X_s \ell^+ \ell^-$  because the charm dependence starts already at one loop, in contrast to the case of the decay  $B \rightarrow X_s \gamma$ . The charm dependence itself leads to an additional uncertainty of  $\sim 7.6\%$  within the partonic quantity (37), if the pole mass is varied,  $m_c^{pole}/m_b^{pole} = 0.29 \pm 0.02$ .

The impact of the NNLL contributions is significant. The large matching scale  $\mu_W$  uncertainty of 16% of the NLL result was removed; the low-scale uncertainty  $\mu_b$  of 13% was cut in half; and also the central value of the integrated low dilepton spectrum (37) was significantly changed by more than 10% due to NNLL corrections. Using the measured semi-leptonic branching ratio  $\mathcal{B}_{exp}^{sl}$ , the prediction for the corresponding branching ratio is given by

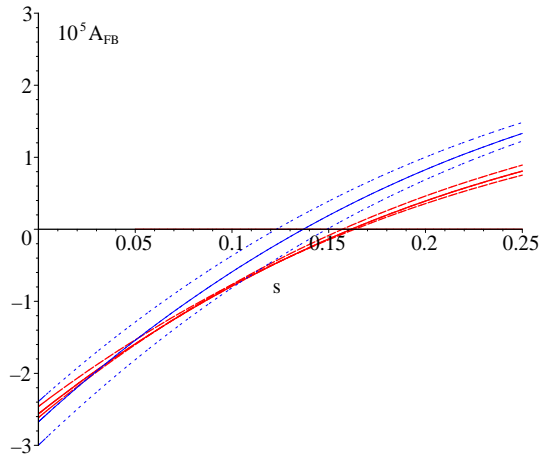
$$\begin{aligned} \mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{Cut: } \hat{s} \in [0.05, 0.25]} &= \quad (38) \\ &= \mathcal{B}_{exp}^{sl} \int_{0.05}^{0.25} d\hat{s} [R_{quark}^{\ell^+ \ell^-}(\hat{s}) + R_{m_b^2}(\hat{s}) + R_{m_c^2}(\hat{s})] \\ &= (1.36 \pm 0.08_{scale}) \times 10^{-6} \end{aligned}$$

$R_{m_b^2}(\hat{s})$  and  $R_{m_c^2}(\hat{s})$  are the non-perturbative contributions scaling with  $1/m_b^2$  and  $1/m_c^2$ . The recent first measurement of BELLE, with a rather large uncertainty [22], is compatible with this SM prediction.

The phenomenological impact of the NNLL contributions on the FB asymmetry is also significant [70,71]. The position of the zero of the FB asymmetry, defined by  $A_{FB}(\hat{s}_0) = 0$ , is particularly interesting to determine relative sign and magnitude of the Wilson coefficients  $C_7$  and  $C_9$  and it is therefore extremely sensitive to possible new physics effects. The previous NLL result, where the error is determined by the scale dependence, is now modified by the NNLL contributions [70,71]:

$$\hat{s}_0^{NLL} = 0.14 \pm 0.02, \quad \hat{s}_0^{NNLL} = 0.162 \pm 0.008. \quad (39)$$

In the NNLL case the variation of the result induced by the scale dependence is accidentally very small (about  $\pm 1\%$ ) and cannot be regarded as a good estimate of missing higher-order effects. Taking into account the separate scale variation of both Wilson coefficients  $C_9$  and  $C_7$ , and the charm-mass dependence, one estimates a conservative overall error on  $\hat{s}_0$  of about 5% [70]. In this  $\hat{s}$  region the non-perturbative  $1/m_b^2$  and  $1/m_c^2$  corrections to  $A_{FB}$  are very small and also under control. An illustration of the shift of the central value and the reduced scale dependence between NLL and NNLL expressions of  $A_{FB}(s)$ , in the

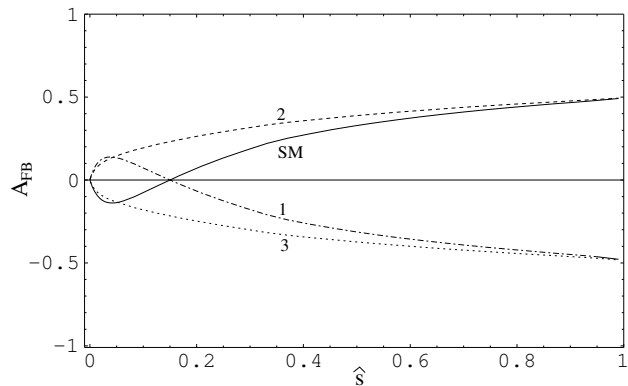


**Figure 4.** Comparison between NNLL and NLL results for  $A_{FB}(s)$  in the low  $s$  region. The three thick lines are the NNLL predictions for  $\mu = 5$  GeV (full), and  $\mu = 2.5$  and  $10$  GeV (dashed); the dotted curves are the corresponding NLL results. All curves for  $m_c/m_b = 0.29$ .

low- $\hat{s}$  region, is presented in fig. 4. The complete effect of NNLL contributions to the FB asymmetry adds up to a 16% shift compared with the NLL result, with a residual error due to higher-order terms reduced at the 5% level. Thus, the zero of the FB asymmetry in the inclusive mode turns out to be one of the most sensitive tests for new physics beyond the SM.

The  $B$  factories will soon provide statistics and resolution needed for the measurements of  $B \rightarrow X_s \ell^+ \ell^-$  kinematic distributions. Correspondingly, the recently calculated new (NNLL) contributions [68–72]<sup>4</sup> have significantly improved the sensitivity of the inclusive  $B \rightarrow X_s \ell^+ \ell^-$  decay in testing extensions of the SM in the sector of flavour dynamics. However, with the present experimental knowledge the decay  $B \rightarrow X_s \gamma$  still leads to the most restrictive constraints as was found in [74]. Especially, the MFV scenarios are already highly constrained and only small deviations to the SM rates and distributions are possible; therefore no useful additional bounds from the semi-leptonic modes beyond what are already known from the  $B \rightarrow X_s \gamma$  can be deduced for the MFV models at the moment. Within the model-independent analysis, the impact of the partial NNLL contributions on the allowed ranges for the Wilson coefficients was already found to be significant. In this analysis, however, only the integrated branching ratios were used to derive constraints. It is clear that one needs measurements of the kinematic distributions of the  $B \rightarrow X_s \ell^+ \ell^-$ , the dilepton mass spectrum and the FB asymmetry in order to determine the exact values and signs of the Wil-

<sup>4</sup>We add here that the three-loop mixing is fully under control. A quite recent calculation of a missing NNLL mixing piece leads to a correction below 2% [73]



**Figure 5.** Four different shapes of the normalized FB asymmetry  $\overline{A}_{FB}$  for the decay  $B \rightarrow X_s \ell^+ \ell^-$ . The four curves correspond to four sample points of the Wilson coefficients that are compatible with the present measurements of the integrated branching ratios.

son coefficients. In fig. 5, the impact of these future measurements is illustrated. It shows the shape of the FB asymmetry for the SM and three additional sample points, which are all still allowed by the present measurements of the branching ratios; thus, even rather rough measurements of the FB asymmetry will either rule out large parts of the parameter space of extended models or show clear evidence for new physics beyond the SM.

#### 4.2 $B \rightarrow K^{(*)} \ell^+ \ell^-$

For completeness, let us briefly comment on the impact of exclusive  $B \rightarrow K^{(*)} \ell^+ \ell^-$  modes. First of all, let us stress that hadronic uncertainties on these exclusive rates are dominated by the errors on form factors and are much larger than in the corresponding inclusive decays. In fact, following the analysis presented in Ref. [74], we see that inclusive modes already put much stronger constraints on the various Wilson coefficients.

Concerning the measurement of a zero in the spectrum of the forward-backward asymmetry, things are different. According to Refs. [75,64] the value of the dilepton invariant mass ( $q_0^2$ ), for which the differential forward-backward asymmetry vanishes, can be predicted in quite a clean way. In the QCD factorization approach at leading order in  $\Lambda_{\text{QCD}}/m_b$ , the value of  $q_0^2$  is free from hadronic uncertainties at order  $\alpha_s^0$  (a dependence on the soft form factor  $\xi_\perp$  and the light cone wave functions of the  $B$  and  $K^*$  mesons appear at NLL). Within the SM, the authors of Ref. [64] find:  $q_0^2 = (4.1 \pm 0.6) \text{ GeV}^2$ . As in the inclusive case, such a measurement will have a huge phenomenological impact.

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